Transmit Selection Diversity for Unitary Precoded Multiuser Spatial Multiplexing Systems with Linear Receivers

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Last Modified: August 17, 2005.

Abstract

Multiuser spatial multiplexing is a downlink transmission technique that uses linear transmit precoding to multiplex multiple users and pre-cancel inter-user interference. In such a system the spatial degrees of freedom are used for interference mitigation and generally come at the expense of diversity gain. This paper proposes two precoding methods that use extra transmit antennas, beyond the minimum required, to provide additional degrees of diversity. The approach taken is to solve for a unitary transmit precoder, under a zero inter-user interference constraint, that minimizes an upper bound on the symbol error rate (SER) for each user. Solutions where all transmit antennas are employed as well as subsets of antennas (to reduce analog components) are described. Numerical results confirm a dramatic improvement in terms of SER and mutual information over single user MIMO methods and static allocation methods. For example, the proposed techniques achieve an SNR improvement of 6-10 dB at an uncoded SER of 10^{-3}, with only one extra transmit antenna.

Index Terms

MIMO systems, Precoding, Diversity methods.

EDICS: 3-ACCS Multiuser and Multiaccess Communication.

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I. INTRODUCTION

Multiple-input multiple-output (MIMO) communication [1]–[7] is widely acknowledged as the key technology for achieving high data rates in bandwidth constrained wireless systems. While initial interest was focused on single user links, more recently there have been a number of investigations in multiuser MIMO communication [8]-[13],[25]-[38]. In this paper, the downlink of a multiuser spatial multiplexing (MUSM) communication system is investigated where antenna arrays are used at both the base transceiver station (BTS) and at K mobile users. Data streams to multiple users, which are pre-multiplied by specially designed precoding matrices, are broadcast to all mobiles simultaneously over the same frequency band. The transmitted data can be jointly detected over all mobile users, or independently at each user subject to interference from co-channel users. This paper considers the latter more practical scenario where no cooperation exists between mobile terminals.

The investigated MUSM system is an example of the MIMO broadcast (MIMO-BC) communication problem, for which the capacity region and maximum achievable sum-rate have been intensively investigated in recent years. Pioneering work on MIMO-BC capacity included [8]–[10] which investigated the special case of a single receive antenna per user, and [11]-[13] where multiple antennas are employed at each mobile terminal. The optimal technique that achieves the maximum sum-rate capacity of the MIMO-BC channel is the so-called "dirty-paper coding" (DPC) approach [14], by which multiuser interference is non-causally cancelled at the transmitter. Despite its significance from the information theoretical point of view, the DPC approach is not considered practical due to the abundance of accurate channel state information (CSI) required instantaneously at the transmitter, and its sensitivity to CSI imperfections. If interference cancellation is not performed at the transmitter as in DPC, then the receivers presumably have to handle the multiuser spatial interference and cancel it in some way. Prior research on interference cancellation of multiuser MIMO systems has primarily focused on the uplink [15]–[16] since complex receivers are really only viable at the BTS and the mobile units must be inexpensive and low-power. This motivates the precoding approach, in which the BTS assists in the interference cancellation process so that simple linear receivers are viable at the mobile units.

Due to the low-complexity realization at the mobile unit, and the large diversity gain, precoding for MIMO systems – also referred to as "closed-loop" MIMO where CSI is known at the transmitter– has been a subject of much recent interest. Extensive results are available for single-user precoder design, e.g. [17]-[24]. Relatively fewer results exist on multiuser MIMO precoding. One category of multiuser precoders allow some inter-user interference and apply beamforming to support multiple users

[25]–[30]. The iterative nature of such algorithms, however, usually results in huge complexity, and the residual co-channel interference (CCI) at mobile still necessitates some CCI cancellation to ensure satisfactory error performance. Perfect interference cancellation requires more transmit antennas and is generally suboptimal in terms of sum capacity [36], but it enables simpler precoder design and allows for low-complexity mobile device [31]-[33], which is a very attractive feature for practical systems. In [32], an iterative joint channel diagonalization (JCD) approach was proposed to avoid the CCI, but only the necessary condition for the existence of channel diagonalization was provided, and the complicated iterative algorithm was not theoretically proved to converge universally. Another CCI cancellation approach is the block diagonalization (BD) method in [31], which diagonalizes the multiuser MIMO channel non-iteratively, followed by a conventional water-filling module to maximize the sum capacity. The BTS must have a minimum number of antennas to ensure complete interference cancellation. These works aimed to achieve the optimal capacity from an information theoretical point of view, subject to zero-interference constraint achieved by different interference cancellation approaches. The diversity gain which is critical for combating fading and link-level error performance, was not addressed.

In this paper we derive linear precoders for MUSM, under the special scenarios where the BTS has more antennas than strictly required for interference avoidance. In contrast to the capacity optimization in the previous work, our work studies the precoder design from the link-level error rate optimization perspective with a fixed number of substreams, since it is also an important factor in practical system besides the Shannon capacity. Two cases are studied, where (1) there are the same number of RF (radio-frequency) units as antenna elements; (2) a limited number of RFs are available and the BTS transmits over a subset of the available antennas. For the first case, a two-step unitary precoder design in the Stiefel manifold framework is proposed, which includes both interference cancellation and symbol error performance enhancement by selection diversity. The first step is to identify a group of unitary downlink precoding matrices at the BTS that perfectly avoid interference at mobile terminals. A QR decomposition based method is proposed to meet the zero-interference constraint, which has lower computational complexity than approaches in [31]-[33]. In the second step, an enhanced space-time precoder with eigenmode selection is proposed to minimize a symbol error rate (SER) upper bound. Based on the signal-to-noise ratio (SNR) bounds in [43][55], eigenmode selection is proposed to optimally bound the SER of each user by performing a secondary singular value decomposition (SVD) and allocating data to the optimal set of eigenmodes. It turns out interestingly that the optimal strategy is similar to the approach in [31], however, it is derived from SER optimization goal instead of the capacity maximization purpose. The SER and capacity performance of the proposed method is carefully compared with the existing methods.

The advantages of MUSM over time division multiple access (TDMA) in terms of asymptotic capacity have been addressed by Sharif *et al.* in [34], Jindal *et al.* in [35], and Yoo *et al.* in [36] and noting that zero-forcing beamforming is a special case of MUSM with single-antenna terminals. In addition, the precoded MUSM system provides a natural framework for multiuser diversity, in which extra users are present and the best subset of users for transmission are scheduled optimally [36]-[38].

If a limited number of RF units are available due to cost constraints, an alternative to the optimal eigenmode selection procedure is to switch appropriately chosen antenna elements from the array to the available RF chains. Since antenna elements are much cheaper than RF devices, performing antenna selection will substantially decrease the system cost. Antenna selection can be performed at either the receiver or the transmitter [39]–[44], in an effort to achieve capacity maximization [39][40], or error performance optimization with practical signaling schemes [43][44]. Prior work on antenna selection has focused on single-user systems, while antenna selection techniques for multiuser systems have not been well studied. In this paper, we propose an antenna selection technique for MUSM in the context of unitary precoding, and extend the single-user selection in [43] to the multiuser scenario. Two selection criterion are proposed, which minimize the SER and maximize the sum capacity, respectively. Even though antenna selection is suboptimal compared with eigenmode selection, simulation results will show that a large portion of the diversity is still obtainable with this low-cost option.

The rest of the paper is organized as follows. In Section II the channel and system model of the MUSM system are introduced. In Section III, the unitary space-time precoder design for interference avoidance is proposed, and in Section IV the eigenmode selection technique for SER optimization with various linear and non-linear receivers is proposed, and performance analysis of the SER and sum capacity is provided. In Section V, the antenna selection criterion for multiuser MIMO is discussed and two selection algorithms are proposed. Numerical results in Section VI demonstrate the performance improvements that can be achieved with the proposed techniques. Conclusions are given in Section VII.

II. Preliminaries and System Model

In this section we describe the notation that will be used throughout this paper. Then we discuss the narrow-band channel model and the multi-user precoding signal model under consideration. All vectors and matrices are in boldface, with matrices capitalized.

A. Notation

- Let $\mathbf{\Phi}$ denote a complex matrix, and $\mathbf{\Phi}^T$, $\mathbf{\Phi}^H$ and $\mathbf{\Phi}^\dagger$ denote the transpose, conjugate transpose and Moore-Penrose pseudo-inverse of $\mathbf{\Phi}$, respectively.
- $\mathbf{\Phi}^{(i,j)}$ denotes the $(i,j)^{th}$ element of matrix $\mathbf{\Phi}$.
- $W_{\mathbf{\Phi}}$ denotes the vector space spanned by the columns of $\mathbf{\Phi}$ and $W_{\mathbf{\Phi}}^{\perp}$ denotes the complementary subspace of $W_{\mathbf{\Phi}}$.
- vec $(\mathbf{\Phi})$ denotes the vector produced by stacking the columns of $\mathbf{\Phi}$ on top of each other.
- diag $\{\phi_1, \phi_2, ..., \phi_n\}$ denotes a $n \times n$ diagonal matrix with diag $\{\phi_1, \phi_2, ..., \phi_n\}^{(i,i)} = \phi_i$.
- $\mathcal{E}_s$ denotes expectation with respect to random variable s.
- The trace of a $m \times m$ square matrix $\mathbf{\Phi}$ is expressed as tr $(\mathbf{\Phi}) = \sum_{i=1}^{m} \mathbf{\Phi}^{(i,i)}$.
- The Frobenius norm of a $m \times n$ matrix $\mathbf{\Phi}$ is $\|\mathbf{\Phi}\|_F^2 = \text{tr}\left(\mathbf{\Phi}\mathbf{\Phi}^H\right) = \sum_{i=1}^{r} |\lambda_i(\mathbf{\Phi})|^2$, where $r =$ rank$(\mathbf{\Phi}) \leq \min(m,n)$ and $\{\lambda_i(\mathbf{\Phi})\}_{i=1}^{r}$ are the singular values of $\mathbf{\Phi}$.
- The singular values $\{\lambda_i(\mathbf{\Phi})\}_{i=1}^{n}$ are non-negative for arbitrary complex matrix $\mathbf{\Phi}$, as shown in [52].
- $\mathbb{U}(n,k)$ is the collection of $n \times k$ complex matrices with unit-norm orthogonal columns, which is commonly known as the Stiefel manifold.

B. Channel Model

Consider the downlink transmission of a point-to-multipoint (PMP) wireless link as illustrated in Fig. 1 with M_T' transmit antennas, M_T RF chains at the BTS, and K mobile users where the k^{th} user has $M_{R,k}$ receive antennas, $k = 1, 2, \ldots, K$. A narrow-band flat-fading channel is assumed, which is satisfied if orthogonal division multiplexing (OFDM) is employed, as is widely expected in future MIMO systems. The channel transfer matrix from the BTS to the k^{th} mobile station (MS) is given by a complex matrix $\mathbf{H}_k \in \mathbb{C}^{M_{R,k} \times M_T}$, where $\mathbf{H}_k^{(i,j)}$ denotes the channel fading coefficient from the j^{th} transmit antenna to the i^{th} receive antenna of user k. We assume that both the BTS and MSs experience sufficient local scattering, thus the entries of $\mathbf{H}_k$ are samples of an i.i.d. (independent identically distributed) zero-mean complex Gaussian process with distribution $\mathcal{CN}(0,1)$. Channel degeneracy due to keyhole channel [58], or extreme correlations are not considered in this paper, and $\mathbf{H}_k$ has full rank (i.e. rank $(\mathbf{H}_k) = \min(M_{R,k}, M_T)$) with probability one. In addition, we also assume that the channels $\{\mathbf{H}_k\}_{k=1}^{K}$ experienced by different MSs are independent, and the composite channel matrix $\mathbf{H} = \begin{pmatrix} \mathbf{H}_1^H & \mathbf{H}_2^H & \cdots & \mathbf{H}_K^H \end{pmatrix}^H$ has full rank.

C. Signal Model

The BTS broadcasts data to all K users simultaneously over the same frequency band. The data from the k^{th} user is demultiplexed into $N_k \leq M_{R,k}$ data substreams, where $M_{R,k}$ upper bounds the maximum number of substreams that can be detected with a linear receiver. At a discrete time instant (we drop the temporal index for simplicity), the spatial multiplexer of the k^{th} data branch generates a N_k-dimensional vector symbol $\mathbf{x}_k = [x_{k,1}, x_{k,2}, \cdots, x_{k,N_k}]^T$, where symbols $x_{k,i}(k = 1, \ldots, K; i = 1, \ldots, N_k)$ are chosen from the same constellation set $\mathcal{S}$. For convenience we assume no error correction coding and a uniform allocation of power across the substreams for each user, i.e. $\mathbf{R}_{\mathbf{x}_k} = \mathcal{E}_{\mathbf{x}}\{\mathbf{x}_k \mathbf{x}_k^H\} = \frac{E_{s,k}}{N_k}\mathbf{I}$, where E_s is the sum power, $E_{s,k} = \frac{N_k}{\sum_{j=1}^{K} N_j} E_s$ is the power allocated to the k^{th} user. As will be shown in the next section, the proposed precoder decomposes the multiuser MIMO channel into multiple parallel single-user MIMO channels, therefore a separate power allocation/bit loading module can be concatenated to the proposed precoder as an outer block for each user. Since the main objective of this paper is to demonstrate the precoder's interference avoidance and diversity enhancing capability, a simple uniform power allocation model is used for brevity. Adaptive power allocation will be addressed Section IV.

At the BTS, the symbol vector for the k^{th} user is multiplied by a $M_T \times N_k$ precoding matrix $\mathbf{T}_k$ and summed with the precoded signals from the other users to produce the composite transmitted vector $\sum_{k=1}^{K} \mathbf{T}_k \mathbf{x}_k$. Each precoding matrix in $\{\mathbf{T}_k\}_{k=1}^{K}$ are chosen from the Stiefel manifold $\mathbb{U}\left(M_T, N_k\right)$. This implies that $\mathbf{T}_k^H \mathbf{T}_k = \mathbf{I}_{N_k}, \forall k$, i.e., $\mathbf{T}_k$ has orthonormal columns, which was also used in [31][33]. The unitary property forces the power per stream to be a constant thus does not alter the uniform power allocation strategy. As discussed above, adaptive power allocation can be achieved by concatenating a power adaptation module to our proposed precoder. In that case, the unitary constraint is generalized to the sum power constraint in [18] and [22], and is discussed in Section IV.

Neglecting symbol timing errors and frequency offsets, the $M_{R,k}$-dimensional received signal $\mathbf{r}_k$ at the k^{th} terminal is a superposition of the K signal branches distorted by channel fading plus additive white Gaussian noise (AWGN)

$$\mathbf{r}_k = \mathbf{H}_k \mathbf{T}_k \mathbf{x}_k + \mathbf{H}_k \sum_{j=1, j\neq k}^{K} \mathbf{T}_j \mathbf{x}_j + \mathbf{n}_k = \mathbf{H}_k \mathbf{T}_k \mathbf{x}_k + \mathbf{z}_k + \mathbf{n}_k. \quad (1)$$

The AWGN noise on the k^{th} user's receive antenna array is given by $\mathbf{n}_k$, which follows the complex i.i.d. Gaussian distribution of $\mathcal{CN}(0, N_o\mathbf{I})$. The CCI component on the k^{th} user is represented as $\mathbf{z}_k$.

Throughout this paper, it is assumed $\left\{\mathbf{H}_k\right\}_{k=1}^{K}$ is perfectly known at the transmitter to design the precoding matrices and perform antenna selection. It is assumed that each receiver only has knowledge of its own channel. The assumption of perfect CSI has been widely used in many existing literature in

MIMO precoding [17]-[22] and multiuser MIMO system [8]-[14],[29]-[36]. It can be fulfilled by channel estimation in time-division-duplex (TDD) systems (e.g. IEEE 802.16, [47]), or feedback in frequency-division-duplex (FDD) systems.

The precision of channel estimation/feedback at the BTS plays an important role in the accuracy of the precoding matrices, and hence the error performance and achievable sum data rate. Factors impacting the accuracy of channel knowledge include Doppler shift, the number of transmit/receive antennas, feedback delay, etc. In TDD systems, channel estimation at the BTS has to be carried out more frequently if the Doppler spread is significant and the channel varies at a higher speed. A larger transmit/receive antenna array also necessitates more precise channel estimation. Similar conclusions can be made for FDD systems. In [48], the performance analysis of multiuser space-time coded MIMO system with unitary precoding was provided, where a SER and BER upper bound were derived. Similar analysis for multiuser spatial multiplexing system is an interesting topic for future research.

III. Transmit Precoding for Interference Cancellation

The goal of multiuser MIMO downlink transmission is to achieve high data rates by using spatial division multiple access (SDMA) to serve multiple users at the same time. Since the data to multiple users are simultaneously transmitted and the spatial channels are not exactly orthogonal, CCI constitutes the major performance impairment. Recent information theoretic results reveal that when the interference is non-causally known at the transmitter, DPC is able to achieve the maximum sum-rate capacity of the MIMO-BC channel, at the expense of a very complicated binning strategy which has to be realized using nested codes [49]. Tomlinson-Harashima precoding, which was originally developed for intersymbol interference pre-cancellation, has been shown able to achieve capacity close to DPC, but it suffers from several shaping and power losses [50]. A combined beamforming and coding technique for known interference to achieve sum data rate of MIMO-BC channel was proposed in [51]. Several transmitter-based CCI pre-cancellation techniques have also been proposed recently, e.g., the BD [31], the JCD [32], and the transmitter pre-processing [33]. The basic idea behind these techniques is to use a large number of transmit antennas to orthogonalize the signal, followed by water-filling to optimize the capacity. In the following section, we briefly review the BD approach [31], and propose the first step of our precoder design that is used as the baseline for the eigenmode selection. The objective of this step, similar to [31]-[33], is to diagonalize the multiuser channel and eliminate CCI. This is implemented with standard QR decomposition that has lower complexity and numerically more stable than previously proposed algorithms, which will be discussed accordingly.

A. BD for Interference Cancellation

The BD approach seeks to find the precoding matrices $\{\mathbf{T}_k\}_{k=1}^{K}$ such that $\mathbf{H}_k\mathbf{T}_j = \mathbf{0}, \forall j \neq k$. For simplicity, denote the congregate interfering channel transfer matrix (CICTM) of user k as $\bar{\mathbf{H}}_k = \begin{pmatrix} \mathbf{H}_1^H & \cdots & \mathbf{H}_{k-1}^H & \mathbf{H}_{k+1}^H & \cdots & \mathbf{H}_K^H \end{pmatrix}^H$. The zero-interference constraint is re-expressed as

$$\bar{\mathbf{H}}_k\mathbf{T}_k = \mathbf{0}, \quad \forall k = 1, \ldots, K. \tag{2}$$

Denote the SVD of $\bar{\mathbf{H}}_k$ as $\bar{\mathbf{H}}_k = \bar{\mathbf{U}}_k\begin{pmatrix} \bar{\mathbf{\Sigma}}_k & \mathbf{0} \end{pmatrix}\begin{pmatrix} \bar{\mathbf{V}}_k^1 & \bar{\mathbf{V}}_k^0 \end{pmatrix}^H$, where $\bar{\mathbf{V}}_k = \begin{pmatrix} \bar{\mathbf{V}}_k^1 & \bar{\mathbf{V}}_k^0 \end{pmatrix} \in \mathbb{U}(M_T, M_T)$. Matrix $\bar{\mathbf{\Sigma}}_k$ is the $\bar{r}_k \times \bar{r}_k$ diagonal matrix containing the $\bar{r}_k$ non-zero singular values of $\bar{\mathbf{H}}_k$, and $\bar{\mathbf{V}}_k^0$ contains the singular vectors corresponding to the zero singular values. Since the columns of $\bar{\mathbf{V}}_k^0$ span the null space of $\bar{\mathbf{H}}_k$, constructing $\mathbf{T}_k$ with N_k columns of $\bar{\mathbf{V}}_k^0$ will automatically satisfy the zero-interference constraints. Assuming that the matrix channel is full rank, which occurs with enough scattering with probability one, N_k such singular vectors exist provided that the transmit array size satisfies $M_T \geq \sum_{j=1,\ j\neq k}^{K} M_{R,j} + N_k$. In case the channels are not full rank, the transmit array constraint will be in terms of channel ranks and is in fact less restrictive. Since this occurs much less frequently we do not elaborate on this condition here. For future reference note that such precoding matrices are not unique, because right multiplication by an arbitrary unitary matrix will also satisfy (2).

B. Multiuser Downlink Precoder

The interference cancellation step of our proposed precoder is implemented by enforcing the orthogonality in the matrix channel of each user, i.e., by projecting the interfering data branches onto the complementary subspace spanned by the desired users' channel $\mathbf{H}_k$. This projection method has also been followed in [31][33] with SVD approach. In this paper we propose to use standard QR decomposition to allow for a quicker solution for interference cancellation.

Note that for a $n \times m$ matrix $\mathbf{\Phi}$ where $n \leq m$, we have $\mathbf{\Phi}\left(\mathbf{I} - \mathbf{\Phi}^\dagger\mathbf{\Phi}\right) = \mathbf{0}$. Hence, we can simply construct $\mathbf{T}_k$ as a linear combination of the column basis vectors of $\left(\mathbf{I} - \bar{\mathbf{H}}_k^\dagger\bar{\mathbf{H}}_k\right)$, which can be obtained by the Gram-Schmidt Orthogonalization (GSO), or the standard QR decomposition which has several numerically stable solutions. Write the QR decomposition of $\mathbf{I} - \bar{\mathbf{H}}_k^\dagger\bar{\mathbf{H}}_k$ as

$$\mathbf{I} - \bar{\mathbf{H}}_k^\dagger\bar{\mathbf{H}}_k = \mathbf{Q}_k\mathbf{R}_k = \begin{pmatrix} \mathbf{Q}_k & \bar{\mathbf{Q}}_k \end{pmatrix}\begin{pmatrix} \mathbf{R}_k \\ \mathbf{0} \end{pmatrix}, \tag{3}$$

where $\mathbf{Q}_k \in \mathbb{U}\left(M_T, M_T - \sum_{j=1,j\neq k}^{K} M_{R,j}\right)$ contains the basis of the complimentary subspace of $W_{\bar{\mathbf{H}}_k^T}^{\perp}$. $\mathbf{R}_k$ is an upper triangular matrix of dimension $\left(M_T - \sum_{j=1,j\neq k}^{K} M_{R,j}\right) \times M_T$. To reflect the fact that right multiplication of unitary matrices preserve both the orthogonalization and unitary properties, write the precoder as

$$\mathbf{T}_k = \mathbf{Q}_k \mathbf{D}_k, \tag{4}$$

where $\mathbf{D}_k \in \mathbb{U}\left(M_T - \sum_{j=1,j\neq k}^{K} M_{R,j}, N_k\right), \ \forall k = 1, \ldots, K$ are unitary eigenmode selection matrices.

When (2) is satisfied, the interference at each mobile receiver is perfectly avoided. Substituting (4) into (1), the received signal at the k^{th} user is obtained as

$$\mathbf{r}_k = \mathbf{H}_k \mathbf{T}_k \mathbf{x}_k + \mathbf{H}_k \sum_{j=1,j\neq k}^{K} \mathbf{T}_j \mathbf{x}_j + \mathbf{n}_k = \tilde{\mathbf{H}}_k \mathbf{x}_k + \mathbf{n}_k, \tag{5}$$

where the $M_{R,k} \times N_k$ matrix $\tilde{\mathbf{H}}_k = \mathbf{H}_k \mathbf{T}_k$ is the equivalent channel transfer matrix to terminal k. Note that the multiuser MIMO channel is decoupled into K parallel non-interfering single-user MIMO links. Each user operates in its corresponding single-user link independently without affecting other links.

C. Complexity Analysis

The complexity of the previously proposed precoder design [31] is based on the SVD of the matrix channel $\bar{\mathbf{H}}_k = \begin{pmatrix} \mathbf{H}_1^H & \cdots & \mathbf{H}_{k-1}^H & \mathbf{H}_{k+1}^H & \cdots & \mathbf{H}_k^H \end{pmatrix}^H$, which has a complexity of $\mathcal{O}\left(\max\left(p^2 q, pq^2, q^3\right)\right)$ (see pp. 254, [52]), where $p = M_T$ and $q = \sum_{j=1,j\neq k}^{K} M_{R,j}$. To completely cancel the interference, the system must satisfy $M_T \geq \max_k \left(\sum_{j=1,j\neq k}^{K} M_{R,j}\right)$ (see [31]), hence the computational complexity turns out to be $\mathcal{O}\left(M_T^2 \max_k \left(\sum_{j=1,j\neq k}^{K} M_{R,j}\right)\right)$.

The complexity of the proposed precoder is mainly determined by the Moore-Penrose pseudo-inverse $\bar{\mathbf{H}}_k^{\dagger} = \bar{\mathbf{H}}_k \left(\bar{\mathbf{H}}_k \bar{\mathbf{H}}_k^H\right)^{-1}$, and the QR decomposition of $\mathbf{I} - \bar{\mathbf{H}}_k^{\dagger} \bar{\mathbf{H}}_k$. The complexity of the most efficient pseudo-inverse operation follows $\mathcal{O}\left(\left(\max_k \sum_{j=1,j\neq k}^{K} M_{R,j}\right)^{\omega}\right)$ where $2 < \omega < 3$ [53]. The complexity of QR decomposition of $\mathbf{I} - \bar{\mathbf{H}}_k^{\dagger} \bar{\mathbf{H}}_k$ is lower than $\mathcal{O}\left(\left(\max_k \sum_{j=1,j\neq k}^{K} M_{R,j}\right)^3\right)$ by a factor of 1.3-1.4 [54]. Since $M_T \geq \max_k \left(\sum_{j=1,j\neq k}^{K} M_{R,j}\right)$, the proposed algorithm has slightly lower computational complexity than the SVD-based approach. In addition, the QR-based method are generally much more stable and accurate numerically [52][65].

The approach in [32] follows an iterative SVD operation of a smaller size interfering matrix, so the computational complexity cannot be directly compared to our approach.

IV. Transmit Precoding with Eigenmode Selection

The previous section reviewed the BD approach [31], which is a transmitter-based interference cancellation technique for multiuser spatial multiplexing, and then proposed a different multiuser precoder design based on QR decomposition. After the interference pre-cancellation at the BTS, each user operates in a single-user MIMO link. In this section, we show how to improve the system error performance by selecting the proper spatial eigenmodes when extra transmit antennas are available. Because the SER performance of single user spatial multiplexing with linear receivers is a function of the effective SNR of each substream after precoding, in this paper, the optimization objective is to maximize the post-processing SNR, which is equivalent to minimizing the SER.

A. SNR Lower Bound

Previous work has shown that the post-processing SNR of single-user spatial multiplexing systems with linear receivers is lower bounded by a monotonically increasing function of the minimum singular value of the equivalent channel [43][55]. For mobile user with zero-forcing (ZF) receiver, the minimum post-processing SNR per stream after decoding is bounded by

$$SNR_{min} \geq \frac{E_{s,k}}{N_k N_o \text{max}_i \left\{ \left(\mathbf{T}_k^H \mathbf{H}_k^H \mathbf{H}_k \mathbf{T}_k \right)^{-1} \right\}^{(i,i)}}, \; i = 1, \ldots N_k. \tag{6}$$

For users with MMSE receiver, the minimum SNR is lower bounded by

$$SNR_{min} \geq \frac{E_{s,k}}{N_k N_o} \left(\lambda_{min}(\mathbf{H}_k \mathbf{T}_k) + \sqrt{\frac{N_k N_o}{E_{s,k}}} \right)^2 - 1, \quad i = 1, \ldots N_k. \tag{7}$$

A similar bound exists for the non-linear successive interference cancellation (SIC) based receivers, e.g. V-BLAST [3]. It decodes the substreams sequentially, where the receiver decodes earlier substreams, subtracts its interference and then decodes the later substreams. The performance of V-BLAST receiver is mainly dependent on the first substream, which has the lowest diversity gain. Narasimhan derived a SNR lower bound for the V-BLAST receiver in [55], which has the same form in (6).

As a result, we can effectively reduce the maximum SER by increasing $\lambda_{min}(\mathbf{H}_k \mathbf{T}_k)$, for a variety of linear and non-linear mobile receivers.

B. Eigenmode Selection

Given that the squared minimum singular value of the equivalent channel $\mathbf{H}_k \mathbf{T}_k$ lower bounds the per stream SNR, the objective of eigenmode optimization is to choose the optimum precoding matrix $\mathbf{T}_{k,opt}$ that maximizes the minimum singular value $\lambda_{min}(\mathbf{H}_k \mathbf{T}_k)$.

Problem Statement:

Based on the bound in (6) and (7), find a precoding matrix

$$\mathbf{T}_{k,opt} = \arg \max_{\mathbf{T}_k \in \mathbb{U}(M_T, N_k)} \lambda_{min} \left(\mathbf{H}_k \mathbf{T}_k \right) \tag{8}$$

subject to the unitary constraint

$$\mathbf{T}_k \in \mathbb{U}(M_T, N_k), \quad k = 1, 2, \ldots, K \tag{9}$$

and the zero-interference constraint

$$\mathbf{H}_i \mathbf{T}_k = \mathbf{0}, \; i, k = 1, 2, \ldots, K, \; i \neq k. \tag{10}$$

Since $\mathbf{T}_k = \mathbf{Q}_k \mathbf{D}_k$ and $\mathbf{Q}_k$ is fixed, the above optimization problem is equivalent to selecting $\mathbf{D}_{k,opt}$ to maximize $\lambda_{min} \left(\mathbf{H}_k \mathbf{Q}_k \mathbf{D}_k \right)$. This problem relates to how the right multiplication of a tall matrix with unit-norm orthogonal columns will affect the minimum singular value of matrix, where the following theorem will prove useful:

Theorem 1: (*Horn & Johnson* [65]) Let $\mathbf{A}_n$ be $n \times n$ Hermitian matrix, and $r \leq n$ be a given integer. Let $\mathbf{U} = [\mathbf{u}_1 \cdots \mathbf{u}_r] \in \mathbb{U}(n, r)$ and $\mathbf{B}_r = \mathbf{U}^H \mathbf{A}_n \mathbf{U} \in \mathbb{C}^{r \times r}$. Arranging the eigenvalues of $\mathbf{A}_n$ and $\mathbf{B}_r$ in decreasing order, then we have

$$\mu_k(\mathbf{A}_n) \geq \mu_k(\mathbf{B}_r) \geq \mu_{k+n-r}(\mathbf{A}_n), \quad k = 1, 2, \ldots, r. \tag{11}$$

The detailed proof is given in [65]. An extension of this theorem is derived in the following corollary.

Corollary 1: Let $\mathbf{\Phi}$ be a $n \times m$ matrix where $n \leq m$, and $r \leq n$ be a given integer. Let $\mathbf{U} = [\mathbf{u}_1 \cdots \mathbf{u}_r] \in \mathbb{U}\left(m, r\right)$ be arbitrary unitary matrix and $\tilde{\mathbf{\Phi}} = \mathbf{\Phi} \mathbf{U} \in \mathbb{C}^{n \times r}$. Arranging the singular values of $\mathbf{\Phi}$ and $\tilde{\mathbf{\Phi}}$ in decreasing order yields

$$\lambda_k(\mathbf{\Phi}) \geq \lambda_k(\tilde{\mathbf{\Phi}}) \geq \lambda_{k+n-r}(\mathbf{\Phi}), \quad k = 1, 2, \ldots, r. \tag{12}$$

Proof: The corollary is proven by denoting $\mathbf{A} = \mathbf{\Phi}^H \mathbf{\Phi}$ and substituting $\mu_k\left(\mathbf{A}\right) = \lambda_k^2\left(\mathbf{\Phi}\right)$, $\mu_k\left(\mathbf{U}^H \mathbf{A} \mathbf{U}\right) = \lambda_k^2\left(\mathbf{\Phi} \mathbf{U}\right)$ into the above theorem, recalling the non-negativity of singular values. $\square$

The benefits of choosing the optimal $\mathbf{D}_{k,opt}$ can be explained as follows. If the system uses more transmit antennas than required for interference cancellation, the equivalent channel after precoding generates $r_k = \text{rank}\left(\mathbf{H}_k \mathbf{Q}_k\right)$ spatial eigenmodes, more than the transmitted data substreams, i.e. $r_k > N_k$. Arranging the eigenmodes with respect to their gains in decreasing order, the matrix $\mathbf{D}_k$ determines which set of eigenmodes are selected and how the power/substreams are allocated. Because the SER performance is upper bounded by $\lambda_{min}\left(\mathbf{H}_k \mathbf{Q}_k \mathbf{D}_k\right)$, a good strategy is to select the N_k eigenmodes with the largest gains. Mathematically, as $\lambda_{min}\left(\mathbf{H}_k \mathbf{Q}_k \mathbf{D}_k\right)$ is a variable and upper bounded by the N_k^{th} largest singular values of $\mathbf{H}_k\mathbf{Q}_k$, selecting the first N_k eigenmodes automatically achieves this upper bound for $\lambda_{min}\left(\mathbf{H}_k\mathbf{Q}_k\mathbf{D}_k\right)$. It is interesting to note that the selection of the optimum set of eigenmodes was also developed in [31], whereas for capacity optimization goal. The number of selected eigenmodes in [31] is a variable determined by water-filling, while we fix N_k and consider the SER optimization.

Accordingly, the benefits of eigenmode selection depend on the number of spatial eigenmodes, which is a function of the system antenna configuration.

Lemma 1: The k^{th} user has

$$r_k = \text{rank}\left(\mathbf{H}_k\mathbf{Q}_k\right) = \min\left(M_{R,k},\ M_T - \sum_{i=1,i\neq k}^{K} M_{R,i}\right) \tag{13}$$

spatial eigenmodes to transmit its N_k data substreams.

Proof: $\bar{\mathbf{H}}_k$ is a matrix of size $\sum_{i=1,i\neq k}^{K} M_{R,i} \times M_T$, and $(\mathbf{I} - \bar{\mathbf{H}}_k^{\dagger}\bar{\mathbf{H}}_k)$ has QR decomposition as $\mathbf{Q}_k\mathbf{R}_k$. Since $(\mathbf{I} - \bar{\mathbf{H}}_k^{\dagger}\bar{\mathbf{H}}_k)$ is a projection matrix with rank $(M_T - \sum_{i=1,i\neq k}^{K} M_{R,i})$, the rank of $\mathbf{Q}_k$ is $(M_T - \sum_{i=1,i\neq k}^{K} M_{R,i})$ as well. Also, because $\mathbf{Q}_k$ is a function of $\bar{\mathbf{H}}_k$ and independent of $\mathbf{H}_k$, the rank of $\mathbf{H}_k\mathbf{Q}_k$ satisfies the condition specified above. □

Lemma 2: To improve the SER performance by using eigenmode selection, the k^{th} user's receive antenna number should satisfy

$$M_{R,k} > N_k \tag{14}$$

$$M_T - \textstyle\sum_{i=1,i\neq k}^{K} M_{R,i} > N_k. \tag{15}$$

Proof: The proof of this lemma is straightforward, as this specification ensures there are more eigenmodes than data streams. □

In this paper, we limit the k^{th} user transmission rate such that the number of data substreams N_k is not more than its receive antenna number $M_{R,k}$, i.e., $N_k \leq M_{R,k}$. Otherwise the MIMO system will be rank-deficient and the performance with linear receivers is significantly degraded. To enable eigenmode selection, the k^{th} user must have at least one additional antenna to satisfy (14), and the BTS must have enough antennas to satisfy (15).

C. Unitary Precoder for Optimal Eigenmode Selection

In the previous section, we presented the eigenmode selection technique and specified a necessary condition to achieve the benefits of eigenmode selection. In particular, when $M_{R,k} > N_k$ and $M_T - \sum_{j=1,j\neq k}^{K} M_{R,j} > N_k$, the k^{th} user has sufficient spatial eigenmodes to perform eigenmode selection. In this case the best strategy is to transmit the data over a set of N_k eigenmodes with the largest channel gains.

Denote the SVD of $\mathbf{H}_k\mathbf{Q}_k$ as

$$\mathbf{H}_k\mathbf{Q}_k = \mathbf{U}_k\mathbf{\Sigma}_k\mathbf{V}_k^H \tag{16}$$

where $\mathbf{U}_k \in \mathbb{U}(M_{R,k}, r_k)$, $\mathbf{V}_k \in \mathbb{U}(M_T - \sum_{j=1,j\neq k}^{K} M_{R,j}, r_k)$, rank r_k is given in (13), and $\mathbf{\Sigma}_k = \text{diag}\{\lambda_{k,1}, \lambda_{k,2}, \ldots, \lambda_{k,r_k}\}$ is the diagonal matrix consisting of all the singular values in descending order. As discussed in the previous section, the design objective is to maximize the minimum singular value of the equivalent channel, i.e., $\lambda_{min}(\mathbf{H}_k\mathbf{Q}_k\mathbf{D}_k)$. According to the theorem and corollary in the last section, $\lambda_{min}(\mathbf{H}_k\mathbf{Q}_k\mathbf{D}_k) \leq \lambda_{k,N_k}$ and the equality holds when

$$\mathbf{D}_{k,opt} \quad = \quad \arg \max_{\mathbf{D}_k \in \mathbb{U}(r_k, N_k)} \lambda_{min}(\mathbf{H}_k\mathbf{Q}_k\mathbf{D}_k) = \mathbf{V}_{k,[1:N_k]} \tag{17}$$

where $\mathbf{V}_{k,[1:N_k]}$ denotes the first N_k columns of $\mathbf{V}_k$. An optimal precoding matrix is given

$$\mathbf{T}_{k,opt} \quad = \quad \arg \max_{\mathbf{T}_k \in \mathbb{U}(M_T, N_k)} \lambda_{min}(\mathbf{H}_k\mathbf{T}_k) = \mathbf{Q}_k\mathbf{V}_{k,[1:N_k]}. \tag{18}$$

Recall that the precoding matrix is not unique since performance is invariant to right multiplication by a unitary matrix. Note that if $M_{R,k} = N_k$ or $M_T - \sum_{j=1,j\neq k}^{K} M_{R,j} = N_k$, it is infeasible to perform eigenmode selection for the k^{th} user, and in such a case the precoding matrix $\mathbf{T}_k$ is chosen as $\mathbf{Q}_k$.

The unitary characteristics of the downlink precoder allow for a straightforward analysis on the link-level SER.

Lemma 3: For a K-user spatial multiplexing system with M_T transmit antennas and $M_{R,k}$ antennas at the k^{th} user, if the MIMO channel $\mathbf{H}_k$ follows i.i.d. complex Gaussian distribution $\mathcal{CN}(0,1)$, then the equivalent MIMO channel $\tilde{\mathbf{H}}_k$ after unitary precoding is also i.i.d. Gaussian distributed $\mathcal{CN}(0,1)$, if eigenmode selection is not performed.

Proof: see the Appendix.

Now we compare the SER performance of the eigenmode selection and work in [31]. Note that for the same number of eigen channels, water-filling is suboptimal in terms of SER than equal power allocation, because water-filling allocates less power to eigen channel with lower gain, and the SER is convex of SNR. Therefore if water-filling in [31] selects $n \geq N_k$ eigen channels, the eigenmode selection outperforms in SER. If $n < N_k$, it is unknown which scheme is better because the water-filling does not provide a closed-form power distribution over the substreams.

Also note that adaptively distributing transmit power to the eigenmodes can further reduce the SER. This improvement, however, is on top of and irrelevant to the proposed eigenmode selection.

TABLE I

SUMMARY OF NOTATIONS

$\mathbf{H}$	$\begin{pmatrix} \mathbf{H}_1^H & \mathbf{H}_2^H & \cdots & \mathbf{H}_K^H \end{pmatrix}^H$
$\bar{\mathbf{H}}_k$	$\begin{pmatrix} \mathbf{H}_1^H & \cdots & \mathbf{H}_{k-1}^H & \mathbf{H}_{k+1}^H & \cdots & \mathbf{H}_K^H \end{pmatrix}^H$
$\tilde{\mathbf{H}}_k$	$\tilde{\mathbf{H}}_k = \mathbf{H}_k\mathbf{T}_k = \mathbf{H}_k\mathbf{Q}_k\mathbf{D}_k$
$\mathbf{Q}_k$	QR decomposition result of $\mathbf{I} - \bar{\mathbf{H}}_k^\dagger\bar{\mathbf{H}}_k$
$\mathbf{D}_{k,opt}$	$\mathbf{V}_{k,[1:N_k]}$
$\mathbf{V}_k$	right singular vector matrix of $\mathbf{H}_k\mathbf{Q}_k$

Because each user is effectively utilizing a single-user MIMO link after the precoding, techniques for single-user MIMO are applicable once the orthogonality is established. The application of eigenmode selection is different from the single user scenario in a number of ways. First, the necessary condition of eigenmode selection for the k^{th} user depends not only on the its own receive antenna and the base station antenna number, but also on the antenna settings of other co-channel users. Second, the effective channel per user after precoding also depends on the channel characteristics of other users, although mutual interference no longer exists. Finally, by using a very limited number of extra antennas (e.g., one additional antenna) we are able to achieve dramatic diversity improvement for *all* the user simultaneously, which is economically attractive in practical systems.

D. Sum-rate Capacity Analysis

Assuming that the transmitted data streams are independently encoded and independently decoded, the sum-rate capacity of the multiuser system is simply the summation of each user's individual channel capacity. Under the uniform power allocation, the sum-rate capacity is given by

$$\begin{aligned} C &= \sum_{k=1}^{K} \log_2 \det\left(\mathbf{I}_{M_{R,k}} + \frac{E_{s,k}}{N_k N_o}\tilde{\mathbf{H}}_k^H\tilde{\mathbf{H}}_k\right) \\ &= \sum_{k=1}^{K}\sum_{i=1}^{N_k} \log_2\left(1 + \frac{E_{s,k}}{N_k N_o}|\lambda_{k,i}|^2\right). \end{aligned} \tag{19}$$

Intuitively, each user creates a group of non-interfering eigenmodes and the system sum capacity is the summation of the capacity of every eigenmode (19).

For the results in (19), the channel knowledge is not used to perform adaptive power allocation on different eigenmodes. As pointed out in Section II, an adaptive power allocation module can be concatenated to the multiuser precoder to adaptively allocate transmit power to multiple spatial eigenmodes, according to their respective eigenmode gain. Similar to the single-user MIMO scenario, the optimum power allocation for capacity optimization follows the well-known water-filling algorithm, where the only difference is that the allocation is conducted in both spatial dimension and user-wise. The sum-rate capacity is

$$\begin{aligned} C &= \max_{\mathcal{E}(\mathbf{x}_k\mathbf{x}_k^H)>0,\ \sum_k \mathcal{E}(\mathbf{x}_k\mathbf{x}_k^H)\le E_s} I\left(\mathbf{x}_1,\ldots,\mathbf{x}_K;\mathbf{r}_1,\ldots,\mathbf{r}_K|\tilde{\mathbf{H}}_1,\ldots,\tilde{\mathbf{H}}_K\right) \\ &= \sum_{k=1}^{K}\sum_{i=1}^{N_k}\log_2\left(1+\frac{1}{N_k N_o}\left(\gamma-\frac{N_o}{|\lambda_{k,i}|^2}\right)_+|\lambda_{k,i}|^2\right), \end{aligned} \tag{20}$$

where

$$(x)_+ = \begin{cases} 0 & x\le 0 \\ x & x>0 \end{cases} \tag{21}$$

and γ is the threshold determined by the sum power constraint

$$E_s = \sum_{k=1}^{K}\sum_{i=1}^{N_k}\left(\gamma_k-\frac{N_o}{|\lambda_{k,i}|^2}\right)_+. \tag{22}$$

In terms of sum-capacity, the eigenmode selection chooses the best N_k eigen channels out of the total r_k eigen channels, while the number of used eigen channels in approaches in [31] is a variable determined by the water-filling. Therefore the eigenmode selection is suboptimal in terms of sum-capacity. This suboptimality, however, results from our deliberate restriction on the number of streams per user (N_k). If water-filling happens to choose the same number of eigenmodes for conventional and our schemes, then our approach has exactly the same capacity as in [31]-[33].

V. Transmit Antenna Selection

The previous section proposed eigenmode selection as an effective transmit selection diversity technique when there are sufficient antennas, at the transmitter side. One drawback of eigenmode selection is that it also requires M_T expensive RF chains to meet the channel rank requirement (13), which leads to a higher system cost. As an alternative, antenna selection can be used to provide transmit diversity at a relatively lower system cost with fewer RF chains, although naturally with some performance loss.

A. Single-user Antenna Selection

Antenna selection refers to choosing a subset of available antennas from the BTS antenna pool and switching them to the available RF units. Extensive research has been conducted on its application in a single-user MIMO system, at the transmitter or the receiver, using either instantaneous or statistical channel knowledge. In [39][40], the authors studied receive antenna selection for spatial multiplexing systems, and proposed the optimal and suboptimal selection algorithms to maximize channel capacity. Transmit antenna selection for link-level error performance optimization of spatial multiplexing system was studied by [43]. For space-time coded MIMO systems, Gore *et al.* studied various selection algorithms [44] with the objective function as the maximized post-processing SNR. A complete overview of MIMO antenna selection technique is provided in [45][46].

Even though only a subset of antennas are used, analysis and simulation show a very interesting result: antenna selection over M_T antennas can achieve the same diversity performance as a full system where *all* M_T' antennas are simultaneously used [40][45]. This implies that there may not be a large penalty for reducing the number of RF chains, as long as $M_T' > M_T$ antenna elements can be deployed.

B. Proposed Multiuser Antenna Selection

The system configuration is different in the context of antenna selection. Suppose there are only M_T RF chains which are exactly the minimum requirement for supporting multiuser downlink precoding, thus eigenmode selection is not feasible. Suppose there are $M_T' > M_T$ BTS antennas available, however, and for each transmission we switch a selected subset of M_T antennas to the RF chains and transmit over the "preferred" antennas. The selected antenna set is indexed by $p \in P$ where P is the available $\binom{M_T'}{M_T}$ sets. The channel matrices $\{\mathbf{H}_k\}_{k=1}^K$ will be indexed by the antenna set p, i.e., $\{\mathbf{H}_{k,p}\}_{k=1}^K$.

Antenna selection can be based on the optimization of SER or the channel capacity. Again, for reasons stated earlier, we first focus on the minimization of the maximum SER, which is an effective upper bound of the average SER. The equivalent channel matrix after unitary precoding depends on both the real channel $\mathbf{H}_{k,p}$ and the precoding matrices $\mathbf{T}_{k,p}$ which is a function of p. Recall (6)(7) and note that the maximum SER of user k is upper bounded by a non-decreasing function of $\lambda_{min}(\mathbf{H}_{k,p}\mathbf{T}_{k,p})$, therefore the maximum system SER is upper bounded by the user with the worst performance, which depends on the minimum of *all* users' minimum singular values. Therefore, one approach for antenna selection is to maximize the minimum of all users' singular values.

Selection Criterion 1 - *Maximum Minimum Singular Value (MMSV)*: For every subset of transmit antennas $p \in P$, compute $\tilde{\lambda}_{p,min} = \min_{k=1,\ldots,K} \lambda_{min}(\mathbf{H}_{k,p}\mathbf{T}_{k,p})$ corresponding to p. To optimize the SER performance, select the antenna set p that maximizes the minimum singular value $\tilde{\lambda}_{p,min}$

$$p_{opt} = \arg\max_{p \in P} \tilde{\lambda}_{p,min}. \tag{23}$$

Antenna selection can also be implemented by choosing the performance metric as the sum-rate capacity and selecting the optimum antenna set.

Selection Criterion 2 - *Maximum Sum-Rate Capacity (MSRC)*: For every subset of transmit antennas $p \in P$, compute the sum capacity without adaptive power allocation as given in (19), or with adaptive power control as given in (20). Select the antenna set p that maximizes sum capacity

$$p_{opt} = \arg\max_{p \in P} R. \tag{24}$$

Selection according to a capacity criterion identifies the optimum antenna subset with the largest sum-rate. This sum-rate is only achieved when there is no restriction on the complexity and length of the coding scheme. Due to the complexity, delay, and modulation constellation constraints in practical system, the actual achievable data rate needs to consider a SNR-gap in the sum rate expression in (19) and (20). Particularly, the achievable data rate is expressed as

$$C = \sum_{k=1}^{K} \sum_{i=1}^{N_k} \log_2 \left(1 + \frac{E_{s,k}}{\Gamma N_k N_o} |\lambda_{k,i}|^2 \right) \tag{25}$$

for uniform power allocation scheme and

$$C = \sum_{k=1}^{K} \sum_{i=1}^{N_k} \log_2 \left(1 + \frac{1}{\Gamma N_k N_o} \left(\gamma - \frac{N_o}{|\lambda_{k,i}|^2} \right)_+ |\lambda_{k,i}|^2 \right) \tag{26}$$

for the water-filling case. The SNR-gap Γ defines the gap between a practical coding and modulation scheme and the Shannon capacity.

C. *Antenna Selection vs. Eigenmode Selection*

Antenna selection and eigenmode selection are two diversity techniques to improve communication link quality by utilizing excess transmit antennas at the BTS. The major differences between these techniques lie in two aspects.

First, at any time, antenna selection involves transmitting over a subset of all BTS antennas, while the eigenmode selection involves transmitting over all BTS antennas. Antenna selection requires fewer RF chains than eigenmode selection, thus has lower cost. Although it naturally has suboptimal performance than eigenmode selection, simulation results in the next section show that it has the same diversity gain as a full system using *all* antennas, similar to the single-user MIMO case. Therefore it provides an effective and low-cost diversity technique for practical wireless systems.

Second, antenna selection has less stringent system configuration requirements than the eigenmode selection. One of the necessary conditions to perform eigenmode selection is that the k^{th} user has more receive antenna than its data substreams, i.e. $N_k < M_{R,k}$. Antenna selection, however, is still feasible even if this requirement is not met, as long as the BTS antenna number M'_T is sufficiently large.

Note that the MMSV and MSRC antenna selection algorithms are based on an exhaustive search over all the available antenna sets, which is a disadvantage relative to eigenmode selection. Selection algorithms with lower complexity are interesting topics for future research.

VI. Numerical Results

Monte Carlo simulation results are presented in this section to demonstrate the performance of the multiuser spatial multiplexing system with the two proposed selection diversity techniques. The first subsection demonstrates their performance improvement in terms of average SER and sum-rate capacity, in i.i.d. complex Gaussian channel and with perfect channel knowledge. Performance evaluation in i.i.d. complex Gaussian channel with channel estimation error, as well as in correlated MIMO channel, is also provided for comparison. 4-QAM modulation with Gray coding is used for all users' data streams, For each configuration, simulation is terminated after 10^6 independent channel realizations are simulated or 100 symbol errors are observed.

A. i.i.d. Gaussian Channel with Perfect Channel Knowledge

Fig. 2 compares the SER performance of a single and multiuser system where each user has three antennas and receives 2 data substreams. Three cases are studied: (a) single-user system with 2 BTS transmit antennas, (b) two-user system with 5 BTS transmit antennas, (c) two-user system with 6 BTS transmit antennas and eigenmode selection. The horizontal axis represents the average SNR per branch (user) per receive antennas. The vertical axis represents the SER averaged among all users. In case (a) and (b), the BTS antenna number is the minimum required to support spatial multiplexing and precoder design, so no eigenmode selection is performed. The multiuser system achieves the same per user performance with a single-user system, which obtains a diversity order of 2 with ZF receiver. By adding a single antenna to the BTS and utilizing eigenmode selection, however, a significant SNR reduction of 8 dB is achieved at SER=10^{-4} for the ZF receiver. Similarly, a SNR reduction of 5 dB is achieved for V-BLAST receiver. The asymptotic slope of SER curve, which is the definition of diversity gain, is larger than scenarios without eigenmode selection. Clearly, the eigenmode selection algorithm achieves a higher diversity order than simple spatial multiplexing and this improvement becomes more significant as more antennas are added at the BTS. It is also observed that by adaptively adjusting the transmit power of substreams, a further 4 dB SNR reduction is achieved at a SER of 10^{-4}.

The sum capacity improvement due to eigenmode selection (19) is shown in Fig. 3. Each user has 3 antennas and receives 2 data substreams. Without eigenmode selection, a capacity of 13 bps/Hz and 26 bps/Hz are achieved for the single-user and two-user systems at a SNR of 20 dB. By adding one extra BTS antenna and performing eigenmode selection, an additional capacity increase of 4 bps/Hz is observed for the two-user system. In addition, it is expected that even larger capacity improvement is achievable as more users are present, given that total transmit power increases correspondingly. The reason is that eigenmode selection is performed for all users, with the cost of only one extra transmit antenna.

The SER of antenna selection for a 2-user system with ZF receiver is given in Fig. 4. Each user has 2 receive antennas and receives 2 substreams. Eigenmode selection is not applicable because condition (14) is violated, so antenna selection is used as a transmit diversity approach. The number of RF chains is set to be $M_T = 4$, the minimum to support spatial multiplexing with linear receivers. The BTS is equipped with M_T' transmit antennas, where M_T' varies from 4 to 10. $M_T' = 4$ is the case without antenna selection, and $M_T' = 5, 6, 8, 10$ corresponds to the cases of $1, 2, 4, 6$ extra antennas for selection. The MMSV selection criterion is used for SER improvement. With only 1 extra antenna, a surprisingly large SNR gain of 10 dB is achieved at a SER of 10^{-3}. Adding another extra antenna brings a further SNR gain of 3 dB. The gain per antenna, naturally, decreases as more antennas are added so one or two extra antennas appears to be sufficient for most practical cases.

The SER gain by antenna selection for a 2-user system where each user is equipped with 3 receive antennas and receives 2 data substreams is demonstrated in Fig. 5. More receive antennas increase the receive diversity and reduce the SER relative to Fig. 4. The BTS has 5 RF units and hence transmits over 5 antennas at any time. Eigenmode selection is not feasible in the system, because the number of transmit RF units are not sufficient. Three scenarios with $M_T' = 6, 8, 10$, corresponding to $1, 3, 5$ extra antennas, are simulated. Similarly, it is observed that adding one extra antenna brings approximately 5 dB SNR gain at a SER of 10^{-3}, and an 8 dB SNR reduction at a SER of 10^{-4}. Additional antennas will introduce further but rapidly decreasing improvement, at the cost of higher computational complexity.

Fig. 6 compares the performance of the eigenmode selection and the antenna selection algorithms, in a 2-user system where each user has 3 receive antennas and receives 2 substreams. The BTS will perform antenna selection if there are $M_T = 5$ RF chains, or perform eigenmode selection if there are $M_T \geq 6$ RF chains. There are 6 transmit antennas in either case. The eigenmode selection method slightly outperforms the antenna selection approach, while both methods achieves the same diversity gain and substantially outperform a system without any selection diversity. This indicates that from a financial point of view, with sufficiently spaced antennas switches are more valuable to system performance than RF chains.

B. i.i.d. Gaussian Channel with Imperfect Channel Knowledge

Perfect channel knowledge at the BTS is typically very hard to acquire, due to channel estimation/feedback error, as well as channel mismatch resulting from feedback delay. This section provides numerical evaluation on the impact of imperfect channel knowledge on the performance of proposed selection diversity approaches.

The channel estimation model in [48][60] is used in this paper, where the channel matrix known at the BTS $\check{\mathbf{H}}_k$ is given by

$$\check{\mathbf{H}}_k = \mathbf{H}_k + \mathbf{E}_k \tag{27}$$

where $\mathbf{H}_k$ is the true channel matrix and $\mathbf{E}_k$ is the error matrix. Entries of $\mathbf{E}_k$ follows i.i.d. complex Gaussian distribution with zero mean and covariance $\sigma^2_{MSE}/2$ per real dimension. The channel knowledge error is denoted as $MSE = 10\log_{10}\sigma^2_{MSE}$dB.

Plotted in Fig. 7 are the curves of SER vs. channel estimation mean square error (MSE) for a two-user system where each user has 3 antennas and receives 2 substreams. In (a), one extra antenna/RF chain is used for eigenmode selection. Intuitively, the SER deteriorates as channel estimation error increases and results in larger channel mismatch. Performance is less sensitive to channel MSE when SNR is in low to moderate range, where the channel noise dominates. For example, when SNR=20 dB, the average SER remains roughly constant for channel MSE ranging from -40 to -20 dB. As SNR increases, the channel error plays a more important role and becomes the major performance dominant factor. Similarly, Fig. 7 (b) shows the SER vs. channel MSE curves for the same system configuration, except that no eigenmode selection is carried out.

The SER curves of antenna selection with channel estimation error for a two-user system where each user has 2 antennas and receive 2 substreams is plotted in Fig. 8 (a). For comparison, the SER curves normalized to the SER at a MSE=-40 dB is plotted in Fig. 8. (b). It is observed that the performance is more sensitive to channel knowledge error when SNR is relatively high, and when more redundant antennas are used for selection purpose.

C. Correlated MIMO Channel with Perfect Channel Knowledge

Two channel models, namely, the exponentially correlated model and IEEE 802.11N model, are used for evaluation. We assume that channel correlation exists between all elements of the transmit antenna array, and between the elements of the receive antenna array of *each* mobile. Correlation between antennas of different users are omitted, due to their well separated geographic locations.

Exponentially Correlated MIMO Model: Denote a flat-faded MIMO channel matrix of user k as $\mathbf{H}_k$. The spatial correlation between the channel matrix elements is modelled as

$$\mathbf{R}_{\mathbf{H}_k} = \mathcal{E}\left(\text{vec}\left(\mathbf{H}_k\right)\text{vec}\left(\mathbf{H}_k\right)^H\right) = \mathbf{R}_{t,k}^T \otimes \mathbf{R}_{r,k}, \tag{28}$$

where $\otimes$ represents the Kronecker product. The $M_T \times M_T$ transmit correlation matrix $\mathbf{R}_{t,k}$ and the $M_{R,k} \times M_{R,k}$ receive correlation matrix $\mathbf{R}_{r,k}$ denote the correlations of the rows and the columns of $\mathbf{H}_k$. The exponentially correlated channel has $\mathbf{R}_{t,k}$ given as $\mathbf{R}_{t,k}^{(i,j)} = \rho_t^{|i-j|}$, where $|\rho_t| \leq 1$. $\mathbf{R}_{r,k}$ follows the same model except that ρ_r replaces ρ_t. This model has been shown to be suitable for many channels [61][62].

IEEE 802.11N Channel Model: This standard builds upon previous 802.11 standards by adding MIMO antenna techniques, allowing for increased data throughput and greater range by exploiting the multipath electromagnetic waves propagation. It provides a deeper perception into the real MIMO channel, taking into account various factors such as the angle of arrival (AOA) and departure (AOD), antenna array fashion, angle spread (AS), etc. We also provide performance evaluation for this channel model because it includes various practical system parameters in building the channel correlation. We consider a B-model in the standard which captures a non-line-of-sight (NLOS) environment. Antenna element spacing is set to half wavelength in this simulation [63].

Fig. 9 gives the SER and sum-rate capacity of a two-user system under exponentially correlated channel, where each user has 3 receive antennas, 2 substreams, 1 extra BTS antenna and performs eigenmode selection. Scenarios where correlation exists at the transmitter, at the receiver, and at both side of the link are investigated, with various ρ_t and ρ_r. Clearly, channel correlation degrades both the SER and capacity performance, due the loss of spatial degrees of freedom in the matrix channel. For example, given $\rho_t = \rho_r = 0.7$, the SER increases by a magnitude of 100, while the sum-rate capacity is reduced by approximately 30% at a SNR=20 dB, compared with the uncorrelated channel. Performance loss due to the correlation is more severe at the transmitter than at the receiver side (e.g, $\rho_t = 0.7, \rho_r = 0.0$ vs. $\rho_t = 0.0, \rho_r = 0.7$), which can be attributed to two facts. Firstly, spatial freedom loss due to correlation is more significant at the transmitter because it consists of more antennas. Secondly, the correlation at the transmitter will affect the performance of all users. The correlation at a $given$ user, however, only decrease its own spatial degrees of freedom, while the degrees of freedom of other users remain unchanged or increased (e.g., see (3)).

The SER and sum-rate capacity curves of antenna selection, in exponentially correlated MIMO channel for a two-user system, are shown in Fig. 10. Each user has 2 receive antennas, 2 substreams, while the

BTS has one extra antenna for selection. Similarly, correlation substantially degrades the performance, and such loss is more sensitive to correlation at transmitter.

For IEEE 802.11N model, the SER and capacity of eigenmode selection and antenna selection are depicted in Fig. 9- Fig. 10. More SER degradation is observed compared to the exponential correlated channel model, in terms of both SER and sum-rate capacity.

VII. CONCLUSIONS

Multiuser spatial multiplexing uses precoding to support multiple users in multi-antenna wireless channels. In this paper, we proposed a novel unitary precoder design for multiuser spatial multiplexing system, which uses additional antennas to improve the diversity advantage for all users simultaneously. Two specific designs were proposed: eigenmode selection and multi-user antenna selection. The principle of eigenmode selection is that every user signals on the best orthogonal basis, according to maximizing the minimum singular value of the effective channel or the sum capacity, and yet maintaining the zero inter-user interference constraint. Multi-user antenna selection operates similarly to eigenmode selection with the additional constraint that only a subset of the available transmit antennas are employed. Multi-user antenna selection requires fewer RF chains and suffers a slight performance penalty versus complete eigenmode selection.

Multi-user spatial multiplexing requires a substantial number of transmit antennas and this makes it prohibitive to support multiple users in the presence of space constraints at the transmitter. More work is needed to investigate the effects of channel correlation and as well as realistic array design on performance. Preliminary work on the effects of different compact array design in MIMO systems is currently under investigation [64].

APPENDIX

After downlink precoding, each mobile user is effectively in a single-user MIMO channel with equivalent channel matrix $\tilde{\mathbf{H}}_k = \mathbf{H}_k\mathbf{T}_k$. Without eigenmode selection, $\mathbf{T}_k$ is a function of the interfering channel $\mathbf{H}_j,\ j \neq k$ and therefore independent of $\mathbf{H}_k$. Since $\mathbf{H}_k$ has i.i.d complex Gaussian entries of zero mean and unit variance, and because linear operations of Gaussian random variables are still Gaussian, $\tilde{\mathbf{H}}_k$ conditioning on $\mathbf{T}_k$ is also Gaussian with zero-mean. Define $\mathbf{\Lambda}_k(\mathbf{T}_k) = \mathcal{E}_{\mathbf{H}_k|\mathbf{T}_k}\left(\text{vec}\left(\tilde{\mathbf{H}}_k\right)\text{vec}\left(\tilde{\mathbf{H}}_k\right)^H\right)$, then

$$\begin{aligned}\mathbf{\Lambda}_k(\mathbf{T}_k) &= \mathcal{E}_{\mathbf{H}_k|\mathbf{T}_k}\left(\text{vec}\left(\tilde{\mathbf{H}}_k\right)\text{vec}\left(\tilde{\mathbf{H}}_k\right)^H\right) \\ &= \mathcal{E}_{\mathbf{H}_k|\mathbf{T}_k}\left(\text{vec}\left(\mathbf{H}_k\mathbf{T}_k\right)\text{vec}\left(\mathbf{H}_k\mathbf{T}_k\right)^H\right) \\ &= \mathcal{E}_{\mathbf{H}_k|\mathbf{T}_k}\left(\mathbf{T}_k^H\otimes\mathbf{I}\cdot\text{vec}\left(\mathbf{H}_k\right)\text{vec}\left(\mathbf{H}_k\right)^H\cdot\mathbf{T}_k\otimes\mathbf{I}\right) \\ &= \left(\mathbf{T}_k^H\otimes\mathbf{I}\right)\cdot\mathcal{E}_{\mathbf{H}_k|\mathbf{T}_k}\left(\text{vec}\left(\mathbf{H}_k\right)\text{vec}\left(\mathbf{H}_k\right)^H\right)\cdot\left(\mathbf{T}_k\otimes\mathbf{I}\right) \\ &= \mathbf{I}.\end{aligned} \tag{29}$$

Therefore, the distrubtion of $\tilde{\mathbf{H}}_k$ conditioning on $\mathbf{T}_k$ is given as

$$f_{\tilde{\mathbf{H}}_k|\mathbf{T}_k}\left(\tilde{\mathbf{H}}_k\right) = (2\pi)^{M_{R,k}N_k}\exp\left(-\frac{1}{2}\text{tr}\left(\tilde{\mathbf{H}}_k\tilde{\mathbf{H}}_k^H\right)\right) \tag{30}$$

which is independent of $\mathbf{T}_k$. As a result, the unconditonal distribution of $\tilde{\mathbf{H}}_k$ is derived as

$$\begin{aligned}f_{\tilde{\mathbf{H}}_k}\left(\tilde{\mathbf{H}}_k\right) &= \int_{\mathbf{T}_k} f_{\tilde{\mathbf{H}}_k|\mathbf{T}_k}\left(\tilde{\mathbf{H}}_k\right)f_{\mathbf{T}_k}\left(\mathbf{T}_k\right)d\mathbf{T}_k \\ &= (2\pi)^{M_{R,k}N_k}\exp\left(-\frac{1}{2}\text{tr}\left(\tilde{\mathbf{H}}_k\tilde{\mathbf{H}}_k^H\right)\right)\int_{\mathbf{T}_k} f_{\mathbf{T}_k}\left(\mathbf{T}_k\right)d\mathbf{T}_k \\ &= (2\pi)^{M_{R,k}N_k}\exp\left(-\frac{1}{2}\text{tr}\left(\tilde{\mathbf{H}}_k\tilde{\mathbf{H}}_k^H\right)\right)\end{aligned} \tag{31}$$

Hence the lemma is proved. □

As a result, the error performance of each user can be easily obtained through existing spatial multiplexing performance analysis methodologies for single-user spatial multiplexing system [56][57]. With eigenmode selection, however, the error performance of each user is dependent on the joint statistical distribution of the selected subset of eigenmodes.

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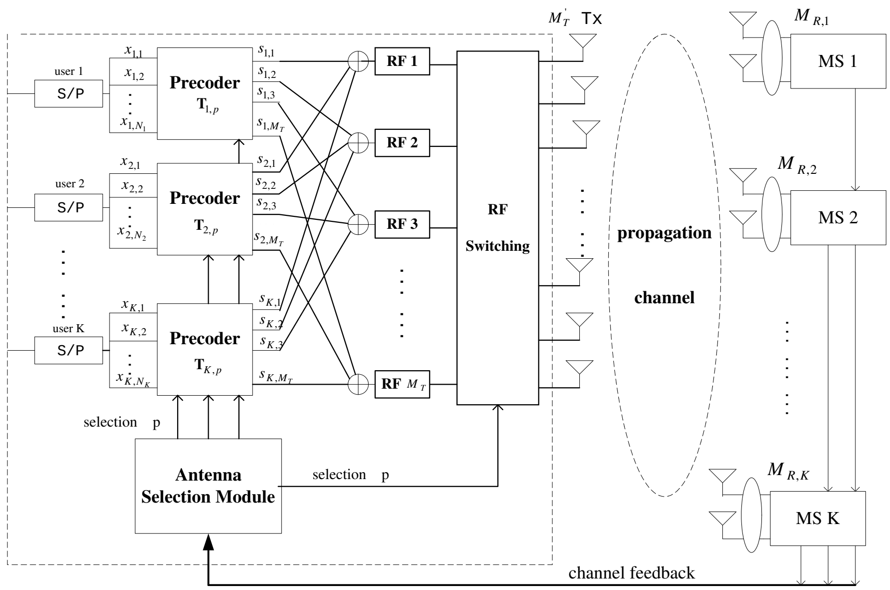


Fig. 1. Block diagram of the MUSM system with precoding: perfect feedback is assumed with $\{\mathbf{H}_k\}_{k=1}^K$ exactly known at the transmitter for precoder design.

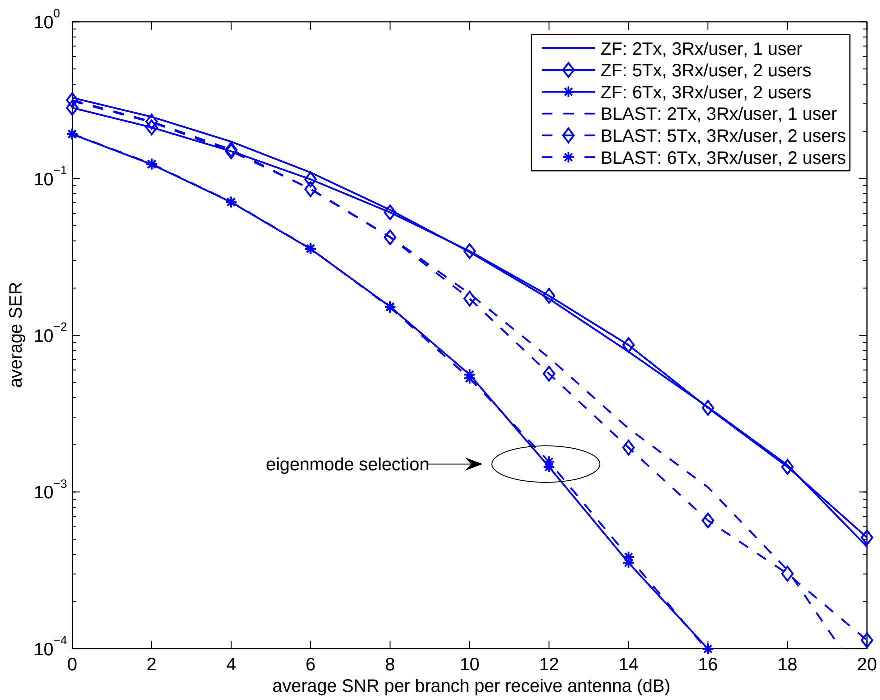


Fig. 2. SER comparison of single user and multiuser spatial multiplexing system with 3 antennas, 2 substreams per user, using ZF and V-BLAST receivers.

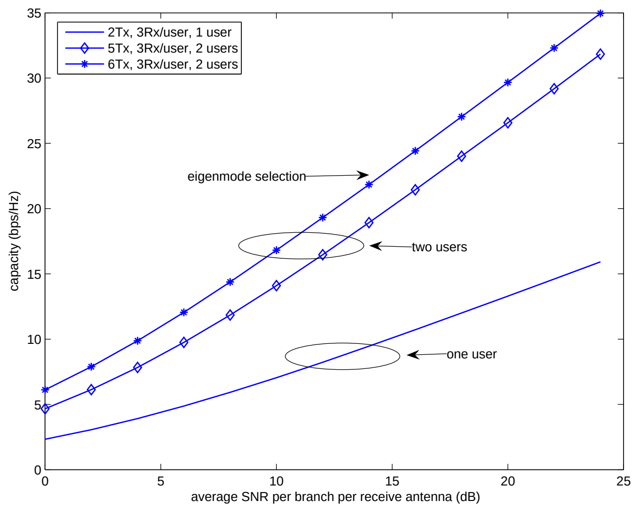


Fig. 3. Sum-rate capacity of single user and multiuser spatial multiplexing system with 3 antennas, 2 substreams per user, with and without eigenmode selection.

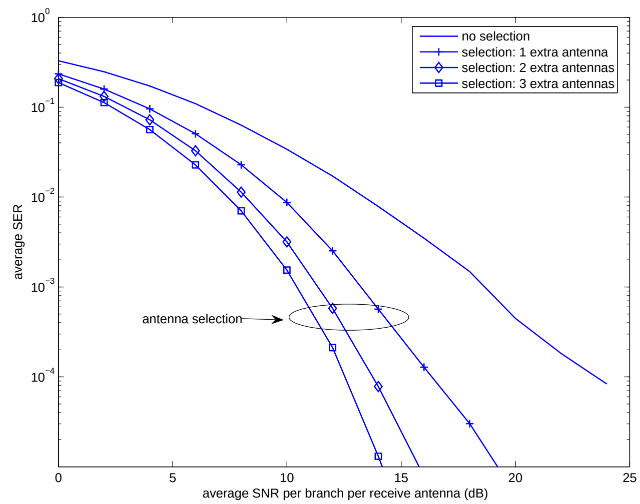


Fig. 5. SER performance of antenna selection with 2 users, 3 receive antennas and 2 data substreams per user, using ZF receiver.

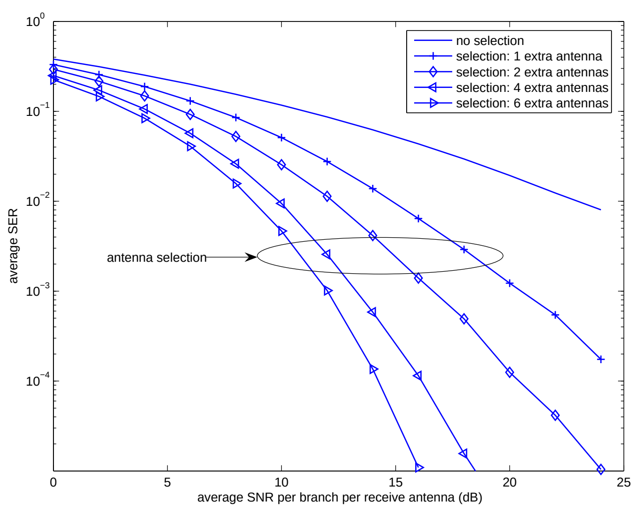


Fig. 4. SER performance of antenna selection with 2 users, 2 receive antennas and 2 data substreams per user, using ZF receiver.

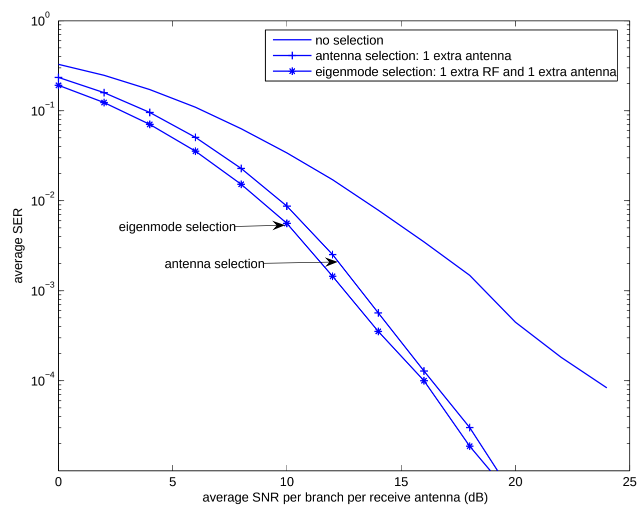
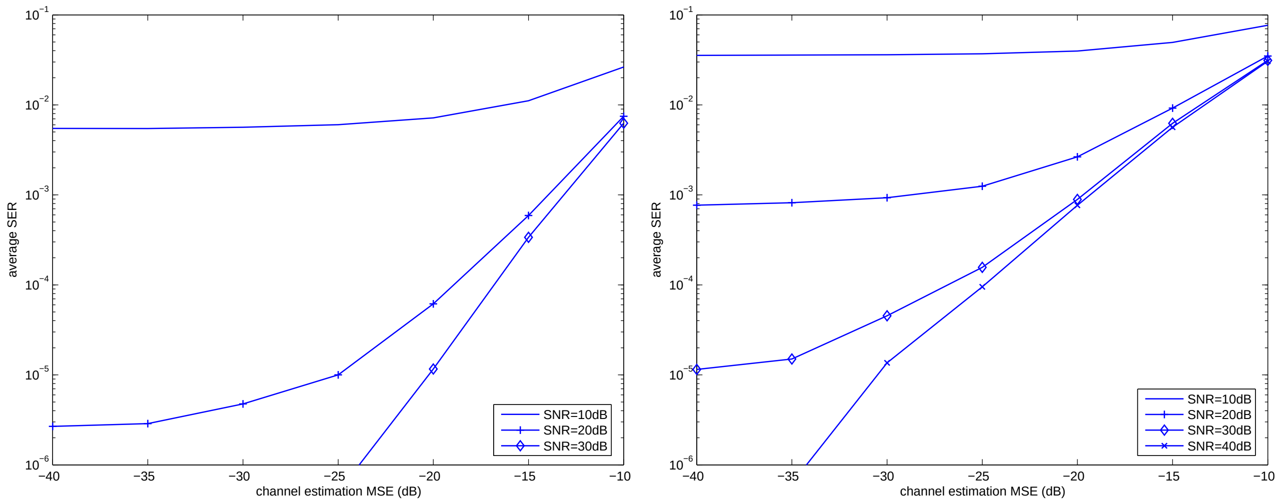


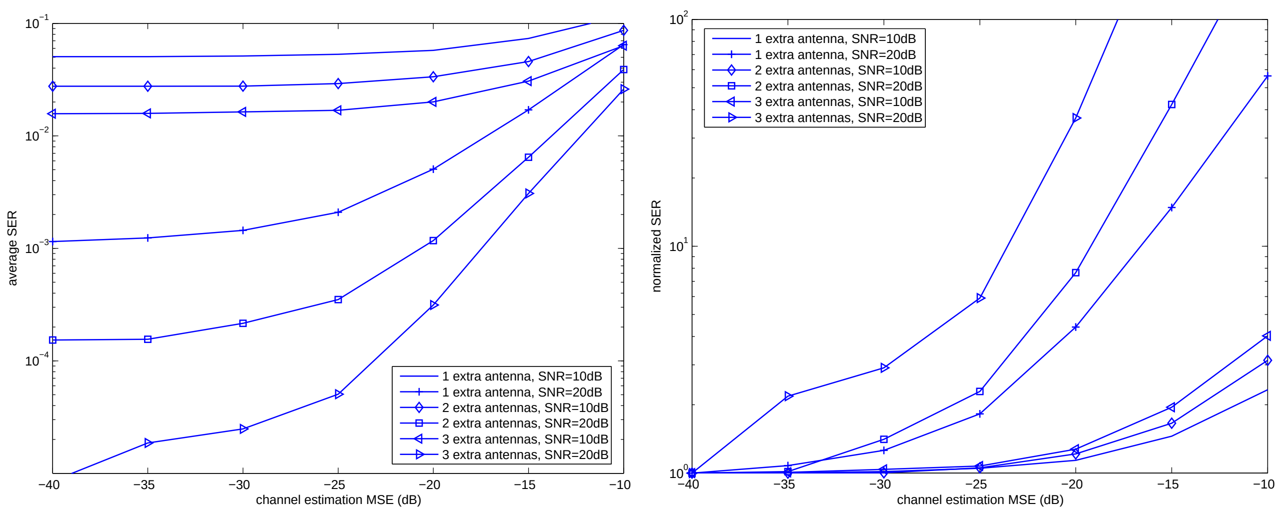
Fig. 6. SER comparison of antenna selection and eigenmode selection with 2 users, 3 receive antennas and 2 data substreams per user, using ZF receiver.



(a) 6 Tx antennas, with eigenmode selection

(b) 5 Tx antennas, without eigenmode selection

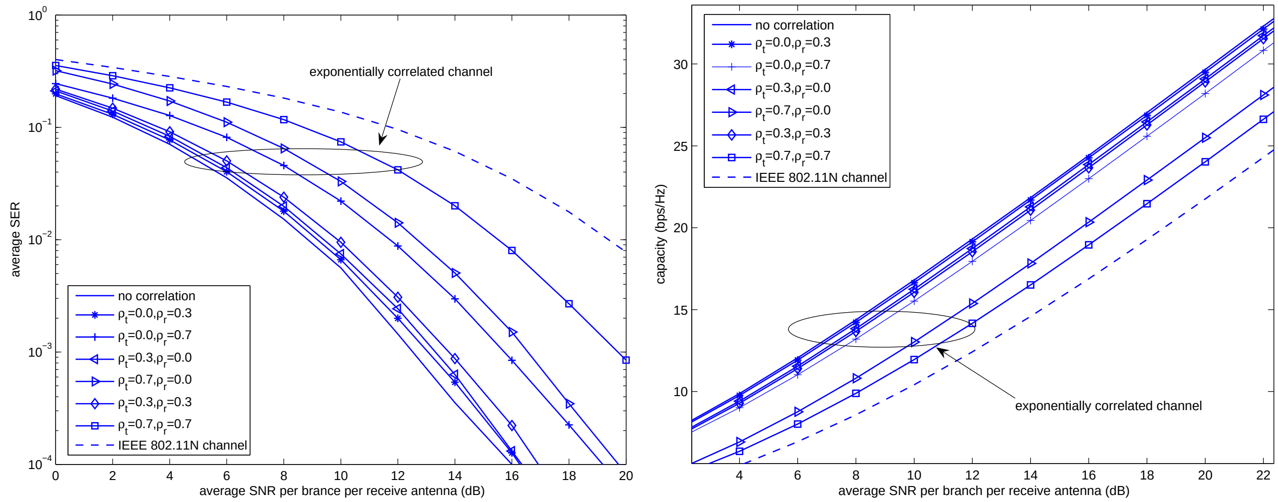
Fig. 7. SER comparison with channel estimation error for two-user system with 3 receive antennas, 2 substreams per user.



(a) average SER

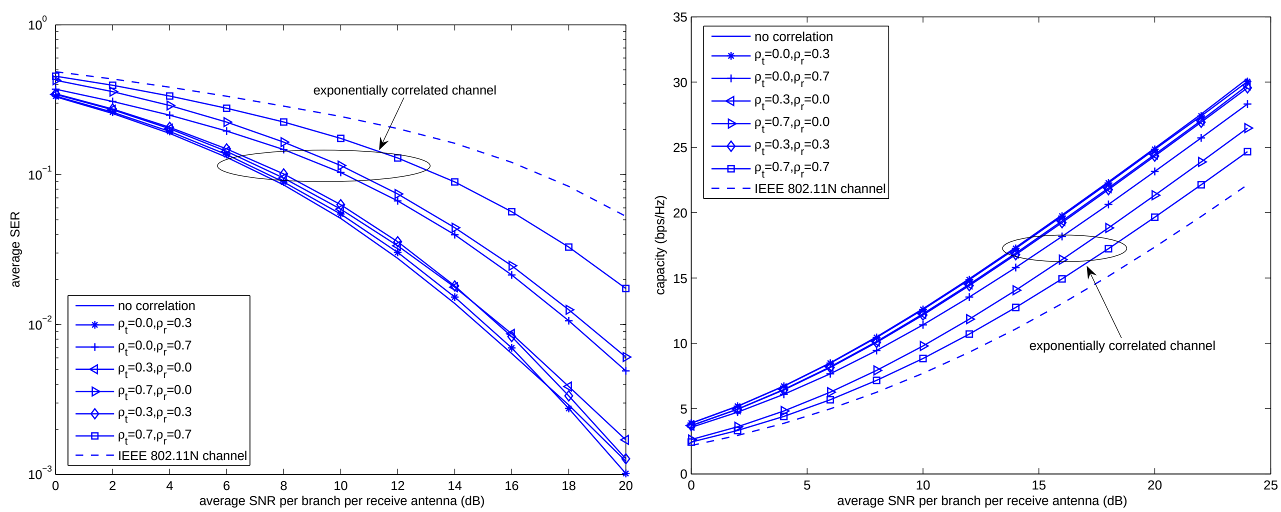
(b) average SER normalized to the SER at MSE=-40 dB

Fig. 8. SER of antenna selection with channel estimation error for two-user system with 2 receive antennas, 2 substreams per user.



(a) average SER (b) sum-rate capacity

Fig. 9. Performance of eigenmode selection in correlated channel for two-user system with 3 receive antennas, 2 substreams per user, 1 extra BTS antenna.



(a) average SER (b) sum-rate capacity

Fig. 10. Performance of antenna selection in correlated channel for two-user system with 2 receive antennas, 2 substreams per user, 1 extra BTS antenna for selection.