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Mathematical Models for Manufacturing Systems Capacity Planning and Expansion – an Overview

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Abstract. When planning a new manufacturing system, the optimal investment in the system capacity is a major decision to make. The problem of capacity planning is not an easy because of the unpredictable character of the market demand and multi-criteria optimization character of the task. Therefore there is still no one complex methodology of the capacity planning and management. In this paper some mathematical models for capacity planning which can be used at the stage of manufacturing system design or expansion are presented.

Introduction

Manufacturers today face more challenges than ever before due to the highly volatile market, which creates large fluctuations in product demand and the frequent arrival of new technologies and new products [1]. To remain competitive, companies must design manufacturing systems that not only produce high-quality products at low cost but also respond to market changes in an economical way [2-5]. Therefore, making investments decisions in new manufacturing systems requires knowledge both in engineering and in finance and economics [6-7].

Particularly, the system design is an iterative process that includes a sequence of following decisions [8]:

1. Decide whether to invest at all in a new production system, and, if to invest, in which type of system.
2. Based on product sale forecasting and estimated capital investment, determine whether to invest in dedicated, flexible, or portfolio capacity.
3. Calculate the cycle time of each operation and the total time needed for the whole process to produce one product.
4. Optimize the system configuration such that a proper line balancing maximizes system throughput, and tooling cost is minimized to reduce capital investment.
5. Find out the buffer capacity that optimizes the system throughput.
6. Determine the projected operations; it is more challenging when flexible system that produce several products are employed.
7. Consider system responsiveness to changing orders of customers; responsiveness impacts the system throughput.
8. Calculate the optimal speed of each machine; it will impact the whole system throughput.

Therefore, when planning for a new manufacturing system to produce one or several products over a planning horizon, the key decision is how to select the optimal system's capacity. In this paper we presents mathematical, optimization models which can be used when planning the capacity level for the company which produces one or more products in the same time.

Capacity Planning and Management Capacity Problem Formulation

As defined in [9], flexible capacities "possess the ability to change over to produce a set of products very economically and quickly". Therefore, flexible systems may alleviate unfavorable effects of demand uncertainties. However, the versatility to produce multiple products often requires higher investment costs compared to dedicated systems that can only produce one type of products.

The sequence of events are as follows: at the beginning of the planning horizon, the company makes a strategic investment decision on the quantity and types of manufacturing systems to purchase. Once initial investment decisions are given, the company continually makes operating decisions every period on how to allocate its resources in the most profitable way across products.

In most countries, capacity investment decisions are made before demand is observed and the optimal capacity choices may vary from one company to another even in the same industry. The investment decision on the amount and type of capacity – dedicated, flexible, or a portfolio of dedicated and flexible systems – is mainly influenced by the following factors [8]:

1. The number of products to be produced simultaneously in the plant – flexible technology can deal with changes or uncertainty in demand mix. It enables to change the mix of products manufactured in a plant, and produce more of highly profitable products when their demand surges. Usually, if a plant produces more than four, five products simultaneously, the decision will be to invest in flexible capacity.
2. Investment cost of dedicated versus flexible systems – when producing large quantities, the investment cost in flexible systems is always larger than that in dedicated lines. The margin may be 10-100% in large machining systems. Flexible capacities are usually favored more when their investment costs are closer to those of dedicated lines. When the manufactured quantities are small, dedicated lines are not cheaper than flexible systems, and the latter is the optimal solution.
3. Product marginal revenues: higher profit margins and higher prices of the product produced warrant a higher investment level, since losing sales of products causes a significant financial loss. Installing flexible capacity in such cases is economically justified.
4. Product demand volatility during the planning horizon – investment in flexible capacity hedges against uncertainty in future demand, since production can be easily shifted from a product with diminishing demand to a product with rising demand. Therefore, when market volatilities are high, an investment in flexible capacity is the right economic choice.
5. The frequency of product changes and the expected lifetime of products – when a company plans to rapidly introduce new product models in the near future and expanding its product scope, the company should invest in flexible capacity.

Generally, the problem of capacity planning must be sold in two stages. First, assuming that strategic investment decision is already given, we compute the maximum possible operating revenue during the entire lifetime of all products (i.e., the planning horizon). Next, we make the strategic capacity decision by choosing the recommended installed capacities that will generate the maximum profit that is corresponding to the highest operating revenue minus investment costs [10-11]. To gain further insights, we provide below a formulation for the optimal capacity investment problems. In particular we present two mathematical optimization models for capacity planning for one- and multiproduct manufacturing processes.

A Model of Capacity Planning for Single Product Manufacturing Process

Let consider a capacity management problem for a company that produces only one type of good over a finite N-unit time horizon under stochastic market demand. It is assumed that no inventory of finished products is allowed in that company. Capacity management is performed by observing the current capacity and the probability distribution of the market demand at each time period and making optimal decisions to change the capacity based on these observations for the next period. The company can belong to an oligopoly market, but the effects of the policy is neglected, and it is assumed that there exists only one decision maker with perfect recall who makes the optimal decisions to manage the capacity of the company based on the centralized information.

The market demand is stochastic with independent distributions. It can be represented by a stochastic sequence of positive independent variables D_k with a priori continuous cumulative probability distribution functions $\psi_k(D_k)$. The general structure of the market demand as described above is shown in Figure 1 where $\varphi_k(D_k)$ are the probability density functions of the stochastic demand process.

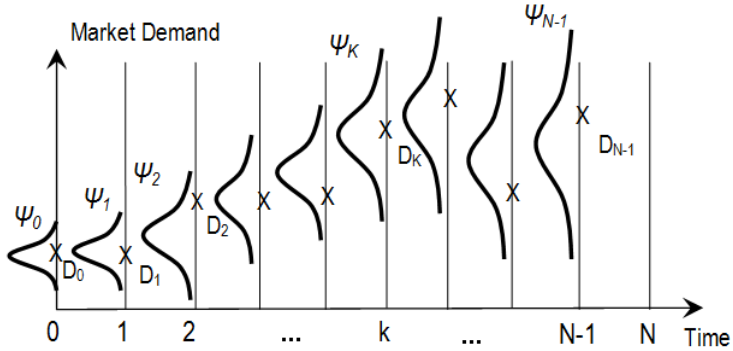

Fig. 1. Distributions of the market demand [12]

The capacity management dynamics evolves in discrete time. It is assumed that there is delay time from when the capacity is ordered until it can be utilized, shown by T. The dynamic capacity evolution is represented by:

$$y_k = \min(C_k; D_k)$$
$$C_{k+1} = C_k + X_{k-T} \tag{1}$$

where C_k represents the capacity level of the company at time k, X_k is the control input which defines the addition of removal of capacity, and y_k represents the sales of the firm. The delay time T is limited to be a multiple of the time increment, k.

The production of the produced good or service costs γ_P per unit to produce, and is sold at a fixed price P per unit with (P - y_k) profit. Unsatisfied market demand has a penalty cost γ_S per unit. The available capacity of the company is C_k at time k, and it takes a proportional holding or overhead cost, γ_H per unit of capacity at each time period, to maintain this level of capacity. The holding cost consist of the costs of maintenance and staffing of the capacity. Effect of all these costs at time period k, represents the one-period expected operating cost function, $G_k(C_k)$ incurring at period $[k,k+1)$ which is evaluated at time k as:

$$G_k(C_k) = E\{(\gamma_P - P)\min(C_K, D_K) + \gamma_S \max(0, D_k - C_k) + \gamma_H C_k\} \tag{2}$$

There is a cost involved with adding capacity to the company. Addition of X units of capacity to the firm costs aX, where a is the proportional ordering cost. Decreasing capacity has a cost rX $(X<0)$ which is a return from selling extra capacity, and r is the reward of selling one unit of capacity. Effect of addition and reduction of capacity represents the management or control cost at time k, $M_k(X_k)$, which is the cost of expanding/subtracting capacity incurred at time k. At the end of the time horizon (i.e., $k=N$) the remaining capacity can be sold for a salvage value γ_N per unit of terminal capacity. The opportunity cost of money for the firm is represented by ρ and the discount factor is represented by $\beta=1/(1+\rho)$.

Given an initial capacity, the problem is to find and optimal decision sequence, or a policy that minimizes the expected discounted cost:

$$\min_{x_0 \dots x_{N-1}} \{-\beta^N \gamma_N C_N + \sum_{k=0}^{N-1} \beta^k [G_k(C_{k+1}) + M_k(X_k)]\} \quad (3)$$

At each time *k,* the decision maker observes the current capacity C_k, and the demand distribution $\psi_k(D_k)$ and makes the decision X_k to generate the new optimal capacity level. The demand realization *Dk* is generated according to the given probability measure, and the operating cost G_k and control cost M_k are incurred and added to the previous costs. The terminal cost is the additional cost, which incurs at time *N* and it will be added to the previous costs. Assume that the company operates at time *k+1*, and it Has a minimal or optimal cost-to-go $V_{k+1}(C_{k+1})$ which represents the cost of the optimal policy to go from time *k+1* to the terminal time *N.* Assuming the optimality of the cost-to-go function $V_{k+1}(C_{k+1})$, one can write the optimal cost-to-go function for the firm at time *k,*

$$V_k(C_k) = \min_{X_k}\{M_k(X_k) + G_k(C_{k+1}) + \beta E V_{k+1}(C_{k+1})\} \quad (4)$$

$$V_N(C_N) = -\gamma_N C_N$$

where: $V_N(C_N)$ represents the final salvage value of the company's capacity at time *N.* Equations (4) are optimality equations for the capacity management problem represented by stochastic dynamic programming. Based on the optimality theorem [13], a Markov policy exists and is optimal if and only if the minimum at (4) is achieved. To obtain the minimum value, it is shown that the optimal cost-to-go $V_k(C_k)$ is convex in C_k and then the functions X_k which make it minimal for $k=0,1,2,\dots N-1$ are obtained.

The structure of the derived optimal policy for this problem is shown in Figure 2. The optimal policy is written based on two optimal thresholds. If the current capacity is lower than the lower optimal threshold *L,* the new capacity level should be chosen to be equal to *L*. If the current capacity is higher than the upper optimal threshold *U,* then the new capacity level should be chosen to be equal to *U.* And finally, for the capacity levels between *L* and *U*, the optimal decision is to maintain the current capacity (i.e. no change).

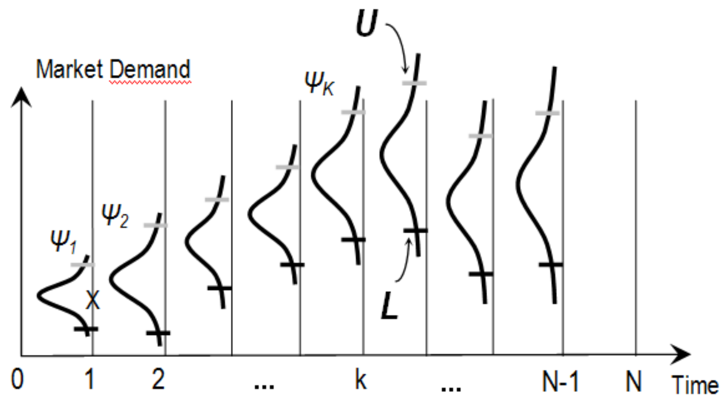

Fig. 2. Optimal Capacity Management Policy [12]

Optimal thresholds levels *L* and *U* are obtained numerically by solving Equations (4). It is also shown that there exists optimal lower and upper limits for *L* and *U* shown by *L*,* and U* and the sufficient condition for a policy to be optimal is to be located within these two limits. Thus, in numerically finding the optimal policy, one only needs to search for an optimal policy in the region between *L*,* and U*. The model was characterized in detail in [12]. Moreover some numerical results were there presented.

A Model of Capacity Planning for Multiproduct Manufacturing Process

Consider a manufacturing company that produces two types of products over a time horizon consisting of several periods. Marginal revenues of p_A and p_B are received for each unit of type A and B product, respectively.

Demands for each product at each period are uncertain. For capacity planning purposes, the company employs demand forecasts for each type of product. These forecasts lead to probability density functions for product demands across periods. As future periods posses higher levels of uncertainty, the forecast accuracy decreases with time. According to the problem defined by Koren [8] in this study we consider a scenario where an existing product A is gradually replaced by a new product – product B. Figure 3 illustrates typical demand distributions for the products where Ψ_i^t and d_i^t denote, respectively, the probability density function and mean demand in period t for product i, i = A,B. For a three-period analysis, we let $d = (d_A, d_B)$ denote the realization of all product demands, where $d_A = (d_A^1, d_A^2, d_A^3)$ and $d_B = (d_B^1, d_B^2, d_B^3)$.

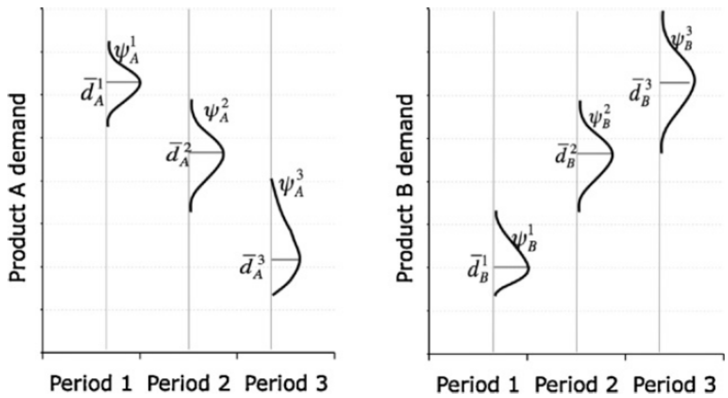

Fig. 3. Demand distributions for products A and B for a planning horizon of three periods [14]

The manufacturing capacity investment decision is carried out at the beginning of the planning horizon when only forecasts for products are available. Let $k = (k_A, k_B, k_{AB})$ denote the variables expressing the size of the capacity, where k_A, k_B and k_{AB} are the dedicated capacities for products A and B, and the flexible capacity AB, respectively. In terms of investment costs, let $c = (c_A, c_B, c_{AB})$ denote the investment cost per unit capacity in dedicated line for product A, dedicated line for product B and flexible (for A and B), respectively. It is assumed that $c_A, c_B \le c_{AB} \le c_A + c_B$. The right term, $c_A, c_B \le c_A + c_B$, gives the upper bound on the cost of flexible system.

We follow a capacity investment cost structure as presented in [8] and [14] and assume that both dedicated and flexible capacities are purchased in discrete batches where the increments of the dedicated capacity are much larger than that of the flexible capacity. In practice, companies may incur additional costs to simultaneously operate and maintain dedicated and flexible systems. Therefore, lower bounds on capacity purchases must be applied; a certain type of capacity below the bound will not be purchased.

Let $k_j \in \{0, S_j\}$ where $S_j = \{k_j + w_j \delta_j | w_j \in Z^+\}$ for j = A, B, AB denote the feasible set of capacity selections for each type of manufacturing system where k_j and δ_j denote the minimum capacity purchase and capacity increment sizes.

In that case, the problem of capacity planning must be solved in two stages. First, assuming that a strategic investment decision is already given, the maximum possible operating revenue during the planning horizon are computed. Next, the strategic capacity decision is made by choosing the recommended installed capacities that will generate the maximum profit that is corresponding to the highest operating revenue minus investment costs.

So, the problem may be formulated as a linear program with an optimization cost index. Cost index $R(d,k)$ expresses the revenue that can be achieved for a given capacity investment decision *k*, and for any fulfillment of product demands *d* over the planning horizon.

$$R(d,k) = \max_{x,y} \sum_{t=1}^{T} \beta^{t-1}[p_A(x_A^t + y_A^t) + p_B(x_B^t + y_B^t)] \quad (5)$$

subject to constraints:

$$x_A^t \le k_A \qquad \forall t = 1,...,T \quad (6)$$

$$x_B^t \le k_B \qquad \forall t = 1,...,T \quad (7)$$

$$y_A^t + y_B^t \le k_{AB} \qquad \forall t = 1,...,T \quad (8)$$

$$x_A^t + y_A^t \le d_A^t \qquad \forall t = 1,...,T \quad (9)$$

$$x_B^t + y_B^t \le d_B^t \qquad \forall t = 1,...,T \quad (10)$$

The decision variables x_A^t and x_B^t denote, respectively, how many units of dedicated capacity A and B are needed to fill period t demand, whereas the decision variables y_A^t and y_B^t denote the optimal allocation of the flexible capacity between products. In addition, β is the discount factor per period that is used to calculate the NPV of the revenues, $\beta = 1/(1+r)$, where *r* is the annual rate of return. Constraints (6)-(10) guarantee that one will assign neither more capacity than the maximum available, nor more capacity than demand (i.e., the production quantities within a period do not exceed available capacity and are bound by the demand).

Having obtained the maximum operating revenue, it is possible now to write the strategic decision problem of determining the optimal capacity investments *k*.

$$\max_{k} \; E_d(R(d,k)) - c * k' \quad (7)$$

In above formulation, E_d[R(d,k)] is expected value of the operative revenue where the expectation is taken over demand distributions and c*k' represents the total investment costs. As was described previously, we have $k_A \in \{0, S_A\}$ where $S_A = \{k_A + w_A.\delta_A | w_A \in Z^+\}$ with $k_B \in \{0, S_B\}$ and $k_{AB} \in \{0, S_{AB}\}$ The company's objective is to maximize E_d. Numerical example exploited above presented model for the firm producing two products over a planning horizon during which product demands possess uncertainties was presented in [14].

Summary

Capacity planning and expansion of manufacturing systems could be seen as a system's feature that might provide a significant increase of potentials for resolving a number of problems in manufacturing systems design and operation. In other words, manufacturing systems capacity might provide further optimization of the manufacturing systems design and operation or enable the development of paradigmatically new manufacturing systems for the sustainability and wellbeing society. Besides functional aspects of capacity planning and expansion, which could be seen as a primarily technical issues, by considering the wider social concerns the capacity feature might also seen as an instrument for value increase (following request for value creation and sustainable society).

Two approaches to the capacity management problem were presented in this paper. First of them is based on Markov decision process and is dedicated for manufacturing systems which are able to produce only one type of product. The second approach is suitable for optimizing the capacity of the system in case of manufacturing more than one type of product. It allows to determine a range of investment cost parameters, product revenues and demand uncertainties influence and finally to select the optimal strategy to whether invest in pure flexible, pure dedicated or a portfolio of both types of systems.

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