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PROBABILITY DISTRIBUTION OF THE INTER-ARRIVAL TIME TO CELLULAR TELEPHONY CHANNELS

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Abstract - This paper is focused on the statistical analysis of the call arrival process in channels of mobile telephony access networks. The approach is fully empirical, and is based on actual activity data, collected from real Base Stations in a working mobile network. The arrival process of the merged "fresh" (new calls) and handover traffic is proved to be smoother than Poisson traffic. This conclusion can be applied when dimensioning the number of channels needed in a BS to achieve a targeted grade of service.

I. INTRODUCTION

The design and performance evaluation of mobile telephony access networks is usually based on queuing theory concepts to guarantee a certain Grade of Service (GOS) to the end user, in terms of low blocking probability or probability of a call being interrupted due to failed handover. To obtain further details about the system performance during the design stage, simulations are often carried out to improve the figures computed analytically.

The accurate knowledge of the statistical properties of the random variables (r.v.) involved in the call process allows to feed analysis and simulations precisely. This knowledge is thus extremely relevant for a successful design, with accurate predictions of the figures related with the GOS.

There is a current trend to use the channel idle time to transmit other services such as short data messages. This makes the statistical knowledge of the r.v related with the call process even more helpful, specially the idle time. This knowledge allows to know whether it possible or not to forecast the next idle time, and which is the error range incurred in the prediction.

When observing a channel in a Base Station (BS), the random variables (r.v.) involved in the call process are described with the help of Figure 1. Obviously the three r.v are related: the inter-arrival is the sum of the channel holding and the idle time. On the other hand, the channel load (utilization) also relates the above-mentioned r.v. according to the following formula, which is accomplished both for the instantaneous and for the average values of the r.v:

$$\rho = \frac{\overline{T}_{ch}}{\overline{T}_{ch} + \overline{T}_{id}} = \frac{\overline{T}_{ch}}{\overline{T}_{\text{int}}} \qquad (1)$$

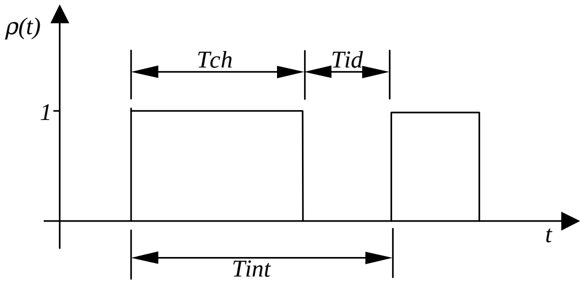

Fig. 1. Random variables of channel occupancy.

The call holding time has been proved to fit combinations of lognormal distributions better than the negative exponential in the case of fixed telephony service [1] and in public cellular mobile systems [2]. The channel holding time in cellular systems depends, besides, on factors such as the Mobile Station (MS) mobility, speed, cell size and area, etc. The statistical properties of the channel holding time in public mobile telephony were examined by using analytical tools and simulation in [3, 4, 5, 6] and under empirical approaches in [7, 8, 9].

Other research works targeted the other half of the problem: inter-arrival or idle time. Although the Poissonian hypothesis is acceptable for the fresh traffic arrivals due to new calls, as it is in fixed telephony, in the case of a cell in a mobile network, handovers from surrounding cells distort the process as explained below in Section IV. In [10, 11] analytical tools and simulation results are used to investigate the arrival process. In [12] the empirical approach is used to show that call arrivals to a Private Mobile Radio system (non-cellular) don't follow the Poisson distribution. In [7, 13] the empirical approach is applied to public cellular systems.

In this paper, further research results which complete the study submitted in [13] are presented. As in [13], a fully empirical approach is undertaken. The paper is organized as follows. In Section II the data acquisition process is briefly described. Section III is focused on the study of possible dependencies among the r.v examined. Section IV presents the statistical modeling along with the probability distributions that best fit the inter-arrival time. In Section V similar results are presented for the channel idle time. The main points and conclusion of the work are summarized in Section VI.

II. DATA ACQUICITION PROCESS

The data used in the study were collected through a scanning receiver controlled by a Personal Computer (PC). The program that controls the scanner generates reports of activity: time at which every communication that uses the channel starts and finishes. The system monitored was a TACS (very similar to AMPS) public cellular system in Barcelona.

TACS uses Frequency Modulation, so the detection of the carrier in the down-link was sufficient for the knowledge of the channel occupancy. The down-link provides a more stable and easier to detect carrier power than the up-link, leading to reports with fewer cuts and interference.

The collected data were pre-processed to minimize the effects of short cuts, interference and noise that remained in the first version of the activity report. To this purpose, activity values under 2 seconds were considered to be caused by noise or interference and thus suppressed. Activities separated by a silence shorter than 1 second were considered to be short cuts due to fading and thus joined. A more detailed description can be found in [9, 13]

With this equipment, several samples were collected for different purposes. To obtain the probability density function (p.d.f.) of the inter-arrival and idle time, samples belonging to the busy hour were used, because they are representative of the worst case and are the right scenario for design purposes. Another reason to use the one hour period is that the arrival process can be assumed to be stationary within one hour periods, and this is a necessary condition for the simple statistical study performed in this work. Three samples belonging to different load levels were considered: Heavy (ρ=0.6), Medium (ρ=0.5) and Light (ρ=0.4). Note that in this environment 0.6 is considered to be heavy load because the targeted blocking probability must be kept very small: blocking probabilities of more than 2% can be considered as poor in public telephone systems. For the correlation study of Section III an additional sample belonging to the whole day (8 in the morning till midnight) was also obtained.

III. CORRELATION STUDY

Given two r.v., the correlation coefficient gives an idea of the level of similarity between them. If the correlation coefficient is near to unity, the two random variables are statistically similar [14]. As in every experimental study, the probability distributions of the observed r.v. are not known beforehand, thus it makes no sense trying to calculate the exact figures of the correlation coefficients. As only a limited number of observations of these r.v. is available, the coefficients must be estimated according to the available data. The information available is a series of channel holding time (*Tch*) along with their following idle time (*Tid*).

The correlation coefficients below examined can be understood with the help of Figure 2. $r_{ch\text{--}id}$ is the correlation coefficient between the channel holding time and its following idle time, while $r_{id\text{-}ch}$ is the correlation coefficient between the idle time and its following channel holding time. $r_{id\text{-}id}$ is the auto-correlation between the idle time and its lag-*i* version (*i*=1 in the figure) and $r_{ch\text{-}ch}$ is the auto-correlation between the channel holding time and its lag-*i* version.

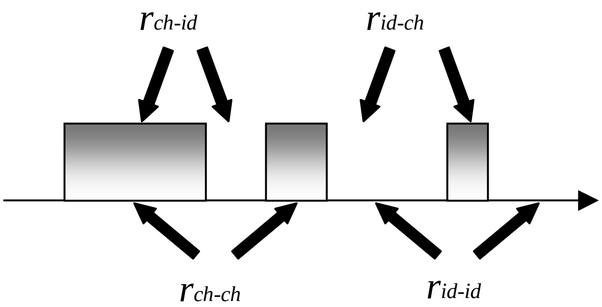

Fig. 2. Correlation coefficients examined.

In Table I the values of the correlation coefficients are presented for two samples corresponding to the Busy Hour (BH) and Whole Day (WD). The coefficients of the BH sample are not small and samples with coefficients from -0.25 to 0.25 were found in different activity samples. This means that certain dependency exists between the idle and its neighbor channel holding time, when the observation period is short. This dependency is always weak and no rule could be established to link the correlation magnitude or sign with other parameters such as the channel load. The coefficients for the WD were very small in all checked samples.

Table I. Correlation coefficients

$r_{id\text{-}ch}$(BH)	$r_{id\text{-}ch}$(WD)	$r_{ch\text{-}id}$(BH)	$r_{ch\text{-}id}$(WD)
-0.08	-0.013	0.17	0.002

This results were somewhat expected. The data belonging to the busy hour are mostly related with professional calls and share features such as mobility pattern, speed, whole call duration, etc. These factors in common can easily lead to the weak dependency found in the BH sample. The same argument can not en applied to the calls occurring during the rest of the day. The WD sample merges occupancies due to professional and personal calls with very different mobility and activity profiles.

In Table II the auto correlation coefficients of both the channel and idle time are presented again for both BH and WD samples, and for lags from 1 to 8. The auto-correlation of the channel holding time series is small for all lags, as expected. This fact is easily explained because every channel sizing is an independent event and therefore its duration must be independent of others. The behavior of the coefficient for these small lag values seems to be random-like.

Table II. Auto-correlation coefficients

Lag	$r_{ch\text{-}ch}$(BH)	$r_{ch\text{-}ch}$(WD)	$r_{id\text{-}id}$(BH)	$r_{id\text{-}id}$(WD)
1	-0,0994	0,0552	-0,0431	0,1687
2	-0,0214	0,0821	0,0552	0,3328
3	0,0543	0,0638	0,1436	0,4598
4	-0,1363	0,0634	-0,0119	0,1718
5	-0,0036	0,0545	0,1194	0,2464
6	-0,0712	0,0648	0,0339	0,2364
7	-0,1050	0,0215	-0,0752	0,2794
8	-0,0965	0,0212	0,1404	0,1807

The auto-correlation coefficients of the idle time series are not negligible. Moreover, far from presenting a random-like aspect they seem to decrease as the lag increases, especially in the WD sample. In Figure 3 values of the auto-correlation for the WD sample and longer lags of up to 50 are displayed. As below shown in Section V the coefficient of variation (*cv*) of the idle time series is smaller than one, while the *cv* of the channel holding time is bigger than one [9]. Thus the idle time is not so spread around its mean as the channel holding time is. In other words, the values of the idle time are more similar to each other than the values of the channel holding time.

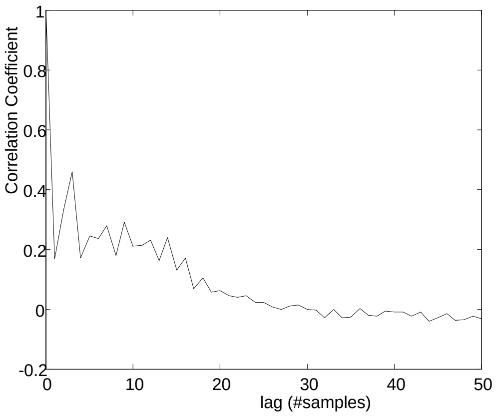

Fig. 3 Auto-correlation of the idle time (WD).

The explanation for the monotonously decreasing behavior is also simple. As the channel load increases the idle time decreases according to Equation (1). Along the day, the channel load increases or decreases smoothly. Idle times separated by less than 1 hour can be assumed to belong to the same or very similar load intervals while lags longer than 3 hours can be assumed to correspond to independent loads. It is obvious the higher dependency for shorter lags that leads to the shape of Figure 3.

IV. INTER-ARRIVAL TIME

In every cell of a Cellular Mobile Telephony network, the offered traffic is the result of two traffic streams. On one hand, one has “fresh” traffic originated inside the limits of the cell (T1). This traffic is caused by calls which start inside the observed cell and finish wherever. On the other hand, there are “handover” attempts due to calls from neighbouring cells (T2) which started wherever and try to get a channel in the observed cell.

The infinite population hypothesis could be accepted for T1 only if a large number of Mobile Stations (MS) were present inside the cell bounds. T2 is traffic which has already been carried in the neighbouring cells, and therefore it comes from a population of no more than the total number of channels of all the neighbouring cells together. The superposition of T1 and T2 is thus smoother than Poisson. This argument is presented in [10] where the same conclusion is also achieved by means of analytical tools and simulation.

In this work samples of actual channel activity were used to experimentally reach this same conclusion. The inter-arrival time to the channel in the measured BS was statistically processed: the mean and coefficient of variation were obtained, and different candidate probability distributions were checked against the empirical distribution. Maximum Likelihood Estimation (MLE) was used to estimate the parameters of the candidate p.d.f. and the Kolmogorov-Smirnov (K-S) goodness-of-fit test was selected to estimate the level of significance α [15]. These estimations were achieved for samples belonging to the three load levels mentioned in Section II: High, Medium and Light loads.

The results of the study are summarised in Table III where statistical figures are presented for the p.d.f. that better fitted the empirical distribution: the Erlang-3,*k*. Note the high significance values, and the fact that in most studies significance of 5% to 10% are enough to accept the theoretical distribution. Other distributions tested were the negative exponential, lognormal and combinations of lognormals, erlang-*n*,*k* (the best significance was always for n=3) and hyper-erlang. Further details, along with the exact figures for all the mentioned distributions, can be found in [13]. In Figure 4 the erlang-3,8 p.d.f. is depicted along with the empirical histogram for a system with ρ=0.6.

Table III: Statistical figures of inter-arrival time and significance of erlang-3,*k*.

ρ	Mean (s.)	cv^2	k	α
0.6	67.77	0.49	8	0.40
0.5	84.25	0.38	6	0.37
0.4	90.59	0.33	5	0.24

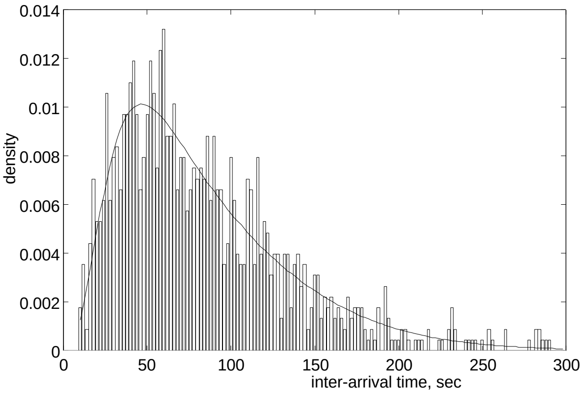

Figure 4. Inter-arrival time distribution: empirical vs. theoretical for ρ=0.6.

In this study the exponential distribution gave negligible significance, in front of [7] where it was accepted. The difference between both studies must not cause surprise, and should be considered as linked with the empirical approach. Note that many factors influence the examined statistics.

It is possible to presume some statistical properties of the inter-arrival time to the pool of channels in the BS, from the available information which belongs to one channel only. As the channels are assigned randomly, the mean inter-arrival time to one channel is C times the mean inter-arrival time to the BS, being C the number of channels allocated to the observed BS. The *cv* should be the same when observed at one single channel and at the BS as explained in [12].

Although blocked arrivals were not included in the empirical sample because they were not seen by the detection equipment, the blocking probability is very small in this scenario, and can only very slightly distort the results presented in Table III. The observation of the *cv* achieved leads to the conclusion that the arrival process is smoother than Poisson.

V. IDLE TIME

The existing relationship among the inter-arrival, channel holding and idle time with the inclusion of the load as in Equation (1), along with the lack of dependency (better say weak dependency) shown in Section III, makes it possible to obtain the properties of one of the three r.v. once known the other two. However, the increasing use of the idle periods for data transmission in mobile networks, makes it worth a direct knowledge of the statistical properties of the idle time.

By using again the MLE and K-S tools, the results presented in Table IV were achieved for the channel idle time. After testing all distributions mentioned in Section IV, the p.d.f. which better fitted the empirical sample was the hyper-erlang-2,2 (a mixture of two erlang-2 with different scale parameter). Note the high significance values and the fact that the significance for the negative exponential was negligible. In Figure 5 the hyper-erlang-2,2 distribution is depicted along with the empirical sample for a load of 60%.

Table IV: Statistical figure of channel idle time and significance of hyper-erlang-2,2.

ρ	Mean (s.)	cv^2	α
0.6	28.75	0.80	0.22
0.5	44.54	0.63	0.37
0.4	57.54	0.58	0.30

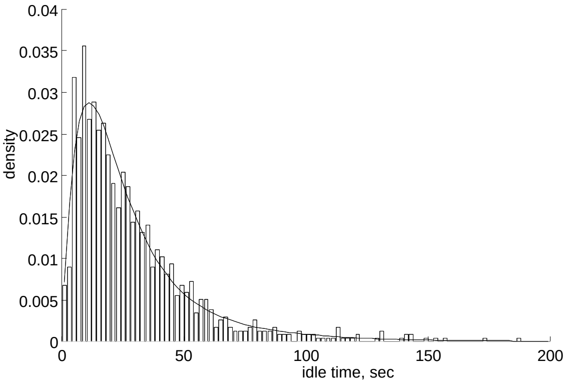

Figure 5. Channel idle time distribution: empirical vs. theoretical for ρ=0.6.

VI. CONCLUSION

The random variables involved with the call process in public cellular networks were analyzed with the use a fully empirical approach. In this work the inter-arrival and idle time of the channel occupancy were focused.

The correlation study showed that the examined r.v. are not related to each other when the observation period is the busy hour. This is the observation period commonly accepted for design purposes. For longer periods the idle time has some non-negligible auto-correlation which could be expected as caused by the slow load variation.

The inter-arrival time to one channel was found to have a *cv* smaller than unity. This leads to the conclusion that in the examined BS the arrivals to the channel were smoother than Poisson traffic. A design assuming Poisson arrivals would cause the system oversize or a GOS better than forecasted. The idle time was proved to have a cv also smaller than unity.

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