

# To Layer or Not To Layer: Balancing Transport and Physical Layers in Wireless Multihop Networks

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**Abstract**—In a wireless ad hoc network with multihop transmissions and interference-limited link rates, can we balance power control in the physical layer and congestion control in the transport layer to enhance the overall network performance, while maintaining the stability, robustness, and architectural modularity of the network?

We present a distributive power control algorithm that couples with the original TCP protocols to increase the end-to-end throughput and energy efficiency of the network. Under the rigorous framework of nonlinearly constrained optimization, we prove the convergence of this coupled system to the global optimum of joint power control and congestion control, for both synchronized and asynchronous implementations. The rate of convergence is geometric and a desirable modularity between the transport and physical layers is maintained. In particular, when the congestion control mechanism is TCP Vegas, that a simple utilization in the physical layer of the router buffer occupancy information suffices to achieve the joint optimum of this cross layer design. Both analytic results and simulations illustrate other desirable properties of the proposed algorithm, including robustness to channel outage and to path loss estimation errors, and flexibility in trading-off performance optimality for implementation simplicity.

**Keywords:** Congestion control, Convex optimization, Cross-layer design, Energy-aware protocols, Lagrange duality, Power control, Network utility, Transport Control Protocol, Wireless ad hoc networks.

## I. INTRODUCTION

In wireless ad hoc networks with multihop transmissions and interference-limited link rates, in order to achieve high end-to-end throughput in an energy efficient manner, congestion control and power control need to be jointly designed and distributively implemented. Congestion control mechanisms, such as those in Transport Control Protocol (TCP), regulate allowed source rates so that the total traffic load on any link does not exceed the available capacity. At the same time, the attainable data rates on wireless links depend on the interference levels, which in turn depend on the power control policy. This paper proposes, analyzes, and simulates a distributed algorithm for jointly optimal congestion control and power control. The algorithm utilizes the coupling between the transport and physical layers to increase end-to-end throughput and energy efficiency in a wireless ad hoc network.

Congestion control mechanisms, including the congestion

avoidance phase in all variants of TCP, have recently been shown to be distributed algorithms implicitly solving network utility maximization problems [17], [19], [20], [21], [25], which are linearly constrained by link capacities that are assumed to be fixed quantities. However, network resources can sometimes be allocated to change link capacities, therefore change TCP dynamics and the optimal solution to network utility maximization. For example, in CDMA wireless networks, transmit powers can be controlled to give different Signal to Interference Ratios (SIR) on the links, changing the attainable throughput on each link.

This formulation of network utility maximization with ‘elastic’ link capacities leads to a new approach of congestion avoidance in wireless ad hoc networks. The current approach of congestion control in the Internet is to avoid the development of a bottleneck link by reducing the allowed transmission rates from all the sources using this link. Intuitively, an alternative approach is to build (in real time) a larger transmission ‘pipe’ and ‘drain’ the queued packets out faster on a bottleneck link. Indeed, a smart power control algorithm would allocate just the ‘right’ amount of power at the ‘right’ nodes to alleviate the bandwidth bottlenecks, which may then induce an increase in end-to-end TCP throughput. But there are two major difficulties in making this idea work: pre-defining which link constitutes a ‘bottleneck’ is infeasible, and changing the transmit power on one link also affects the data rates available on other links, due to the interference in wireless CDMA networks. Increasing the attainable throughput on one link reduces the attainable throughputs on other links. We need to find an algorithm that distributively detects the ‘bottlenecks’ and optimally ‘shuffles’ the bottlenecks around in the network.

We make this intuitive approach precise and rigorous in this paper. After reviewing the background in section II and specifying the problem formulation in section III, we propose in section IV a distributed power control algorithm that couples with the original TCP algorithm to solve the joint problem of congestion control and power control. The joint algorithm can be distributively implemented on a multihop ad hoc network, despite the fact that the data rate on a wireless link is a global function of all the interfering powers (and violates the assumption in the examples in related work [31]). Interpretations in

terms of demand-supply coordination through shadow prices is presented, as well as numerical examples illustrating that end-to-end throughput and energy efficiency of the network can indeed be significantly increased.

It is not unexpected that performance can be enhanced through a cross layer design in wireless ad hoc networks. The more challenging task is to analyze the algorithm rigorously and to make it attractive according to other design criteria [16]. First, we need to obtain the benchmark that establishes the limit of possible performance gains obtainable through this cross layer design. Since the overall system may not remain stable if two subsystems are allowed to interact, we need to prove stability of the cross layer interaction and robustness to fluctuations of the underlying parameters. Cross layer designs usually improve performance at the expense of higher complexity in communication and computation, making complexity reduction an important issue. Finally, even after crossing the layers, a high degree of architectural modularity is desirable for practical implementation and future network evolution.

Our algorithm performance analysis in sections V and VI contains the following results:

- In section V, we prove that, under very mild conditions, the proposed algorithm converges to the joint and global optimum of the nonlinear congestion-power control.
- Furthermore, at equilibrium, the available data rate on each logical link will be exactly fully utilized by all the sources using the link. This result is proved not just for joint congestion control and power control, but for any cross layer design between congestion control and physical layer resource allocation.
- This desirable convergence is achieved as power control uses the same shadow prices that are already generated by TCP for regulating distributed users. Performance enhancement is achieved without modifying the existing TCP protocol stack.
- In subsection VI.A, we provide the sufficient conditions under which convergence to the global optimum is maintained despite the errors in path loss estimation or packet losses due to channel outage.
- In subsection VI.B, we propose a suite of simplified versions of the optimal algorithm to flexibly trade-off performance with complexity.
- In subsection VI.C, we prove that the algorithm will still converge under any finite asynchronism in practical implementation, and characterize the conditions under which asynchronous implementation does not induce a reduction in convergence speed.
- In subsection VI.D, we show that the rate of convergence of the algorithm is geometric, and provide a simple bound on the convergence speed. Further suggestions on choosing algorithm parameters and achieving convergence speedup are made in subsection VI.E.

While we try to formulate and answer the question of ‘to layer or not to layer’ for the case of layers 1 and 4 from a util-

ity maximization perspective, it is worth emphasizing that the idea of ‘layering’ in communication network design is often motivated by considerations on architectural modularity, evolvability, and scalability. This paper investigates the motivating question in the title only from one perspective out of several important ones.

## II. BACKGROUND AND RELATED WORK

Both power control in CDMA wireless networks and congestion control in the Internet are extensively researched topics:

- Changing the transmit power on one link will affect the quality of service, such as attainable data rates, on other links. Many power control algorithms (*e.g.*, an iterative one in [12]) have been proposed, but the effects of user demand regulation through end-to-end congestion control are usually ignored.
- TCP is the predominant protocol responsible for congestion control in the Internet and is being extended to wireless networks. Optimization-theoretic analysis is recently conducted for variants of TCP (*e.g.*, based on network utility maximization in [17], [19], [20], [21], [25] and based on a different nonlinear programming formulation in [2]), but an underlying assumption is that each communication link is a fixed-size transmission pipe provided by the physical layer.
- Although utility maximization jointly over rates and powers have been studied for cellular networks (*e.g.*, [8]), the inter-dependency and coupling effects between source rate control and link capacity regulation in wireless ad hoc networks have not been systematically investigated, and form the focus of this paper.

Kelly [17], [18] analyzed rate allocation through congestion control as a distributive solution of network utility maximization. The congestion avoidance phase of different versions of TCP has recently been analyzed as approximated primal-dual algorithms solving appropriately formulated utility maximization problems. Since we will be referring to TCP Vegas as an example in this paper and using it as the congestion control mechanism in the simulations, we now briefly review TCP Vegas.

TCP Vegas is a sliding window based protocol that distributively regulates the allowed source rates in a mesh network [6]. Let  $d_s$  be the propagation delay for the path originating from source  $s$ , and  $D_s$  be the propagation plus queuing delay. Obviously  $d_s = D_s$  when there is no congestion along all the links used by source  $s$ . The window size  $w_s$  is updated depending on whether the difference between the expected rate  $\frac{w_s}{d_s}$  and the actual rate  $\frac{w_s}{D_s}$  is smaller than a parameter  $\alpha_s$ :

$$w_s(t+1) = \begin{cases} w_s(t) + \frac{1}{D_s(t)} & \text{if } \frac{w_s(t)}{d_s} - \frac{w_s(t)}{D_s(t)} < \alpha_s \\ w_s(t) - \frac{1}{D_s(t)} & \text{if } \frac{w_s(t)}{d_s} - \frac{w_s(t)}{D_s(t)} > \alpha_s \\ w_s(t) & \text{else.} \end{cases} \quad (1)$$

The end-to-end throughputs are the allowed source rates  $x_s(t) = \frac{w_s(t)}{D_s(t)}$ , which are the primal variables of a utility maximization problem [21]. The associated dual variables (or shadow prices)  $\lambda_l$  for TCP Vegas are shown [21] to be the queuing delays along each link  $l$ , updated as follows:

$$\lambda_l(t+1) = \left[ \lambda_l(t) + \frac{\gamma}{c_l} \left( \sum_{s:l \in L(s)} x_s(t) - c_l \right) \right]^+ \quad (2)$$

where  $\gamma > 0$  is a constant,  $L(s)$  denotes the set of links traversed by the connection originating from source  $s$ , and the term  $\frac{1}{c_l}(\sum_{s:l \in L(s)} x_s(t) - c_l)$  represents the queuing delay as the ratio between packet backlog and link capacity  $c_l$ .

Cross layer designs in communication networks have attracted the attention of various researchers recently (overviews in *e.g.*, [29]). A partial list of some of the papers that focus on OSI layers 1, 2 and 3 includes joint routing and resource allocation based on different routing models in [23], [26], [31], joint routing and pricing in [24], joint routing and data compression in [28], joint resource allocation and pricing in [22], joint medium access control and physical layer diversity in [27], [32], joint resource allocation and scheduling in [33], and joint power control and scheduling in [10], [11]. There is another collection of recent work focusing on design across physical and application layers, especially for multimedia transmissions, and a collection of work focusing on modification of protocols in layers 3 and 4 by utilizing certain parameters in layers 1 and 2.

This paper complements the above studies in several ways. By extending the framework of network utility maximization to allow for elastic link capacities, we extend the optimization-theoretic analysis of TCP [20] to provide a quantitative framework of co-design across layers 1 and 4, under which theorems of global convergence can be proved for nonlinearly coupled dynamics. The resulted jointly optimal congestion control and power control algorithm enhances end-to-end throughput and energy efficiency in wireless ad hoc networks. Echoing the concern on cross layer designs in [16], we also put special emphasis on the practical implementation issues of robustness, asynchronism, complexity, and rate of convergence.

### III. PROBLEM FORMULATION

Consider a wireless ad hoc network with  $N$  nodes and an established logical topology, where some nodes are sources of transmission, and a sequence of connected links  $l \in L(s)$  forms a route originating from source  $s$ . Let  $x_s$  be the transmission rate of source  $s$ , and  $c_l$  be the capacity, in terms of the attainable data rate, on logical link  $l$ . Note that each physical link may need to be regarded as multiple logical links.

The standard formulation of network utility maximization for elastic traffic source [17] is to maximize the sum of individual sources' utilities represented through continuously differentiable, increasing, and strictly concave functions  $U_s(x_s)$ , sub-

ject to the link capacity constraint:

$$\begin{aligned} & \text{maximize} && \sum_s U_s(x_s) \\ & \text{subject to} && \sum_{s:l \in L(s)} x_s \leq c_l, \quad \forall l, \\ & && \mathbf{x} \succeq 0 \end{aligned} \quad (3)$$

where the variables are  $\mathbf{x}$ . Note that link capacities  $\mathbf{c}$  are assumed to be fixed parameters.

It has recently been shown (*e.g.*, in [17], [19], [20], [21], [25]) that congestion control mechanisms can be viewed as distributed solution methods for this utility maximization problem. As each source updates its allowed rate (the primal variable) through a TCP congestion avoidance algorithm, each link updates a congestion indicator (the dual variable, which can be interpreted as the 'shadow price' of using the link) through a queue management algorithm, and implicitly feeds it back to all the sources using this link. In particular, TCP Vegas is shown [21] to be implicitly solving (3) for logarithmic utility functions:  $U_s(x_s) = \alpha_s d_s \log x_s$ , using queuing delays as the dual variables.

However, the scope of (3) for a CDMA-based wireless ad hoc network is limited, because the data rates attainable on the logical links are not fixed, and instead can be written (for a large family of modulations) as a global and nonlinear function of the transmit power vector  $\mathbf{P}$ :

$$c_l(\mathbf{P}) = \frac{1}{T} \log(1 + K \text{SIR}_l(\mathbf{P}))$$

where  $T$  is the symbol period,  $K = \frac{-\phi_1}{\log(\phi_2 \text{BER})}$  [13] where  $\phi_1, \phi_2$  are constants depending on the modulation and BER is the required bit error rate.  $\text{SIR}_l$  is the signal to interference ratio for link  $l$  defined as  $\text{SIR}_l = \frac{P_l G_{ll}}{\sum_{k \neq l} P_k G_{lk} + n_l}$  for a given set of path losses  $G_{lk}$  (from the transmitter on logical link  $k$  to the receiver on logical link  $l$ ) and a given set of noises  $n_l$  (for the receiver on logical link  $l$ ). The  $G_{lk}$  factors incorporate propagation loss, spreading gain, and other normalization constants. With reasonable spreading gain, SIR is much larger than 1 and  $c_l$  can be approximated as  $\frac{1}{T} \log(K \text{SIR}_l)$ . Without loss of generality, let  $T$  be 1 time unit.

We have now specified the following network utility maximization with 'elastic' link capacities:

$$\begin{aligned} & \text{maximize} && \sum_s U_s(x_s) \\ & \text{subject to} && \sum_{s:l \in L(s)} x_s \leq c_l(\mathbf{P}), \quad \forall l, \\ & && \mathbf{P}, \mathbf{x} \succeq 0 \end{aligned} \quad (4)$$

where the optimization variables are both source rates  $\mathbf{x}$  and transmit powers  $\mathbf{P}$ . The key difference from the standard utility maximization (3) is that each link capacity  $c_l$  is now a function of the new optimization variables: the transmit powers  $\mathbf{P}$ .

Problem (4) may be modified by adding simple constraints on maximum transmit powers allowed at each node:  $P_i \leq P_{i,max}$ , and by augmenting the objective function with a cost term  $-\sum_i P_i$  of total powers used. It turns out these two modifications do not lead to new technical challenges in designing an optimal distributed algorithm. For simplicity of presentation, we

will focus on (4) that captures the essence of the problem and challenges. However, note that even without the local power upper bounds or the total power cost term, arbitrary increase in powers do not lead to higher network utility, because all link rates are interference limited.

The nonlinearly constrained optimization (4) may be solved through centralized computation using the recently developed geometric programming technique in [9], [15]. However, in the context of wireless ad hoc networks, new distributive algorithms are needed to solve (4). The primary challenge is that there are two global dependencies in (4):

- Source rates  $\mathbf{x}$  and link capacities  $\mathbf{c}$  are globally coupled across the network, as reflected in the range of summation  $\{s : l \in L(S)\}$  in each of the constraints in (4).
- Each link capacity  $c_l(\mathbf{P})$ , in terms of the attainable throughput under a given power vector, is a global function of all the interfering transmit powers.

Our first goal in this paper is to distributively find the joint and globally optimal solution  $(\mathbf{x}^*, \mathbf{P}^*)$  to (4) by breaking down these two global dependencies.

#### IV. ALGORITHM, INTERPRETATIONS, AND NUMERICAL EXAMPLE

We first present the following distributive algorithm and will later prove that it converges to the joint and global optimum of (4) and possesses several other desirable properties of a cross layer design. We first present the ideal form of the algorithm, assuming no propagation delay and allowing significant message passing overhead. Some of these practical issues will be investigated in section V. We emphasize that items 3 and 4 in the algorithm can couple with any TCP congestion control mechanisms to solve the corresponding network utility maximization [20]. To make the algorithm and its analysis concrete, we will focus on TCP Vegas (as reflected in items 1 and 2 below) and the corresponding logarithmic utility maximization.

##### Jointly Optimal Congestion-control and Power-control (JOCP) Algorithm

- 1) During time slot  $t$ , at each intermediate node, queuing delay  $\lambda_l$  is implicitly updated:

$$\lambda_l(t+1) = \left[ \lambda_l(t) + \frac{\gamma}{c_l(t)} \left( \sum_{s:l \in L(s)} x_s(t) - c_l(t) \right) \right]^+ . \quad (5)$$

- 2) At each source, total delay  $D_s$  is measured and used to update the TCP window size, and consequently source rate  $x_s$ :

$$w_s(t+1) = \begin{cases} w_s(t) + \frac{1}{D_s(t)} & \text{if } \frac{w_s(t)}{d_s} - \frac{w_s(t)}{D_s(t)} < \alpha_s \\ w_s(t) - \frac{1}{D_s(t)} & \text{if } \frac{w_s(t)}{d_s} - \frac{w_s(t)}{D_s(t)} > \alpha_s \\ w_s(t) & \text{else.} \end{cases} \quad (6)$$

$$x_s(t+1) = \frac{w_s(t+1)}{D_s(t)}.$$

- 3) Each transmitter  $j$  calculates a message  $m_j(t) \in \mathbf{R}_+$  based on locally measurable quantities, and pass the message to all other transmitters through a flooding protocol:

$$m_j(t) = \frac{\lambda_j(t) \text{SIR}_j(t)}{P_j(t) G_{jj}}.$$

- 4) Each transmitter updates its power based on locally measurable quantities and the received messages, where  $\kappa > 0$  is a constant:

$$P_l(t+1) = P_l(t) + \frac{\kappa \lambda_l(t)}{P_l(t)} - \kappa \sum_{j \neq l} G_{lj} m_j(t). \quad (7)$$

We first present some intuitive arguments on this algorithm before proving the convergence theorem and discussing the practical implementation issues. Taking in the current values of  $\frac{\lambda_j(t) \text{SIR}_j(t)}{P_j(t) G_{jj}}$  as the messages from other transmitters indexed by  $j$ , the transmitter on link  $l$  adjusts its power level in the next time slot in two ways: first increase power directly proportional to the current shadow price (e.g., queuing delay in TCP Vegas) and inversely proportional to the current power level, then decreases power by a weighted sum of the messages from all other transmitters, where the weights are the path losses  $G_{ij}$ . Intuitively, if the local queuing delay is high, transmit power should increase, with more moderate increase when the current power level is already high. If queuing delays on other links are high, transmit power should decrease in order to reduce interference on those links.

Note that to compute  $m_j$ , the values of queuing delay  $\lambda_j$ , signal-interference-ratio  $\text{SIR}_j$ , and received power level  $P_j G_{jj}$  can be directly measured by node  $j$  locally. This algorithm only uses the resulted message  $m_j$  but not the individual values of  $\lambda_j$ ,  $\text{SIR}_j$ ,  $P_j$  and  $G_{jj}$ . To conduct the power update,  $G_{ij}$  factors are assumed to be estimated through training sequences. In practical wireless ad hoc networks,  $G_{ij}$  are stochastic rather than deterministic, mobility of the nodes changes the values of  $G_{ij}$ , and path loss estimations can be inaccurate. The effects of the fluctuations of  $G_{ij}$  will be discussed in subsection VI.A.

We also observe that the power control part of the joint algorithm can be interpreted as the selfish maximization of a local utility function of power by the transmitter of each link:

$$\text{maximize}_{P_l} U_l(P_l)$$

where  $U_l(P_l) = \lambda_l c_l - \beta_l P_l$  and  $\beta_l = \sum_{j \neq l} \left( \frac{G_{lj} \text{SIR}_j}{G_{jj}} \right) \frac{\lambda_j}{P_j}$ . This complements the standard interpretation of congestion control as the selfish maximization by each transmitter of a local utility function  $U_s(x_s)$  of its source rate.

The unmodified source algorithm (6) and queue algorithm (5) of TCP, together with the new power control algorithm (7), form a set of distributed, joint congestion control and resource allocation in wireless ad hoc networks. As the transmit powers

change, SIR and thus data rate also change on each link, which in turn change the congestion control dynamics. At the same time, congestion control dynamics change the dual variables  $\lambda(t)$ , which in turn change the transmit powers. Figure 1 shows this nonlinear coupling of ‘supply’ (regulated by power control) and ‘demand’ (regulated by congestion control), through the same shadow prices  $\lambda$  that are used by TCP Vegas to regulate distributed demand:  $\lambda$  now serves the second function of cross layer coordination in the JOCP Algorithm.

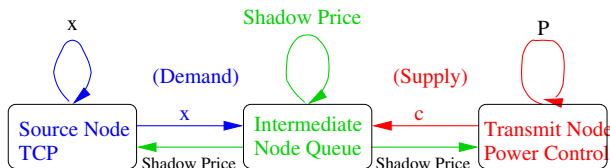


Fig. 1. Nonlinearly coupled dynamics of joint congestion and power control.

It is important to note that there is no need to change the existing TCP congestion control and queue management algorithms. All that is needed to achieve the joint and global optimum of (4) is to utilize the values of queue length in designing power control algorithm in the physical layer. This approach is complementary to some recent suggestions in the Internet community to pass physical layer information to better control routing and congestion in upper layers.

Using the JOCP Algorithm (5,6,7), we simulated the above joint power and congestion control for various wireless ad hoc networks with different topologies and fading environments. The advantage of such a joint control can be captured even in a very small illustrative example, where the logical topology and routes for four multi-hop connections are shown in Figure 2. The path losses  $G_{ij}$  are determined by the relative physical distances (which we vary in different experiments).

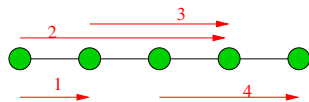


Fig. 2. The logical topology and connections for an illustrative example.

Transmit powers, as regulated by the proposed distributed power control, and source rates, as regulated through TCP Vegas window update, are shown in Figure 3. The initial conditions of the graphs are based on the equilibrium states of TCP Vegas with fixed power levels. With power control, it can be seen that transmit powers  $P$  distributively adapt to induce a ‘smart’ capacity  $c$  and queuing delay  $\lambda$  configuration on the overall network, which in turn lead to increases in end-to-end throughput as indicated by the rise in all the allowed source rates. Notice that some link capacities actually decrease while the capacities on the bottleneck links rise to maximize the total network utility. This is achieved through a distributive adaptation of power, which lowers the power levels that cause most interference on the links that are becoming a bottleneck in the dynamic demand-supply matching process. Confirming our intuition, such a ‘smart’ allocation of power tends to reduce the

spread of queuing delays, thus preventing any link from becoming a bottleneck. Queuing delays on the four links do not become the same though, due to the asymmetry in traffic load on the links and different weights in the logarithmic utility objective functions.

We indeed achieve the primary goal of this co-design across the physical and transport layers. The end-to-end throughput per watt of power transmitted, *i.e.*, the Throughput Power Ratio (TPR), is 82% higher with power control. A series of simulations are conducted based on different fading environments and TCP Vegas parameter settings. Based on the resulted histogram of TPR, We see that power control (7) increases TCP throughput and TPR in all experiments, and in 78% of the instances, energy efficiency rises by 75% to 115%, compared to TCP without power control. Power control and congestion control, each running distributively and coordinated through the dual variables of queuing delay, work together to increase the energy efficiency of multi-hop transmissions across wireless ad hoc networks.

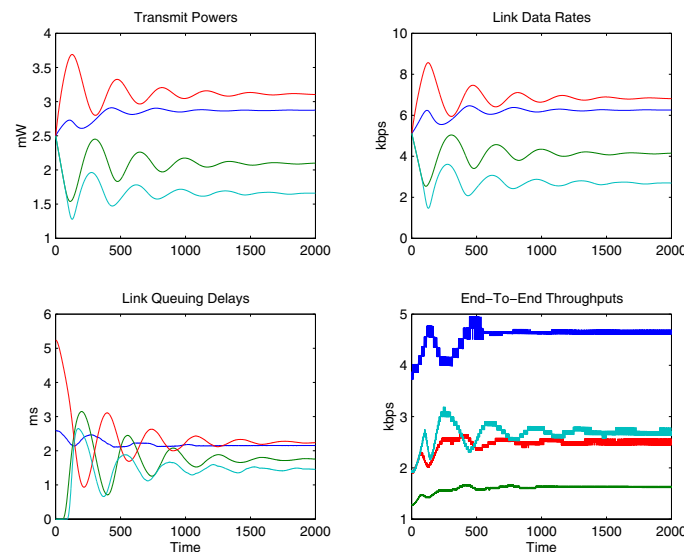


Fig. 3. Numerical example of joint TCP Vegas congestion control and power control. The top left graph shows the primal variables  $P$ . The lower left graph shows the dual variables  $\lambda$ . The lower right graph shows the primal variables  $x$ , *i.e.*, the end-to-end throughput.

## V. PERFORMANCE EVALUATION: CONVERGENCE THEOREM AND EQUILIBRIUM STATE

It is not too surprising that allowing cross layer interactions improves the performance of wireless ad hoc networks. The rest of this paper is devoted to the more challenging task of proving that the JOCP Algorithm has the following desirable properties: global convergence to the joint optimum and a desirable equilibrium, robustness to parameter perturbation and asynchronism, graceful tradeoff between complexity and performance, and geometric rate of convergence.

We first show that convergence of the nonlinearly coupled system, formed by the JOCP Algorithm and shown in Figure 1,

is guaranteed, as long as link data rates are strictly positive and link queuing delays are finite. These are reasonable engineering assumptions under normal operations of a network, since a link with zero data rate is essentially disconnected, and a queue with a finite buffer cannot support infinite queuing delay. To make the algorithm concrete, we again focus on the case of TCP Vegas. But the proof technique is applicable to the interaction between any TCP variants and the new power control algorithm (7).

*Theorem 1:* Assume that transmit power  $P_l$  are within a range between  $P_{l,min} > 0$  and  $P_{l,max} < \infty$  for each link  $l$ , and link queuing delays  $\lambda$  are finite. For small enough constants  $\gamma$  and  $\kappa$ , the distributed JOCP Algorithm (5,6,7) converges to the global optimum of the joint congestion control and power control problem (4).

*Proof:* We first associate a Lagrange multiplier  $\lambda_l$  for each of the constraints  $\sum_{s:l \in L(s)} x_s \leq c_l(\mathbf{P})$  in (4). Using the KKT optimality conditions from optimization theory [3], [5], solving problem (4) is equivalent to satisfying the complementary slackness condition and finding the stationary points of the Lagrangian.

Complementary slackness condition states that at optimality, the product of the dual variable and the associated primal constraint must be zero. This condition is satisfied since the equilibrium queuing delay must be zero if the total equilibrium ingress rate at a router is strictly smaller than the egress link capacity.

We now proceed to find the stationary points of the Lagrangian:  $I_{system}(\mathbf{x}, \mathbf{P}, \boldsymbol{\lambda}) = (\sum_s U_s(x_s) - \sum_l \lambda_l \sum_{s:l \in L(s)} x_s) + (\sum_l \lambda_l c_l(\mathbf{P}))$ . By linearity of the differentiation operator, this can be decomposed into two separate maximization problems:

$$\begin{aligned} \text{maximize}_{\mathbf{x} \geq 0} \quad & \sum_s U_s(x_s) - \sum_s \sum_{l \in L(s)} \lambda_l x_s, \\ \text{maximize}_{\mathbf{P} \geq 0} \quad & I_{power}(\mathbf{P}, \boldsymbol{\lambda}) = \sum_l \lambda_l c_l(\mathbf{P}). \end{aligned}$$

The first maximization is already implicitly solved by the congestion control mechanism (such as TCP Vegas for the case of  $U_s(x_s) = \alpha_s d_s \log x_s$ ). But we still need to solve the second maximization, and use the Lagrange multipliers  $\boldsymbol{\lambda}$  as the shadow prices to allocate exactly the right power to each transmitter, thus increasing the link data rates and reducing congestion at the network bottlenecks. For scalability in ad hoc networks, this power control must also be implemented distributively, just like the congestion control part. Since the data rate on each wireless link is a global function of all the transmit powers, the power control problem cannot be nicely decoupled into local problems for each link as in [31]. However, we show that distributed solution is still feasible, as long as an appropriate set of limited information is passed among the nodes.

But we first need to establish that, if the algorithm converges, the convergence is indeed toward the global optimum. We will establish that the partial Lagrangian to be maximized  $I_{power}(\mathbf{P}) = \sum_l \lambda_l \log(\text{SIR}_l(\mathbf{P}))$  is a concave func-

tion of a logarithmically transformed power vector. Let  $\tilde{P}_l = \log P_l$ ,  $\forall l$ , we have  $I_{power}(\tilde{\mathbf{P}}) =$

$$\begin{aligned} & \sum_l \lambda_l \log \frac{G_{ll} e^{\tilde{P}_l}}{\sum_k G_{lk} e^{\tilde{P}_k} + n_l} \\ & = \sum_l \lambda_l \left[ \log(G_{ll} e^{\tilde{P}_l}) - \log \left( \sum_k G_{lk} e^{\tilde{P}_k} + n_l \right) \right] \\ & = \sum_l \lambda_l \left[ \log(G_{ll} e^{\tilde{P}_l}) - \log \left( \sum_k \exp(\tilde{P}_k + \log G_{lk}) + n_l \right) \right]. \end{aligned}$$

The first term in the square bracket is linear in  $\tilde{\mathbf{P}}$ , and the second term is concave in  $\tilde{\mathbf{P}}$  because the log of a sum of exponentials of linear functions of  $\tilde{\mathbf{P}}$  is convex, as verified below.

Taking the derivative of  $I_{power}(\tilde{\mathbf{P}})$  with respect to  $\tilde{P}_l$ , we have

$$\begin{aligned} \nabla_l I_{power}(\tilde{\mathbf{P}}) & = \lambda_l - \sum_{j \neq l} \frac{\lambda_j G_{jl} e^{\tilde{P}_l}}{\sum_{k \neq j} G_{jk} e^{\tilde{P}_k} + n_j} \\ & = \lambda_l - P_l \sum_{j \neq l} \frac{\lambda_j G_{jl}}{\sum_{k \neq j} G_{jk} P_k + n_j}. \end{aligned}$$

Taking derivatives again, for each of the nonlinear  $-\lambda_l \log \left( \sum_k \exp(\tilde{P}_k + \log G_{lk}) + n_l \right)$  terms in  $I_{power}(\tilde{\mathbf{P}})$ , we obtain the Hessian:

$$\mathbf{H}^l = \frac{-\lambda_l}{(\sum_k z_{lk} + n_l)^2} \left( \left( \sum_k z_{lk} + n_l \right) \mathbf{diag}(\mathbf{z}_l) - \mathbf{z}_l \mathbf{z}_l^T \right)$$

where  $z_{lk} = \exp(\tilde{P}_k + \log G_{lk})$  and  $\mathbf{z}_l$  is a column vector  $[z_{l1}, z_{l2}, \dots, z_{lN}]^T$ .

Matrix  $\mathbf{H}^l$  is indeed negative definite: for all vectors  $\mathbf{v}$ ,

$$\mathbf{v}^T \mathbf{H}^l \mathbf{v} = \frac{-\lambda_l \left( (\sum_k z_{lk} + n_l) (\sum_k v_k^2 z_{lk}) - (\sum_k v_k z_{lk})^2 \right)}{(\sum_k z_{lk} + n_l)^2} < 0. \quad (8)$$

This is because of the Cauchy Schwarz inequality:  $(\mathbf{a}^T \mathbf{a})(\mathbf{b}^T \mathbf{b}) \geq (\mathbf{a}^T \mathbf{b})^2$  where  $a_k = v_k \sqrt{z_{lk}}$  and  $b_k = \sqrt{z_{lk}}$  and the fact that  $n_l > 0$ . Therefore,  $I_{power}(\tilde{\mathbf{P}})$  is a strictly concave function of  $(\tilde{\mathbf{P}})$ , and its Hessian is a negative definite block diagonal matrix  $\mathbf{diag}(\mathbf{H}^1, \mathbf{H}^2, \dots, \mathbf{H}^L)$ .

Coming back to the  $\mathbf{P}$  solution space instead of  $\tilde{\mathbf{P}}$ , it is easy to verify that the derivative of  $I_{power}(\mathbf{P})$  with respect to  $P_l$  is

$$\nabla_l I_{power}(\mathbf{P}) = \frac{\lambda_l}{P_l} - \sum_{j \neq l} \frac{\lambda_j G_{jl}}{\sum_{k \neq j} G_{jk} P_k + n_j}.$$

Therefore, the logarithmic change of variables (that provides the needed concavity property of the maximization) simply scales each entry of the gradient by  $P_l$ :  $\nabla_l I_{power}(\mathbf{P}) = \frac{1}{P_l} \nabla_l I_{power}(\tilde{\mathbf{P}})$ .

We now use the gradient method [5], with a constant step size  $\kappa$ , to maximize  $I_{power}(\mathbf{P})$ :

$$\begin{aligned} P_l(t+1) &= P_l(t) + \kappa \nabla_l I_{power}(\mathbf{P}) \\ &= P_l(t) + \kappa \left( \frac{\lambda_l(t)}{P_l(t)} - \sum_{j \neq l} \frac{\lambda_j(t) G_{jl}}{\sum_{k \neq j} G_{jk} P_k(t) + n_j} \right) \text{where} \end{aligned}$$

Simplifying the equation and using the definition of SIR, we can write the gradient steps as the following distributed power control algorithm with message passing [7]:

$$P_l(t+1) = P_l(t) + \frac{\kappa \lambda_l(t)}{P_l(t)} - \kappa \sum_{j \neq l} G_{lj} m_j(t)$$

where  $m_j(t)$  are messages passed from node  $j$ :

$$m_j(t) = \frac{\lambda_j(t) \text{SIR}_j(t)}{P_j(t) G_{jj}}$$

This is exactly items 3 and 4 in the JOCP Algorithm.

It is known [3] that when the step size along the gradient direction is optimized, the gradient-based iterations converge. Such an optimization of step size  $\kappa$  in (7) would require global coordination in a wireless ad hoc network, and is undesirable or infeasible. However, in general gradient-based iterations with a constant step size may not converge.

By the descent lemma [3], convergence of the gradient-based optimization of a function  $f(\mathbf{x})$ , with a constant step size  $\kappa$ , is guaranteed if  $f(\mathbf{x})$  has the Lipschitz continuity property:  $\|\nabla f(\mathbf{x}_1) - \nabla f(\mathbf{x}_2)\| \leq L \|\mathbf{x}_1 - \mathbf{x}_2\|$  for some  $L > 0$ , and the step size is small enough:  $\epsilon \leq \kappa \leq \frac{2-\epsilon}{L}$  for some  $\epsilon > 0$ . Since  $\nabla f(\mathbf{x}_1) - \nabla f(\mathbf{x}_2) = \nabla^2 f(\mathbf{x})(\mathbf{x}_1 - \mathbf{x}_2)$  for all  $\mathbf{x}$  that are convex combination of  $\mathbf{x}_1$  and  $\mathbf{x}_2$ , by the Cauchy Schwarz inequality, we have

$$\|\nabla f(\mathbf{x}_1) - \nabla f(\mathbf{x}_2)\| \leq \|\nabla^2 f(\mathbf{x})\| \|\mathbf{x}_1 - \mathbf{x}_2\|,$$

i.e.,  $f(\mathbf{x})$  has the Lipschitz continuity property if it has a Hessian bounded in  $l_2$  norm.

The Hessian  $\mathbf{H}$  of  $\sum_l \lambda_l c_l(\mathbf{P})$  can be verified to be

$$H_{ll} = \sum_{j \neq l} \lambda_j \left( \frac{G_{jl}}{\sum_{k \neq j} G_{jk} P_k + n_j} \right)^2 - \frac{\lambda_l}{P_l^2}, \quad (9)$$

$$H_{li} = \sum_{j \neq l, i} \frac{\lambda_j G_{jl} G_{ji}}{\left( \sum_{k \neq j} G_{jk} P_k + n_j \right)^2}, \quad i \neq l. \quad (10)$$

Since  $\lambda$  are finite, it is obvious that  $\|\mathbf{H}\|_2$  is upper bounded. The upper bound can be estimated by the following inequality:

$$\|\mathbf{H}\|_2 \leq \sqrt{\|\mathbf{H}\|_1 \|\mathbf{H}\|_\infty}$$

where  $\|\mathbf{H}\|_1$  is the maximum column-sum matrix norm of  $\mathbf{H}$ , and  $\|\mathbf{H}\|_\infty$  is the maximum row-sum matrix norm.

Therefore, the power control part (7) converges for a small enough step size  $\kappa$ :

$$\epsilon \leq \kappa \leq \frac{2-\epsilon}{L'}$$

$$\begin{aligned} (L')^2 &= \max_i \left( \sum_l \sum_{j \neq l, i} \frac{\lambda_j G_{jl} G_{ji}}{\left( \sum_{k \neq j} G_{jk} P_k + n_j \right)^2} \right. \\ &\quad \left. + \left| \sum_{j \neq l} \lambda_j \left( \frac{G_{jl}}{\sum_{k \neq j} G_{jk} P_k + n_j} \right)^2 - \frac{\lambda_l}{P_l^2} \right| \right) \\ &\quad \times \max_l \left( \sum_i \sum_{j \neq l, i} \frac{\lambda_j G_{jl} G_{ji}}{\left( \sum_{k \neq j} G_{jk} P_k + n_j \right)^2} \right. \\ &\quad \left. + \left| \sum_{j \neq l} \lambda_j \left( \frac{G_{jl}}{\sum_{k \neq j} G_{jk} P_k + n_j} \right)^2 - \frac{\lambda_l}{P_l^2} \right| \right) \end{aligned}$$

and  $\epsilon$  can be any small positive number  $\leq \frac{2}{1+L'}$ .

It is known [21] that TCP Vegas converges for a small enough step size  $0 < \gamma \leq \frac{2\alpha_{min} d_{min} c_{min}}{L_{max} S_{max} x_{max}^2}$ , where  $\alpha_{min}$  and  $d_{min}$  are the smallest TCP source parameters  $\alpha_s$  and  $d_s$  among the sources, respectively,  $x_{max}$  is the largest possible source rates,  $c_{min}$  is the smallest link data rate,  $L_{max}$  is the largest number of links a path can have, and  $S_{max}$  is the largest number of sources sharing a link.

Convergence of TCP Vegas assumes that  $c_{min} \neq 0$ . Since  $\text{SIR}_l$  is lower bounded by  $\frac{P_{l,min} G_{ll}}{\sum_{j \neq l} P_{j,max} G_{lj} + n_l}$ , each  $c_l$  is lower bounded by a strictly positive number. (In fact, the formulation in (4) assumes high SIR in the first place.) Consequently, TCP Vegas (6.5) also converges. The JOCP Algorithm converges as the congestion control and power control parts both converge and are coupled by vector  $\lambda$  that converges.

Since  $c_l$  can be turned into a concave function in  $\tilde{\mathbf{P}}$ , each constraint  $\sum_{s:l \in L(s)} x_s - c_l(\mathbf{P}) \leq 0$  in (4) is an upper bound constraint on a convex function in  $(\mathbf{x}, \tilde{\mathbf{P}})$ . So problem (4) can be turned into maximizing a strictly concave objective function over a convex constraint set. The established convergence is thus indeed toward a unique global optimum. ■

In addition to convergence guarantee, total network utility  $\sum_s U_s(x_s)$  with power control can never be smaller than that without power control, because by allowing power adaptation, we are optimizing over a larger constraint set. Note that an increase in network utility  $\sum_s U_s(x_s)$  is not equivalent to a higher total throughput  $\sum_s x_s$ , since  $U_s$  can be any increasing, strictly concave functions of  $x_s$ . However, empirical evidence from simulation suggests that at least in the logarithmic utility case of TCP Vegas, both throughput and energy efficiency will indeed rise significantly after power control (7) regulates bandwidth supply, and dual variables  $\lambda$  balance demand with supply.

Now that the existence and joint optimum of the equilibrium point  $(\mathbf{x}^*, \mathbf{P}^*)$  of the JOCP Algorithm is established, we need to address the following question about the equilibrium point:

will the power control part ‘produce’ more link capacities than needed by the sources? We answer this question in the following general theorem that covers TCP Vegas under any type of elastic link capacities, where link capacity  $c_l(\boldsymbol{\theta})$  can be any concave function of some physical layer resources  $\boldsymbol{\theta}$ :

$$\begin{aligned} & \text{maximize} && \sum_s \alpha_s d_s \log x_s \\ & \text{subject to} && \sum_{s:l \in L(s)} x_s \leq c_l(\boldsymbol{\theta}), \quad \forall l, \\ & && \boldsymbol{\theta}, \mathbf{x} \succeq 0 \end{aligned} \quad (11)$$

There can be different algorithms to implement resource allocation in the physical layer and congestion control in the transport layer. Coordinating these two layers through the Lagrange dual variables  $\boldsymbol{\lambda}$  leads to a desirable equilibrium:

*Theorem 2:* The global optimum of joint TCP Vegas congestion control and resource allocation (11) is such that the capacity on each link becomes fully utilized by all the sources that traverse through it:

$$c_l^*(\boldsymbol{\lambda}^*) = \sum_{s:l \in L(s)} x_s^*(\boldsymbol{\lambda}^*), \quad \forall l.$$

*Proof:* We first form the partial Lagrangian  $I_{congestion}(\mathbf{x}, \boldsymbol{\lambda})$  of (11):  $\sum_s \alpha_s d_s \log x_s - \sum_l \lambda_l \sum_{s:l \in L(s)} x_s$ . Consider its maximization over  $\mathbf{x}$ :

$$\text{maximize}_{\mathbf{x}} \left[ \sum_s \alpha_s d_s \log x_s - \sum_s \sum_{l \in L(s)} \lambda_l x_s \right],$$

which can be solved by differentiating with respect to  $x_s$  to obtain the optimal source rates as a function of  $\boldsymbol{\lambda}$ :

$$x_s^*(\boldsymbol{\lambda}) = \frac{\alpha_s d_s}{\sum_{l \in L(s)} \lambda_l}. \quad (12)$$

Substituting this back into  $I_{congestion}(\mathbf{x}, \boldsymbol{\lambda})$ , the optimized value of the partial Lagrangian is

$$\sum_s \alpha_s d_s \log \left( \frac{\alpha_s d_s}{\sum_{l \in L(s)} \lambda_l} \right) - \sum_s \alpha_s d_s.$$

At the same time, for a fixed  $\boldsymbol{\lambda}$ , resource  $\boldsymbol{\theta}$  is being optimized over to maximize the other partial Lagrangian  $I_{resource}(\boldsymbol{\theta}) = \sum_l \lambda_l c_l(\boldsymbol{\theta})$ , resulting in the optimized link capacities  $\mathbf{c}^*(\boldsymbol{\lambda})$ .

Now we add the optimized values of both partial Lagrangians to obtain the Lagrange dual function  $g(\boldsymbol{\lambda}) = I(\mathbf{x}^*, \boldsymbol{\theta}^*, \boldsymbol{\lambda}) = I_{congestion}(\mathbf{x}^*, \boldsymbol{\lambda}) + I_{resource}(\boldsymbol{\theta}^*, \boldsymbol{\lambda})$ . By strong duality, solving (11) is equivalent to minimizing  $g(\boldsymbol{\lambda})$  over  $\boldsymbol{\lambda} \succeq 0$ , *i.e.*, we need to optimize

$$- \sum_s \alpha_s d_s \log \left( \sum_{l \in L(s)} \lambda_l \right) + \sum_l \lambda_l c_l^*(\boldsymbol{\lambda}) \quad (13)$$

over  $\boldsymbol{\lambda}$ . At optimality, the derivative of (13) with respect to  $\lambda$  must be 0, *i.e.*,

$$c_l^*(\boldsymbol{\lambda}^*) = \sum_{s:l \in L(s)} \beta_s \frac{1}{\sum_{l' \in L(s)} \lambda_{l'}^*}, \quad \forall l$$

where the right hand side is equal to  $\sum_{s:l \in L(s)} x_s^*(\boldsymbol{\lambda}^*)$  by (12). Therefore,

$$c_l^*(\boldsymbol{\lambda}^*) = \sum_{s:l \in L(s)} x_s^*(\boldsymbol{\lambda}^*), \quad \forall l. \quad \blacksquare$$

This theorem shows that at equilibrium, resources will be allocated and source rates adjusted such that the the resulted link capacities are just enough to accommodate the traffic flows. Roughly speaking, supply meets demand ‘tightly’. The Lagrange multipliers are effective enough in coordinating the transport layer with the physical layer that link capacities will not be ‘produced’ more than needed. The focus of this paper on wireless ad hoc networks with power control is certainly a special case covered by Theorem 2.

## VI. SOME PRACTICAL ISSUES: ROBUSTNESS, COMPLEXITY REDUCTION, ASYNCHRONOUS IMPLEMENTATION, AND CONVERGENCE SPEED

In this section, we present results on some practical issues related to the proposed JOCP Algorithm.

### A. Robustness

We start with the robustness properties of the JOCP Algorithm, focusing on the following aspects:

- 1) The effects of wrong estimates of path losses at various nodes. Even with an accurate estimation, mobility of the nodes and fast variation of the fading process may lead to a mismatch between the  $G_{ij}$  used in the power update algorithm and the  $G_{ij}$  that actually appear in the link data rate formula.
- 2) The effects of packet loss due to wireless channel outage during deep fading.

First, it is assumed in the power control algorithm (7) that the pass loss factors  $G_{ij}$  are perfectly estimated by the receivers. It is useful to know how much error in the estimation of  $G_{ij}$  can be tolerated without losing the convergence of joint power control and TCP congestion control.

Denoting the error in the estimation of  $G_{ij}$  at time  $t$  as  $\Delta G_{ij}(t)$ , and suppressing the time index on  $\boldsymbol{\lambda}(t), \mathbf{P}(t), \mathbf{SIR}(t), \Delta G_{ij}(t)$ , we provide a sufficient condition in the following

*Proposition 1:* Convergence to the global optimum of (4) is achieved through the JOCP Algorithm (5,6,7) with  $G_{ij}$  estimation errors, if there exists a  $T$  such that for all times  $t \geq T$ , the following inequality holds:

$$\begin{aligned} & \sum_l \sum_{j \neq l} \sum_{k \neq l} (G_{jl} G_{kl} - \Delta G_{jl} \Delta G_{kl}) \frac{\lambda_j \lambda_k \mathbf{SIR}_j \mathbf{SIR}_k}{P_j P_k G_{jj} G_{kk}} \\ & > 2 \sum_l \sum_{j \neq l} \frac{\lambda_l \lambda_j G_{jl}}{P_l P_j G_{jj}} \mathbf{SIR}_j - \frac{\lambda_l^2}{P_l^2}. \end{aligned}$$

*Proof:* In minimizing a function  $f(\mathbf{x})$  through the gradient iterations, it is easy to show that if there is an error  $\mathbf{e}$  in



the gradient and the search direction becomes  $\nabla f(\mathbf{x}) + \mathbf{e}$ , then convergence to the correct stationary point is maintained in the region  $\{\mathbf{x} \mid \|\nabla f(\mathbf{x})\| \geq \mathbf{e}\}$ .

In our optimization problem, an error in  $G_{ij}$  produces an error  $\mathbf{e}$  in the gradient vector where

$$e_l = \sum_{j \neq l} \Delta G_{lj} \frac{\lambda_j \text{SIR}_j}{P_j G_{jj}}.$$

The region of convergence  $\{\mathbf{P} \mid \|\nabla I_{power}(\mathbf{P})\| \geq \mathbf{e}\}$  with the above error can be calculated and then simplified to be the expression in the above proposition. ■

While Proposition 1 gives an analytic test of convergence under wrong estimates of  $G_{ij}$  for any network, empirical experiments can be carried out in simulations where the  $G_{ij}$  factors in (7) are perturbed randomly within a range. Results of one typical experiment is shown in the lower left graph in Figure 4, for the same network topology and logical connections as in Figure 2. In this simulation, the  $G_{ij}$  factors are generated at random between +25% and -25% of their true values. It can be seen that the algorithms converge to the same global optimum after a longer and wider transient period.

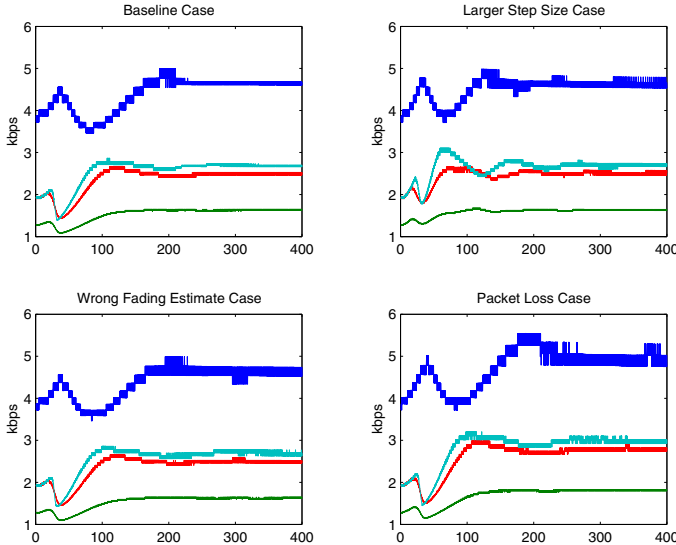


Fig. 4. Robustness of joint power control and TCP Vegas. Top left case is the baseline performance of the four end-to-end throughput. Top right case shows that a larger step size in the algorithm accelerates convergence but also leads to larger variance. Bottom left case shows that the algorithm is robust to wrong estimates of path losses. Bottom right case shows robustness against packet losses on links with wireless channel outage.

Another peculiar feature of wireless transmissions is that during deep fading, SIR on a link may become too small for correct decoding at the receiver. This channel outage induces packet losses on the link. Consequently the queue buffer sizes become smaller than they should have been. Analysis of TCP in such lossy environment has been carried out, for example in [1]. In our framework of nonlinear optimization, since queuing delays are implicitly used as the dual variables  $\lambda$  in TCP Vegas, such

channel variations lead to incorrect values of the dual variables. Sources will mistake the decreases in total queuing delay as indications of reduced congestion levels, and boost their source rates through TCP update accordingly. Having incorrect pricing on the wireless links may thus prevent the joint system from converging to the global optimum.

Following the proof for Proposition 1, we have the following sufficient condition for convergence, with outage-induced packet loss on link  $l$  denoted as  $\Delta y_l$ :

*Proposition 2:* Convergence to the global optimum of (4) is achieved through the JOCP Algorithm (5,6,7) with packet losses, if there exists a  $T$  such that for all times  $t \geq T$ , the following inequality holds:

$$\sum_l \left[ \frac{1}{P_l^2} \left( \lambda_l^2 - \left( \frac{\Delta y_l}{c_l} \right)^2 \right) + \sum_{j \neq l} \left( \frac{G_{jl} \text{SIR}_j}{G_{jj} P_j} \right)^2 \left( \lambda_j - \left( \frac{\Delta y_j}{c_j} \right)^2 \right) \right] > 2 \sum_l \sum_{j \neq l} \left( \lambda_j \lambda_l - \frac{\Delta y_l \Delta y_j}{c_l c_j} \right) \frac{G_{jl} \text{SIR}_j}{G_{jj} P_l P_j}.$$

Because the chance of having simultaneous channel outage at all links is small, it is reasonable to expect that only few  $\Delta y_l$  are nonzero at any time. We again empirically experiment with channel outage induced packet loss on various links, and a typical result is shown in the lower right graph in Figure 4 where the underlying outage probability is 20%. The convergence is slower but still maintained toward the same optimal solution.

## B. Complexity reduction

Another practical issue concerning the JOCP Algorithm is the tradeoff between performance optimality and implementation simplicity. The increases in TCP throughput and energy efficiency have been achieved with a rise in the communication complexity of message passing and in the computational complexity of power update. There can be many terms in the  $\sum_{j \neq l} G_{lj} m_j(t)$  sum in (7) as the number of transmitters increases. Fortunately, those transmitters far away from transmitter  $l$  will have their messages be correspondingly multiplied by a smaller  $G_{lj} \propto d_{lj}^{-\alpha}$  (where  $d_{lj}$  is the distance between node  $l$  and node  $j$  and  $\alpha$  ranges between 2 and 6). Their messages  $m_j$  will therefore be given much smaller weights in the power update.

This leads to a simplified power control algorithm, where only messages from a small set  $J_l$  of other transmitters are passed to the transmitter on link  $l$ . Naturally, if there are  $V$  elements in set  $J_l$ , they should correspond to the nodes with the  $V$  largest  $G_{lj}$  toward node  $l$ . The power update equation becomes:

$$P_l(t+1) = P_l(t) + \frac{\kappa \lambda_l(t)}{P_l(t)} - \kappa \sum_{j \in J_l} G_{lj} m_j. \quad (14)$$

Following the proof for Proposition 1, the following sufficient condition of convergence with the simplified algorithm can be shown:

*Proposition 3:* Convergence to the global optimum of (4) is achieved through the simplified version of the JOCP Algorithm (5,6,14), if there exists a  $T$  such that for all time  $t \geq T$ , the following inequality holds:

$$\sum_l \sum_{j \in J_l} \left( \frac{G_{jl} \lambda_j \text{SIR}_j}{G_{jj} P_j} \right)^2 > 2 \sum_l \sum_{j \neq l} \frac{\lambda_l \lambda_j G_{jl}}{P_l P_j G_{jj}} \text{SIR}_j - \frac{\lambda_l^2}{P_l^2}. \quad (15)$$

The reduction in complexity can be measured by the ratio

$$\Delta \text{COM} = \frac{\sum_l |J_l|}{N(N-1)}$$

where  $N$  is the total number of transmitters in the network. Obviously,  $0 \leq \Delta \text{COM} \leq 1$ , and a smaller  $\Delta \text{COM}$  represents a simpler and less optimal message passing and power update. The effectiveness of complexity reduction through partial message passing depends on the path loss matrix  $\mathbf{G}$ . While the intuition is clear: the reduced-complexity versions do not work well for network topologies where nodes are evenly spread out, we do not yet have an analytic characterization on the trade-off between  $\Delta \text{COM}$  and energy efficiency enhancement or the maximized network utility.

### C. Asynchronous implementation

The algorithmic analysis thus far has been limited to the case where propagation delay is insignificant and all the local clocks are synchronized, which is not practical in large wireless ad hoc networks. In this subsection, we investigate the stability of the algorithm under asynchronous implementation, either due to variable propagation delays or clock asynchronism.

Suppose each source updates  $x_s$  and each transmitter updates  $P_l$  at asynchronous time instances, using possibly outdated variables such as  $\lambda_l$  and  $m_j$  in their update. At least one local update is carried out sometime within a window of  $D$  time slots, and the variables used in the update can be delayed up to  $D$  time slots. We have the following

*Proposition 4:* The asynchronous JOCP Algorithm converges if and only if  $D$  is finite.

*Proof:* When  $D = \infty$ , the diagonal dominance condition [4] must be satisfied for the Hessian  $\mathbf{H}$  of the partial Lagrangian  $I_{\text{power}}(\mathbf{P})$ , i.e.,  $H_{ll} \geq \sum_{i \neq l} |H_{li}|$ ,  $\forall l$ , which in general is not true, based on the formula of  $\mathbf{H}$  in (9).

For a finite  $D$ , convergence of gradient-based method is maintained if the function being optimized are non-negative and has the Lipschitz continuity property, and the constant step sizes are smaller than  $\frac{1}{D}$  [4]. Nonnegativity and Lipschitz continuity are indeed satisfied for both partial Lagrangians  $I_{\text{congestion}}(\mathbf{x})$  and  $I_{\text{power}}(\mathbf{P})$ . Thus convergence is maintained if

$$\kappa \leq \min \left\{ \frac{1}{D}, \frac{2-\epsilon}{L'} \right\} \quad (16)$$

$$\gamma \leq \min \left\{ \frac{1}{D}, \frac{2\alpha_{\min} d_{\min} c_{\min}}{L_{\max} S_{\max} x_{\max}^2} \right\}. \quad (17)$$

This result shows that the proposed algorithm is able to support asynchronous implementation as long as the constants  $\kappa, \gamma$  are small enough. The effect of asynchronism is to reduce the maximum step sizes allowed if convergence is to be maintained, thus reducing the convergence speed. However, in the case of sufficiently small asynchronism:

$$D \leq \min \left\{ \frac{L'}{2-\epsilon}, \frac{L_{\max} S_{\max} x_{\max}^2}{2\alpha_{\min} d_{\min} c_{\min}} \right\},$$

delay in message passing and asynchronism in rate-power updates become the loose constraints in (16,17), and do not cause any reduction in step sizes and convergence rate.

### D. Rate of convergence

So far we have focused on the equilibrium behaviors of the JOCP Algorithm. This subsection provides a preliminary analysis on its rate of convergence. Rate of convergence for any distributive algorithm on wireless ad hoc networks is particularly important because the network topology are dynamic and source traffic may exhibit a low degree of stability. A key question for practical implementation of the proposed cross layer design is whether the coupled nonlinear dynamics between TCP and power control can proceed reasonably close to the equilibrium before the network topology, routing, and source characteristics change dramatically.

Convergence analysis for distributive nonlinear optimization can take several different approaches. We focus on the more practical local analysis approach, which investigates the rate of convergence after the algorithm reaches a point reasonably close to the optimum. Because our algorithm nonlinearly depends on the path loss matrix  $\mathbf{G}$ , exact and closed-form results on the rate of convergence is very difficult to obtain. Nonetheless, the following result on the geometric convergence property and a bound on the convergence speed can be proved.

Let  $U^{(k)}$  be the network utility at the  $k$ th iteration of the JOCP Algorithm, and  $U^*$  be the maximized network utility. Let  $e^{(k)} = |U^{(k)} - U^*|$  be the error. Let  $\mathbf{P}^{(k)}$  be the power vector at the  $k$ th iteration, and  $\mathbf{P}^*$  be the optimizer. Denote by  $M^{(k)}$  the largest eigenvalues of the  $k$ th iteration Hessian of  $I_{\text{power}}(\mathbf{P}^{(k)})$ , and  $m^{(k)}$  the smallest eigenvalue. Let  $M = \limsup_{k \rightarrow \infty} M^{(k)}$  and  $m = \limsup_{k \rightarrow \infty} m^{(k)}$ , and assume  $M, m \in \mathbf{R}$ . Let  $\mathbf{H}$  be the limit of the Hessian derived in (9) as  $k \rightarrow \infty$ , and the entries of  $\mathbf{H}$  be denoted as  $H_{ij}$ .

*Proposition 5:* The joint congestion control and power control algorithm converges geometrically, i.e., there exist  $q > 0$  and  $\beta \in (0, 1)$  such that for all  $k$ ,  $e^{(k)} \leq q\beta^k$ . With an appropriate constant parameter  $\kappa$ , the rate of convergence (of the power control part) is at least  $\frac{M'-m'}{M'+m'}$ , where

$$M' = \max_i \left( H_{ii} + \sum_{j \neq i} |H_{ij}| \right)$$

$$m' = \min_i \left( H_{ii} - \sum_{j \neq i} |H_{ij}| \right).$$

*Proof:* It is known [3] that the convergence of a gradient-based maximization of a nonlinear function  $f(\mathbf{x})$  is geometric if the convergent point  $\mathbf{x}^*$  is not singular, *i.e.*, the Hessian of  $f$  at  $\mathbf{x}^*$  is negative definite. As shown in the convergence proof of Theorem 1, the Hessian of  $I_{power}(\tilde{\mathbf{P}})$  is indeed negative definite. Interestingly, the Hessian would have been only negative semidefinite (and may be singular) had there not been the strictly positive noise terms  $n_l > 0$  in (8).

From the local analysis results in [3], it can be shown that  $\limsup_{k \rightarrow \infty} \frac{\|\mathbf{P}^{(k+1)} - \mathbf{P}^*\|^2}{\|\mathbf{P}^{(k)} - \mathbf{P}^*\|^2}$  equals  $\limsup_{k \rightarrow \infty} \max\{|1 - \kappa m^{(k)}|^2, |1 - \kappa M^{(k)}|^2\}$ . Therefore, choosing  $\kappa = \frac{2}{M+m}$  gives a rate of convergence that is at least  $\frac{M-m}{M+m}$ . Finding the exact values of the largest and smallest eigenvalues of  $\mathbf{H}$  in closed form is very difficult. However, by Gersgorin's theorem [14], all eigenvalues of  $\mathbf{H}$  must lie in the following union:

$$\bigcup_i \{z : |z - H_{ii}| \leq \sum_{j \neq i} |H_{ij}|\}.$$

Therefore, the largest value an eigenvalue can assume is  $M' = \max_i (H_{ii} + \sum_{j \neq i} |H_{ij}|)$ , and the smallest value an eigenvalue can assume is  $m' = \min_i (H_{ii} - \sum_{j \neq i} |H_{ij}|)$ . The worst case convergence rate is when  $M = M'$  and  $m = m'$ . The proposition follows. ■

A similar result holds for the rate of convergence of the congestion control part. However, we add the cautionary note that the above lower bound on the rate of convergence is based on the worst case scenario and can be orders of magnitude loose. Depending on the path loss environment in an ad hoc network, our numerical simulations show that the actual convergence speed is often much faster than the bound in Proposition 5.

### E. Further algorithmic enhancements

In concluding our performance analysis of the JOCP Algorithm, we briefly outline a couple of algorithmic enhancements that can be readily accomplished.

It is desirable to choose a constant step size that is neither so large that the algorithm diverges (violating the conditions in sections V and VI.C) nor so small that the convergence is too slow (as bounded in subsection VI.D). One way to accomplish this is to let each source and each transmitter autonomously decrease the step sizes at each time slot  $t$  according to the following rule:

$$\gamma(t) = \kappa(t) = \frac{1}{t}.$$

Such a diminishing sequence of step sizes also makes the algorithm even more robust: errors in queuing delays  $\lambda$  and path losses  $\mathbf{G}$  that are proportional to the magnitudes of  $\lambda$  and  $\mathbf{G}$

can be tolerated without losing the algorithm's convergence to the joint and global optimum  $(\mathbf{x}^*, \mathbf{P}^*)$ .

It is also possible to speed up the convergence of the algorithm by diagonally scaling the distributed gradient method:

$$P_l(t+1) = P_l(t) + \kappa \mathbf{W} \nabla_l I_{power}(\mathbf{P})$$

where  $\mathbf{W}$  ideally should be the inverse of the Hessian  $\mathbf{H}$  of  $I_{power}(\mathbf{P})$ . Since forming this inverse will require extensive global coordination and centralized computation, we approximate the inverse by letting

$$\mathbf{W} = \mathbf{diag}(H_{ii}^{-1}).$$

Substituting the expression for  $H_{ii}$  in (9) and simplifying the expressions, we arrive at the following accelerated algorithm:

$$P_l(t+1) = P_l(t) + \kappa \frac{\frac{\lambda_l(t)}{P_l(t)} - \sum_{j \neq l} G_{lj} m_j(t)}{\frac{\lambda_l(t)}{P_l^2(t)} - \sum_{j \neq l} \frac{(G_{lj} m_j(t))^2}{\lambda_j(t)}}. \quad (18)$$

Therefore, by passing an additional message: the explicit value of shadow price  $\lambda_j(t)$  from node  $j$ , the jointly optimal congestion control and power control algorithm can converge faster.

## VII. LIMITATIONS AND EXTENSIONS

This paper is limited in a number of aspects:

- Our analysis has focused on the equilibrium state of joint congestion control and power control. We only have limited results on the rate of convergence, and very little understanding of the transient or stability properties of the algorithm.
- We have assumed that the time scale of power control and congestion control is longer than the time scale needed for channel coding and modulation to achieve  $c_l = \log(KSIR_l)$ , and shorter than the time scale of dynamic changes in network topology and routing.
- Our physical layer model assumes some given codes and modulations under high SIR. When SIR is comparable to 1, the constraint  $\sum_{s:l \in L(s)} x_s \leq c_l(\mathbf{P})$  may not be convertible into an upper bound constraint on a convex function in  $(\mathbf{x}, \mathbf{P})$ , and the partial Lagrangian solution may only provide a performance bound.
- Our transport layer model is accurate only for the 'elephant' traffic using long-lived TCP flows, but not for short TCP sessions.
- There are physical layer design variables other than power, *e.g.*, coding parameters, interleaver depth, and modulation types, that can be adapted to change the supportable data rates on a wireless link.

In addition to rate and power controls, two other obvious mechanisms to reduce bottleneck congestion are scheduling over different time slots and routing through alternate paths. Neither has been investigated jointly with the algorithm in this

paper. We are extending the framework of nonlinearly constrained optimization to incorporate other layers in wireless ad hoc networks (or in wired networks *e.g.*, as recently done in [30]), where the optimization variables represent design parameters in each layer being considered, the constraint functions model the physical or economic constraints, and the objective function can be utility functions of source rates and also other system parameters. Similar to the results in this paper, we then need to answer the questions on equilibrium state behavior, suitable decomposition for distributed solution, convergence and performance of the algorithms, and tradeoff among optimality, complexity, and robustness. Optimization-theoretic framework may provide a rigorous approach to answer other versions of the question ‘to layer or not to layer’ from different perspectives.

## VIII. CONCLUSION

We present a distributed power control algorithm that couples with the original TCP algorithms to increase end-to-end throughput and energy efficiency of multihop transmissions in wireless ad hoc networks. No modification to the existing TCP protocols is needed to achieve the optimal balancing between bandwidth demand (regulated through TCP) and supply (regulated through power control). We prove that the nonlinearly coupled system converges to the global optimum of the joint congestion control and power control problem. The convergence is geometric and can be maintained under any finite asynchronism. The proposed algorithm is robust to wireless channel variations and path loss estimation errors. Suboptimal but much simplified versions of the algorithm are presented for scalable architectures. Throughout the paper, we expand the scope of network utility maximization methodology to handle nonlinear, elastic link capacities. This extension enables us to rigorously prove that the proposed JOCP Algorithm has the above desirable properties in achieving the optimal balance between the transport and physical layers in wireless ad hoc networks.

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