Preprints of the 8th IFAC/IFIP/IFORS Symposium on Transportation Systems, Chania, Greece, June 1997, pp. 69–74.

STRING STABILITY PROPERTIES OF AHS LONGITUDINAL VEHICLE CONTROLLERS

Jennifer Eyre Diana Yanakiev
Ioannis Kanellakopoulos

UCLA Electrical Engineering, Los Angeles, CA 90095-1594

Abstract: A recently proposed strategy for evaluating the string stability properties of longitudinal vehicle controllers used in Automated Highway Systems (AHS) is to consider the platoon as a mass-spring-damper system. This simplified analysis framework results in linear closed-loop systems, yielding transfer functions which characterize the spacing error response of the platoon. These transfer functions are then used to compare the string stability properties of a variety of longitudinal vehicle controllers, sometimes with unexpected results.

Keywords: IVHS, Interconnected systems, Stability analysis, Error transfer functions, Impulse responses, Vehicles

1. INTRODUCTION

Automated vehicles comprising platoons must exhibit both individual stability and stability as a group, a property referred to as "string stability". String stability is typically defined as the requirement that spacing errors (the difference between the actual and desired intervehicle spacing) are attenuated as they propagate through the platoon, thus eliminating the so-called "slinky effect" and reducing the likelihood of collisions. Unfortunately, string stability analysis is complicated by the presence of severe nonlinearities in realistic vehicle models. Linearized models are thus often used for this purpose, since for small deviations from the nominal operating conditions they retain much of the information contained in the nonlinear model. Yanakiev and Kanellakopoulos (1996) proposed a simplified framework in which the platoon is viewed as a (linear) mass-spring-damper system. This framework combines the tractability of linear analysis with the physical intuition of mechanical systems, and yields transfer functions which characterize the spacing error response of the platoon. Analysis of these transfer functions can be used to determine the string stability properties of a platoon operating under a given control scheme. Of course, the conclusions drawn from this linear approximation are to be used as guidelines, rather than rules, for longitudinal controller selection in AHS.

Spacing error attenuation is generally viewed as the only requirement for string stability. However, there is another issue to be addressed. Consider, for example, the following scenario. In a platoon with one meter (1 m) nominal intervehicle spacing, the lead vehicle accelerates and generates an 8 m positive spacing error between itself and its follower. This error is then propagated as a negative spacing error of 1.5 m between the second and third vehicles, and a collision occurs. This example of unacceptable platoon performance illustrates the fact that guaranteeing spacing error attenuation does not eliminate the possibility that a large positive spacing error may generate a smaller, but negative, error upstream. The issue here is not one of avoiding position overshoot during platoon braking maneuvers, which is impossible. Rather, it is one of avoiding overshoot in response to lead vehicle acceleration. To eliminate this possibility, one must ensure that the impulse response of the spacing error (the inverse Laplace transform of the spacing error transfer function) remains positive.

It is worth mentioning here that, although AHS-specific inputs do not take the form of impulse functions, position overshoot has been observed in systems that have impulse response undershoot, even during routine longitudinal maneuvers. This leads to the conclusion that a positive impulse response is not too stringent a requirement if one wishes to eliminate any possibility of overshoot during acceleration maneuvers.

As shown in (Swaroop *et al.*, 1994), requiring a positive impulse response has an additional benefit: it makes possible the use of frequency-domain analysis for string stability. If $G(s)$ is the spacing error transfer function and $g(t)$ its inverse Laplace transform, then

$$\|g * z\|_\infty \;\le\; \|g\|_1 \, \|z\|_\infty \,, \tag{1}$$

where $z(t)$ is the input spacing error and

$$\|g\|_1 \;=\; \int_0^\infty |g(t)| \, dt \,. \tag{2}$$

Therefore, the necessary and sufficient requirement for spacing error attenuation is that the corresponding linear operator's L_∞-induced norm is less than one, i.e., that

$$\|g\|_1 \;\le\; 1 \,. \tag{3}$$

From linear systems theory, it is known that

$$|G(0)| \;\le\; \|G\|_\infty \;=\; \max_\omega |G(j\omega)| \;\le\; \|g\|_1 \,. \tag{4}$$

Using the definition of the Laplace transform,

$$|G(0)| \;=\; \left|\int_0^\infty g(t)\, dt\,\right| \;\le\; \int_0^\infty |g(t)| dt \;=\; \|g\|_1 \,. \tag{5}$$

If the impulse response $g(t)$ remains positive, then

$$\|G\|_\infty \;=\; |G(0)| \;=\; \|g\|_1 \,. \tag{6}$$

This is the only case in which the L_1 norm of $g(t)$ can be evaluated in the frequency domain. In this case, the condition $\|g\|_1 \le 1$ is replaced by the equivalent condition

$$\|G\|_\infty \le 1 \,, \quad g(t) \ge 0 \;\forall\, t \,. \tag{7}$$

It is important to note that, if the impulse response is not positive, frequency-domain analysis will guarantee string stability only in the L_2 sense, since $\|G\|_\infty$ is the frequency-domain equivalent of the L_2-induced norm. Under this condition, ensuring L_∞ string stability requires analysis in the time domain.

String stability analysis reveals how spacing errors are propagated through the platoon, but does not offer much insight into the behavior of the initial spacing error generated by applying a control input to the lead vehicle. The framework for analysis used here lends itself to analyzing this initial error, by allowing computation of the transfer function relating the input force applied to the system to the spacing error between the first two vehicles. Again, it is desirable that this transfer function should result in a positive impulse response to avoid a negative spacing error. As will be shown, the relationship between the input force and the initial spacing error may reveal important qualitative information about the platoon's performance that is not contained in the spacing error transfer function.

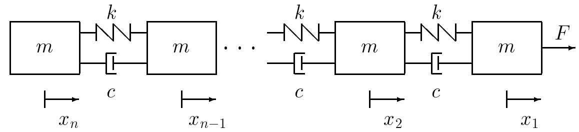

Fig. 1. Mass-spring-damper system.

2. ANALYSIS

2.1 *Analytical Framework*

In the analysis presented here, the platoon is viewed as a mass-spring-damper system (Yanakiev and Kanellakopoulos, 1996). The vehicles are represented as masses, and the electronic couplings between them are considered to be springs and dampers. The spring and damping constants represent the control gains on the relative position and velocity, respectively. Transfer functions describing the propagation of spacing errors are generated by deriving a state-space representation of the system using the position of each vehicle as a state, and then translating this representation into error coordinates.

The two major classes of controllers considered here are unidirectional (forward-looking only), and bidirectional (forward and backward-looking), originally proposed as a means to improve platoon performance and safety (Yang and Tongue, 1996). For the bidirectional controller, each mass is considered to be coupled by springs and dampers to both the preceding and following masses, allowing it to use information about the relative distance and velocity of both the immediately preceding and immediately following vehicles. In the unidirectional control scenario, each mass is considered to be connected only to its immediate predecessor without being affected by its follower. This is not representative of a physical mass-spring-damper system, but is still useful for analysis of electronically coupled vehicle strings.

Within the mass-spring-damper framework we may consider several intervehicle spacing policies, each of which has differing implications for string stability.

2.2 *Constant Intervehicle Spacing*

The first controller we will consider employs constant intervehicle spacing. In the unidirectional scenario, it is well-known that string stability cannot be achieved for autonomously operating vehicles with constant spacing, so we will focus on the stability properties of the bidirectional controller. Using bidirectional control, the transfer function relating spacing errors between adjacent vehicles changes based on the position of the vehicle in the platoon. This is because each vehicle feels the combined effects of all the preceding and following vehicles. Starting from the end of the platoon and assuming a platoon of size n, the following transfer function is obtained:

$$G_1(s) = \frac{z_{n-1}(s)}{z_{n-2}(s)} = \frac{\frac{c}{m}s + \frac{k}{m}}{s^2 + \frac{2c}{m}s + \frac{2k}{m}} , \qquad (8)$$

where z_{n-1} is the spacing error between the last two vehicles, z_{n-2} is the spacing error between the $n-1^{\text{st}}$ vehicle and its predecessor, c is the damping factor, k is the spring constant, and m is the vehicle mass (for the derivation of transfer functions, the reader is referred to (Yanakiev and Kanellakopoulos, 1996)). In general, a necessary condition for achieving a positive impulse response is that the dominant pole of the system is real and lies to the right of the dominant zero (Swaroop and Niemann, 1996). In the above transfer function, the poles cannot be moved relative to the zero in a way which satisfies this condition; the poles and zeros are coupled and cannot be placed independently. Hence, the impulse response of this system always crosses the zero axis in some finite positive time t, regardless of parameter choice. By moving the dominant pole (and zero) closer to the $j\omega$-axis, the magnitude of the undershoot of the impulse response can be made arbitrarily small. This improvement in platoon performance comes at the expense of individual vehicle performance, however, and thus some tradeoff must be made between the percentage of undershoot acceptable (if any) and controller performance. One can analytically determine the necessary and sufficient conditions for which the magnitude of this transfer function is less than one for all frequencies:

$$|G_1(j\omega)| < 1 \;\; \forall\, \omega \quad \text{iff} \quad \frac{c^2}{km} > 0.179 \, . \qquad (9)$$

Since the impulse response is not positive, however, this criterion guarantees only L_2 stability.

Moving forward through the platoon, we arrive at the following iterative formula for computing the spacing error transfer function:

$$G_i(s) = \frac{z_i(s)}{z_{i-1}(s)} = \frac{G_1}{1 - G_{i-1}G_1} \, . \qquad (10)$$

It should be clear from inspection of the form of $G_i(s)$ that, for platoons with more than three vehicles, the analysis becomes quite difficult. It is not possible to analytically determine conditions under which the magnitude is less than unity; we must instead resort to numerical methods. It can be shown, however, that the requirement will always be of the form $\frac{c^2}{km} > C$, where C is a constant that increases with platoon size. Analysis has indicated that there is no upper bound on C; as one would suspect, stability continues to becomes more difficult to achieve as vehicles are added to the platoon. Figure 2 plots the increasing value of C as a function of platoon size.

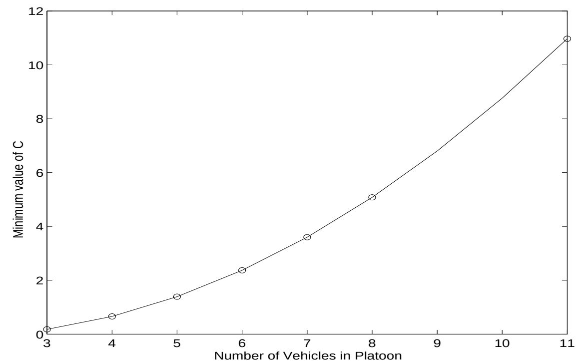

Fig. 2. Minimum value of C versus platoon size.

For a 3-vehicle platoon, the transfer function relating the input force $F(s)$ to the spacing error $z_1(s)$ between the first two vehicles is the following:

$$\frac{z_1(s)}{F(s)} = \frac{s^2 + \frac{2c}{m}s + \frac{2k}{m}}{m(s^2 + \frac{c}{m}s + \frac{k}{m})(s^2 + \frac{3c}{m}s + \frac{3k}{m})} \, . \qquad (11)$$

By careful choice of parameters c and k, it is possible to ensure a positive impulse response and hence avoid position overshoot between the first two vehicles.

2.3 *Speed-dependent Intervehicle Spacing*

2.3.1. *Unidirectional controller*

Speed-dependent spacing has been suggested for use in the unidirectional scenario as a method of achieving string stability for autonomously operating vehicles. For a platoon with constant time headway h, the spacing error transfer function becomes

$$G(s) = \frac{z_i(s)}{z_{i-1}(s)} = \frac{\frac{c}{m}s + \frac{k}{m}}{s^2 + (\frac{c+kh}{m})s + \frac{k}{m}} \, . \qquad (12)$$

The introduction of the time headway term allows the poles of the system to be moved independently from the zero, resulting in a positive impulse response if $h > \frac{m}{c}$. Previous analysis (Yanakiev and Kanellakopoulos, 1996) has shown that the magnitude of the transfer function is less than one when

$$c > \frac{2m - kh^2}{2h} \, . \qquad (13)$$

In this case, the condition leading to a positive impulse response is more restrictive than that for magnitude attenuation. Therefore, string stability can be guaranteed if $h > \frac{m}{c}$.

For this controller, the relationship between the input force and the initial spacing error is

$$\frac{z_1(s)}{F(s)} = \frac{1 - \frac{ch}{m}}{s^2 + (\frac{c+kh}{m})s + \frac{k}{m}} \ . \qquad (14)$$

Because it has the same poles as (12) and no zeros, this system has the advantage of having a positive impulse response when $G(s)$ does.

2.3.2. *Bidirectional Controller*

In the bidirectional scenario, speed-dependent spacing has several possible implementations. If a vehicle's desired separation from its predecessor varies as a function of its own velocity, the spacing error transfer function (again, starting from the end of the platoon) becomes the following:

$$\bar{G}_1(s) = \frac{z_{n-1}(s)}{z_{n-2}(s)} = \frac{\frac{c}{m}s + \frac{k}{m}}{s^2 + (\frac{2c+kh}{m})s + \frac{2k}{m}} \ . \qquad (15)$$

As in the previous case, the impulse response of this system will be positive when $h > \frac{m}{c}$. Closed-form analysis of the magnitude of the transfer function is only possible for the case in which there are three vehicles in the platoon, when we obtain the following:

$$|\bar{G}_1(j\omega)| \ < \ 1 \ \forall \ \omega \text{ iff}$$
$$\frac{c}{m} > -\frac{2}{3}\frac{hk}{m} + \frac{1}{3}\sqrt{\left(\frac{hk}{m}\right)^2 + 1.608\frac{k}{m}} \ . \qquad (16)$$

If $\frac{c^2}{km} > .179 (= \frac{1.608}{9})$, which is the condition derived for magnitude attenuation when a constant spacing policy is used, this requirement will be satisfied for any $h \geq 0$. In addition, for any choice of system parameters there exists an $h > 0$ for which this condition will be satisfied. The requirements for achieving error attenuation, therefore, are less restrictive than those derived for the bidirectional controller with constant intervehicle spacing. Figure 3 illustrates the effect of increasing h on the magnitude of the transfer function. Parameters selected from the region to the left of the curve will result in magnitudes of greater than one, while in the region to the right of the curve the magnitude will be less than one. It is clear that, as one would assume, increasing h expands the region of stability.

The iterative formula for this spacing policy is of the same form we have seen earlier:

$$\bar{G}_i(s) = \frac{\bar{G}_1(s)}{1 - \bar{G}_1(s)\bar{G}_{i-1}(s)} \ . \qquad (17)$$

If, in addition to the forward-looking speed-dependent spacing term considered above, the

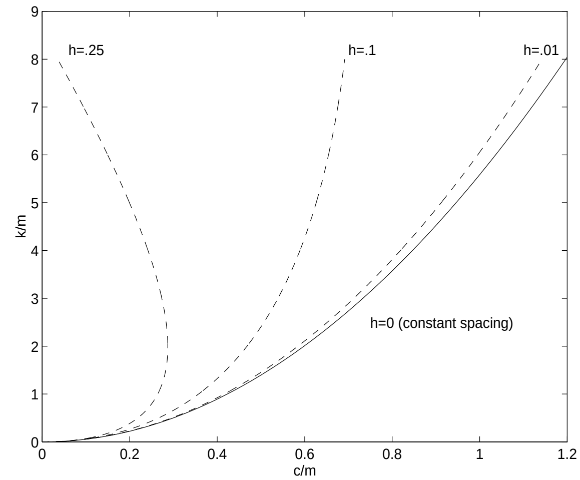

Fig. 3. Regions of error attenuation for 3-vehicle platoon under bidirectional control

vehicle's desired separation from its follower depends on the following vehicle's velocity, it can be shown that $\hat{G}_1(s) = \bar{G}_1(s)$. The iterative formula, however, is no longer of the familiar form. It is modified to:

$$\hat{G}_i(s) = \frac{\hat{G}_1(s)}{1 - \left(\hat{G}_1(s) + \frac{\frac{kh}{m}s}{s^2 + \frac{2c+kh}{m}s + \frac{2k}{m}}\right)\hat{G}_{i-1}(s)} \ . \qquad (18)$$

One may suspect, after comparing $\bar{G}_i$ to $\hat{G}_i$, that error attenuation is more easily achieved with the forward-looking only time headway approach. For $i > 1$, these transfer functions are not amenable to closed-form analysis, but we have been able to confirm the verity of our intuition for a 4-vehicle platoon using numerical analysis, and expect similar results for larger platoons. Figure 4 shows the regions of error attenuation for 4-vehicle platoons using zero time headway, forward-only time headway, and forward and backward time headway. Again, choice of parameters in the area to the right of the curve results in transfer function magnitudes of less than unity. These curves were generated by varying the parameters $\frac{c}{m}$ and $\frac{k}{m}$ and evaluating the magnitude of the transfer function over the range of frequencies for which it is maximized. Points at which the magnitude became equal to one are plotted as small circles, with a curve interpolated between them.

2.4 *Intervehicle Communication*

Achieving string stability for a unidirectional controller with constant intervehicle spacing requires the introduction of some form of intervehicle communication. The string stability properties of controllers using several possible communication schemes will be examined here.

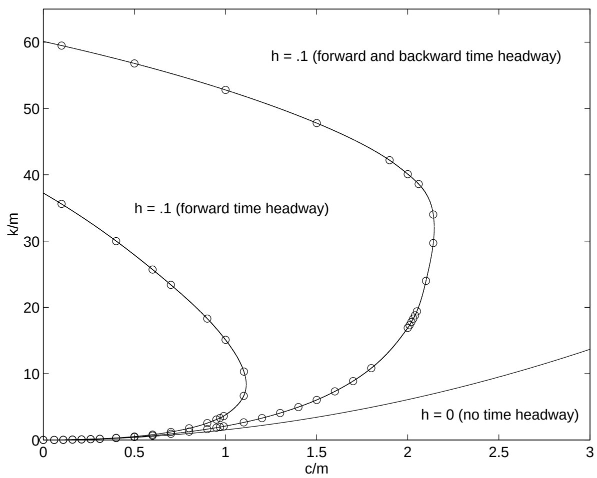

Fig. 4. Regions of error attenuation for 4-vehicle platoon under bidirectional control

The first method considered is one in which the platoon leader broadcasts its current velocity to other platoon members. In the mass-spring-damper framework, this can be represented by connecting an additional unidirectional damper between each vehicle and the platoon leader. Following vehicles, then, would still be able to use information about the position and velocity of their predecessor while also having access to information about the platoon leader's speed. This controller results in the following spacing error transfer function

$$G(s) = \frac{z_i(s)}{z_{i-1}(s)} = \frac{\frac{c}{m}s + \frac{k}{m}}{s^2 + \frac{c+c_d}{m}s + \frac{k}{m}} \ , \tag{19}$$

where c_d represents the additional damping with respect to the platoon leader.

In addition,

$$\frac{z_1(s)}{F(s)} = \frac{\frac{1}{m}}{s^2 + \frac{c+c_d}{m}s + \frac{k}{m}} \ , \tag{20}$$

which, as in the previous case, has the benefit of having no zeros and the same poles as (19).

We can also consider the possibility of disregarding the velocity of each vehicle's predecessor (while still using relative position information) and referencing only the platoon leader's speed (Shladover, 1978). Although at first glance it may seem counterintuitive, ignoring the preceding vehicle's velocity actually improves both the impulse response and the magnitude response of the linear system by removing the zero from the transfer function. Simulations of this controller using the full nonlinear longitudinal platoon model confirm that it results in a string-stable platoon. It should be noted, however, that the mass-spring-damper analysis framework is a simplified representation of a complex physical system, and while useful for qualitative analysis and comparison of control methods it may not illuminate all the possible disadvantages of ignoring the preceding vehicle's speed. This control method corresponds to removal of the damper between each vehicle and its predecessor, while retaining the damper connection to the platoon leader. The resulting transfer function is

$$G(s) = \frac{\frac{k}{m}}{s^2 + \frac{c_d}{m}s + \frac{k}{m}} \ . \tag{21}$$

Since this is a second-order system without zeros, the impulse response will be strictly positive when the poles are real.

An alternative communication scheme (Shladover, 1978; Yanakiev and Kanellakopoulos, 1996) is for the platoon leader to transmit its desired rather than actual velocity, which would presumably result in less frequent intervehicle transmissions and hence reduced bandwidth. Intuitively, it is expected that this controller will result in smoother transitions, because all vehicles in the platoon are given preview information about their desired velocity. This scheme is represented with the addition of a "virtual" mass traveling in front of the platoon at the desired speed, to which every other mass (including the leader) is connected via a unidirectional damper. Again, it is possible to either use or disregard the preceding vehicle's speed in the controller design. An interesting result of our analysis is that this control scheme yields spacing error transfer functions identical to those derived for the case in which the actual velocity of the platoon leader is transmitted. One would be tempted to conclude, then, that the benefits of transmitting the desired rather than actual velocity are minimal. However, analysis of the initial spacing error reveals an important advantage to the former method: the initial spacing error is always zero. Mathematically,

$$\frac{z_1(s)}{F(s)} = 0 \ , \tag{22}$$

regardless of the choice of input force, $F(s)$. The fact that the initial spacing error is zero implies that, in the absence of disturbances, all subsequent spacing errors will also be zero. Therefore, once the platoon is in steady state, a control input applied to the lead vehicle will not generate spacing errors. An even more interesting result is that there will be zero spacing errors even when the effects of actuator delays are included, assuming there are no communication delays and that all vehicles are identical.

3. CONCLUSION

In this paper, it has been demonstrated that the mass-spring-damper platoon representation can be used to qualitatively compare the string stability properties of longitudinal vehicle controllers.

From analysis of both impulse and magnitude responses of various controllers, one can conclude that the introduction of time headway improves both responses, and that its exclusion eliminates the ability to avoid position overshoot in response to lead vehicle acceleration. It has also been shown that communication of desired velocity has an important benefit; namely, that the initial spacing error will be zero regardless of the control input applied to the lead vehicle. Somewhat surprisingly, analysis of the transfer functions resulting from the intervehicle communication methods considered here indicates that there may actually be an advantage to ignoring the immediately preceding vehicle's velocity, and instead referencing only the desired (or actual) velocity of the platoon leader.

ACKNOWLEDGMENT

This work is supported by the California Department of Transportation (CalTrans) under PATH MOUs 240 and 293. The contents of this paper reflect the views of the authors who are responsible for the facts and the accuracy of the data presented herein. The contents do not necessarily reflect the official views or policies of the State of California. This paper does not constitute a standard, specification or regulation.

4. REFERENCES

Shladover, S. E. (1978). Longitudinal control of automated guideway transit vehicles within platoons. *ASME Journal of Dynamic Systems, Measurement, and Control.*

Swaroop, D. and D. Niemann (1996). Some new results on the oscillatory nature of impulse and step responses for linear time-invariant systems. *Preprint.*

Swaroop, D., J.K. Hedrick, C.C. Chien and P. Ioannou (1994). A comparison of spacing and headway control laws for automatically controlled vehicles. *Vehicle Systems Dynamics* **23**, 597–625.

Yanakiev, D. and I. Kanellakopoulos (1996). A simplified framework for string stability analysis in ahs. *Preprints of the 13th IFAC World Congress* **Q**, 177–182.

Yang, Y. and B. Tongue (1996). A new control approach for platoon operations during vehicle exit/entry. *Vehicle Systems Dynamics* **25**, 305–319.