Integrating Preventive Maintenance Planning and Production Scheduling for a Single Machine

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***Summary & Conclusions*—Preventive maintenance planning, and production scheduling are two activities that are inter-dependent but most often performed independently. Considering that preventive maintenance, and repair affect both available production time, and the probability of machine failure, we are surprised that this inter-dependency seems to be overlooked in the literature. We propose an integrated model that coordinates preventive maintenance planning decisions with single-machine scheduling decisions so that the total expected weighted completion time of jobs is minimized. Note that the machine of interest is subject to minimal repair upon failure, and can be renewed by preventive maintenance. We investigate the value of integrating production scheduling with preventive maintenance planning by conducting an extensive experimental study using small scheduling problems. We compare the performance of the integrated solution with the solutions obtained from solving the preventive maintenance planning, and job scheduling problems independently. For the problems studied, integrating the two decision-making processes resulted in an average improvement of approximately 2% and occasional improvements of as much as 20%. Depending on the nature of the manufacturing system, an average savings of 2% may be significant. Certainly, savings in this range indicate that integrated preventive maintenance planning, and production scheduling should be focused on critical (bottleneck) machines. Because we use total enumeration to solve the integrated model for small problems, we propose a heuristic approach for solving larger problems. Our analysis is based on minimizing total weighted completion time; thus, both the scheduling, and maintenance problems favor processing shorter jobs in the beginning of the schedule. Given that due-date-based objectives, such as minimizing total weighted job tardiness, present more apparent trade-offs & conflicts between preventive maintenance planning, and job scheduling, we believe that integrated preventive maintenance planning & production scheduling is a worthwhile area of study.**

***Index Terms*—Minimal repair, optimization, preventive maintenance, production scheduling, renewal, Weibull distribution.**

***Acronyms*[1]**

PM	preventive maintenance
WSPT	weighted shortest processing time

Manuscript received January 1, 2004; revised September 16, 2004 and November 13, 2004. Associate Editor: J. Rupe.

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Digital Object Identifier 10.1109/TR.2005.845967

[1]The singular and plural of an acronym are always spelled the same.

Notation

n	number of jobs to be scheduled
p_j	processing time of job j
P_{max}	maximum job processing time
w_j	weight of job j
x_{ij}	job sequencing decision variable
$p_{[i]}$	processing time of i^{th} job in the sequence
$w_{[i]}$	weight of i^{th} job in the sequence
$c_{[i]}$	completion time of i^{th} job in the sequence (deterministic case)
$C_{[i]}$	completion time of the i^{th} job in the sequence (stochastic case)
T	time to machine failure
β	Weibull shape parameter for probability distribution of T
η	Weibull scale parameter for probability distribution of T
$F(t)$	Cdf(T)
$\overline{F}(t)$	$1 - F(t)$
$z(t)$	hazard function of T
$a_{[0]}$	age of the machine prior to job sequencing-PM planning
$a_{[i]}$	age of the machine after the i^{th} job in the sequence
$\overline{a}_{[i-1]}$	age of the machine immediately prior to the i^{th} job in the sequence (after PM)
t_r	time required to repair the machine
t_p	time required to perform PM on the machine
τ	PM interval for the machine
τ^*	optimal value of τ
$N(\tau)$	number of machine failures in τ time units of machine operation
$m(\tau)$	E$[N(\tau)]$
$A(\tau)$	steady-state machine availability
$y_{[i]}$	PM decision variable
$y^*_{[i]}$	optimal value of $y_{[i]}$

I. INTRODUCTION

PRODUCTION scheduling, and preventive maintenance (PM) planning are two areas that have received tremendous attention in both the manufacturing industry, and the manufacturing systems & operations research literature. In practice, these activities are typically performed independently despite the clear relationship that exists between them. PM activities take time that could otherwise be used for production, but delaying PM for production may increase the probability of machine failure. Hence, there are trade-offs, and conflicts between PM planning, and production scheduling. Our contention is that manufacturing system productivity could be improved by integrating these decisions. We investigate this contention using an integrated PM planning & job scheduling model.

Similar to the situation in practice, these areas are typically treated independently in the production systems, and operations research literature. There is an extensive amount of research in the production/machine scheduling literature, but several review papers, and two recent books cover the majority of the advancement in the area [1], [2]. Production scheduling models tend to be deterministic optimization models designed to maximize some measure of customer satisfaction. Solution methodologies vary from traditional integer programming, and associated Branch-Bound techniques to Lagrangean relaxation, and optimization-based heuristics. These models & techniques have been implemented in a variety of manufacturing systems.

Hundreds of papers on the use of mathematical modeling for analysing, planning, and optimizing maintenance actions can be found in the literature. Fortunately, several authors have reviewed the literature in this area [3]–[9]. Preventive maintenance planning models are typically stochastic models (either mathematical or simulation) accompanied by optimization techniques designed to maximize equipment availability, or minimize equipment maintenance costs.

One can argue that the models used in production scheduling, and preventive maintenance planning are designed with an implicit common goal of maximizing equipment productivity. Despite this common objective, production scheduling models typically either ignore equipment failure, or treat it as a random event. Therefore, existing studies in the stochastic scheduling literature seem to take either a reactive approach, or a robustness-based approach. In the former, one tries to update the preplanned schedule in the face of machine failures [10], [11]. The latter tries to find a schedule that is rather insensitive to the disruptions [12], [13]. Likewise, preventive maintenance planning models tend to ignore the potential disruptions in production resulting from PM actions. Those that consider job schedules tend to ignore the possibility of revising a previously-determined production schedule based on machine availability considerations [14], [15].

There is only limited literature on models that attempt to combine preventive maintenance planning, and production scheduling. Some of these studies focus on the process industry (e.g. chemical plants), and provide case study results showing the effects of equipment failures on the schedule robustness [16]–[18]. There are several studies that test the effectiveness of simple preventive maintenance policies using discrete-event simulation, rather than optimizing them along with scheduling decisions. These can be viewed as natural extensions of other studies that rely on simulation for comparing scheduling rules. While some papers in this category consider limited maintenance resources in traditional job shop environments [19], [20]; others focus on the interaction of scheduling, and maintenance policies, and assume unlimited resources [21]. There are also studies that extend the simple machine scheduling models by considering the maintenance decisions as given, or constraints, rather than integrating them [22]. Weinstein & Chung [23] investigate strategic level maintenance planning rather than operational level in the context of hierarchical production planning. Cho, Abad, & Parlar [24] consider the effects of age-related quality problems, and take into account maintenance policies that improve system performance.

There are only a few studies that explicitly try to integrate preventive maintenance & scheduling decisions, and to optimize them simultaneously. Ashayeri, Teelen, & Selen [25] propose a discrete-time multi-machine integrated model, but production decisions are determining the production quantities (lot sizing) rather than scheduling distinct jobs, and they consider discrete probabilities of failure instead of defined probability distributions. Graves & Lee [26] consider a single-machine scheduling problem with total weighted completion time as the objective function just as we do, but they schedule only one maintenance activity during the planning horizon. They show some complexity results depending on the length of the planning horizon. Lee & Chen [27] extend this to parallel machines, but still with only one maintenance action. Qi, Chen, & Tu [28] consider a similar single-machine problem with possibly multiple maintenance actions, but they do not explicitly model the risk of not performing maintenance, which is explicitly captured in our analysis.

In this paper, we develop a mathematical model which incorporates production scheduling, and preventive maintenance planning for a single machine. Through a simple example, we demonstrate a procedure for identifying optimal scheduling, and PM decisions. We then provide insights gained from studying the model using numeric examples. Finally, we describe the benefits of integrating the two activities into a single decision-making process.

II. The Production Scheduling Problem

Consider a single machine in a manufacturing system that is required to process a set of n jobs, and suppose that preempting one job for another is not permitted. The purpose of production scheduling (for this particular problem) is to choose an optimal sequence for the jobs. Let

$$x_{ij} = \begin{cases} 1 & \text{if the } i^{\text{th}} \text{ job performed is job } j \\ 0 & \text{otherwise} \end{cases} \qquad i = 1, 2, \ldots n \quad j = 1, 2, \ldots n \tag{1}$$

Suppose our objective is to minimize the total weighted completion time for the jobs, and we ignore the possibility of machine failure. Then

$$p_{[i]} = \sum_{j=1}^{n} p_j x_{ij} \tag{2}$$

$$w_{[i]} = \sum_{j=1}^{n} w_j x_{ij} \tag{3}$$

$$c_{[i]} = \sum_{k=1}^{i} p_{[k]} \tag{4}$$

TABLE I
EXAMPLE SCHEDULING PROBLEM PARAMETERS

Job	Processing Time	Weight	Ratio
1	25	3	0.12
2	7	7	1
3	42	9	0.21

The optimal sequence is the one which minimizes the total weighted completion time

$$\sum_{i=1}^{n} w_{[i]} c_{[i]} \tag{5}$$

The resulting mathematical programming formulation of this job scheduling problem is given by

$$\text{Minimize} \quad \sum_{i=1}^{n} \left(\sum_{j=1}^{n} w_j x_{ij} \right) \left(\sum_{k=1}^{i} \sum_{j=1}^{n} p_j x_{kj} \right) \tag{6}$$

$$\text{subject to} \sum_{j=1}^{n} x_{ij} = 1 \quad i = 1, 2, \ldots n \tag{7}$$

$$\sum_{i=1}^{n} x_{ij} = 1 \quad j = 1, 2, \ldots, n \tag{8}$$

$$x_{ij} \text{ binary} \quad i = 1, 2, \ldots, n \quad j = 1, 2, \ldots, n \tag{9}$$

The two sets of functional constraints, (7), and (8), ensure that each position in the schedule receives one job, and each job is assigned to one position in the schedule, respectively.

The global optimal solution to the mathematical program is easy to obtain using the weighted shortest processing time (WSPT) rule [2]. For each job, the ratio

$$\frac{w_j}{p_j} \tag{10}$$

is computed. Jobs are then sequenced in descending order based on this ratio. For example, suppose a machine is required to process three jobs. The processing times, weights, and ratios for these jobs are presented in Table I. The job sequence which minimizes the total weighted completion time for these three jobs is 2-3-1.

III. The Preventive Maintenance Planning Problem

Suppose the machine used to process the jobs is subject to failure, and the time to failure for the machine, is governed by a Weibull probability distribution having shape parameter greater than 1. When the machine fails, we assume it is minimally repaired, i.e. the machine is restored to an operating condition, but machine age is not altered. This implies that, upon machine failure, the machine operator does just enough maintenance to resume machine function. Because $\beta > 1$, it may be practical to perform preventive maintenance on the machine in order to reduce the increasing risk of machine failure. We assume that PM restores the machine to a "good as new" condition, such that the machine's age becomes zero. This implies that PM is a more comprehensive action than repair, perhaps corresponding to the replacement of several key components in the machine.

Given the failure, repair, and PM characteristics of the machine, a reasonable question is "How often should PM be performed on the machine?" We assume an age-based PM policy is applied, i.e. PM is performed on the machine after τ time units of operation. Assuming our objective is to maximize machine availability, we can use mathematical modeling to determine an optimal value for τ.

Because we assume PM restores the machine to a "good as new" condition, we can model the operation & maintenance of the machine as a renewal process, where the renewal points are:

(1) the initiation of machine operation, and
(2) the end of each PM activity.

Because we assume repair is minimal, we can model the occurrence of failures during each "cycle" of the renewal process using a nonhomogeneous Poisson process. Then, the expected value of $N(\tau)$ is given by

$$m(\tau) = \int_0^\tau z(t)dt = \int_0^\tau \frac{\beta}{\eta^\beta} t^{\beta-1} dt = \left(\frac{\tau}{\eta}\right)^\beta \tag{11}$$

where $z(t)$ corresponds to the hazard function of the underlying Weibull probability distribution. So, the "average" cycle consists of an "uptime" period of τ time units of operation; and a "downtime" period of $m(\tau)$ repairs of length t_r, and a PM action of length t_p. Therefore, the resulting steady-state availability of the machine, expressed as a function of the PM interval, is given by

$$A(\tau) = \frac{\tau}{\tau + m(\tau)t_r + t_p}. \tag{12}$$

Differentiation, and algebraic analysis yields an optimal PM interval of

$$\tau^* = \eta \left[\frac{t_p}{t_r(\beta - 1)} \right]^{\frac{1}{\beta}} \tag{13}$$

For example, suppose a machine's failure, repair, and PM characteristics are such that $\beta = 2$, $\eta = 100$, $t_r = 15$, and $t_p = 5$. Evaluation of (13) indicates that PM should be performed on this machine after 57.7 time units of operation.

IV. The Integrated Problem

Suppose a machine possesses the production requirements defined in Section II; and the failure, repair, and PM characteristics described in Section III. Furthermore, assume jobs are not preempted for PM, and jobs interrupted by failure can be resumed after repair without any additional time penalty. Because both production scheduling, and PM planning are designed to maximize the effective use of the machine, it may be advantageous to solve the production scheduling, and PM planning problems for this machine simultaneously. In addition to choosing a job sequence, one must also decide whether or not to perform PM prior to each job. The integrated problem is further complicated by the fact that completion times for the jobs are stochastic, because the machine may or may not fail during each job, and PM decisions change the stochastic process governing machine failure.

The completion time for a job is a random variable that depends on

- the age of the machine prior to processing the job;
- the completion time for previous jobs;
- the time to complete PM, and the PM decision;
- the job's processing time; the repair time; and
- the number of machine failures during the job.

Let

$$y_{[i]} = \begin{cases} 1 & \text{if PM is performed prior to the } i^{\text{th}} \text{ job} \\ 0 & \text{otherwise} \end{cases} \quad i = 1, 2, \ldots, n \qquad (14)$$

Because we assume that PM renews the machine, and repair is minimal

$$\overline{a}_{[i-1]} = a_{[i-1]} \left(1 - y_{[i]}\right) \qquad i = 1, 2, \ldots, n \qquad (15)$$

$$a_{[i]} = \overline{a}_{[i-1]} + p_{[i]} \qquad i = 1, 2, \ldots, n \qquad (16)$$

The expected value of $C_{[i]}$ is given by

$$\mathrm{E}\left(C_{[i]}\right) = \sum_{k=1}^{i} \left\{t_p y_{[k]} + p_{[k]} + t_r \left[m\left(a_{[k]}\right) - m\left(\overline{a}_{[k-1]}\right)\right]\right\} \qquad i = 1, 2, \ldots, n \qquad (17)$$

which includes time spent on all PM actions performed before the i^{th} job, all processing times up to & including the i^{th} job, and the expected value of the time spent on repairs that occur before or during the i^{th} job. Our modified objective is to identify the PM actions & job sequence that minimize the total weighted expected completion time

$$\sum_{i=1}^{n} w_{[i]} \mathrm{E}\left(C_{[i]}\right) \qquad (18)$$

The resulting mathematical programming formulation of the integrated problem is given by

$$\text{Minimize} \sum_{i=1}^{n} w_{[i]} \mathrm{E}\left(C_{[i]}\right) \qquad (19)$$

$$\text{subject to} \sum_{j=1}^{n} x_{ij} = 1 \quad i = 1, 2, \ldots n \qquad (20)$$

$$\sum_{i=1}^{n} x_{ij} = 1 \quad j = 1, 2, \ldots, n \qquad (21)$$

$$x_{ij} \text{ binary} \quad i = 1, 2, \ldots, n \;\; j = 1, 2, \ldots, n \qquad (22)$$

$$y_{[i]} \text{ binary} \quad i = 1, 2, \ldots, n \qquad (23)$$

V. Solving the Integrated Problem

We solve the integrated problem using total enumeration. To demonstrate this enumerative procedure, we make use of the example defined in Sections II & III. Suppose $a_{[0]} = 88$. The first step in our solution procedure is to enumerate the set of $n!$ feasible job sequences. The six feasible sequences for this example are enumerated in Table II.

The second step in our solution procedure is to identify the optimal set of PM decisions for each feasible job sequence. Each of the 2^n feasible sets of PM decisions is applied to each job sequence. The objective function value is computed for each set of PM decisions, and the set with the smallest objective function value is identified as optimal for that sequence. This analysis for the 1-2-3 sequence is presented in Table III. Note that the optimal PM decisions for this sequence are to perform PM only before the first job. The job sequence-PM decisions with the overall minimum objective function value are identified as the global optimal solution. The results for this example are presented in Table II. Note that the optimal solution is to use the job sequence 2-3-1 with PM performed prior to job 3 (the second job in the sequence).

TABLE II
INTEGRATION EXAMPLE RESULTS

Job Sequence	$y^*_{[1]}$	$y^*_{[2]}$	$y^*_{[3]}$	Objective Function Value
1 – 2 – 3	1	0	0	1147.5
1 – 3 – 2	1	0	0	1411.9
2 – 1 – 3	0	1	0	970.9
2 – 3 – 1	***0***	***1***	***0***	***852.5***
3 – 1 – 2	1	0	0	1293.5
3 – 2 – 1	1	0	0	1111.7

TABLE III
1-2-3 PM ANALYSIS

$y_{[1]}$	$y_{[2]}$	$y_{[3]}$	Objective Function Value
0	0	0	1307.3
0	0	1	1216.2
0	1	0	1221.1
0	1	1	1258.2
1	***0***	***0***	***1147.5***
1	0	1	1156.2
1	1	0	1190.7
1	1	1	1227.8

VI. Solution Analysis

Investigating the implications & benefits of integrating the job scheduling & PM planning decisions can be summarized with the following questions.

- How does the optimal integrated job sequence compare to the WSPT sequence?
- How does the optimal integrated job-PM sequence compare to the WSPT sequence combined with the independently-obtained optimal PM interval?
- How does the minimum integrated objective function value compare to the objective function value for the WSPT sequence combined with the independently-obtained optimal PM interval?

First, we address these questions for the example considered in Section V. Then, we summarize the answers to these questions with more numeric examples.

For the Section V example, the optimal job sequence is the same as the WSPT sequence (2-3-1). However, the optimal job sequence-PM decisions (2-3-1 with PM prior to job 3) are different from the WSPT sequence combined with the independently-obtained optimal PM interval. Recall that the optimal PM interval for this machine is 57.7 time units. Using this interval would mandate performing PM prior to the first job (job 2) because $a_{[0]} = 88$. After job 2, the age of the machine would be 7

TABLE IV
EXPERIMENTAL DESIGN

Trial	β	t_p	t_r	P_{max}	τ^*
1	2	5	15	50	58
2	2	5	15	100	58
3	2	5	25	50	45
4	2	5	25	100	45
5	2	10	15	50	82
6	2	10	15	100	82
7	2	10	25	50	63
8	2	10	25	100	63
9	3	5	15	50	55
10	3	5	15	100	55
11	3	5	25	50	46
12	3	5	25	100	46
13	3	10	15	50	69
14	3	10	15	100	69
15	3	10	25	50	58
16	3	10	25	100	58

$(p_2 = 7)$. Thus, no PM would be performed prior to the second job (job 3). After job 3, the age of the machine would be 49 $(p_3 = 42)$. Thus, no PM would be performed prior to job 1.

For the Section V example, if the WSPT job sequence is combined with the independently-obtained optimal PM interval, the objective function value is 864.6. Thus, the minimum objective function value of 852.5 represents a 1.4% savings over WSPT with the optimal PM interval. We refer to this as *Savings*.

In an attempt to gain insight into the answers to the three questions posed above, a numeric experiment was designed. In all experimental trials, the machine was required to process three jobs, and $\eta = 100$. For each experimental trial, 1000 individual problems were randomly generated using Monte Carlo simulation of the initial age of the equipment, the weights of the jobs, and the processing times for the jobs. The initial age of the equipment was modeled as a discrete uniform random variable over the integers 1 through 100. The weight of a job was modeled as a discrete uniform random variable over the integers 1 through 10. The processing time of a job was modeled as a discrete uniform random variable over the integers $1, 2, \ldots, P_{max}$, where P_{max} is a controlled factor for the experiment. In addition to P_{max}, the other controlled factors were β, t_p, and t_r. A 2^4 factorial design was used to generate the 16 experimental trials. The factor values, and the optimal PM interval for each trial are given in Table IV.

More than half (60.2%) of the problems analyzed indicate some positive Savings, and the average Savings is 2.2%. So, the results indicate that integrating job scheduling, and PM planning is superior to solving the two problems independently. Almost all (97.7%) of the integrated solutions utilize the WSPT sequence. This similarity, and the relatively small average Savings result from the fact that the total weighted completion time, and machine availability performance measures both encourage processing shorter jobs first. It is worthwhile to note that changes to the WSPT sequence typically occur when two or more jobs have weight to processing time ratios that are very close. In fact, some of the 2.3% that differ from WSPT occur when two or more job ratios tie, and the job sequencing problem has multiple optimal solutions. In these cases, the integrated problem breaks the tie based on PM considerations. Note that less than half (39.8%) of the problems studied have an integrated solution equivalent to the WSPT sequence with the independently-obtained optimal PM interval. Therefore, the benefit of this integrated problem is more effective PM planning.

VII. Larger Problems

The results described in Section VI demonstrate that there is potential benefit to be realized from integrating job sequencing, and PM planning decisions. However, the examples considered in the experiment consider only 3 jobs. In practice, job sequencing problems consider a larger number of jobs, and the jobs to be scheduled evolve in a dynamic fashion. We investigated solving larger problems conducting additional experiments.

The results of the 3-job experiment indicate that the WSPT rule is valid for almost all of the integrated problems. Therefore, we propose the following heuristic:

Step 1. Identify the WSPT job sequence.
Step 2. Identify the PM decisions which minimize total expected weighted completion time for the WSPT job sequence.

We applied this heuristic to the 16 000 problems studied in Section VI. The heuristic yields the optimal solution for 97.9% of these problems; and for those problems that are not optimized by the heuristic, the average deviation from the optimal objective function value is less than 0.005%.

Next, we applied our experimental design to 4-job, and 5-job problems. For each experiment, we randomly generated 500 4-job problems having $\eta = 125$, and 200 5-job problems having $\eta = 150$. We solved these problems using both total enumeration, and the heuristic. For the 4-job problems, the heuristic yields the optimal solution for 97.3% of the problems, and for those problems that are not optimized by the heuristic, the average deviation from the optimal objective function value is less than 0.003%. For the 5-job problems, the heuristic yields the optimal solution for 96.0% of the problems; and for those problems that are not optimized by the heuristic, the average deviation from the optimal objective function value is less than 0.002%.

Therefore, our recommendation is as follows. The enumerative solution procedure works well for a small number of jobs (8 or less). However, because for n jobs the procedure requires the evaluation of $n!$ job sequences, and 2^n sets of PM decisions for each sequence; this procedure is not effective for larger problems. Given that the heuristic can handle much larger problems (up to 20 jobs), it should be used for preliminary planning purposes. Then, as job requirements evolve over time, the job-PM schedule can be updated over shorter periods using the enumerative approach.

VIII. Concluding Remarks and Future Research

The model presented lends itself to a number of meaningful extensions. Objective functions that lead to greater conflict between job sequencing, and PM planning, could be used. For example, a due-date-based function, such as total weighted expected tardiness, does not necessarily emphasize processing shorter jobs earlier in the sequence. Our contention is that integrating PM & job scheduling will undercover true trade-offs between conflicting goals of individual decisions. Assumptions regarding the failure, repair, and PM characteristics of the machine could be modified or eliminated. For example, repair may restore the equipment to a "good as new" condition; or it may be possible to interrupt a job for PM. Multiple machines and/or job shops could be considered. We intend to explore these extensions as well as improved exact, and heuristic solution procedures in future work.

In some production environments, the equipment used is highly reliable. As a result, PM schedules for such equipment may be weekly, monthly, or even semi-annual. In these environments, the use of a job-to-job PM planning tool is unnecessary. However, the models we propose to develop can be applied in these scenarios. Rather than integrating job sequencing & PM decisions, the models could be used as an aggregate planning tool for integrating lot scheduling & PM planning decisions.

Acknowledgment

The authors wish to thank the anonymous reviewers, and the associate editor for their comments and insights which greatly improved the content of the paper.

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