CHAPTER NINETEEN

Assessing Spatial Similarity in Geographic Databases

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19.1 INTRODUCTION

One of the main functions of spatial information systems such as GIS is the unification and integration of different data sets and making them available for coherent manipulation and analysis by different applications. Integrating data in spatial information systems involves the integration of diverse types of information drawn from a variety of sources requiring effective matching of similar entities in these data sets and information consistency across data sets. Typically, spatial information can be provided in different forms by a number of sources. Data sources in GIS can include maps, field surveys, photogrammetry and remote sensing. Data sets may be collected at different scales or resolutions at different times. They may be collected in incompatible ways and may vary in reliability. Some details may be missing or undefined. Incompatibilities between different data sets can include incompatibilities between the spatial entities for which data are recorded, including differences in dimension, shape and positional accuracy.

For example, it may be required that a schematic representation of a certain region in space be stored in a GIS besides a more faithful representation (a schematic representation can be useful as an interactive tourist map). The two data sets are different. Many objects may be omitted from the schematic representation. The positional accuracy of the objects may not be maintained. However, both data sets hold the same relative position and orientation for the common subset of objects they hold.

A pre-requisite for the effective use and manipulation of several diverse spatial data sets is the understanding of the contents of the data sets and how they compare to each other.

In this paper, a systematic approach is proposed for studying spatial similarity of geographic data sets. The approach involves the following steps:

- Analysing the different aspects of equivalence between the data sets. A range of spatial equivalence classes are identified which can be checked in isolation.
- Studying measures of spatial equivalence which can be applied to every class. Different levels of equivalence are proposed, namely, total, partial, conditional and inconsistent. Data sets can then be ranked as being consistent in which class to which level. This provides the flexibility for two data sets to be integrated without necessarily being totally consistent in every aspect.
- Developing methods for checking and representing explicitly the different equivalence classes and levels in the spatial database.

- Explicit representation of ambiguity or uncertainty resulting from the inconsistency of the data sets studied.

A qualitative representation scheme is proposed where the spatial content of the data sets is encoded. A simple scheme is first presented for handling topological information which is then extended for handling both topological and orientation information.

The use of a common representation scheme for different sets allows for the direct (qualitative) comparison of those sets and for the detection of any (qualitative) inconsistencies among them. This approach can, in some cases, alleviate the need for the expensive, error-prone, operation of transformation of data sets from one form to another (e.g. from raster to vector) which is the process commonly used for comparing spatial data sets. Automatic spatial reasoning techniques can be incorporated for the derivation of spatial relationships which are not explicitly represented.

The rest of this paper is structured as follows. Section 2 gives an overview of related work. In section three the different aspects of spatial equivalence are identified and classified between object-based and relation-based types. Section 4 presents a simple approach to the explicit representation of topological equivalence which is also extended to handle orientation equivalence. Conclusions are given in section 5.

19.2 RELATED WORK

Methods for checking consistency in spatial databases have been limited to checking topological consistency of pairs of spatial objects and not to whole map scenes (Kuijpers, 1995, 1997, Egenhofer and Sharma 1992, Egenhofer *et al*, 1994).

In (Egenhofer and Sharma, 1992), consistency networks were used to check the consistency of a spatial scene containing regions with holes. In (Tryfona, 1997), consistency of topological relations between multiple representations of objects,

specifically between parts and aggregate representations, is given. Approaches to the qualitative representation of images or maps can be classified into two categories. In the first category, spatial relations are studied and defined between pairs of objects, e.g. defining relationships between two simple regions or two linear objects, etc. In the second category, approaches attempt to describe continuous spaces by describing sets of objects and relationships in these spaces.

In the first category, several methods were proposed, namely, the work of Cohn *et al* (1993a, 1993b, 1996) and the work of Egenhofer *et al* (1990, 1993a, 1993b) and Jen and Boursier (1994). The set of topological relations between two spatial objects, e.g. convex regions, are first defined. Then those are used to define relationships between more complex objects such as regions with holes. In (Egenhofer and Sharma, 1992), eight topological relations between simple regions were used to represent composite and non-composite fields using a method similar to consistency networks.

In the second category, the main approaches proposed defines spatial scenes using symbolic arrays and minimum bounding rectangles (Papadias, 1994, Chang *et al*, 1987, Glasgow, 1990). However, it is recognised that approximating objects by their minimum bounding rectangles may produce misleading results. In a different approach, Lundell (1996) used graphical illustrations to represent the adjacency between composite and non-composite physical fields. Composite fields are represented by drawing connected lines between the different representations of data layers or themes. The representation of change is depicted through a sequence of diagrams. A computational model for this method is however not directly envisaged. Glasgow and Papadias (1995) showed how a symbolic array can represent whole map scenes schematically.

19.3 ASPECTS OF SPATIAL EQUIVALENCE

In checking the similarity of two geographic data sets which relate to the same area in space, two consecutive steps are needed,

1. Object matching: where corresponding objects in both sets are identified using some equivalence tests. The result of this procedure is the identification of which objects in both sets can be considered to be the same, for example, matching two sets of land parcels in an old and up to date map or matching two road networks in maps with different scales, etc. Note that those objects could differ with regard both to positional information and geometric structure.
2. Spatial Equivalence representation: where the explicit representation of the relationship between the data sets is needed to allow for the intelligent manipulation of both sets by the system and to project to the user a clear view of the nature of the data used.

The equivalence of two representations of a spatial object can be studied from three points of view: relative to fixed frame of reference, relative to the principal object studied, or, with reference to relationships with other objects in the data sets. Thus spatial equivalence can be studied using an absolute frame of reference, an object-based frame of reference and a relation-based frame of reference. Three classes of spatial equivalence can therefore be identified as follows.

Positional Equivalence

Objects are represented by the specific coordinates describing their spatial extents. Under this reference, two objects from two different data sets match only if their representative sets of coordinates match exactly and two data sets can be considered as locationally consistent if any position (x,y,z) corresponds to the same object in both sets.

19.3.2 Object-Based Equivalence Classes

A spatial data set consists of the spatial properties of a set of objects in a defined space. These properties include a description of spatial extent, from which the dimension and the shape of the object can be derived. An object in the data set can be composite, i.e. consisting of, or containing other objects. Object-based consistency can be classified using the above properties. Two spatial data sets can be said to be object-based consistent of a certain class if for each object in both sets this consistency is achieved.

1. Object Existence Equivalence:

Two data sets are existentially equivalent if all the object classes and instances in one data set exist in the other data set.

2. Object Dimension Equivalence:

Two data sets are equivalent with reference to object dimension, if every object in one set has the same spatial dimension as that of the corresponding object in the other set.

3. Object Shape Equivalence:
Equivalence based on object shape can be as flexible as needed. On a strict level object shapes can be defined using equations of the curve or set of curves defining its boundary. On a less precise level object shapes can approximate well-known geometric shapes, for example a circle, a square, a T shape, zig-zag, etc. Two data sets are said to be equivalent with reference to object shape if every object in the set can be described as shape equivalent to the corresponding object in the other set.

4. Object Size Equivalence:
Several measures of size exist including, length of boundaries, areas and volumes of shapes. Two data sets may be considered as equivalent with reference to object size if every object in one set has a similar size to the corresponding object in the other set.

5. Spatial Detail Equivalence:
Objects in the data sets may be composite, i.e. containing other objects or made up of several connected or non-connected objects. Two data sets can be considered to be equivalent with reference to object detail if corresponding composite objects in both sets can be considered to be equivalent.
An example of object-based equivalence is shown in figure 19.1.

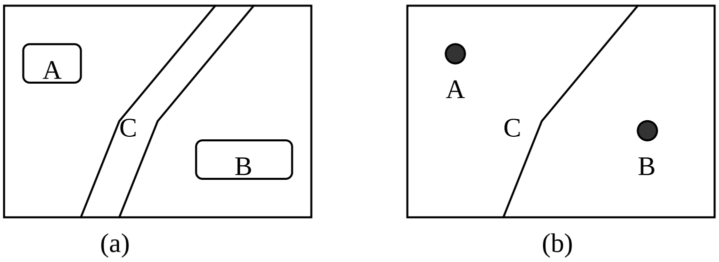

Figure 19.1 Non-consistent data sets with regerence to object dimension.

Interdependency between Equivalence Classes
Other classes of object-based equivalence may exist. The above set of classes are possibly the most important from a general point of view. Note that the above classes may not be mutually exclusive. In particular, the positional consistency implies every other type of consistency and is by default the strictest measure of spatial equivalence. Shape and size imply dimension and all equivalence classes imply existence equivalence. Shape equivalence may imply spatial detail consistency if the object is composed of non-connected sets, etc. Also, it is assumed that a certain degree of inaccuracy can be acceptable in the measurement of some of the properties, for example, size and shape. However, this depends on the applications intended for these data sets. Note, that non-spatial equivalence is assumed here. Measuring non-spatial equivalence is part of the overall problem and is not discussed in this paper.

19.3.3 Relation-Based Equivalence Classes

The third type of consistency measures is based on the spatial relationships between objects in the data sets considered. Three classes of equivalence can be classified according to the types of spatial relationships (Abdelmoty and Williams, 1994, Abdelmoty and El-Geresy, 1994).

1. Topological Equivalence:

Two data sets can be regarded as topologically consistent if the set of topological relationships derived from one set are the same as those derived from the other.

For example, the two sets in figure 19.2 are not topologically consistent.

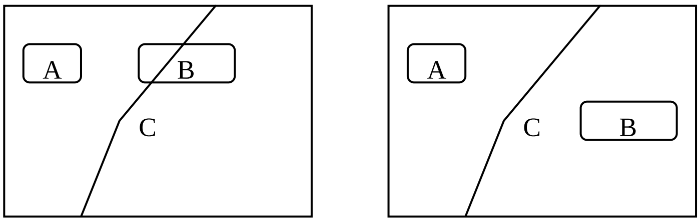

Figure 19.2 Topological inconsistency. (a) Object B crosses object C. (b) Object B is disjoint from C.

2. Direction or Orientation Equivalence:

Two data sets can be regarded as directionally consistent if the relative direction relationship in one set is the same as the other set.

3. Relative Size Equivalence:

Two data sets can be regarded as consistent with reference to relative size relationships if the qualitative size relations of larger and smaller are maintained between corresponding sets of objects in the two sets.

19.3.4 Different Levels of Spatial Consistency

Two spatial data sets can be consistent in more than one class of those defined above.

For example, the data sets can be topologically and dimensionally equivalent, or consistent with reference to dimension, detail and category, etc. As noted earlier some consistencies do assume others. For example, topological equivalence may assume spatial detail. Up till now, the discussion is based on one level of consistency, namely, when all objects in the data sets conform to the consistency class studied. In reality, this is not always the case. Ranking the level of consistency for the different classes identified is important as it would provide the user of the GDB an initial measure of the nature of the data sets in his use. Further processing of this ranking would be used to identify how the data sets compare and which parts of the data sets are consistent, i.e. the nature of such consistency. Let S1 and S2 represent the set of knowledge present in two data sets. This knowledge consists of all the different types of information that can be derived from every data set. It can be classified according to the object-based and relation-based classes. Let S1i and S2i represent the subsets of the set of knowledge S1 and S2 respectively, which belong to a certain class i, e.g. shape properties or directional or topological relationships, etc. Four different levels of consistency can be identified.

A. Total Consistency

Two data sets S_{1} and S_{2} can be said to be totally consistent with reference to a certain consistency class i, if $ {S_{1}}_i \cap {S_{2}}_i = {S_{1}}_i \vee {S_{2}}_i$, i.e. ${S_{1}}_i = {S_{2}}_i$. In this case a query to the GIS involving only properties of class i shall return identical results if posed to either S_{1} or S_{2}.

B. Partial Consistency

Two data sets S_{1} and S_{2} can be said to be partially consistent with reference to a certain consistency class i, if $ {S_{1}}_i \cap {S_{2}}_i = C_i$ and $C_i \subset {S_{1}}_i \wedge C_i \subset {S_{2}}_i$. In this case only part of class i knowledge is consistent in the two sets. If the two data sets are to be used together, then it is important to know which subsets of the different classes of knowledge can be manipulated interchangeably between sets.

C. Conditional Consistency

Two data sets S_{1} and S_{2} are said to be conditionally consistent with reference to a certain consistency class i, if there exists a set of functions F which when applied to ${S_{1}}_i$ makes it totally consistent with ${S_{2}}_i$, i.e. ${S_{2}}_i = F({S_{1}}_i)$. This can also represent the case where ${S_{1}}_i$ is consistent with ${S_{2}}_i$ but ${S_{2}}_i$ is not consistent with ${S_{1}}_i$, i.e. (${S_{1}}_i \cap {S_{2}}_i = {S_{1}}_i) \wedge ({S_{1}}_i \subset {S_{2}}_i$), (an asymmetric consistency).

The set of functions F must be non-ad-hoc, i.e. predefined. For example, the set of cartographic generalisation rules used to produce maps at different scales or a set of predefined rules used to produce a schematic from a faithful representation of a map.

D. Inconsistency Level

Two data sets S_{1} and S_{2} can be said to be inconsistent with reference to a certain consistency class i, if $ {S_{1}}_i \cap {S_{2}}_i = \phi$, i.e. they do not share any piece of knowledge from that class. In this case a query to the GIS involving properties of class i shall return non-identical results if posed to S_{1} and S_{2}.

In most cases the data sets studied relate to a combination of classes and levels. For example, two data sets can be partially consistent in terms of shape and dimension but are totally consistent topologically, or are conditionally consistent with respect to object detail as well as partially consistent topologically.

Figure 19.3 shows the integration of different sets of knowledge which are consistent in different classes and levels.

19.4 REPRESENATION OF DIFFERENT LEVELS OF CONSISTENCY FOR DIFFERENT CLASSES

Determining the class and level of consistency between two data sets involves the extraction and comparison of the set of properties or relationships for that class.

Although it is useful for the user and the system to be informed of the class and level of consistency in general, it may not be enough for certain application domains. In those cases explicit representation of the consistent set of knowledge is needed. A closer look at the different classes of consistency reveals that they are mostly qualitative measures (apart from location, size and shape). Hence, the common set of spatial knowledge between data sets can be represented qualitatively. A structuring mechanism can be envisaged which can be applied on a geographic data set to allow the explicit representation of some of the qualitative properties and relationships and the derivation of others. Multiple spatial representations can exist for the same geographic objects. However, properties and relationships are always related to objects and not to their underlying representations. Hence the structuring mechanism envisaged should be based on the geographic objects level and not on the geometrical representations. This structure can then be built for any data set irrespective of its underlying form of spatial representation.

Manipulation of such qualitative structure could make use of spatial reasoning techniques (Egenhofer, 1994, Cui, *et al*, 1993, El-Geresy, 1997, Hernandez, 1994). For example it would be possible to store only some of the topological relationships and derive others using composition tables for similar and mixed types of spatial relations.

Explicit representation of this knowledge would allow comparisons between data sets, seamless manipulation of existing sets, integration of new sets and consistent update of existing ones. In developing the proposed structuring mechanism, several questions need to be answered, including,

- What are the types of knowledge that can be represented explicitly and which can be derived?
- How can the different classes of knowledge be structured?

In this section, the representation of the class and level of topological consistency is first given and then extended to include orientation relationships.

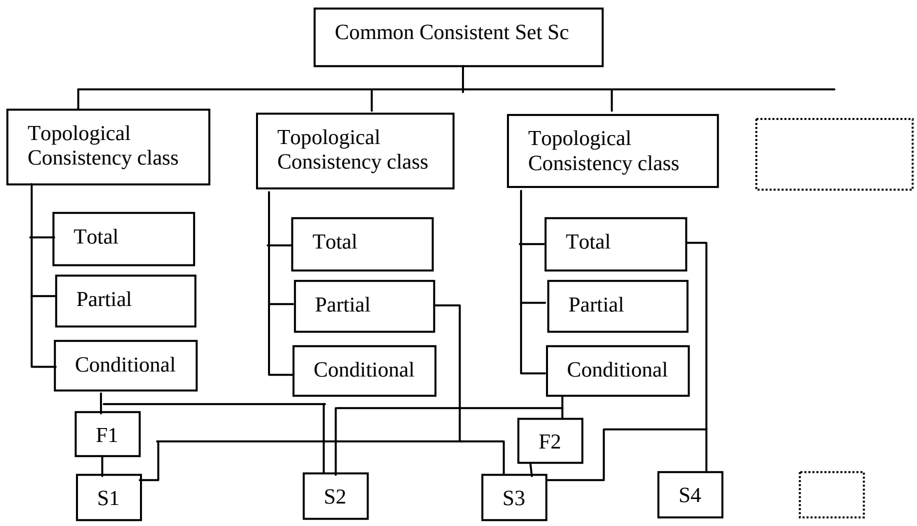

Figure 19.3 Integrating different data sets with different classes and levels of consistency to produce a common set of consistent knowledge. F1 and F2 represent sets of predefined functions for conditional consistency.

19.4.1 Representation of Topological Equivalence with Adjacency Relationships

Checking topological equivalence between two geographic data sets is the process of checking that the same set of topological relationships between objects in one set exist for the corresponding objects in the other set. This process involves the explicit extraction and representation of topological relationships. Several approaches to checking the topological consistency of two spatial scenes have been proposed (Kuijpers *et al*, 1995, 1997, Egenhofer and Sharma, 1992, Egenhofer *et al*, 1994). However, they do not consider the issue of integrating both scenes and hence do not provide means of representing the common set of consistent knowledge. In this section, a simple structure for storing the adjacency relationships between objects in the data sets is proposed from which topological relationships can be derived. The structure can then be used to represent the common set of consistent knowledge between data sets as well as the ambiguity or uncertainty in the knowledge derived from both sets. The structure is based on the following assumptions.

19.4.2 Assumptions

Given a space S and a set of spatial entities $O_1, \cdots, O_n$ embedded in it.

- Space S is dense and infinite.
- The spatial entities are connected. If an entity is not connected, each of its components will be considered separately.
- The entities jointly cover the whole space, i.e.$S = O_1 \cup \cdots O_n \cup S_0$, where S_0 is the complement of the entities in space S. The inclusion of S_0 is necessary for two reasons: a) to avoid mis-interpretation of space topology and b) to provide an explicit representation of the edges of the scene (or map).
- The spatial entities don't overlap, i.e. $O_i \cap O_j = \phi$ for all $1 \leq i \neq j \leq n$.

19.4.3 Capturing Topology- The Adjacency Matrix

The adjacency matrix is a qualitative spatial structure which captures the adjacency relations between different spatial objects. The adjacency relation (a binary symmetric relation) can be used for capturing the topological distribution of objects. In figure 19.4(a) a map is shown with five entities A, B, C, D and E. In 19.4(b) the adjacency between the entities are encoded in a matrix. The fact that two entities are adjacent is represented by a (1) in the matrix and by a (0) otherwise.
For example, A is adjacent to B, C and D but not to E, and D is adjacent to all others. Since adjacency is a symmetric relation, the resulting matrix will be symmetric around the diagonal. Hence, only half the matrix is sufficient for the representation of the space topology and the matrix can be collapsed to the structure in figure 19.4©. The complement of the objects in question shall be considered to be infinite. The suffix 0 (S_{0}) is used to represent this component. As seen in the figure, the map edges are represented explicitly by the adjacency relations of S_0 (complement of objects in S). Objects B and E do not touch any of the map edges.

Figure 19.4 (a) Space containing five objects. (b) Adjacency matrix for the scene in (a). (c) Half the symmetric adjacency matrix is sufficient to capture the scene representation.

Checking Topological Equivalence

The adjacency matrix can be used to check the topological equivalence of two scenes. Figure 19.5 shows a different data set of the same geographical area as figure 19.4 and its corresponding adjacency matrix. There are two differences between the two scenes as can be seen from the structures. These are: in 19.4 object A is connected to C while it is not in 19.5, and object E in 19.4 does not exist in 19.5.

Figure 19.5 (a) Different data set for the same area in figure \ref{matrix}. (b) Its corresponding adjacency matrix.

The only relationship stored explicitly in the above structures is *adjacency* and other topological relationships can be derived simply. For example, in 19.4, object E is adjacent only to D and hence it is topologically inside D.
Also, the relationship for complex objects can be realised from the grouping of relationships between its constituting parts, and so on. Hence, using this structure alone we can redraw the topological equivalences of the two scenes (obviously the exact shape of each object is not meant to be represented here). The adjacency structures can be organised in a tree structure representing different levels of detail in the data sets.
Also, an explicit reference to object dimension will enable a (schematic) reproduction of the topological equivalence of the data sets. However, object dimension in both data sets need not be consistent.

Representing the Common Consistent Set of Knowledge

The scenes in 19.4 and 19.5 are partially topologically consistent. The set of common knowledge in both data sets can be grouped in an adjacency structure as shown in figure 19. The structure in 19.6 is informative of the consistent topological common knowledge between the two data sets. In this case, the adjacency between objects A and C is unknown, represented by a (-), and object E does not exist in both data sets and hence it is deleted from this set. Using this structure one can recreate the common knowledge in both scenes with the ambiguity of the relation between A and C.

Figure 19.6 The adjacency structure representing the common set of consistent knowledge in the structures of figures 19.4and 19.5.

19.4.4 Capturing Orientation: The Matrix Map

The adjacency matrix captures the topology of space under consideration. Orientation relations can be added to the cells of the matrix. Orientation relations have converses and therefore half the matrix is still enough to capture these relations. The matrix can be kept compact by exploiting the transitive property of the relations by qualitative reasoning.
Thus those relations shall be explicitly defined between adjacent spatial entities only.
Other relations between non-adjacent entities can then be deduced using qualitative reasoning. The convention of orientation relations is R(column,row). For example, in figure 19.7, West(A,B) and South(A,C). Different granularities of the orientation relations can be defined, e.g. south-west(A,D). Consider the example in figure 19.7. The following orientation relations are defined between adjacent objects.

\begin{math}
W(A,B), N(A,C), S(A,D) \wedge SW(A,D), \\
S(B,D), W(B,E), W(C,D),W(D,E)
\end{math}.[19.1] The matrix in figure 19.7(b) contains the orientation relations between adjacent objects only.
The rest of the orientation relations can then be derived using the rules:
\begin{math}
W(A,B) \wedge W(B,E) \rightarrow W(A,E) \\
S(A,D) \wedge W(D,E) \rightarrow S(A,E) \vee SW(A,E) \vee W(A,E) \\ SW(A,D) \wedge W(D,E) \rightarrow W(A,E) \vee SW(A,E) \\ \end{math}

[19.1] S denotes South, W denotes West, etc

Note that more than one reasoning path exists. For example, from the above rules we conclude that $S(A,E) \vee W(A,E) \vee SW(A,E)$. If an object is surrounded partly or fully by another object, such as in the case of part-whole relations, a notation is used to represent both relations, e.g. $IE(A,B)$ denotes the relations $Inside(A,B) \wedge East(A,B)$ as shown in figure 19.8. The matrix structure is given in 19.8(b) and the converse relations are used if the order of objects is reversed in the matrix as in 19.8 (c). Whether A is totally inside B or shares its boundary can be inferred by examining the rest of the matrix cells for A and B. (A is totally inside B if it is only adjacent to B, i.e. its corresponding row and column contain the value 1 only with object B).

Figure 19.7 (a) Set of adjacent regions. (b) Corresponding adjacency matrix including orientation relations between adjacent regions only.

Figure 19.8 (a) Representing part-whole relationships Inside(A,B) $\wedge$ East(A,B). (b) its matrix representation, (c) the matrix with the order of the objects reversed.

The combined adjacency and orientation relations and the explicit edge representation can be denoted the **Matrix Map**. A sketch map can be recreated from the matrix.
The matrix map can be further enriched with the size relations by specifying an ordered set of size relations between objects. The set D > A > B > C will capture the complete size relations between objects in the map.

Example
Consider the data sets in figure 19.9. The difference between the two sets is only apparent when their matrix maps are considered. The two data sets are totally equivalent topologically but partially equivalent directionally. Note that in defining the orientation relations, a specific consistent frame of reference has to be adopted. Different approaches exist for the representation of orientation relations (Hernandez, 1994, Abdelmoty and Williams, 1994). In this example, a simple conic division of the orientation space is adopted.

Figure 19.9 (a) and (d) Two data sets of the same geographic area. (b) and (e) Their corresponding, equivalent, adjacency matrices. (c) and (f) Their corresponding (different) matrix maps.

19.5 CONCLUSIONS

In this paper a study of the nature of equivalence between spatial data sets is presented. The proposed approach can be summarised as follows:

- Equivalence of data sets is broken down into two main categories: comparison of basic properties of objects and relationships between those objects. Difference equivalence classes were identified which can be checked in isolation.
- For every class identified, data sets can be equivalent to a certain level or degree. Four levels of equivalence are proposed, namely, total, partial, conditional and inconsistent. Data sets can be ranked according to those levels, for example, totally

consistent topologically but partially consistent with reference to object dimension and so on.

- Explicit representation of the different equivalent classes and levels of consistency is needed in the spatial database when different data sets are to be used together.
- The common set of consistent knowledge in the data sets needs to be expressed explicitly. A qualitative structure is proposed to hold different types of knowledge on the geographic feature or object level (as opposed to the geometric level).

As an example, the representation of topological equivalence is presented using a simple structure which stores adjacency relationships. Topological relationships can be derived from the structure and ambiguity in the relationships can be derived. It was also shown how the structure can be extended to incorporate orientation relationships. Further work needs to be done for devising representation methods for the different consistency classes and for their coherent integration. The work in this paper was done in the context of an ongoing research project which aims at the development of methods for the modelling and manipulation of hybrid data sets in a GIS (Jones *et al*, 1996).

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