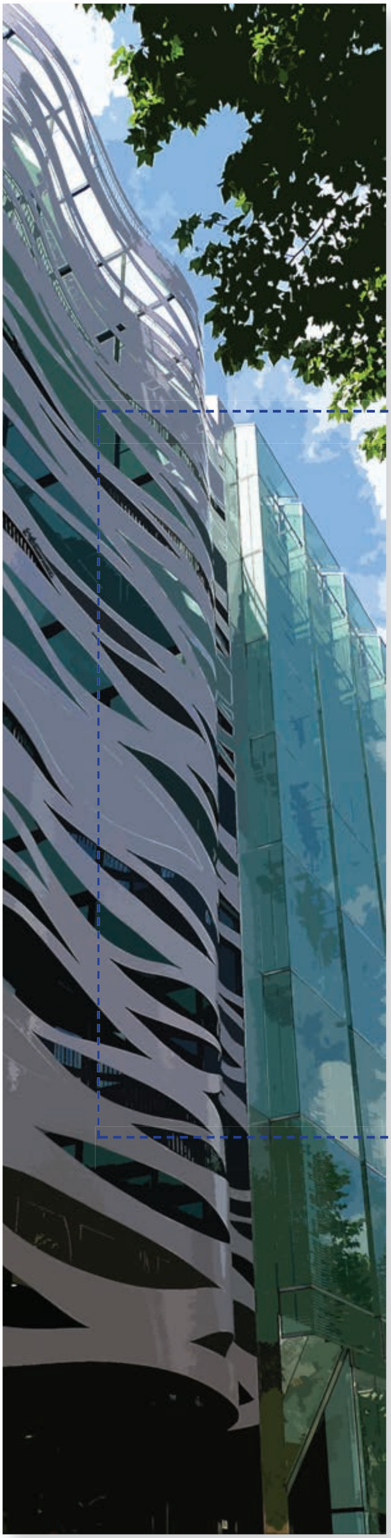

Ole Jonny Klakegg, Kari Hovin Kjølle,
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Asmamaw T. Shiferaw, Ruth Woods
(editors)

PROCEEDINGS FROM 7TH NORDIC CONFERENCE ON

CONSTRUCTION ECONOMICS AND ORGANISATION 2013

GREEN URBANISATION – IMPLICATIONS FOR VALUE CREATION

TRONDHEIM
12–14 JUNE 2013

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Norwegian University of
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CREON

STATSBYGG

SKANSKA

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7th Nordic Conference on Construction Economics and Organisation, Trondheim 12.-14. June 2013

FOREWORD

The first Nordic Conference on Construction Economics and Organisation was held in Gothenburg at Chalmers University of Technology back in 1999. Since then, the conference has been held biannually (with the exception of 2005) in Sweden (4 times), on Iceland and in Denmark. Now it is Norway's turn to host the conference, and Finland is scheduled to take over the baton next time. We are very pleased to be carrying on the tradition, and we hope to live up to the expectations created by previous conferences.

In 2011 in Copenhagen an initiative was taken that marked a shift in the organization of this series of Nordic conferences: CREON was founded. The first general assembly was held during the 6th Nordic conference. The CREON network is a voluntary, non-profit association for people who study, work, teach and do research about all aspects of management and construction. The CREON network aims to promote collaboration across Nordic knowledge institutions and this series of conferences is an important activity for CREON. NTNU and SINTEF, as local organizers, are proud to present the 7th Nordic conference on behalf of CREON.

We, the organizers, had two specific ambitions when we started preparing for this conference: Firstly, we wanted this conference to be acknowledged as a high quality academic conference. We have therefore put a lot of effort in the review process. Three rounds of blind reviews is a lot of work, but now when we see the result – it was worth it. The close collaboration with Akademika Publishing makes sure publication points can be awarded to the authors. The papers are presented in two parallel sessions over three days here at the NTNU Gløshaugen campus.

Secondly, we wanted to establish a closer connection with the construction industry. We therefore put together a very strong Program Committee, comprising of prominent representatives from the Norwegian Construction Industry, who identified the main topic: Green Urbanization – Implications for Value Creation. We realized that it was not realistic to turn an academic conference into a popular construction industry event, so we have chosen to collaborate with NTNU in marking their new initiative for improving knowledge about the building process. Thus the idea for the Building Process Day was born – we will spend half a conference day together with distinguished guests from the Norwegian construction industry. The building process day will also be the scene for another conference innovation: Statsbygg awards for best paper and best young researcher. Enjoy!

Ole Jonny Klakegg, Kari Hovin Kjølle, Cecilie G. Mehaug , Nils O.E. Olsson, Asmamaw T. Shiferaw, Ruth Woods (Editors).

INTRODUCTION, CONTEXT AND SUMMARY

The construction industry plays an important role in society. Construction forms our physical surroundings and creates the infrastructure we need to develop society. Physical infrastructure and buildings represent approximately 70 per cent of Norway's Real Capital. Public investments in infrastructure constitute half of all infrastructure investments in Norway. It is also a major factor in the society’s economy, representing a substantial share of the GNP, and, for example, it represents approximately 30% of the employment in Norway. According to Statistics Norway the construction sector is the third largest industry in Norway, employing 350,000 workers in more than 75,000 enterprises, and has a high turnover; over NOK 308 billion in 2011, approximately the same level as 2008 which was a top year. The Confederation of Norwegian Enterprise (NHO) states that the construction industry in Norway provides 10% of the total value creation. The construction industry is truly a cornerstone of our society.

On the other hand the dwellings and construction industry is also mentioned as “the 40% industry” by the Ministry of Local Government and Regional Development. This is a reminder that the construction industry uses approximately 40% of the total energy in our society, 40% of the materials, and produces about 40% of the waste that goes into landfills. This indicates the industry’s importance in relation to climate and other environmental challenges. If there is one industry that really can make a difference, it is probably construction.

Furthermore, the construction industry has a reputation of being conservative, having a low degree of innovation, and low productivity. It is not known to be the first industry to implement sustainable solutions. The construction industry does use low-tech solutions and employ low skilled workers, but it does also include highly advanced New Tech solutions to technical problems and engage some of the most qualified engineers in our society. The truth about this industry is as complex as the problems it is trying to solve on behalf of society.

In the next ten years, growing globalization will promote an already increasing trend of competition among international construction companies according to The Federation of Norwegian Construction Industries (BNL). Additionally, Norway has the following challenges ahead:

- Growing population, expected to surpass 7 million by 2060, up from today’s 5 million
- Increasing trend towards centralisation
- Growing elderly population with needs for health care and housing
- More pressure on transport infrastructure
- An ever increasing immigrant workforce

- Long cold winters and harsh climate, worsened by climate change which may lead to more floods, landslides and frequent winter storms

All these challenges will lead to:

- High demand for new dwellings
- Need for higher investment in low energy buildings
- Need for more robust buildings and infrastructure
- Need for more investment in transport infrastructure
- Need for a larger workforce and recruitment in all sectors
- Need for good integration programmes, development of expertise and training in relevant areas for new migrants and unskilled labour.

These are the sort of challenges that the Program Committee saw when they discussed the profile for this event back at the beginning of 2011. They called it Green Urbanization. The situation calls for new solutions, new knowledge, new thinking. Both small steps and huge leaps help as long as they lead in the right direction. Is the construction industry ready for it?

The sector is fragmented and contains many small enterprises. Thus, large companies account for a smaller share of the construction output in Norway than in most other countries. Small companies with highly specialized competence indicate a fragmented industry. The typical construction project is also said to be one-of-a-kind at a hectic pace. It is obviously hard to optimize process and solutions in such an environment.

Although to a lesser degree than other countries, the Norwegian construction industry is currently facing the challenges that have followed the 2009 financial crisis; small enterprises lost competence due to temporary redundancy and the investments were at a minimum level. Therefore, the diffusion of new knowledge and investments was also at a minimum. To what degree is the construction industry equipped to meet challenges ahead? And to what degree is the academic community able to help this industry overcome its challenges? These are questions that deserve to be asked, and perhaps some answers or indications may be found among the contributions to this conference? Are the academic resources ready for it?

This introduction, its examples and identified challenges are chosen from the Norwegian context, in full awareness of the current peculiarities of the Norwegian situation. We do have a special and advantageous position, but Norway is still clearly a distinct part of the Nordic context. We are also deeply embedded in the bigger international economy and global community. Therefore, the conference profile and the Nordic conference setting feel highly relevant in 2013.

The contributions span a wide range of issues, organized in three tracks with three major themes in each:

Sustainable Development of the Urban Environment	**Organizing for Execution**	**Efficiency in Construction**
The Sustainability Perspective	Governance and Strategy Implementation	The Human Aspect in Construction
Sustainable Design	Decision Making and Relations	Productivity and Quality
Sustainability and People	Learning from Construction Projects	Supply Chains and Planning

The first track; **Sustainable Development of the Urban Environment** is the signature track of this conference. It relates directly to the challenges addressed by the program committee back in 2011. The invitation to authors included contributions on sustainability in a wide sense – the concept of sustainability, the framework conditions defined by government and international agreements, the built environment, both the upgrading of existing buildings and finding solutions for future built environments. As the papers of this track shows, the authors cover these issues from several perspectives and cover a wide range of issues as intended. The track provides a varied and thought provoking approach to the term "sustainable"; one of the most oft-used terms in the construction industry today, but which also continues to be one of the most important issues.

Key issues addressed by the papers are; different challenges in combining urbanization and environment respect, the role and use of green certification systems, the role of sustainability in project management, passive house building, renovation and retrofitting from a sustainable perspective and the development of new technology to the deal with climate and age related problems in building materials. Green has become an important issue and two papers look at the role of green certification and policy in stimulating company activity. It can on the one hand, as one paper suggests, become a catalyst, stimulating more green certified buildings. On the other hand, green may mean, as the second example shows, following the market rather than focusing on policies which benefit clients and society. Encouraging a sustainable build is a theme which may be understood as central in this track; it is present in the aforementioned papers and also plays a role in the papers which focus on retrofitting, project management and the building of passive houses. Further issues are exploring the difference between project management success and project success; analyzing collaborative working and experienced effects on the energy performance of a building project; an analysis of existing Norwegian retail development and their impact on local energy consumption; and the effects of user involvement in the briefing and design of a workplace. Scandinavian and particularly Norwegian examples dominate the papers, but there are also case stories from USA and China and contributions from the Netherlands and the UK.

The second track; **Organizing for Execution** represents a combination of new and classic issues around governance, decision making and learning. It covers issues with a wide perspective and long-ranging consequences for the organisations involved. Key issues are governance mechanisms, strategy implementation, decision making, relations and learning. Several papers discuss aspects of governance and how organisations may implement processes and structures in order to improve their value creation and value for money in investments. Examples presented here are the governments in the Netherlands and Norway, as well as several anonymous companies associated with the construction industry. This has a lot to do with designing purposeful decision making processes and using the right criteria for prioritizing and choice of projects. Other perspectives are how to implement necessary transformations of the organization in a changing environment. This is an important issue in a world of increasing globalization, competition and new technologies.

One major topic in several papers is the clarity and better understanding of roles and responsibilities in project organisations and between the project and its mother organization, as well as other stakeholders. These relational issues include communication, motivation, emotions and trust, just to mention some important aspects. The most fundamental topic in these papers is perhaps learning. Learning from cases and accumulating experiences in organisations in construction has been argued a particularly challenging thing to do. Several papers look into these challenges.

The types of organisations represented in these papers range from large public agencies, via industrial companies down to facilities management companies. The projects range accordingly from large infrastructure investments via large building design and development processes down to small and medium sized renovation and upgrading projects in existing buildings. All in all, this track comprises discussions on some of the major issues engaging the research community on construction projects in recent years. The picture is clearly Nordic in the sense that most of the cases reported are documented in the Nordic region, but extended to include Poland, France and the UK.

The third track; **Efficiency in construction** is the original core area of construction economics and organisation, internationally perhaps better known as construction management. It covers both qualitative and quantitative aspects of efficiency in construction. The majority of the papers address the human aspect in construction, but in different ways. Innovation, learning, daily life, scheduling, BIM, productivity, quality, procurement, contracts and supply chains are addressed, among other issues. Roles and interfaces between different stakeholders in a construction project are addressed in several papers.

Innovation is a key topic. It is addressed both explicitly in some papers, and implicitly in many more papers. Innovation in the construction sector is an important topic. It is mainly illustrated through cases. The construction sector is characterised by cooperation between many stakeholders. Design, planning and execution are typically carried out by project-oriented organizations. Deliveries of building components and materials are carried out by manufacturing companies. Interestingly, we also have comparisons between the construction sector and other sectors, as well as the use of analytical models used in other industries but here applied in a construction context.

Contracts and supply chain are addressed in several papers. The contractual relationships in the construction industry are illustrated, with special focus on incentives and stakeholder relations. Planning is addressed in a quantitative way, but from different perspectives. We also have a terminology overview related to planning.

The track includes examples of technology advancement in the construction industry, including Building Information Modelling (BIM).The track includes BIM approaches in a life cycle perspective.

Cases and data come from a wide array of countries, and are not limited to the Nordic region.

The research approaches represent an interesting mix of theoretical work in the form of literature reviews and conceptual papers, development of decision models and understanding of observed performance in real situations, as well as documenting learning from cases and demonstration projects. The empirical side is not surprisingly dominated by document studies and interviews. Several papers are based on case studies. Some papers have a more theoretical approach, while others are very empirical and data driven. In total these proceedings represent a good cross section of contemporary research in the field of construction economics and organization in 2013.

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SINGULARITY FUNCTIONS FOR INTEGRATING TEMPORAL AND FINANCIAL CONSTRAINT MODEL OF CONSTRUCTION PROJECTS

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Abstract. *The performance of construction projects exist within a multitude of constraints of temporal, financial, or other nature that their managers must consider and satisfy. The interdependent nature of constraints has made it difficult to efficiently explore different options during planning and execution so that the desired performance is reached and maintained. There exists a strong need to develop a new comprehensive numeric modeling approach for such managerial aspects of construction projects. Singularity functions have the ability to provide the desired integrated model. They are unique in that they are defined as range-based case distinctions, so that they can be 'switched on and off' as needed, but will remain continuous. This class of mathematical functions, known by the names of their inventors, Macaulay and Föppl, was first used in structural engineering analysis to calculate the effects of variable load profiles on structural members. This transformational research can newly apply singularity functions to problems in construction engineering and management. Various types of temporal and financial constraints are reviewed, including the timing and duration of activities on the one hand, and costs and payments on the other. Conditional constraints, e.g. cash discounts for early payments, the cost of capital from retained earnings or financing interest, and bonuses or penalties for performance deviations, are of special interest. Such components are vital to achieve realism in a mathematical model. An example of a small construction project with several different types of constraints illustrates its new application. The paper concludes with recommendations to broaden this innovative modeling scheme to other yet even more complex constraints. This can also enhance constraint satisfaction with optimization and increase the efficiency of future construction management.*

KEYWORDS: Construction project; constraints; scheduling; finance; singularity functions.

1 INTRODUCTION

All construction projects are subject to constraints, and the final project can be considered to be a single valid solution to a constrained system of interacting parameters from among the various 'dimensions' of project management, i.e. its well known and often cited aspects time, cost, scope (Kerzner, 2009), resources, safety, quality, sustainability, etc. Therefore project managers must be keenly aware of which specific constraints exist, what values they have, and how they impact the particular project. Based on such analysis, they must find efficient and effective ways of steering their projects to remain not only within the region of feasible solutions, but also within it to minimize some parameters and maximize others, depending on their nature. Project management thus can be seen as a multi-objective optimization problem.

The question of how to model constraints becomes a significant challenge if one considers that they have very different behaviors, may interact with positive or negative correlation but are not necessarily directly proportional, may be absolute or variable in terms of how strictly they must be followed, and their values may be fixed points or distributed ranges in form of discrete or continuous numbers. Together these factors complicate an accurate and detailed yet intuitive and flexible modeling and analyzing different types of constraints in construction management. Existing approaches for this non-trivial task suffer from being disjointed and unnecessarily complex. This paper therefore presents a novel approach toward modeling such temporal and financial constraints in an integrated manner by using singularity functions.

2 IMPORTANCE OF CONSTRAINTS

Constraints establish limitations to the feasible states of a system, e.g. a project schedule. They are expressed mathematically as a set of inequality or equality conditions for individual values or ranges of permissible inputs or outputs. Constraints can affect any independent (input) or dependent (output) variable of such system. Together, they create the boundaries within which all solutions of the system must remain to be considered valid in the analysis.

Without identifying and obeying all or most constraints it would be impossible to perform any mathematically based planning of construction projects. In other words, all project plans can be viewed as a set of constraints, which often are interrelated. In general, constraints must be quantifiable numerically, i.e. measurable in binary values (i.e. for yes/no statuses), integers (e.g. time resolution in full workdays), rational numbers (if derived as a ratio of two integers, e.g. levels of crew productivity), or real numbers to be usable in an analysis or optimization.

Additionally, constraints can be treated as either ‘hard’ or ‘soft’ in terms of their relative importance. Hard constraints are inviolable and must be obeyed under all circumstances. Reasons for such a classification include that they are technically impossible to circumvent (due to laws of nature or currently available technology), legally required by governing laws, carry a significant financial penalty, or will create actual harm to participants (that may range from physical damages to intangibles such as the professional reputation of a company) if they are not followed. From this one end of a spectrum extends the range of soft constraints, which in practice include negotiable terms or conditions, desirable features, and a plethora of other items, which can be thought of as having a hierarchical order according to which they can be disobeyed if necessary to still meet project performance objectives. Besides the fact that some constraints may be disobeyed under special circumstances lies the additional area of constraints whose values are not fixed but random to a certain degree, i.e. are probabilistic in nature. Such constraints can arise when being uncertain about a realistic value (e.g. activity duration), modeling unforeseen site conditions (e.g. soil types that require specific treatment), or exploring future options that are yet to be decided (e.g. installation of different possible finishes). For brevity, the issues of hard versus soft and of deterministic versus probabilistic constraints are not treated in detail in this paper but left to be explored under future research.

3 TYPES OF CONSTRAINTS

3.1 Temporal constraints

Temporal constraints, beyond the trivial point or range for a single value, arise primarily as acyclic dependencies between activities in project management. Event refers to a single point in time; activity refers to a work process that occurs during a range across time. Events and activities can be combined, e.g. to model a milestone when an ongoing process must have produced a specific intermediate quantity of work product, or a deadline when it must have finished. The general case of two activities will be examined in this paper, which can always be reduced to the case of activity and event by setting the start and finish of the latter equal.

Exactly three temporal relationships exist between two events, $A < B$, $A = B$, and $B < A$, but a significantly larger number of possible temporal interval relationships, due to the many ways in which they can overlap to varying degrees. It is therefore necessary to explore how many different relationships can theoretically exist. Consider two uninterrupted activities A and B. Their complete possible array of permutations is obtained by shifting two intervals along each other until any element within an equality or inequality expression will change. The durations of two intervals can be such that $dur(A) < dur(B)$, $dur(A) = dur(B)$, or $dur(A) >$

dur(*B*). For all possible relationships it holds that $A_S < A_F$ and $B_S < B_F$. Moreover, *A* can occur before, during, or after *B*. If *A* exactly touches *B* (either before or after it), a finish-to-start relationship exists. The most variety of cases arises from concurrency of *A* and *B*: They can partially overlap with different scenarios as to whether or not pairs of starts or finishes are unequal or equal; they can be exactly concurrent; or one can encase the other. Tables 1 and 2 together list all thirteen possible permutations that exist for exactly two activities, which were originally explored by Allen (1983) to create an interval-based algebra for computer science, after initially identifying only nine types (Allen, 1981) of $A < B$, $A = B$, $A > B$, *A* meets *B*, *A* met-by *B*, *A* overlaps *B*, *A* overlapped-by *B*, *A* during *B*, and *A* contains *B*. *A* and *B* are not treated interchangeably and $A = B$ creates the uneven total count in the otherwise symmetric constellations. These thirteen types require that the relationships are broadened from merely reviewing the intervals *A* and *B* themselves to establishing equality or inequality relationships between their starts and finishes, A_S, A_F, B_S, and B_F. This refinement of interval logic already arises in most such cases, e.g. *A* meets *B* (equal to $A_F = B_S$) or *A* met-by *B* (equal to $A_S = B_F$).

Table 1 : Types of point-based temporal constraints (adapted from Freksa, 1992)

Constraint	$A_F < B_S$	$A_F = B_S$	$A_S = B_F$	$A_S > B_F$
Description	*A* before *B*	*A* meets *B*	*A* met-by *B*	*A* after *B*

Table 2 : Types of interval-based temporal constraints (adapted from Freksa, 1992)

Constraint	$A_S < B_S$	$A_S = B_S$	$A_S > B_S$
$A_F > B_F$	*A* contains *B*	*A* started-by *B*	*A* overlapped-by *B*
$A_F = B_F$	*A* finished-by *B*	*A* equals *B*	*A* finishes *B*
$A_F < B_F$	*A* overlaps *B*	*A* starts *B*	*A* during *B*

The traditional scheduling technique that is widely used for construction projects in North America, the critical path method (CPM) (Galloway, 2006), is based originally upon a single link type, finish-to-start. Allowing that any start or finish of *A* can be linked with any start or finish of *B* in the precedence diagramming method (Fondahl, 1964) extends this to four types: *Finish-to-start* (F-S), *start-to-start* (S-S), *start-to-finish* (S-F), *and finish-to-finish* (F-F). This obviously significantly increases the number of possible permutations like a ‘combinatorial explosion’ of options. It suffices for complete sequencing and schedule calculations if a pair of activities is linked by at least one link of any type. Moreover, permutations increase if one considers that activities can be linked with *not just one but multiple links* between their starts and finishes; for example S-S and F-F links are commonly combined to enforce concurrency between the two activities. It is assumed that no truly redundant links exist (e.g. connecting activities *A* and *C*, which are already connected via *B*), or that they have been removed via a redundancy identification algorithm (Bashir, 2010). Furthermore, the permutations increase when allowing *durations on the links themselves*, which specify a (minimum) temporal offset to always be maintained between origin and terminus of said links. Such durations often are called *lead or lag* (Crandall, 1973), which indicates a delay or overlap between the two points in time. Specifically, lead refers to a positive value for the temporal offset from predecessor to successor activity, while lad refers to a negative offset. Crandall (1973) noted that the S-F link is rarely used in practice, even more so with a lead or lag, but it will be still listed in the following discussion. Recall also that relative activities durations can be equal or unequal per the conditions $dur(A) < dur(B)$, $dur(A) = dur(B)$, or $dur(A) > dur(B)$. Combining all of these factors generates a plethora of possible permutations already for just one pair of activities.

Note that even further constellations would be possible in two additional cases that would add more realism to a schedule, but are excluded here for brevity: (1) Allowing relationships to *originate and terminate at any point in time* during ongoing activities, not just their starts and finishes. And (2) that each activity can be composed of multiple segments with different durations. In general, both cases can be reduced via *activity splitting* (Son and Mattila, 2004) into the already discussed ones – in the former case, converting it into two sub-activities A_1 and A_2 with a F-S link, from whose connection the link to B emerges; and in the latter case, dividing it into *n* segments A_i with a F-S link, where the counting index *i* is from 1 to *n*. Yet even this seemingly increasingly complex perspective of temporal constraints is still limited in that it only considers point-to-point relationships, not range-based relationships. As will be seen in the following model based on singularity functions, activities in reality relate to each other throughout their execution, and interact on an ongoing basis. Under such productivity-oriented view, their starts and finishes merely function as markers of their 0% and 100%, but by themselves are insufficient to plan, control, and manage the detailed progress over time.

Naturally, unless activities must be executed strictly sequentially for technical reasons, due to limited resource availability, or because they would create a physical interference or spatial conflict, they should be scheduled with maximum concurrency to minimize the total project duration and achieve an efficient productive process. Therefore the major question arises for project planners: *Which one of this bewildering array of possible overlap options to chose?*

The following diagrams systematically group possible interval relationships for the three relative activity durations, four link types, and durations on said links as lead, lag, or zero. Its notation is that links always originate at the predecessor *A* and terminate at its successor *B*. Figure 1 shows the permutations for all four different link types, assuming equal durations of *A* and *B*. Applying an approach similar to Allen (1983), the seven different types of overlaps of the nine from Table 2 that apply (*A* during *B*, and *A* contains *B* require unequal durations) create 4 · 7 = 28 permutations. They are arranged in the diagram in rows by link type and in columns by temporal interval relationships. Note that each of the four link types can create identical overlaps between the two activities, but obviously requires different leads or lags.

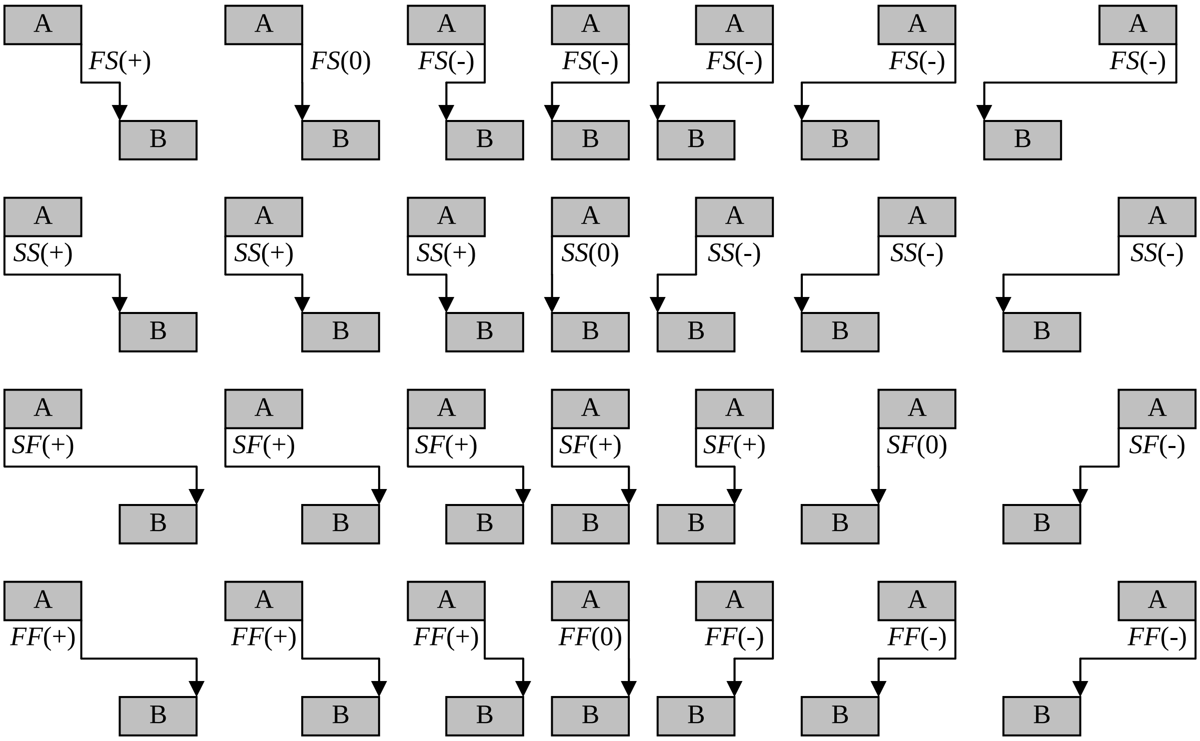

Figure 1: Permutations for four link types and equal durations

Figure 2 shows the remaining permutations, now assuming unequal durations of *A* and *B*. For brevity, the middle row combines two cases within each diagram. Note that none of the permutations of Figure 1 are repeated: For *A* before *B*, *A* after *B*, *A* meets *B*, *A* met-by *B*, *A* overlaps *B*, and *A* overlapped-by *B* it is irrelevant whether $dur(A) < dur(B)$, $dur(A) = dur(B)$, or $dur(A) > dur(B)$; and *A* equals *B* obviously assumes equal durations. They have the factors 2 for $A > B$ or $A < B$; 3 for the left, left-right, or right overlap as denoted by *A* finished-by *B*, *A* contains *B*, or *A* started-by *B* (for $A > B$ columns in the left half of Figure 2), or *A* finishes *B*, *A* during *B*, or *A* starts *B* (for $A < B$ columns in the right half of Figure 2) of Allen's (1983) classification; and 4 for the four link types. This generates $2 \cdot 3 \cdot 4 = 24$ permutations, for a total of 52. Of course, the seven columns in Figure 1 plus the six columns in Figure 2 yield the thirteen original temporal interval relationships. This discussion has extended them with all four link types and allowing leads and lags. Comparing the verbal notation with these diagrams, it is found that the former only captures the temporal interval relationship, not its reason in form of the underlying constraints from one or multiple links with leads or lags.

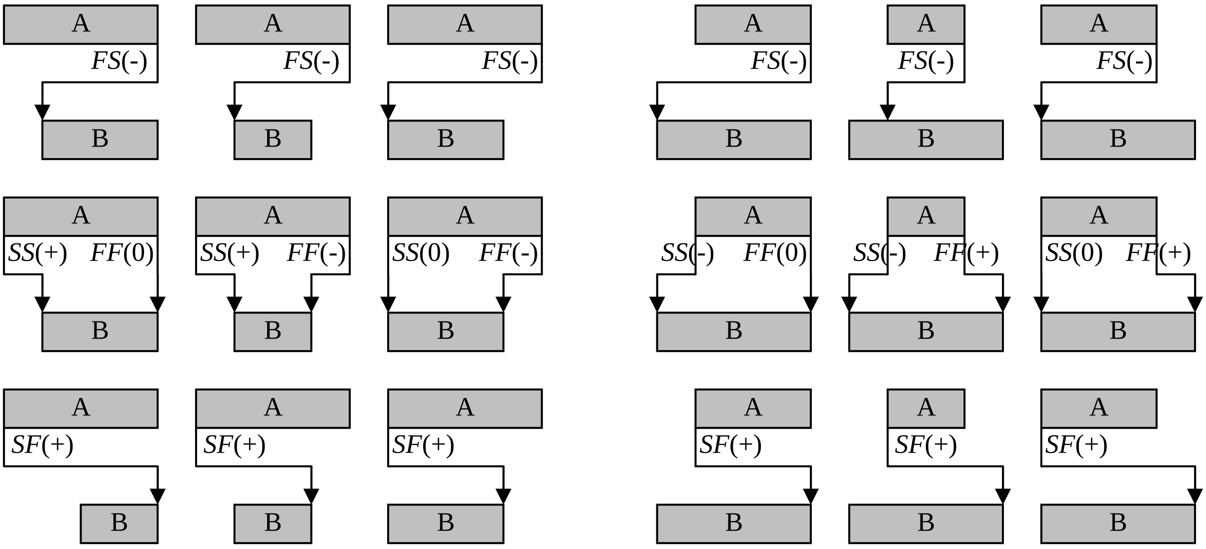

Figure 2: Permutations for four link types and unequal durations

Reviewing the many permutations, a major disconnect exists between the area of computer science and its abstract software-oriented concepts, and the area of construction management, where project managers must plan, schedule, and control their projects to satisfy the goals of remaining on schedule, within budget, and delivering the facility safely and at quality. But it is a major challenge for schedulers to select the appropriate link type for each relationship in a schedule, because defaulting to F-S links precludes any overlap and thus is very inefficient in many cases. Despite their technical or administrative sequencing constraints, in practice many activities can and should be performed at least partially concurrently. This minimizes the total project duration, down to the value when any interference or conflict will occur on the construction site. Selecting the proper link type for pairwise activities of known durations is often guided by information about the means and methods. Yet this challenge is attenuated by the fact that for each link type there exist thirteen possible temporal interval relationships, not including that appropriate leads or lags must still be chosen for the specific final overlap.

Multiplying this with the number of activities in typical construction projects, which range from the hundreds to the thousands, yields an enormous amount of possible schedules for the particular project. Considering further that a baseline schedule will be regularly updated at a monthly if not weekly frequency with as-built progress or modified with as-needed mitigation of changes, interruptions, or delays, network schedules suffer from an *exponential explosion* of permutations. This creates significant potential for inefficiencies and even errors to be built into networks, because it is difficult if not virtually impossible to select the optimum option for a project when linking its many interacting activities. Network schedules have inherent conceptual drawbacks due to their representation of temporal interval relationships, which limit how one can reason about, understand, and influence their realized behavior in practice.

3.2 Financial constraints

Financial constraints on the surface list only monetary values, but upon closer examination cannot easily be separated from an underlying time component. For example, depending on a user's perspective, a budget can be treated as an absolute value or a target to be achieved by a certain date or maintained during a certain duration. Considering inflation on costs, assessing interest on financing, and other *Time Value of Money* concepts illustrate this issue. Whether or not financial constraints also act in a temporal manner is not an matter of formal definition but of practical use. Same as all constraints, they establish minimum, equality, or maximum conditions for individual values or ranges of values. Financial phenomena are best analyzed in chronological sequence in which they occur in business transactions. This sequence of cash outflows and inflows generates a cash flow profile, which – in somewhat of an analogy to the point-to-point representation of temporal constraints – had been labeled "difficult to model" (Kenley 2003, p. 168). Examples of financial transactions are payments (inflows) or costs (outflows) at a point in time, or costs that are accrue across a range of time, as is a commonly assumed in modeling cash flows at the project and program levels (Abido & Elazouni, 2011).

Their sequence start with costs, accumulating them into bills, paying them less a retainage that is withheld, assessing interest, determining bonuses or penalties, releasing the retainage, and calculating the final profit (Lucko, 2011). Contractors may analyze financial implications of different options to schedule activities and minimize their overhead, inventory costs, and exposure to internal and external risk factors while maximizing production of billable goods and services. They may use a mixture of debt financing (subject to terms and conditions) and retained earnings, consider whether or not to pay their suppliers early if a discount is offered, and seek to be paid quickly by the owner, all in support of the goal to maximize their profit or net present value (if options exist for durations or interest rates) within the overall constraint framework (Smith-Daniels & Smith-Daniels, 1987). Owners seek to minimize their project duration and cost, using similar analyses, but from the payer's view instead of the payee's.

Broadening the view beyond mere project activities, a plethora of economic phenomena emerges at the micro and macro levels. Financial markets with their valuation based on future revenue potential represent an enormously complex system of constraints within the overall economy. Issues on such scale may arise from e.g. financing mechanisms that may be subject to liquidity constraints due to variable interest rates, purchasing options or futures to hedge against risks such as undesirable upswings in commodity prices for construction materials and supplies, or offering performance-based incentive payment and remunerations. Another consideration is how the legal structure of the various project participants impacts financial constraints, e.g. a public owner using bonds to finance a major new infrastructure project.

Construction projects encounter a multitude of different types of constraints, which cover all of their aforementioned managerial 'dimensions' of time, cost, scope, resources, safety, quality, sustainability, etc. For brevity, only temporal and financial constraints are discussed; the mathematical model of in this paper can already express other types insofar as their direct impacts on time and costs are concerned by treating them similar to non-productive activities. Many real-world phenomena still exceed the presented modeling assumptions and analytical capabilities and require future research. Any model of reasonable size and complexity that is suitable for practical use will thus necessarily include only a limited set of factors to simplify reality, of which the extrinsic ones, i.e. those that have little or no relationship with the nature and performance of the construction project, are especially difficult to capture appropriately.

4 SINGULARITY FUNCTIONS

Singularity functions are capable of expressing different types of constraints with just one functional operator. They were used in structural engineering (Föppl, 1924; Macaulay, 1919) based on an analytical approach to simplify differential equations by Clebsch (1862). Yet it was noted that even in that area "its use has not been as widespread as one would expect... Perhaps the reason is that it makes use of a rather special representation of the Macauley [*sic*] and the singularity functions" (Chicurel-Uziel, 1999, p. 281). Equation 1 is referred to as the basic term, using the notation by Wittrick (1965), which provides a simple case distinction between two ranges on a continuous horizontal *x*-axis, one from – ∞ to just before the cutoff *a*, the other from including a to + ∞. In the former, the value of the term is defined as exactly zero; in the latter the term takes on a value from evaluating its brackets normally. The factor *s* (for strength) designates the intensity of the term once it has become active. Its meaning depends on the value of the exponent *n*: For $n = 0$ the basic term describes a step of the constant height *s*; for $n = 1$ it is a ramp of the linear slope *s*; and for $n = 2$ a parabola. Any higher order terms are possible, yet in practical applications it often suffices to only use basic terms of zero and first order, i.e. modeling the phenomena of interest as piecewise segmental.

$$y(x)_{basic\ term} = s \cdot \langle x - a \rangle^n = \begin{cases} 0 & if\ x < a \\ s \cdot (x - a)^n & if\ x \geq a \end{cases} \tag{1}$$

A singularity function, as the name indicates, describes singularities, i.e. points of interest along the *x*-axis where the behavior of the phenomenon changes in any of its aspects, whether they are value, growth rate, curvature, or other. All that is necessary is creating a customized basic term for each change, specifying an appropriate *a* to 'switch it on' at its location on the *x*-axis, and adding all of them into the complete singularity function. Since it encompasses a theoretically unlimited number of basic terms acting together per Equation 2, it can describe any complicated behavior, provided that it remains a true function that uniquely maps *x* onto $y(x)$. Changes can be continuous or discontinuous in their nature, i.e. impact the value of $y(x)$ with vertical steps or mere slope changes. The basic term itself is defined as right-continuous.

$$y(x)_{sng\ fnc} = \sum_{i=1}^{m} s_i \cdot \langle x - a_i \rangle^{n_i} \tag{2}$$

The case distinctions in singularity functions creates conditionally varying values $y(x)$ that can be modeled as a series of IF commands in software programming languages. Singularity functions can be added and subtracted, multiplied (which combines multiple conditions) and divided, integrated and differentiated, and scaled like regular functions. Extending Equation 1 by e.g. replacing *x* with a function $f(x)$ such as a rounding operator $\lfloor\ \rfloor$ or $\lceil\ \rceil$ (Iverson 1962) to create stepped or repeating patterns, incorporating non-polynomial functions, or inverting the case distinction to $s \cdot \langle a - x \rangle^n$ so that it changes from active to inactive at its cutoff, can be done with relative ease, but is not explored here for brevity. Practical use of Equations 1 and 2, e.g. manual evaluation, should follow three principles (Lucko and Peña Orozco 2009):

- Sorting: Multiple terms within Equation 2 should be sorted by their cutoff *a*, exponent *n*, and strength *s* from left to right as is common in regular polynomial function for clarity;
- Simplification: If multiple terms within Equation 2 have identical *a* and *n*, their *s* should be added to minimize the number of basic terms that are required in a singularity function;
- Superposition: Complex behaviors $y(x)$ can be modeled by additively overlapping multiple

basic terms within a single singularity function, whose length is theoretically unlimited.

5 MODELING CONSTRAINTS

5.1 Temporal constraints with singularity functions

All of the previously discussed temporal interval relationships as shown in Figures 1 and 2 can be modeled with singularity functions. Consider two activities *A* and *B*, each containing a single segment of known slope, i.e. constant productivity, per Equation 3. Their intercepts s_0 are listed first to indicate that their curves can be plotted by first locating their *y*-values and installing slope s_1 from that coordinate. They must have a sequence {*A*, *B*} and produce four units of work each. A time buffer of 1.5 workdays is required after *A* throughout its duration.

$$y(x)_{act} = s_0 \cdot \langle x-0 \rangle^0 + s_1 \cdot \langle x-0 \rangle^1 \tag{3}$$

Assuming a productivity of 4 work units per 3 workdays for *A*, its slope s_1 is $^3/_4$ in Figures 3a through 3d due to the vertical time axis. This axis configuration is used because the linear scheduling analysis minimizes the output *y*(*x*), i.e. the total project duration. A ‘stacking and consolidation’ algorithm (Lucko 2009) can perform the desired scheduling using two steps:

- Stacking: Determine the maximum *y*(*x*) for any *x* of the predecessor singularity function, use this value as the intercept for successor singularity function to remain on the safe side;
- Consolidation: Subtract successor minus predecessor singularity functions, determine its minimum $\Delta y(x) = min\{\mathrm{y}(x)_B - y(x)_{\mathrm{buffer}\,A}\}$, and subtract it from the successor intercept s_1.

In this example, the singularity function for the buffer of *A* must be used for the predecessor. Equation 4 performs the stacking by calculating a tentative intercept of $s_0 = 4.5$ for *B*, keeping its slope unchanged at $^4/_4$, which Equation 5 then consolidates to the final intercept of $s_0 = 1.5$ for *B* as shown in Figure 3a. This algorithm guarantees the minimum total project duration.

$$\max\{y(x)_{buffer\,A}\} = \max\{1.5 \cdot \langle x-0 \rangle^0 + 3/4 \cdot \langle x-0 \rangle^1\} = 4.5 \Rightarrow y(x)_{stack\,B} = 4.5 \cdot \langle x-0 \rangle^0 + \ldots \tag{4}$$

$$\begin{gathered} \min\{y(x)_{stack\,B} - y(x)_{buffer\,A}\} = (4.5-1.5) \cdot \langle x-0 \rangle^0 - (4/4-3/4) \cdot \langle x-0 \rangle^1 = 3\,at\,x=0 \\ \Rightarrow y(x)_{consolidate\,B} = (4.5-3) \cdot \langle x-0 \rangle^0 + 4/4 \cdot \langle x-0 \rangle^1 = 1.5 \cdot \langle x-0 \rangle^0 + 4/4 \cdot \langle x-0 \rangle^1 \end{gathered} \tag{5}$$

Importantly, *only two inputs are required to generate a complete linear schedule*. Inputs are user-selected productivities for *A* and *B* (or equivalently durations and quantities of work product) and their sequence {*A*, *B*}. Temporal constraints by links are marked as dotted lines in Figures 3a through 3d. Note that a S-S link has automatically emerged in Figure 3a, as can be proven by setting equal the final singularity functions for *B* and buffer of *A* and solving for the unknown *x*, here $x = 0$. Concurrency of *A* and *B* exists between the minimum y-value of *B* and maximum *y*-value of *A*, here $y_{\mathrm{overlap}} = \{1.5, 3\}$. The lead on the S-S link is the intercept of *B* minus the intercept of *A*, S-S = 1.5 – 0 = 1.5. In other words, the detailed type of temporal interval relationship that emerges is *A* overlaps *B* created by a S-S link of lead 1.5 workdays.

An analogous calculation can be performed for Figure 3b to yield its F-F relationship. To complete this comparison of the four link types, Figure 3c shows how the most common type and *de facto* default in network schedules, a F-S link, is actually an inefficient link type with zero overlap in linear scheduling. To occur automatically it either has to strictly enforce its constraint as shown by the horizontal dotted line (as if performing stacking only, but omitting consolidation), or in the case the workface of *B* moving into the opposite direction than *A*, marked as a dashed line. The final link type, S-F, is admittedly rare in network schedules already and downright unusual when shown in the linear schedule of Figure 3d. *Importantly, the exact link type and its lead or lag duration per the 52 possible permutations of Figures 1* *and 2 are generated as the output of the linear schedule analysis with singularity functions.* Two other insights emerge from Figures 3a through 3d: The activities in optimally efficient schedules have parallel slopes, i.e. identical productivities, and develop range relationships (Lucko 2008), not point-to-point ones that are the only possible ones in network scheduling.

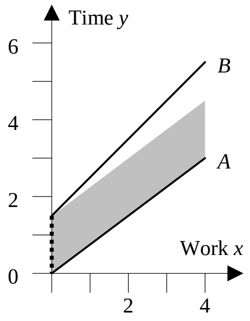

Figure 3a: Start-to-start

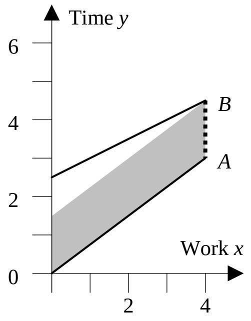

Figure 3b: Finish-to-finish

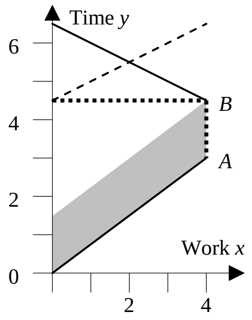

Figure 3c: Finish-to-start

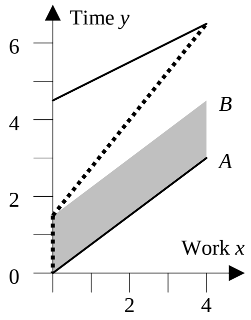

Figure 3d: Start-to-finish

5.2 Financial constraints with singularity functions

The complete sequence of financial phenomena and their constraints can be modeled with singularity functions. Time is assigned to the y-axis, which takes on the role of independent (input) variable, while money is the dependent (output) variable on the z-axis. The individual steps – cost, bill, pay, etc. – are modeled, here for a single activity, as a set of transformations starting with Equation 6. It adds the cost per period as a slope c at the start of an activity and subtracts it at its finish. For brevity, billing and payment that are separated by a time delay b in practice are combined into Equation 7, which thus performs the following transformations: Increasing the cost c by a factor $(1 + p)$ for the profit margin, aggregates growing costs into stepped payments by applying the aforementioned rounddown operator $\lfloor\ \rfloor$ to y, decreasing the payment by a factor $(1 – r)$ for the retainage, and shifting the payment by b after billing. Individual cost items, e.g. mobilization, can simply be modeled as basic term of order $n = 0$, as can bonuses, penalties, or released retainage, which are subject to conditions of whether or not a specific project duration y_{dur} or balance z_{bal} is exceeded. The former temporal constraint activates at y_{dur}, for the latter financial constraint its basic term is multiplied with another one that checks the balance condition per Equation 8. Equation 9 models a discount e for early payment, which modifies the amount due depending on how soon after invoicing it is paid.

$$z(y)_{cost} = c \cdot \left[\left\langle y - a_{start} \right\rangle^1 - \left\langle y - a_{finish} \right\rangle^1 \right] \tag{6}$$

$$z(y)_{bill\ pay} = (1-r) \cdot (1+p) \cdot c \cdot \left[\left\langle \lfloor y \rfloor - a_{start} - b \right\rangle^1 - \left\langle \lfloor y \rfloor - a_{finish} - b \right\rangle^1 \right] \tag{7}$$

$$z(y)_{cond\ cost} = \left\langle z(y) - z_{bal} \right\rangle^0 \cdot c \cdot \left\langle y - a \right\rangle^0 \tag{8}$$

$$z(y)_{discount} = c \cdot (1-e) \cdot \left\langle y - 0 \right\rangle^0 + e \cdot \left\langle y - a_{discount} \right\rangle^0 \tag{9}$$

Starts and finishes of activities are variables and can be updated during planning to model different options or during execution to model various delays. These can be external delays of predecessors (which impact both a_{start} and a_{finish}) or internal delays of the activity itself (which impacts only a_{finish}). Adding cash outflows per Equations 6 and 8 and inflows per Equation 7 provides a plain cash flow profile before financing. Financing fees are added at each period end, here using a common approximation (Abido & Elazouni, 2011) of assessing interest on the balance at the previous period end plus one half of the additional debt, if any, during the current period, and only if the balance is negative. A more comprehensive model has been derived by Lucko & Thompson (2010), but is omitted for simplicity. It is assumed that the financing has one source at a fixed interest rate. Assume a small example of Figure 4, which consists of an activity from $a_{\text{start}} = 0.25$ time units to $a_{\text{finish}} = 2.75$ time units, creates \$50 cost per period, assumes that retainage r and profit p are zero, has a billing-to-payment delay $b = 1$ period, and an interest rate i = 5% per period. Its principal can be simplified from $z(y-1) + ½ \cdot [z(y) - z(y-1)]$ to $½ \cdot [z(y-1) + z(y)]$. This yields the cost function $z(y)_{\text{cost}} = \$50 / \text{period} \cdot [\langle y - 0.75\rangle^1 - \langle y - 2.75\rangle^1]$ and the payment function $z(y)_{\text{pay}} = \$50 / \text{period} \cdot [\langle\lfloor y\rfloor - 1.75\rangle^1 - \langle\lfloor y\rfloor - 3.75\rangle^1]$. Their difference creates the cash flow profile shown in Figure 4, which has a non-trivial shape due to its partial start and finish date. Equations 6 and 7 model its respective partial costs and payments correctly. Calculating its interest fees at each period end directly yields the following values: $z(1)_{\text{int}} = 5\% \cdot ½ \cdot (0 - 12.5) = - \$0.3125$; $z(2)_{\text{int}} = 5\% \cdot ½ \cdot (-12.5 - 62.5) = - \$1.875$; $z(3)_{\text{int}} = 5\% \cdot ½ \cdot (-50 - 87.5) = - \$3.4375$; and $z(4)_{\text{int}} = 5\% \cdot ½ \cdot (-37.5 - 37.5) = - \$1.875$. Their sum is - \$7.50. However, this approach creates the problem of an extra required calculation step; if balances are changing sign during any period, the exact location of such crossing would have to be identified so that interest would only be charged on actual negative balances. Decomposing interest into separate charges on activity costs and earnings on progress payments (at the same interest rate) eliminates this potential problem. Equation 10 can be used for both the negative and positive interest. The rounddown operator ensures that the interest only changes at period ends, and ε is an infinitesimally small value to ensure that the value just prior to a period end is used, because singularity functions are right-continuous. Applying Equation 9 to Equation 6 yields the respective values of – \$0.3125, – \$1.875, – \$4.0625, and – \$5.00 at times $y = \{1, 2, 3, 4\}$ for charges of – \$11.25 on activity costs; and to Equation 7 it yields \$0.00, \$0.00, \$0.625, and \$3.125 for earnings of \$3.75 on progress payments, and an overall interest of – \$7.50, which verifies the previous result. Note that Equation 9 provides direct interest per period only, which must be added to yield its total.

$$z(y)_{int} = i \cdot 1/2 \cdot [z(\lfloor y \rfloor - \varepsilon) + z(\lfloor y \rfloor - 1)] \tag{10}$$

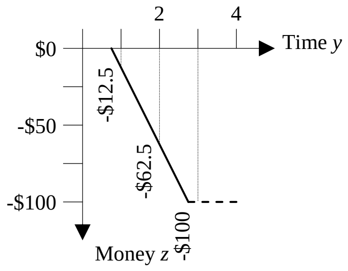

Figure 4a: Costs

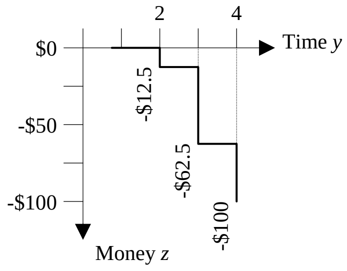

Figure 4b: Payments

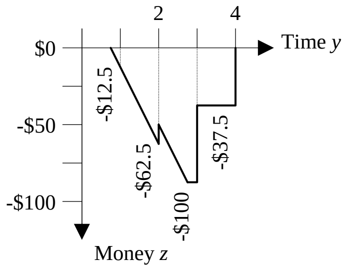

Figure 4c: Balance

Singularity functions can thus model various financial constraints and create equations for all steps in financial transactions, at single points in time or for ranges across time. The costs, bills, retainage, payments, released retainage, and financing, as well as bonuses or penalties, liquidated damages, credit limit ($z(y)$ of a constant negative value), and other phenomena can all be expressed with the same integrated mathematical expression (Lucko, 2011). This has a number of significant advantages, including that (a) all elements can be directly derived from a minimum amount of input values, (b) if needed they can be conditional with multiplicative terms per Equation 1 that activate only if a condition is fulfilled, and (c) essential variables and their constraints, i.e. time, work quantity, resources, etc., can be linked seamlessly so that construction project managers can plan, measure, and control them in a single holistic model.

5.3 Opportunities for integrated and automated constraint modeling

While computational effort to calculate starts and finishes is proportional to the number of activities and links in a schedule, evaluating its possible permutations will create a significant computational complexity. This may negatively affect the speed of running algorithms on the software side, e.g. for scheduling, and consume energy on the hardware side, e.g. in modern mobile computing devices. Singularity functions provide an integrated model wherein time, work, and cost can all be written as the same type of mathematical expression. This provides opportunities toward seamless conversions and transformations between them, modeling and studying their interactions in sensitivity analysis, and serves as inspiration for treating other variables in an analogous manner. Singularity functions can easily be computerized as `IF` statements. The approach presented in this paper reduces the number of temporal constraints that are required as scheduling input and simplifies them to sequence information, as exact link types and lead or lag durations emerge as output in the linear schedule analysis. This is a step toward automating the creation of constraints in computer representation of construction projects. It can also allow streamlining algorithms that perform solution or optimization tasks.

6 CONCLUSIONS

Temporal and financial constraints reside at the core of the planning and control tasks that construction project managers must perform. It is therefore very desirable to reduce or even eliminate the effort in creating and handling them. Singularity functions applied to the linear scheduling method are providing a novel approach to efficiently model constraints. Such a representation can to some degree reduce the combinatorial explosion that would be caused by (a) using an unsuitable representation that leads to an excess of constraints that have to be included, and (b) evaluating the multitude of permutations how the project can be performed in practice. This approach could also support novel computational strategies. Future research should therefore explore if and how computational efficiency can be improved by integrating singularity functions that can express different types of constraints with modern algorithms.

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