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An Extended Analytical Approach to Credit Risk Management

ALEXANDRE KURTH – HADLEY TAYLOR – ARMIN WAGNER

Among the 'reduced form models' for measuring the credit risk of a bank's portfolio is CreditRisk+, which provides a closed-form solution for calculating the portfolio loss distribution based on an actuarial approach. The limitations of this model are well known, but they are often misinterpreted as being deeply embedded within the model. Dismantling the mathematical components of the model allows one to modify and extend it in several ways while remaining within an analytical approach. One of the most unattractive features is the orthogonality of the background factors or sectors as it hinders any resemblance to real-world macroeconomic indexes or industrial sectors and geographical areas. Among other extensions, which we mention briefly, we present in more detail how the original model can be amended to consider correlations among default risk sectors and among severity risk segments. These extensions are applied to real-life data, based on mortality rate data produced by the Italian Central Bank.

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1. Introduction

In the approach to credit risk modelling, there are some key factors which have a strong influence on the practical implementation of the theoretical models. One of these is the existence of a formal analytical representation of the risks involved and, specifically, of the relation between these risks.

The possibility to describe a closed form solution has some advantages over models for which statistical simulation is necessary: the modeller and the user can clearly understand the deep interrelation among variables and parameters and can therefore understand better *a priori* the consequences of applying variations to the model. Extending a model based on Monte Carlo or historical simulation implies empirical judgement on the completeness of the experiments necessary to prove the soundness of modifications applied to the model. In addition, an analytical solution is typically computed with lower effort.

The actuarial approach to credit risk evaluation, which has been applied extensively to insurance portfolios, can be viewed as a typically analytical two-state approach (default/non-default) to estimating the portfolio loss distribution (reduced form model). In contrast, for models following a mark-to-market approach, the changes in profit & loss enter directly into the determination of the loss distribution. This is often performed with a simulation technique as in CreditMetrics (Gupton *et al.*, 1997), and requires further data, such as rating migrations. Another difference between the approaches is the modelling of dependencies among the elements of the portfolio. This can be performed either at the level of individual obligors or at the aggregated level (e.g. industry segments).

Contrary to a simulation model, an analytical approach preserves the tractability of the model and allows an efficient computation of the full loss distribution – a clear advantage over simulation approaches for performing scenario and sensitivity analyses. In addition, one needs only a limited data set, which makes such an approach easier to implement within a bank's overall risk framework.

An often applied approach for actuarial models is CreditRisk+ (Credit Suisse Financial Products, 1997). Here a natural approach is chosen to consider correlation among factors in the first place, and to consider the relation between the single elements of the portfolio and these background factors in a second step. However, criticism is often raised in the financial industry concerning the limitations of the model: the improper treatment of the risk inherent in recovery rate (BIS, 1999, p. 37); further, the ignorance to rating migrations; and, finally, concerning the treatment of correlations. We elaborate on these perceived shortcomings below.

This paper focuses on CreditRisk+, and takes advantage of the analytical properties of the model to show that it is extendable in several ways (section 2). Section 3 concentrates on one of its main limitations – namely how correlation is considered in the model – and describes how to extend it to derive correlation among default rates from correlation among background factors and, even more interestingly, correlation among recovery rates, which in the original model are considered deterministic. Finally, in section 4, we show some results of applying statistical estimation to such correlations using historical data produced for the Italian market by Italy's Central Bank.

2. CreditRisk+

This section very briefly outlines the main aspects and relations of the CreditRisk+ model (Credit Suisse Financial Products, 1997). The starting point of every credit portfolio model is the equation for the random variable of portfolio loss L over all obligors A:

$$L = \sum_A I_A \cdot \nu_A \tag{1}$$

where I_A denotes the indicator variable describing the default event of A, being 1 if A defaults and 0 otherwise, and ν_A is the exposure net of recovery. The dependence between obligors is incorporated by a common risk factor Γ, which later will be assumed to be gamma-distributed.[1] The variable Γ is of mean equal to 1 and its volatility is denoted by σ. Let us denote the (unconditional) probability of default by p_A. Then conditional on Γ, the default probability of A is Γp_A. This default scaling factor Γ describes the relative number of default events in the economy (Bürgisser *et al.*, 1999). The expected loss EL and the standard deviation UL can now be derived (Bürgisser *et al.*, 2001):

$$EL = \sum_A p_A \nu_A$$

$$UL^2 = \sigma^2 EL^2 + \sum_A [p_A - (1 + \sigma^2) p_A^2] \nu_A^2 \tag{2}$$

There is a slight difference to the original formula in CreditRisk+; see Credit Suisse Financial Products (1997, eqn (118)). This is caused by the assumption that the default is modelled by a Poisson event, while here defaults are Bernouilli events.

To get a hand on the loss distribution, and not just the first and second moment, we make use of the probability generating function, which is defined as a power series of the form

$$G(z) = \sum_{n \geqslant 0} p(n) \cdot z^n \tag{3}$$

where $p(n)$ is the probability of losing the amount n and z is a formal variable. To compute the coefficients $p(n)$, we make three assumptions:

- banding the exposures (net of recovery)[2]
- the default event being approximated by a Poisson variable
- the common risk factor Γ being gamma-distributed.

It then turns out that the probability generating function of the portfolio loss fulfils the relation:

$$G(z) = \left(\frac{1}{1 - \sigma^2 (Q(z) - Q(1))} \right)^{1/\sigma^2} \tag{4}$$

[1] At this first stage, we assume that there is only one sector, i.e. one Γ.
[2] i.e., the exposures (net of recovery) ν_A can be assumed to be integer values

where

$$Q(z) = \sum_A p_A z^{\nu_A}$$

which we denote as the portfolio polynomial; see Credit Suisse Financial Products (1997, eqn (68)) and Bürgisser *et al.* (2001, eqn (13)). Equation (4) can be used to derive a recursive condition on the coefficients $p(n)$ which is due to Panjer:

$$(5) \qquad p(n) = \frac{1}{n(1 + Q(1)\sigma^2)} \sum_{j=1}^{\min(m,n)} \left(\sum_{\nu_A = j} p_A \right) [\sigma^2 n + (1 - \sigma^2) j] p(n - j)$$

where $m = \deg(Q)$ is the largest exposure in the portfolio, and the starting value is given by $p(0) = (1 + Q(1)\sigma^2)^{-1/\sigma^2}$; see Credit Suisse Financial Products (1997, eqn (77)) and Bürgisser *et al.* (2001, eqn (14)).

These equations can be extended to the case of several segments; the economy is then described by a set of systematic risk factors $\Gamma, \ldots, \Gamma_N$. More specifically, under the assumption of independent segments, the extension for (5) is given in CreditRisk+ Credit Suisse Financial Products (1997, eqn (77)). An extension of the model, where the correlation structure of the segments is taken into account is shown below.

2.1. Advantages

One of the main advantages exhibited by the CreditRisk+ model is due to its concise analytical form. It allows calculation of the loss distribution through a recursive procedure which requires less computational effort than simulation-based models.

It also requires a limited amount of data compared to other models such as CreditMetrics™ and definitely lower quantities of information compared to structural models (such as KMV™) which require an estimation of the firm's assets. In the latter models, the evaluation of corporate debt, especially for non-quoted firms, results are impractical and must be approximated with simplifications which eventually lead to the reduced form approach anyway (Knoch and Rachev, 2001). As oppposed to these models, CreditRisk+ derives default information from statistics of historical default rates applying such information to clusters of obligors. In this sense, the valuation is characterized by an actuarial approach which differs substantially from an intensive evaluation of each obligor.

The actuarial connotation of the model can itself be viewed as an advantage if considered from the point of view of the risk manager, as the culture and expertise in applying this approach is made available by the consolidated experience developed in the insurance field.

The evaluation process is not related to market prices or market credit spreads, which is another distinguishing feature of the approach. This is why it is sometimes erroneously labelled as intrinsically apt only to book, as opposed to market, evaluations of the credit portfolio. It is certainly a method which allows the evaluation of illiquid portfolios, i.e. non quoted debt (this feature is hardly attributable to structural models and more generally to so-called mark-to-market approaches). This does not imply, though, that the estimation process of the CreditRisk+ model cannot be applied to marked-to-market exposures.

2.2. Problems

On the other hand, the CreditRisk+ model is considered in general to be inappropriate from a corporate governance perspective. In other words, if a credit portfolio model is to be used as the basis for determining, managing and maintaining capital allocation policies throughout the bank, it must fulfil several requirements which this model does not seem to satisfy.

Default Mode vs Mark to Market

It is hardly acceptable to use separate models for evaluating the banking book and the trading book. Being able to evaluate retail and small business positions, for which this model does not require market prices is, of course, essential; but having to resort to different methodologies or models for evaluating financial products priced on wholesale secondary markets raises important coherence questions, especially when it becomes necessary to consider diversification effects across the bank's positions.

Therefore, an essential feature of an ideal credit portfolio model is the possibility, not of considering market values, but of considering loss due to change in market value, where this change is given by variation in the credit status of the issue. This feature is not present in the standard version of CreditRisk+, and this limits its application to only non-tradable positions typically excluding the bank's investment portfolio.

Conditional vs Unconditional

There is a strong debate among practitioners in credit risk modelling about the appropriateness of considering portfolio loss conditioned by specific external hypothesis such as macroeconomic scenarios or whether an unconditioned distribution of portfolio losses should be used to remain unbiased by specific assumptions.

At first glance, it might seem obvious that losses should be conditioned by external factors and that these must enter in the evaluation. On the other hand, what must be avoided is considering the same factors more than once (Savona and Sironi, 2000). Macroeconomic influences, for example, could have already been considered in the rating process, i.e. when the obligor or the asset has been assigned to a specific risk category.

In any case, stress analysis and scenario-based evaluations remain nevertheless an important complementary tool to general credit risk modelling and these, of couse, imply considering evaluations conditional on these hypothesis.

CreditRisk+, in its original formulation, is an unconditional model, where the gamma distribution is chosen to consider all possible values that the default rate can take on. The Poisson distribution used to estimate the distribution of default events is then mixed with the gamma distribution and combined with the exposures to provide the distribution of losses for the portfolio. The result is an unconditional model for estimating portfolio losses.

The choice of the gamma distribution is important for maintaining a neat and tractable closed-form solution: abandoning this hypothesis threatens the analytical properties of the model. In other words, unconditionally seems embedded in the model.

Correlations

The capability for a risk model to consider diversification effects is essential to the implementation of active portfolio management policies. CreditRisk+ considers sectors as statistically independent entities to which exclusive components of a single obligor's systematic risk are allocated. Such independent entities are more a mathematical concept than a realistic background factor and, unfortunately, have no resemblance to realistic sectors such as industries or geographical entities, for which banks have access to vast amount of data, but which are sometimes evidently influenced by strong correlations.

Recovery Rates

Concerning the risk associated with the degree of success attainable by recovery procedures over defaulted positions CreditRisk+ takes the simplest possible view: the recovery process is riskless. Recovery rates are a constant deterministic factor applied to exposures at the input stage.

In a realistic model, not only would we want to consider the variability of recovery rates, but also, and in this respect all state-of-the-art models are deficient, dependencies among recovery rate categories or, even more interestingly, among severity and default risk.

2.3. Extensions

The problems listed above are important reasons which have strongly inhibited part of the international banking community from applying the CreditRisk+ model to real-world situations.

While, on one hand, the fact that the model does not require asset evaluation information (which is sometimes not available or often based on low quality data) is certainly appealing; on the other hand, the original model is made particularly unattractive by the fact that valuable information which is often available cannot be used (e.g. sector correlation data).

We wish to highlight, however, that this and other problems mentioned above are not intrinsic in the approach underlying the CreditRisk+ model. We just briefly mention here possible paths for overcoming the pitfalls described above; then, section 3 describes in detail a solution for what is probably the main shortcoming of the model: correlations among systematic default and severity risk factors.

Mark to Market

Not being able to cope with changes in the value of credit positions beyond the default case, i.e. considering intermediate values for the exposure between the default and the current (book or marked-to-market) value of the position, is clearly a strong limitation. What is not clear and straightforward is whether the possibility exists for the CreditRisk+ model to do just that, while maintaining certain characteristic and useful properties such as the analytical framework of the model.

Such an attempt has been investigated by Rolfes and Broeker (1998). The clustering process of the original model is maintained in this extension and the introduction of a transition matrix of probabilities is applied to the initial position, producing a portfolio of upgrades and a separate portfolio of downgrades. The CreditRisk+ model is applied to each of these, producing two loss distributions which are then juxtaposed in a convoluted distribution of the total portfolio value which is actually comparable to the CreditMetrics$^{\mathrm{TM}}$ result. The proposed solution, though, remains in an analytical framework avoiding simulation processes.

Conditionality

Obtaining the flexibility of considering conditional evaluations from the CreditRisk+ model seems possible only at the cost of abandoning the analytical approach for a more resource-consumptive simulation approach.

Actually, the choice is not so devastating. A conditional model can be obtained from the CreditRisk+ model by unbundling the mixing process and substituting the gamma distribution, used to describe the default rate on which the Poisson distribution is based, with specific scenarios where the default rate takes on particularly chosen values. This is typically done with an averaging (weighting) process over a set of specific scenarios where every single scenario's loss distribution is derived by a conditional recursion as described in CreditRisk+ (Credit Suisse Financial Products, 1997, eqn (25)).

The issue of appropriateness in the context of a general credit risk management framework (as discussed in section 2.2) remains open, but the choice does not imply having to choose among completely different methodologies from the one underlying the CreditRisk+ model, allowing to retain the advantages of a single overall analytical approach.

The last two points – correlations and recovery rates – are the object of section 3.

3. Extending the Correlation Concept in CreditRisk+

The following subsections show how to extend the conventional CreditRisk+ model to account for

(a) correlated default events, driven by different factors, and
(b) for stochastic variations in severity

and for correlations in these between different collateral types.

3.1. Default Correlations

CreditRisk+ as presented in section 2 (one-sector approach) is appropriate to model losses if defaults are driven by the same systematic factor. However, if for instance, loans are to corporates in different industries that are driven by a separate factor each, one has to extend the model. One approach, presented in CreditRisk+ is to apportion the credit exposure of obligors to different sectors, which are modelled independently. However, such an approach is difficult to realize in practice, especially since there are typically large interdependencies between industries to be considered. Another approach is presented in Bürgisser *et al*. (1999). There, dependencies are taken into account by the first two moments of the loss distribution. More specifically, the *EL* and *UL* of the loss distribution are calculated analytically and independent of any distributional assumptions, taking into account the covariance matrix of the industry default rates. The multi-segment *UL* is then required to match to the variance of the loss distribution for a single systematic factor, whose variance (σ^2) can then be extracted from the following condition:

$$UL^2_{\text{single sector}} \stackrel{!}{=} UL^2_{\text{multi-segment}}$$

$$\sigma^2 EL^2 = \sum_k \sigma_k^2 \cdot EL_k^2 + \sum_{\substack{k,\ell \\ k \neq \ell}} \rho_{k,l} \cdot \sigma_k \sigma_\ell \cdot EL_k \cdot EL_\ell \tag{6}$$

where EL_k denotes the expected loss in segment k, for details see Bürgisser *et al*. (1999). The variable σ_k denotes the standard deviation of the default rate, normalized to its mean, for degment k. The correlations between these normalized default rates are denoted ρ_{kl}. Deriving the overall loss distribution then proceeds with the usual recursion formula from CreditRisk+, using the mean (EL) and variance (σ^2) of the single sector approach just determined above, as the parameters determining the gamma distribution.

The approach is chosen

(a) because there is no unique representation of a multi-dimensional gamma distribution, and
(b) due to the scarcity of data that would be needed to fully specify multi-dimensional distributions.

The next subsection specifies how one can also include the risk of the collateral value, which is neglected in most models.

3.2. Severity Conditions

Most of the energy for publicly avalilable credit risk models has been devoted to model default risk. Collatoral as a risk factor is mostly neglected, although it has been pointed out as a potential risk in the recent literature (BIS, 1999; Frye, 2000a). Indeed, especially for banks active in mortgage lending or in the middle market segment, where typically a large fraction of the loans is covered by real estate or other pledged assets, collateral risk is important.

Therefore, we have extended CreditRisk+, in analogy to default risk, by using a scaling factor, to model the risk inherent in collateral value, where we assume that severity varies independently of default.[3] In analogy to (1), the loss variable is modelled as

$$L = \sum_A I_A \cdot \nu_A \cdot \Lambda_A \cdot \Lambda \tag{7}$$

where Λ_A is a random variable that describes the obligor specific variation of the loss severity and Λ describes the market variation of the loss severity – common to all collateral of the same type. The variables Λ_A and Λ are independent, and both are mean equal to one. The volatility of the idiosyncratic Λ_A is denoted by σ_A and the volatility of Λ is given by σ. In a more general

[3] For an extension, where this assumption is relaxed, see Bürgisser *et al*. (2001).

case, this approach can also be extended to different types of collateral, see Bürgisser *et al*. (2001). The EL and UL of the loss variable can be calculated straight away by the sum of the 'within' and 'between' variances, yielding the formula

$$UL^2 = \underbrace{\sigma^2 EL^2 + \sigma^2\delta^2 EL^2 + \delta^2 EL^2}_{\equiv UL^2_{\text{syst}}}$$

$$\underbrace{+ (1+\delta^2)\cdot\sum_A [(1+\sigma_A^2)p_A - (1+\sigma^2)p_A^2]\nu_A^2}_{\equiv UL^2_{\text{div}}} \tag{8}$$

The latter expression is due to the statistical nature of default and severity and is diversified away for large portfolios. There, only the first three terms matter, which we call systematic or non-diversifiable risk. Now, in addition to (2), UL_{syst} contains contributions from severity risk. Also note, that the systematic component of UL is fully symmetric with respect to either default or severity risk.

Since, as shown in Bürgisser *et al*. (2001, §8.2), idiosyncratic severity variations can, for most cases, be neglected, we focus on systematic severity variations in the following. To calculate the full loss distribution, we proceed by averaging over all conditional loss distributions, given a state of the economy represented by the pair (Γ, Λ), with weights according to the density functions f_Γ and f_Λ of the risk drivers:

$$f(x) = \int\int f_x(x|\Gamma=\gamma, \Lambda=\lambda) f_\Gamma(\gamma) f_\Lambda(\lambda) \mathrm{d}\gamma\, \mathrm{d}\lambda \tag{9}$$

Here, as said above, severity and default are assumed to be independent. This averaging process can now be performed in two steps: first for Γ and then for Λ. Since we have already computed the average with respect to Γ, yielding the discrete distribution $p(n)$ – see equation (5) – the full loss distribution can be calculated from a multiplicative convolution of this distribution with the severity variable Λ. The resulting cumulative loss distribution $F(n)$ can then be expressed as

$$\begin{aligned} F(n) &= \text{prob}\{\nu\lambda \leqslant n\} \\ &= \text{prob}\{\nu = 0\} + \sum_{\nu>0} p(\nu)\,\text{prob}\{\lambda \leqslant n/\nu\} \\ &= p(0) + \sum_{\nu>0} p(\nu) F_\Lambda\{n/\nu\} \end{aligned} \tag{10}$$

where $F_\Lambda(\lambda)$ denotes the cumulative probability distribution of the variable Λ

alone. To actually compute the probabilities $F(n)$ and the corresponding percentiles, we replace this infinite sum by a finite summation over ν, with the upper summation bound depending on n. The resulting approximation error can be controlled as demonstrated in Bürgisser *et al.* (2001, App A.2).

It would now be natural to extend the model to account for both different industry sectors as well as different collateral types that are driven by different factors. However, with the same arguments as given already in section 3.1, we follow a more pragmatic approach in which we reduce a multivariate structure to a single segment with the same *EL* and *UL*. Thereby, we extract the relevant parameters σ and δ, which are used to calculate the full loss distribution as in the single segment case. The exact procedure is given in Bürgisser *et al.* (2001).

It is worthwhile noting, that in the limit of a large portfolio, which is typical for large banks with loans of the order of 10 000 or more, the loss distribution resembles the assumed behaviour of the economy, modelled by the distributions f_Γ and f_Λ. In mathematical terms, the loss distribution is given by the multiplicative convolution of f_Γ and f_Λ. Details of the portfolio only enter via the relative expected losses across the segments. As a conclusion, the focus in modelling credit risk for large portfolios should not be on choosing the correct parameters for single obligors, but on the distributional form of the econometric risk drivers.

3.3. Joint Default and Severity Variations

Another extension concerns the assumption of independence between default and severity variations, which can be bypassed using the technique of conditional independence. In such a model, we calculate the loss density function conditional on the states Γ, Λ (Γ for the systematic factor of the relative number of default events, and Λ the systematic factor influencing the severity). Such realizations of Γ, Λ can be taken either from historical data, from judgement or from a linear factor model. The unconditional loss distribution is then obtained by averaging over all conditional loss distributions with the corresponding weights of occurence of the set Γ, Λ. We thereby include any dependence structure between the risk drivers, without distributional assumptions, at the cost of computational efficiency. This concept is well known in market risk.

More specifically, for a linear factor model, the possible underlying macroeconomic variables for modelling default rates by industry and severities by collateral types are interest rates, gross domestic product, real estate price indices, unemployment rates, exchange rates and others. The dependence between default and severity can be modelled by relating them to common factors, e.g., interest rates. Frye (2000a,b) applies this concept with default modelled by a Merton approach, whereas, here, the model uses stochastic default rates.

4. Results Using Default Rates from Banca d'Italia

This section discusses an example where the CreditRisk+ methodology has been extended with the refinement for default correlations as given in subsection 3.1, cf. Bürgisser *et al*. (1999). Further examples where the severity risk is included are found in Bürgisser *et al*. (2001, §8).

The Italian Central Bank has collected an extensive amount of data on mortality rates related to bank's defaulted investments with corporate counterparties. The survey's results used in our study refer to a period of fifteen years, from 1985 to 1999, and cover 24 different industrial sectors and 5 geographical areas comprising the whole Italian territory (Banca d'Italia, 2000).

As expected, correlations among these geo-industrial sectors are not absent and, sometimes, are substantial. To consider such information in the original CreditRisk+ model would require a very complicated process of orthogonalization of the statistical data on the correlated geo-industrial sectors. This process would produce a different number of statistically independent sectors to which the portfolio elements would then have to be allocated in a non overlapping way (Credit Suisse Financial Products, 1997, A12.2).

Apart from the inevitable approximations incurred and the difficulty of proving the overall soundness of the orthogonalization process, the usefulness of the results would certainly be strongly questionable from the risk manager's point of view, given the problematical interpretation of the mathematical sectors involved in the evaluation.

The extension described in subsection 3.1 allows instead the use of correlations directly computable from the geo-industrial data mentioned above.

In the examples below, the two minor geographical areas South and Islands (Sicily and Sardinia) have been merged into one. The results are based on segment correlations among all geographical regions and all 24 industrial sectors but are here represented by geographical region only, for brevity purposes. Note that correlations are typically in the range of 25−65% and are, therefore, important to consider.

We look at two different ways to determine the risk parameters. In the basic model example of Table 1 all clients are in one single segment such that all individual volatilities add up to the sector default rate's volatility, cf. Credit Suisse Financial Products (1997, eqn (77)), which means that it is implicitly asumed that the sectors are fully correlated. The second example of Table 1 is based on the segmentation and correlations mentioned above. The segment volatilities are again obtained by adding up the individual volatilities of the corresponding clients (Credit Suisse Financial Products, 1997, eqn (77)). Correlations enter into the model as described in subsection 3.1 (Bürgisser *et al*., 1999).

The estimated loss, at the 99.95 percentile, has clearly decreased in the calculation with correlations. This might seem the obvious result of the diversification effect, but the portfolio has not changed from the extended to

Table 1: Risk Contributions to VaR at the 99.95-percentile (Absolute and in % of Exposure at Default)

	Exposure At Default	Expected Loss	Contribution Risk	C. Risk%
Basic model without considering correlation => one sector				
North-West (2014)	101.0	3.72	11.1	11.0%
North-East (2018)	101.2	36.62	10.8	10.6%
Centre (1975)	100.0	3.58	10.7	10.7%
South (272)	13.7	0.49	1.5	10.6%
Total corporate clients (6279)	315.8	11.41	34.0	10.8%
Industry + geographical region correlation				
North-West (2014)	101.0	3.72	9.1	9.1%
North-East (2018)	101.2	36.62	8.4	8.3%
Centre (1975)	100.0	3.58	7.4	7.4%
South (272)	13.7	0.49	0.9	6.6%
Total corporate clients (6279)	315.8	11.41	25.8	8.2%

Notes: All absolute numbers are given in millions of €. Numbers in parenthesis represent the amount of elements in the (sub)portfolio(s).

the basic case. It is the estimation of the variance of the portfolio's loss that has changed. If the portfolio unexpected loss were estimated through observations, it would remain unchanged and the difference in the results would only be at the marginal contribution level: the marginal VaR[4] (or contribution risk) would be reallocated as a consequence of considering the correlations.

What the results highlight is that, if the variance of the portfolio is instead derived from the input data, as suggested in the original documentation (Credit Suisse Financial Products, 1997), the uninformed version (which does not consider correlations) is implicitly assuming the highest correlation possible among all sectors, clearly overestimating the overall risk. In other words, the extension on correlations really allows a more precise evaluation of the portfolio credit risk when the variance is not observable and is estimated through the model itself.

Data in Table 2 emphasizes the result showing that the correlation model produces a decrease in the overall risk by 24%, and that this difference is even more marked in the smaller subportfolios.

Table 2: Risk Contributions to VaR at the 99.95-percentile in % of Exposure at Default

	NW Italy	NE Italy	C. Italy	S. Italy	Total
One sector	11.0%	10.6%	10.7%	10.6%	10.8%
Industry + Geographical correlation	9.1%	8.3%	7.4%	6.6%	8.2%
Difference	−17.5%	−22.1%	−30.8%	−37.8%	−24.0%

[4] Marginal Var is computed as marginal unexpected loss, as in Credit Suisse Financial Products (1997) and Bürgisser *et al.* (2001), multiplied by a scaling factor.

5. Conclusion

The properties of an analytical approach to credit risk modelling are commonly identified with an exaggeratedly simplistic model, which cannot be used in practice within a bank's corporate governance framework, where credit risk modelling does not seem amenable without falling back to simulation approaches which can capture the complexity of a realistic estimation of all the variables involved in a bank's portfolio management.

This paper analyses the best-known analytical approach to credit risk evaluation – namely CreditRisk+ – and shows that important limitations which might seen embedded in the model can be overcome by extending the model without disrupting its nice closed-form structure.

While we only mention some extensions, we describe in some detail how to consider correlated sectors in the model, both for default risk and, drastically improving on the original model, on severity risk.

These possibilities suggest that the analytical approach underlying the CreditRisk+ model, appropriately extended, is a powerful tool which could be extensively applied in several contexts such as active portfolio management on liquid trading positions, collateral and securitization evaluations, stress and scenario-based analysis, capital allocation policy definition and management and, finally, fulfilment of forthcoming Central Bank Regulations; all deeds for which the original model is usually considered inadequate.

Disclaimer

Hadley C. Taylor is Consulting Director of the Banking Business Unit at AISoftw@re S.p.A. Alexandre Kurth and Armin Wagner are in the Credit Risk Control department at 'UBS Switzerland Division' of UBS AG. Opinions expressed herein are the authors' and do not necessarily reflect the opinions of UBS or AISoftw@re.

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Non-technical Summary

Efficient portfolio management techniques, of foremost importance within financial institutions, must consider the presence of different risk-drivers and the relation among these. For commercial banks, Credit Risk is by far the greatest component among the risk factors which must be assessed, controlled, managed and possibly hedged, and for this reason receives the highest attention by Regulators and practitioners and, in the end, by investors in the Banking and Financial sectors.

In recent years, the industry has produced several apparently highly differing approaches to modelling risk in a credit portfolio. Actually, some classifications such as *structural* (where default risk is estimated based on the value of obligors' assets) vs *reduced-form models* (where default risk is modelled by the theoretical probability of default or bankruptcy) have helped to boil down the differences to only the main distinctions. In addition, several

studies have shown that models which seem very different, are, under specific circumstances in fact quite similar, even from a mathematical standpoint.

Nonetheless, some important distinctions remain. Among these is the analytical tractability of a model which can be represented by a mathematical closed-form solution vs approaches which resort to Monte-Carlo or historical simulation to cope with the mathematical intractability of the model. Other distinctions concern actuarial (typically clustering) approaches vs mark-to-market approaches (typically based on a single exposure basis) or the ways used to consider recovery rates on the exposures and, last but not least, the way to represent correlations among the default probability of the obligors.

Some clear advantages come with analytical models, but the complexity of credit risk management at portfolio level seems too complex to be appropriately modelled by anything but simulation-based methods. Here, we propose the revisitation of the most well-known actuarial approach for modelling default risk at portfolio level, namely CreditRisk+, which, in its original form, follows an analytical approach. The key aspect of this revisitation is that, while we show the several ways in which the original model can be extended to overcome the problems mentioned above, we never depart from the analytical paradigm. We suggest that what are considered to be the main shortcomings of the model are not intrinsic in the underlying methodology and therefore we maintain that the extended model can become the backbone of an enterprise-wide credit risk management system. In particular, we discuss an extension of the original model referring to what is probably considered the main limitation of the model, i.e. its approach to considering correlations.

The CreditRisk+ model allows for the representation of background factors, which are totally uncorrelated: they can be viewed mathematically as orthogonal axes which define the space in which the obligor's default probabilities lay. A dependence structure among the obligors is achieved by apportioning the exposures to the orthogonal factors, a process which is rather arbitrary, and therefore which creates an additional source of uncertainty. These factors cannot be used to represent industrial sectors or geographical areas etc., which typically incorporate significant correlations between them, unless resorting to some very complicated process of ortogonalization of correlated factors which is difficult to derive, and most importantly, very difficult to interpret for users of the model.

In the main sections of the paper, we propose a change to the original model which allows for consideration of correlated data in the CreditRisk+ model in a rather straightforward manner. In this way, we obtain two important results:

1. We define an analytical actuarial approach which can consider data on geo-industrial sectors to represent systematic default risk with correlations.
2. We similarly apply the new approach to consider data on severity risk segments to obtain a stochastic representation of recovery rates in

CreditRisk+, which moreover considers correlations among systematic severity risk segments.

We then proceed to show how real-world geo-industrial data can be used in the model and discuss results obtained. The data is taken from mortality rates produced by the Bank of Italy referring to a period of 15 years and is relative to 24 industrial sectors and 5 geographical areas, which cover the whole of the Italian territory.