An Improved Q-learning Algorithm for Path-Planning of a Mobile Robot

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ABSTRACT

Classical Q-learning requires huge computations to attain convergence and a large storage to save the Q-values for all possible actions in a given state. This paper proposes an alternative approach to Q-learning to reduce the convergence time without using the optimal path from a random starting state of a final goal state, when the Q-table is used for path planning of a mobile robot. Further, the proposed algorithm stores the Q-value for the best possible action at a state, and thus save significant storage. Experiments reveal that the acquired Q-table obtained by the proposed algorithm helps in saving turning angles of the robot in the planning stage. Reduction in turning angles is economic from the point of view of energy consumption by the robot. Thus the proposed algorithm has several merits with respect to classical Q-learning. The proposed algorithm is constructed based on four fundamental properties derived here and the validation of the algorithm is studied with Khepera-II robot.

Keywords*:* Q-learning; Reinforcement learning; Motion planning, Mobile Robot, Energy**.**

1.INTRODUCTION

Machine learning is often used in mobile robots to make the robot aware about its world map. In early research on mobility management of robots, supervised learning was generally employed to train a robot to determine its next position in a given map from the sensory readings about the environment. Supervised learning is a good choice for mobility management of robots in fixed maps. However, if there is a small change in the robot’s world, the acquired knowledge is no longer useful to guide the robot to select its next position. A complete training of the robot with both old and new sensory data-action pairs is then needed to overcome the said problem.

Reinforcement learning is an alternative learning policy, which rests on the principle reward and punishment. No prior training instances are presumed in reinforcement learning. A learning agent here does an action on the environment, and receives a feedback from the environment based on its action. The feedback provides an immediate reward for the agent. The learning agent here usually adapts its parameter based on the current and cumulative (futuristic) rewards. Since the exact value of the futuristic reward is not known, it is guessed from the knowledge about the robot’s world map. The primary advantage of reinforcement learning lies in its inherent power of automatic learning even in presence of small changes in the world map.

Motion planning is one of the important tasks in intelligent control of a mobile robot. The problem of motion planning is often decomposed into path planning and trajectory planning. In path planning, we need to generate a collision-free path in an environment with obstacles and optimize it with respect to some criterion [8], [9]. However, the environment may be imprecise, vast, dynamical and partially non-structured [7]. In such environment, path planning depends on the sensory information of the environment, which might be associated with imprecision and uncertainty. Thus, to have a suitable planning scheme in a cluttered environment, the controller of such kind of robots must have to be adaptive in nature. Trajectory planning, on the other hand, considers time and velocity of the robots, while planning its motion to a goal. It is important to note that path planning may ignore the information about time/velocity, and mainly considers path length as the optimality criterion. Several approaches have been proposed to address the problem of motion planning of a mobile robot. If the environment is a known static terrain and it generates a path in advance, it is said to be off-line algorithm. It is called on-line, if it is capable of producing a new path in response to environmental changes.

Many studies have been carried out on path planning for different types of mobile robots. The works in [5] involve mapping, navigation, and planning tasks for Khepera II mobile robot. In these works, the computationally intensive tasks, such as the planning and mapping tasks were not performed onboard. They were performed on a separate computer. The sensor readings and the motor commands are communicated between the robot and the computer via serial connection.

Usually, the planning involves an action policy to reach a desired goal state, through maximization of a value function [1]-[3], which designates sub-objectives and helps choosing the best path. For instance, the value function could be the shortest path, the path with the shortest time, the safest path, or any combination of different sub-objectives. The definition of a task in this class may contain, besides the value function, some *a priori* knowledge about the domain, e.g., environmental map, environmental dynamics, goal position. Such knowledge allows a robot planning, while the lack of such knowledge obliges the robot either to learn it previously or to make use of heuristic strategies, such as moving to goal direction while avoiding obstacles.

Our research applies reinforcement learning techniques to real-world robots. Reinforcement learning has been tested in many simulated environments [3]- [11] but on a limited basis in real-world scenarios. A real-world environment poses more challenges than a simulated environment, such as enlarged state spaces [2], increased computational complexity, significant safety issues (a real robot can cause real damage), and longer turnaround times for results. This research measures how well reinforcement-learning technique, such as Q-learning, can apply to the real robot for navigational problem, and in our research we modified the classical Q-learning algorithm, hereafter called improved Q-learning for increasing its performance one in the path-planning problem.

In Extended Q-learning [17], every state has got fixed action that has to be performed in order to reach the given goal. Naturally, if the environment is changed after learning then the agent has to learn the changed environment again. This problem is overcome in the algorithm proposed here. In the Extended Q-learning, sometime the robot needs more energy because the robot needs to rotate in selecting the best action. In the newly proposed algorithm, this problem is rectified.

The rest of the paper is organized as follows. Classical Q-learning is introduced in section II. Some properties of the Q-learning based on the concept of locking of states are derived in section III. The algorithm for the Improved Q-learning is given in section IV. The algorithm for the path planning is given in section V. Computer Simulation is shown in section VI. Experimental details are included in section VII. Conclusions are listed in section VIII.

2.THE CLASSICAL Q-LEARNING

In classical Q-learning, all possible states of an agent and its possible action in a given state are deterministically known. In other words, for a given agent A, let S_0, S_1, S_2, ..., S_n, be n-possible states, where each state has m possible actions a_0, a_1, a_2, ..., a_m. At a particular state-action pair, the specific reward that the agent acquires is known as immediate reward. For example r(S_i, a_j) denotes the immediate reward that the agent A acquires by executing an action a_j at state S_i. An agent selects its next state from its current states by using a policy. This policy attempts to maximize the cumulative reward that the agent could have in subsequent transition of states from its next state. For example, let the agent be in state S_i and is expecting to select the next best state. Then the Q-value at state S_i due to action of a_j is given in (1).

$$Q(S_i, a_j) = r(S_i, a_j) + \gamma \underset{a'}{Max}\ Q(\delta(S_i, a_j), a') \quad (1)$$

where $\delta(S_i, a_j)$ denotes the next state due to selection of action a_j at state S_i. Let the next state selected be S_k. Then $Q(\delta(S_i, a_j), a') = Q(S_k, a')$. Consequently selection of a' that maximizing $Q(S_i, a_j)$ is an interesting problem. One main drawback for the above Q-learning is to know the Q-value at a state S_k for all possible action a'. As a result, each time it accesses the memory to get Q-value for all possible actions at a particular state to determine the most appropriate next state. So it consumes more time to select the next state. Since the action a' for which $Q(S_k, a')$ is maximum needs to be evaluated, we can remodel the Q-learning equation by identifying the a' that moves the agent closer to the goal.

In the proposed Improved Q-learning, we have only one field for storing the Q-value and one for storing lock variable of the particular state. In this way the space requirement for storing the Q-table gets reduce.

3. PROPERTIES OF THE IMPROVED Q-LEARNING

Let for any state S_k, the distance between the goal state and the next feasible state of S_k are known. Let the next feasible state of S_k be $S \in \{S_a, S_b, S_c, S_d\}$. Let G be the goal and the city block distance between S_a, S_b, S_c, S_d and G be d_{aG}, d_{bG}, d_{cG} and d_{dG} respectively. Let the distance in order be $d_{bG} < d_{aG} < d_{cG} < d_{dG}$. Then the agent should select the next state S_b from its current state S_k. If the Q-value of the state S_b is known. We can evaluate the Q-value of state S_k by the following approach.

$$Q(S_k, a') = r(S_k, a') + \gamma \underset{a''}{Max}\ Q(\delta(S_k, a'), a'')$$
$$= 0 + \gamma \underset{a''}{Max}\ Q(\delta(S_k, a'), a'') \quad (2)$$

Now $\delta(S_k, a') = S_a \mid S_b \mid S_c \mid S_d$, where $\mid$ denotes OR operator.
Therefore,

$$\underset{a''}{Max}\ Q(\delta(S_k, a'), a'')$$
$$= \underset{a''}{Max}\ Q\{S_a \mid S_b \mid S_c \mid S_d, a''\}$$
$$= Q(S_b, a''). \ (\because d_{bG} < d_{aG} < d_{cG} < d_{dG})$$

Combining (2) and (3) we have : (3)

$$Q(S_k, a') = 0 + \gamma Q(S_b, a'').$$

Thus if the next state having the shortage distance with the goal is known, and the Q-value of this state is also known, then the Q-value of the current state is simply γ×Q-value of the next state.

Let S_p, S_n and S_G be the present, next and the goal states respectively. Let Q_p and Q_n be the Q-value at the present and the next states S_p and S_n respectively. Let d_{xy} be the city block distance between the states S_x and S_y. We use a Boolean variable Lock: L_x to indicate that the Q_x value of a state is fixed permanently. We set lock L_n=1 if the Q-value of the state n is fixed, and won't change further after L_n is set to 1. The Lock variable for all states except the goal will be initialized zero in our proposed Q-learning algorithm. We observe four interesting properties as indicated below.

Property 1: If L_n=1 and $d_{pG} < d_{nG}$ then $Q_p = \gamma \times Q_n$ and set *L_p = 1*.

Proof. Let the neighborhood state of S_p be $S \in \{S_a, S_b, S_c, S_n\}$, and the agent selects S_n as the next state as $d_{nG} < d_{xG}$ for $x \in \{a, b, c, n\}$.

Now,

$$Q_p = Q(S_p, a)$$
$$= r(S_p, a) + \gamma \underset{a'}{Max} Q(\delta(S_p, a), a')$$
$$= 0 + \gamma \underset{a'}{Max} Q(S_a \mid S_b \mid S_c \mid S_n, a')$$
$$= \gamma \times Q(S_n, a') \ (\because d_{nG} \le d_{xG} \text{ for } x \in \{a, b, c, n\})$$
$$= \gamma \times Q_n.$$

Since L_n =1, and $d_{pG} > d_{nG}$, $\therefore Q_p < Q_n$, and thus $Q_p = \gamma \times Q_n$ for $0 < \gamma < 1$ is the largest possible value of Q_p, and Q_p should not be updated further. So, *L_p = 1* is set.

Property 2: If L_n=0 and $d_{pG} > d_{nG}$ then $Q_p = Max(Q_p, \gamma \times Q_n)$.

Proof. Let the neighborhood state of S_p be $S \in \{S_a, S_b, S_c, S_n\}$, and the agent selects S_n as the next state as $d_{nG} \le d_{xG}$ for $x \in \{a, b, c, n\}$.

Now,

$$\begin{aligned}
Q_p &= Q(S_p, a) \\
&= r(S_p, a) + \gamma \underset{a'}{Max}\, Q(\delta(S_p, a), a') \\
&= 0 + \gamma \underset{a'}{Max}\, Q(S_a \mid S_b \mid S_c \mid S_n, a') \\
&= \gamma \times Q(S_n, a') \quad (\because d_{nG} \le d_{xG} \text{ for } x \in \{a,b,c,n\}) \\
&= \gamma \times Q_n. \qquad (4)
\end{aligned}$$

Since $d_{pG} > d_{nG}$, $Q_p < Q_n$. So, $Q_p = \gamma \times Q_n$ is logically satisfied. However, as L_n=0, Q_n is not yet stable at the current iteration. So, Q_p is also not stable yet. Let t be the current iteration. Then

$$\left.\begin{aligned}
Q_p(t) &= \gamma \times Q_n(t) \text{ if } Q_p(t-1) \le \gamma \times Q_n(t). \\
&= Q_p(t-1), \text{ otherwise.}
\end{aligned}\right\} \qquad (5)$$

The two alternatives in (5) can be put together as in (6).

$$Q_p(t) = Max(Q_p(t-1), \gamma Q_n(t)) \qquad (6)$$

Ignoring the notion of time t from (6), we obtain $Q_p(t) = Max(Q_p, \gamma \times Q_n)$.

□

Property 3: If $L_p = 1$ and $d_{nG} > d_{pG}$ then $Q_n = \gamma \times Q_p$ and set $L_n = 1$.

Proof. Let the neighborhood of the next state S_n be $S \in \{S_a, S_b, S_c, S_p\}$. Since $d_{pG} \le d_{xG}$ for $x \in \{a, b, c, p\}$, the agent will select the state S_p during its transition form S_n.

Now,

$$\begin{aligned}
Q_n &= Q(S_n, a) \\
&= r(S_n, a) + \gamma \underset{a'}{Max}\, Q(S_a \mid S_b \mid S_c \mid S_p, a') \\
&= 0 + \gamma \underset{a'}{Max}\, Q(S_a \mid S_b \mid S_c \mid S_p, a') \\
&= \gamma \times Q(S_p, a') \quad (\because d_{pG} \le d_{xG}, \forall x). \\
&= \gamma \times Q_p.
\end{aligned}$$

Since $d_{nG} > d_{pG}$, $Q_n < Q_p$. So, $Q_n = \gamma \times Q_p$ is logically acceptable for $0 < \gamma < 1$. Further, as L_p=1, and S_n is the nearest state to S_p with respect to given distance metric, therefore L_n is set to 1.

Property 4: *If* $L_p=0$ and $d_{nG} > d_{pG}$ then $Q_n = Max(Q_n, \gamma \times Q_p)$.

Proof. Let the neighborhood of state S_n be $S \in \{S_a, S_b, S_c, S_p\}$. Let $d_{pG} > d_{xG}$ for $x \in \{a, b, c, p\}$. Then the agent will select S_p during its transition from S_n,

Now,

$$\begin{aligned}
Q_n &= Q(S_n, a) \\
&= r(S_n, a) + \gamma \underset{a'}{Max}\, Q(\delta(S_n, a), a') \\
&= r(S_n, a) + \gamma \underset{a'}{Max}\, Q(S_a \mid S_b \mid S_c \mid S_p, a') \\
&= 0 + \gamma \times Q(S_p, a') \quad (\because d_{pG} \le d_{xG}, \forall x) \\
&= \gamma \times Q_p.
\end{aligned}$$

Since $d_{nG} > d_{pG}$, $\therefore Q_n < Q_p$. So, $Q_n = \gamma \times Q_p$ for $0 < \gamma < 1$ is logically satisfactory. Now, as L_p=0, Q_p is not fixed until this iteration; So Q_n is also not fixed in the current iteration. Now, adding the notion of iteration t in Q_n and Q_p, we have;

$$\left.\begin{aligned}
Q_n(t) &= \gamma \times Q_p(t), \text{ if } Q_n(t-1) < \gamma \times Q_p(t), \\
&= Q_n(t-1), \text{ otherwise.}
\end{aligned}\right\} \qquad (7)$$

Combining the expressions under (7) by a single expression, we write:

$$Q_n(t) = Max(Q_n(t-1), \gamma \times Q_p(t)) \qquad (8)$$

Now, eliminating t from both sides of (8), we obtain,

$$Q_n = Max(Q_n, \gamma \times Q_p). \qquad (9)$$

4. THE IMPROVED Q-LEARNING ALGORITHM

In the new Q-learning algorithm to be proposed, only two fields for each grid are required, one is used to store Q-value and other is used to store the Lock variable. In the new algorithm every state stored only the Q-value and not the best action to reach the goal state. The best path between the current state to the goal is given by the city-block distance; therefore there will be more than one best path to reach the goal. As a result selection of the best action is deferred till the path planning by doing so the energy required by the robot can be minimized. The newly proposed algorithm has four steps. Initialization of Q-table is done in step 1, in step 2, value of γ and starting state of the robot are initialized. Q-table is updating only after the robot reaches the goal state for the first time. Thus the robot is allowed to come to the goal state without updating Q-table. This is done in step 3. The updating of the Q-table is done in step 4 using the properties introduced in section III.

Pseudo Code for Improved Q-learning

```
1. Initialization
   For all $S_i$, i = 1 to n except $S_i = S_G$
   {set $L_i = 0; Q_i = 0;$
   $L_G$ (for goal $S_G$) = 1;
   $Q_G$ (for goal $S_G$) = 100;
2. Assign $\gamma$ in (0,1) and initial state= $S_p$;
3. Repeat
    {
       Select $a_i$ from $A = \{a_1, a_2, ..., a_m\}$;
    } Until $S_P = S_G$;
4. Update Q – table:
     Repeat
     {
        a) Select $a_i$ from $A = \{a_1, a_2, ..., a_m\}$;
        b) Determine $d_{nG}$ and $d_{pG}$;
       If ($d_{nG} < d_{pG}$)
      Then if $L_n = 1$
             Then if $L_p = 0$
                    Then {$Q_p = \gamma \times Q_n; L_p = 1;$}
             Else if ($Q_p < \gamma \times Q_n$)
                  Then { $Q_p = \gamma \times Q_n$};
       Else   if $L_p = 1$
             Then if $L_n = 0$
                    Then {$Q_n = \gamma \times Q_p; L_n = 1;$}
             Else if ($Q_n < \gamma \times Q_p$)
                  Then {$Q_n = \gamma \times Q_p$}
     } Until $L_i = 1$ for all i;
```

Space-Complexity: In classical Q-learning, if there are n states and m action per state, then the Q-table will be of $(m \times n)$ dimension. In the Improved Q-learning, for each state 2 storages are required, one for storing Q-value and other for storing value of the lock variable of a particular state. Thus for n number of states, we require a Q-table of $(2 \times n)$ dimension. The saving in memory in the present context with respect to classical Q thus is given by mn – 2n = n(m – 2).

Time-Complexity: In classical Q-learning, the updating of Q-values in a given state requires determining the largest Q-value, in that cell for all possible actions. Thus if there are m possible actions at a given state, maximization of m possible Q-values, require m −1 comparison. Consequently, if we have n states, the updating of Q values of the entire Q table by classical method requires n(m −1) comparisons. Unlike the classical case, here we do not require any such comparison to evaluate the Q values at a state S_p from the next state S_n. But we need to know whether state n is locked i.e., Q-value of S_n is permanent and stable. Thus if we have n number of states, we require n number of comparison. Consequently, we save n(m −1) – n = nm – 2n= n(m–2).

5. PATH PLANNING ALGORITHM

In newly proposed Improved Q-learning, best action at each state is selected dynamically. This help in reducing the energy required by the agent while moving. In the planning algorithm the best action is selected by comparing the Q-value of the 4-neighbour states. If there are more than one state with equal Q-value than the action that requires minimum energy for turning is selected.

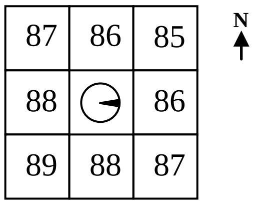


Fig. 1: Portion of environment with Q-value and robot at center.

In Fig. 1 the robot is at the centre facing toward east. Next feasible states are the 4-neighbour states and corresponding Q-value are given in each state. There are two states with maximum Q-value of 88. If the next state is located at south then the robot has to rotate toward right by 90^0 and if the next state is located at west then the robot has to rotate by 180^0. Therefore, the presumed next state will be the state at south as the robot requires less energy to move to that state. During path planning the robot selects one out of several best actions at a given state. If an obstacle is present in the selected direction then next optimal path is selected. If there is no action to be selected then a signal is passed to the robot to backtrack one step.

Pseudo Code for Path Planning

```
$BEGIN$
   1. $CURRENT = starting - state$;
   2. Get next state $NEXT$, where Q - value of the $NEXT \geq$ Q - value in
      the neighbourhood of $CURRENT$ and $NEXT$ is Obstacle - free;
   3. Go to $NEXT$;
   4. $CURRENT = NEXT$;
   5. Repeat from step 2 until $NEXT = GOAL$;
$END$
```

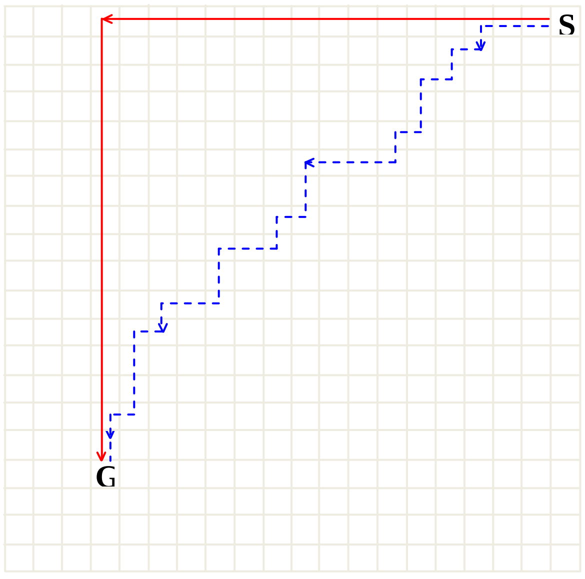
6. COMPUTER SIMULATION

We designed an environment of 20×20 grids. Every grid is given a state number. For a state at (x, y),

$$state\ no. = (x - 1) \times rowsize + y \tag{10}$$

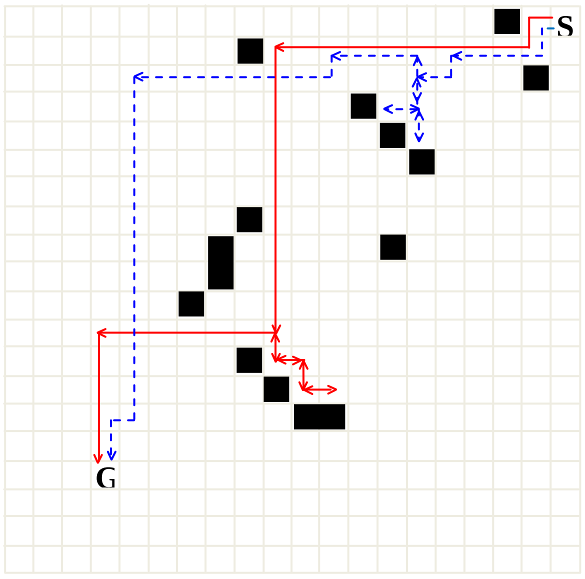
where x is the no of block in x direction
y is the no of block in y direction
$rowsize$ is the length of the row

We make the agent learn the environment with the proposed new algorithm and the original classical Q-learning algorithm for verifying the applicability of the proposed algorithm. We stop the classical Q-learning after 50 thousand iterations. For the new algorithm we allow to learn till all the Lock variables are set. Total numbers of iteration varies from 11 thousand to nearly 25 thousand as the position of the goal changes. Learning is done without any obstacle. After completion of learning we have kept the agent, facing left, in different environments and allow traversing to the goal using the stored Q-table by both the algorithm. The results are shown in the Fig. 2-Fig. 6 and Table 1.



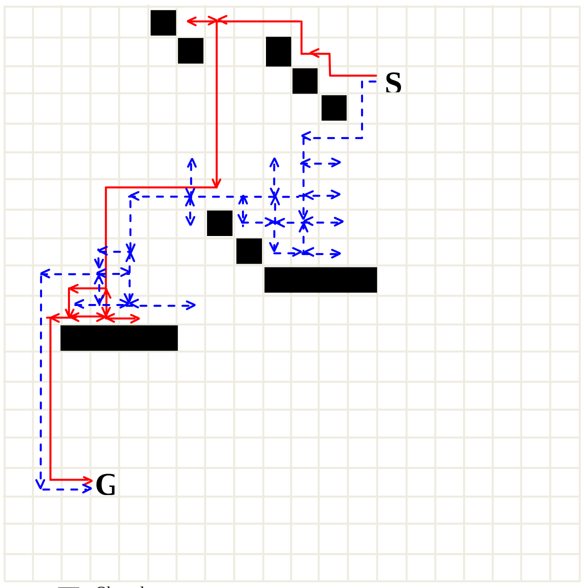
- - - Path taken by the robot with stored Q-table by Classical Q-learning
—— Path taken by the robot with stored Q-table by Improved Q-learning

Figure 2: Environment 1 without obstacle.



■ Obstacle
- - - Path taken by the robot with stored Q-table by Classical Q-learning
—— Path taken by the robot with stored Q-table by Improved Q-learning

Figure 3: Environment 2 with obstacles.

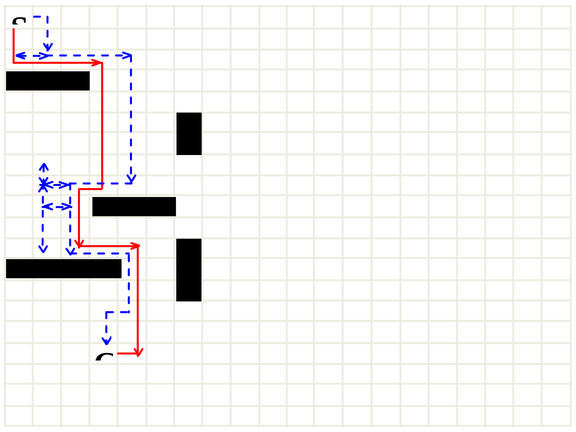


■ Obstacle
- - - Path taken by the robot with stored Q-table by Classical Q-learning
—— Path taken by the robot with stored Q-table by Improved Q-learning

Fig. 4: Environment 3 with obstacles.

TABLE I. **COMPARISON OF TIME TAKEN BY THE ROBOT AND NO. OF 90^0 TURN REQUIRED BY THE ROBOT IN DIFFERENT ENVIRONMENT BY CLASSICAL Q-LEARNING (CQL) AND IMPROVED Q-LEARNING (IQL)**

S.No	Environment	Time Taken in sec		No. of 90^0 Turn	
		IQL	CQL	IQL	CQL
1	1	22.13	25.65	1	19
2	2	30.98	35.32	17	25
3	3	32.35	66.40	24	70
4	4	17.85	32.24	10	27
5	5	22.85	33.72	11	31



Obstacle
Path taken by the robot with stored Q-table by Classical Q-learning
Path taken by the robot with stored Q-table by Improved Q-learning

Fig. 5: Environment 4 with obstacles.

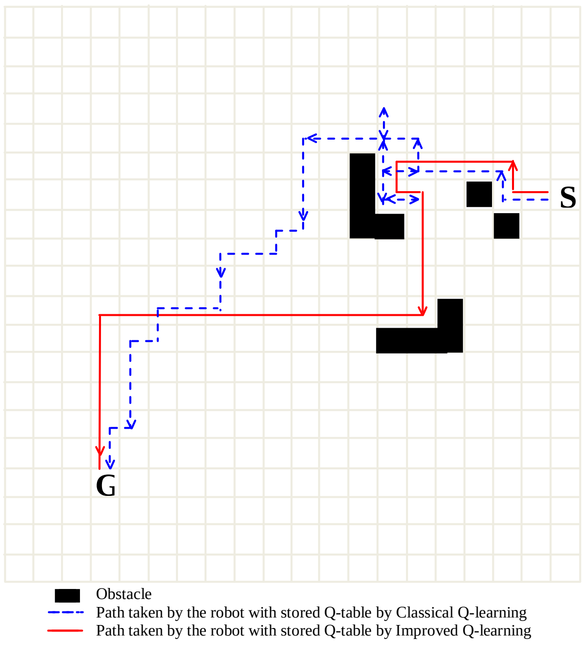

S
G
Obstacle
Path taken by the robot with stored Q-table by Classical Q-learning
Path taken by the robot with stored Q-table by Improved Q-learning

Fig. 6: Environment 5 with obstacles.

7.EXPERIMENTS WITH KHEPERA II ROBOT

Khepera II (Figure 7) is a miniature robot (diameter of 8 cm) equipped with 8 built-in infrared range and light sensors, and 2 relatively accurate encoders for the two motors. The range sensors are positioned at fixed angles and have limited range detection capabilities. We numbered the sensors clockwise from the leftmost sensor to be sensor 0 to sensor 7(Figure 8). Sensor values are numerical ranging from 0 (for distance > 5 cm) to 1023 (approximately 2 cm).The onboard Microprocessor has a flash memory size of 256KB, and the CPU of 8 MHz. Khepera can be used on a desk, connected to a workstation through a wired serial link. This configuration allows an optional experimental configuration with everything at hand: the robot, the environment and the host computer.

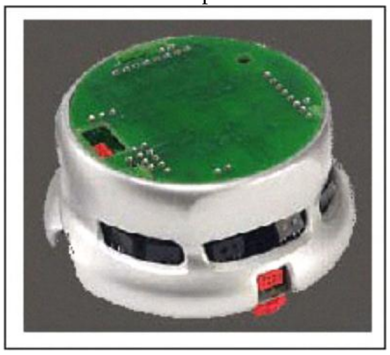


Fig. 7: The Khepera II Robot

Experimental world map with and without obstacle is shown in Fig 9 and Fig 10. The Improved Q-learning algorithm is used in the first phase to learn the movement steps from each grid in the map to its neighbor. After the learning phase is over, the path planning algorithm is executed. The snapshots with and without obstacles are indicated in Fig. 11 and Fig. 12 respectively.

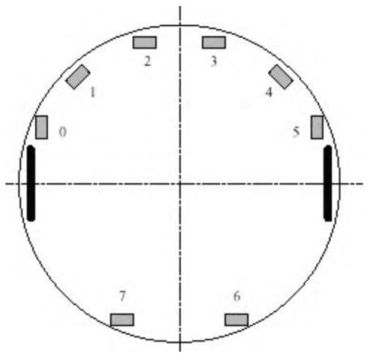


Fig. 8: Position of the sensors of Khepera II

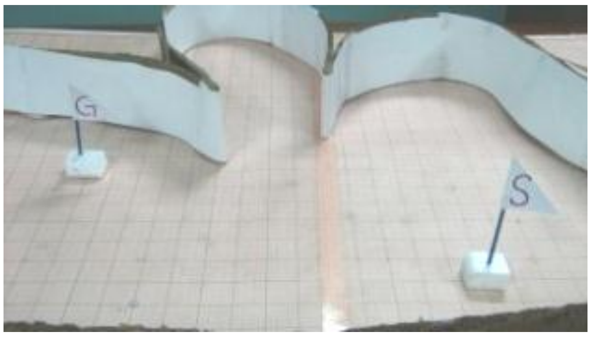


Fig. 9. Experimental world map without obstacle.

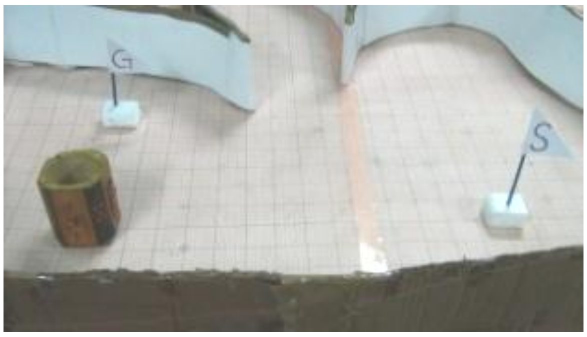


Fig. 10. Experimental world map with obstacle.

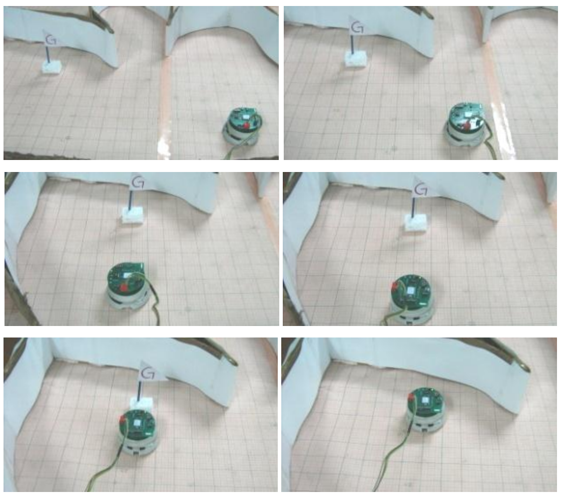


Fig. 11. Snapshots of experimental planning (without obstacle) instances in order.

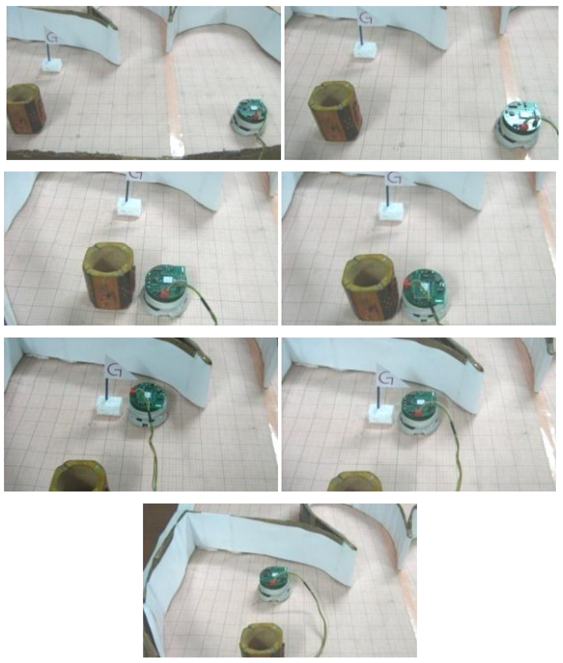


Fig. 12. Snapshots of experimental planning (with one obstacle) instances in order.

8. CONCLUSIONS

The paper proposes a novel approach to reduce both time – and space-complexity of the Classical Q-learning algorithm. A mathematical foundation of the proposed algorithm indicates the correctness, and the experiments on simulated and real platform validate without losing the optimal path in path-planning applications of a mobile robot. The algorithm is coded in C-language and tested on IBM Pentium based simulation environment. The field study is performed on Khepera-II platform. Simulation results also confirm less number of 90^0 turning of the robot around its z-axis with respect to classical Q-learning, and the proposed algorithm helps in energy saving of the robot in path-planning application.

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