Decentralized Power Control for Random Access with Successive Interference Cancellation

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Abstract—This paper is concerned with the decentralized power allocation problem in random access systems. We propose a scheme that is especially suitable for systems requiring high throughput but with difficulty in establishing centralized control, such as cognitive radio environments. Specifically, we assume successive interference cancellation (SIC) at the receiver for multi-packet reception (MPR). We consider a decentralized random power transmission strategy where each user selects its transmitted power level randomly according to a power distribution conditioned on its own channel state. Our focus is on the design of this distribution such that the system packet throughput is maximized under rate and power constraints. We start from a two-user system. A main finding of this paper is that the supports of the optimal power distributions are of discrete nature. This finding greatly simplifies the distribution optimization problem. We also discuss a sub-optimal solution to systems with more than two users. Numerical results demonstrate that the proposed scheme can achieve noticeable performance improvement compared with conventional single-user detection (SUD) based ones and offer a flexible tradeoff between the system throughput and power consumption.

Index Terms—Cognitive radio, random access, multi-packet reception (MPR), successive interference cancellation (SIC).

I. INTRODUCTION

IN A RANDOM access scheme such as ALOHA, a collision
may occur when multiple users transmit simultaneously [1]. N A RANDOM access scheme such as ALOHA, a collision Packets involved in a collision are conventionally assumed unrecoverable and discarded. This is, however, a pessimistic assumption. In practice, it is possible to recover some or all packets from a collision. This phenomenon is captured by the multi-packet reception (MPR) model [2]–[11].

Single-user detection (SUD) has been assumed by many researchers working on MPR [8]–[11]. In this case, the signal of each user is detected by regarding the signals from the others as noise. Each received packet can be successfully recovered as long as the corresponding signal to interferenceplus-noise ratio (SINR) exceeds a certain threshold. Such

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events may happen naturally in fading environments [4], [5]. Proper transmission design can be adopted to increase the probability of such events. For this purpose, a channel-aware ALOHA protocol has been proposed in [6], [7], in which the transmission probability of each user is optimized based on its own channel state. Random power control methods have also been studied in [10], [11] for SUD-based systems, where each user randomly selects a transmitted power level according to a certain distribution denoted by $f(.)$. It is conjectured in [10] that *the optimal power distribution may be of a discrete nature*. However, no rigorous proof is available so far.

Multi-user detection (MUD) techniques such as successive interference cancellation (SIC) have been widely studied for multiple access systems [12]–[14]. With SIC, successful packet recovery in a collision is not determined by the individual SINR values of the involved packets, but by their joint power profile [12]–[14]. Optimizing this power profile can potentially enhance SIC performance. Centralized power control methods have been studied in [12], [15] for fading channels. For decentralized power control, game theory has been studied for MPR, but only with limited gain [16].

This paper is concerned with an ALOHA-type random access technique with SIC. We study a random-power transmission strategy in which each user selects its transmitted power level randomly according to a certain power distribution. We start with a special case of only two users. We take an information theoretical approach and prove that the support of the optimal power distribution is discrete for a SIC receiver. This finding greatly simplifies the optimization problem for the power distribution. We extend the results to fading channels with more than two users. To circumvent the complexity problem, we adopt a suboptimal approach, in which optimization is performed to resolve collisions only involving two users.

The proposed scheme is especially suitable for a cognitive radio system with multiple secondary users transmitting opportunistically over spectrum holes [17]–[20]. In such a system, the time overhead to set up centralized control is a problem, as a spectrum hole may only last a short duration. With decentralized control, on the other hand, the collision probability can be very high if each secondary user always has packets to transmit. The proposed scheme can be used to increase throughput by resolving collisions to a great extent in such an environment, as demonstrated by the numerical results in this paper. It also offers a flexible tradeoff between the system throughput and power consumption, which is desirable in cognitive radio environments for maximizing the throughput

of the secondary users while limiting the interference to the primary users within a tolerable range.

The proposed scheme can also be applied to more general random access systems other than cognitive radio. The related advantage becomes noticeable only when collision probability is high. However, a conventional random access system usually operates in a low to medium collision probability range; otherwise a centralized control scheme (e.g., time-division multiple-access (TDMA)) is a better option. Therefore the proposed scheme is most attractive to applications with heavy loading but also with difficulty in establishing centralized control, such as cognitive radio.

II. PRELIMINARIES

A. Cognitive Radio Systems with Random Access

Consider a certain spectrum hole in a cognitive radio system that is detected by K secondary users. The spectrum hole is shared by these secondary users in a random access manner. Our discussions in this paper are based on the following basic assumptions for simplicity.

- (i) Each secondary user always has a packet to transmit, i.e., the system is fully loaded. Conventional ALOHA is not efficient in this case due to the high collision probability.
- (ii) Each secondary user has no knowledge of the transmission state (i.e., the instantaneous channel state information and transmitted power) of the others. It randomly draws a power level for every transmission. The power control mechanism is decentralized. This will be elaborated in Subsection II-D.
- (iii) Each user has perfect channel state information at the transmitter (CSIT) of its own channel. (We will return to this briefly in Section IV.)

B. SIC Feasible Region

We first consider a two-user non-fading channel, in which the received signal y is given by:

$$
y = \sum_{k=1}^{2} \sqrt{e_k} x_k + \eta \tag{1}
$$

where x_k is the transmitted signal of user k ($k = 1, 2$) with now x_k is a corresponding transmitted now as and x_k unit power, e_k the corresponding transmitted power, and η a complex additive white Gaussian noise (AWGN) sample with mean zero and variance N_0 .

For simplicity, we assume the same rate constraint R for both users. The corresponding SIC feasible region, denoted by S, is defined as the closure of all power pairs (e_1, e_2) that support successful transmissions for both users in (1). For the ideal coding case, S is derived as follows [14]. Assume that the signal of user 1 is decoded first by regarding that from user 2 as interference. Applying the Shannon formula to user 1, we have

$$
\log_2\left(1 + e_1/(e_2 + N_0)\right) \ge R.
$$
 (2)

Upon successful decoding, the signal of user 1 is subtracted from y. Applying the Shannon formula again to the residual signal, we have

$$
\log_2(1 + e_2/N_0) \ge R.
$$
 (3)

Fig. 1. The SIC feasible region of a two-user system with ideal coding and $R \ge 1$ bit/symbol. $E_1 = (2^R - 1)N_0$ and $E_2 = (2^R - 1)(E_1 + N_0)$.

Combining (2) and (3), we obtain

$$
\mathcal{A} = \{ (e_1, e_2) | e_2 \ge (2^R - 1)N_0, e_1 \ge (2^R - 1)(e_2 + N_0) \}.
$$
\n(4)

Alternatively, if user 2 is decoded first, by symmetry, we have

$$
\mathcal{B} = \{ (e_1, e_2) | e_1 \ge (2^R - 1)N_0, e_2 \ge (2^R - 1)(e_1 + N_0) \}.
$$
 (5)

The SIC feasible region is formed by $S = A \cup B$. In the case of $R \ge 1$ bit/symbol, A and B do not intersect. See Fig. 1 for an example.

C. Randomized Power Control

Let the power levels e_1 and e_2 in (1) be randomly drawn based on two probability density functions (PDFs) f_1 and f_2 , respectively. The average system sum power is given by

$$
E(f_1, f_2) = \int e_1 f_1(e_1) de_1 + \int e_2 f_2(e_2) de_2.
$$
 (6)

For the SIC receiver, we say that a power pair (e_1, e_2) succeeds if it falls in the SIC feasible region S . The joint success probability of a PDF pair (f_1, f_2) is then defined as

$$
P_{2U}(f_1, f_2) = \iint_{(e_1, e_2) \in S} f_1(e_1) f_2(e_2) de_1 de_2. \tag{7}
$$

Here the subscript "2U" emphasizes that the feasibility is regarding both users, i.e., a transmission is regarded as successful only when both users are successful. Later in Subsection III-B, we will consider the success probability for an individual user.

Definition 1 below is a criterion to compare two PDF pairs. *Definition 1:* We say that (f_1^*, f_2^*) is better than or equivalent to (f_1, f_2) , denoted by (f_1^*, f_2^*) b.e. (f_1, f_2) , if the inequalities $E(f_1^*, f_2^*)$ \leq $E(f_1, f_2)$ and $P_{2U}(f_1^*, f_2^*) \ge P_{2U}(f_1, f_2)$ hold simultaneously. Furthermore, we say that (f_1^*, f_2^*) is better than (f_1, f_2) , denoted by $(f_1^*, f_2^*) \underbrace{b}{\longrightarrow} (f_1, f_2)$ if at least one inequality above is replaced by strict inequality.

In Definition 1, the relative superiority between two PDF pairs is measured by two parameters. One is the joint success probability $P_{2U}(f_1, f_2)$ in (7), which is related to the system throughput. The other is the average sum power $E(f_1, f_2)$ in (6), which determines the sum power consumption. Naturally, a PDF pair with a higher throughput and less power is preferred.

It can be verified that the relation defined above is transitive, i.e., if $\frac{f_1^{**}, f_2^{**}, b.e.}{f_1^{**}, f_2^{**}, f_3^{**}, f_4^{**}}$ and (f_1^*, f_2^*) b.e., (f_1, f_2) , then (f_1^*, f_2^*) $\frac{f_1^{**}, f_2^{**}}{f_1^{**}, f_2^{**}, f_3^{**}, f_4^{**}, f_5^{**}, f_6^{**}}$ If we further have (f_1^{**}, f_2^{**}) \underline{b} , (f_1^*, f_2^*) or (f_1^*, f_2^*) \underline{b} , (f_1, f_2) , then $(f_1^{**}, f_2^{**}) \underline{b} \rightarrow (f_1, f_2)$.

D. Decentralized Power Control

In (1), the two users experience the same channel condition. If $f_1 \neq f_2$, a centralized control mechanism is required to allocate them between the two users.

Definition 2: We say that a distribution pair (f_1, f_2) is decentralized if $f_1 = f_2 = f$.

Example 1: Consider $f_1 = f_2 = f = 0.5\delta(e) + 0.5\delta(e - E_1)$, where $E_1 = (2^R - 1)N_0$ and $\delta(\cdot)$ is the Dirac delta function. Each user selects power level 0 or E_1 with equal probability. This produces four power pairs marked in Fig. 1, i.e., $(0, 0)$, $(0, E_1)$, $(E_1, 0)$, and (E_1, E_1) , each with probability 0.25. The successful transmissions are related to $(E_1, 0)$ for user 1, and $(0, E₁)$ for user 2. The corresponding joint success probability is zero in this case.

Example 1 can be viewed as the conventional ALOHA [1]. Its performance can be improved by optimizing the related probability profile. Such a scheme is generally sub-optimal since the power distribution is optimized over two power levels 0 and E_1 only. The focus of this paper is to design more general distributions with multiple power levels.

Definition 3: We say that f^* is optimal if there is no f such that $(f, f) \xrightarrow{b} (f^*, f^*)$.

III. TWO-USER SYSTEMS WITHOUT FADING

In this section, we focus on the distribution optimization problem for the two-user system without fading in (1). We will discuss more general cases in the next two sections.

A. Support for the SIC Scheme

We first consider the situation with $R \geq 1$ bit/symbol. As shown in Fig. 2, the related SIC feasible region consists of two un-overlapped sub-regions. Sub-region A is bounded by functions $e_2 = E_1$ and $e_1 = \phi(e_2)$ where

$$
\phi(e) = (2^R - 1)(e + N_0). \tag{8}
$$

Similarly, B is bounded by $e_1 = E_1$ and $e_2 = \phi(e_1)$. The boundary function $\phi(\cdot)$ is monotonically increasing, based on which we can define a set $\mathcal{E} = \{E_n | n = 0, 1, 2, \ldots\}$ as

$$
E_n = \begin{cases} 0 & n = 0, \\ \phi(E_{n-1}) & n > 0. \end{cases}
$$
 (9)

Here E_n is the minimum power level that guarantees successful decoding of one user when the interference power level from the other user is E_{n-1} , as illustrated in Fig. 2.

Fig. 2. Illustrations of some power levels in the set $\mathcal E$ for the SIC feasible region with $R \geq 1$ bit/symbol.

Theorem 1: The support of the optimal power distribution f for the system in (1) with decentralized power control and a SIC receiver is a subset of \mathcal{E} .

The detailed proof of Theorem 1 can be found in the Appendix. The rationale behind Theorem 1 is as follows. From (9), any power level $E' \in (E_0, E_1)$ is unnecessary since E_1 is the minimum power for successful detection without interference. Provided that there is no interference power level in (E_0, E_1) , any signal power level $E' \in (E_1, E_2)$ is also unnecessary since E_2 is the minimum power for successful detection when the interfering packet has power E_1 (so replacing E' by E_1 will not degrade performance, as shown in Remark 2 in the Appendix). This reasoning is generalized in the Appendix to show that $E' \in (E_n, E_{n+1})$ is unnecessary for any n.

The discrete nature of the optimal distribution support shown in Theorem 1 greatly simplifies the distribution design problem, as detailed below.

B. Throughput Optimization

The system throughput is defined as the average number of successfully decoded packets per available slot in a spectrum hole, which can be calculated as

$$
T = Pr\{\text{only one user is successful}\}
$$

+ 2 \cdot Pr\{\text{both users are successful}\} (10a)
= P_{1U}(f) + 2P_{2U}(f)

where $P_{1U}(f)$ and $P_{2U}(f)$ are, respectively, the probabilities that only one user is successful and both users are successful in an available slot.

Note that the optimality derived in Theorem 1 is with respect to $P_{2U}(f)$, which is related to the event that a power pair falls in the feasible region for two-user concurrent transmissions. The power pairs related to $P_{1U}(f)$ is for successful

transmission of only one user, but it still contributes to the throughput. It can be shown that Theorem 1 still holds when $P_{1U}(f)$ is considered. Denote by p_n the probability that each user transmits with power level E_n . We can compute $P_{1U}(f)$ and $P_{2U}(f)$ as follows:

$$
P_{1U}(f)
$$

 $= Pr$ {user 1 selects E_0 and user 2 selects nonzero power} + Pr {user 2 selects E_0 and user 1 selects nonzero power} $= 2p_0(1 - p_0)$ (10b)

$$
P_{2U}(f) = Pr\{\text{both users select nonzero power}\}\
$$

$$
- Pr\{\text{both users have the same power}\}\
$$

$$
= (1 - p_0)^2 - \sum_{n \ge 1} p_n^2
$$
 (10c)

From (10), the system throughput (packets per available slot in a spectrum hole) is given by

$$
T = P_{1U} + 2P_{2U} = 2\left(1 - p_0 - \sum_{n\geq 1} p_n^2\right). \tag{11}
$$

Thus we have the following throughput maximization problem

$$
\max_{\{p_n\}} \qquad 2\left(1 - p_0 - \sum_{n=1}^N p_n^2\right) \tag{12a}
$$

s.t. $\sum_{n=0}^{N} p_n = 1$, (12b)

$$
\sum_{n=0}^{N} E_n \cdot p_n \le \bar{e},\tag{12c}
$$

$$
0 \le p_n \le 1, \forall n. \tag{12d}
$$

where \bar{e} is the average power constraint of each user and N is a properly chosen integer to meet the maximum power constraint E_{max} (i.e., N should be selected to satisfy $E_N \leq$ $E_{\text{max}} < E_{N+1}$). It can be verified that the problem in (12) is convex [21].

C. The Case of R < 1 *bit/symbol*

When $R < 1$ bit/symbol, the two sub-regions A and B overlap (See Fig. 3). The boundary functions $e_1 = \phi(e_2)$ and $e_2 = \phi(e_1)$ intersect with each other at a certain point denoted by $Q=(E_Q, E_Q)$ with E_Q being the solution to equation $e = \phi(e)$. In this case, the transmission of one user with power $e \geq E_Q$ is always successful regardless of the power level of the other. This indicates that reducing a power level e $(e>E_Q)$ to E_Q does not affect the system throughput. Thus we have the following.

Remark 1: For the system in (1) with decentralized power control and a SIC receiver, the support of the optimal f when $R < 1$ bit/symbol is confined within $[0, E_{Q}]$.

Remark 1 can also be verified from the fact that the power levels ${E_n}$ in the set $\mathcal E$ defined in (9) has the following property:

$$
E_n \in [0, E_{\mathbf{Q}}), \forall n, \text{ and } \lim_{n \to \infty} E_n = E_{\mathbf{Q}}.
$$
 (13)

Following a similar procedure as that in the Appendix, we can verify that Theorem 1 still holds in the case of $R < 1$ bit/symbol, i.e., the support of the optimal f is again confined within $\mathcal E$. The related distribution can be optimized in a similar

Fig. 3. The SIC feasible region for a two-user system with ideal coding and $R < 1$ bit/symbol.

way as (12) by noting the following two distinctions: i) the power pair $(E_{\rm Q}, E_{\rm Q})$ also supports successful transmissions for both users and should be included in the throughput calculation, and ii) letting N be a finite number may incur sub-optimality to the solution obtained accordingly, but the loss is marginal when N is sufficiently large.

IV. TWO-USER SYSTEMS WITH FADING

We now consider a two-user system with fading. The received signal is given by

$$
y = \sum_{k=1}^{2} \sqrt{g_k} \sqrt{e_k} x_k + \eta \tag{14}
$$

where the channel gains of both users, ${g_k|k = 1,2}$, are assumed to be independent and identically distributed. Assume that the instantaneous channel gain for user k is given by $g_k = g$, which is known to user k before transmission. Our aim is to optimize the conditional distribution $f_T(e_T | g)$, with e_T the transmitted power, such that the system throughput is maximized.

As a reference, a channel-aware ALOHA scheme is proposed in [6], [7] based on the following special form of $f_T(e_T|q)$,

$$
f_T(e_T|g) = p(g) \cdot \delta(e_T - E_T) + (1 - p(g)) \cdot \delta(e_T) \quad (15)
$$

where E_T is a predetermined power value and $p(g)$ the probability of transmitting with power level ^E*T* when the channel gain is g. An optimization procedure for $p(g)$ is developed in [7]. This scheme is based on a single non-zero transmitted power level E_T . In the following, we will show that the system throughput can be significantly enhanced by allowing multiple power levels at the transmitter and multiuser detection at the receiver.

The basic assumption above is that each secondary user knows its channel gain. This can be accomplished in different ways, depending on the system structure. If time division duplex (TDD) mode is adopted with channel reciprocity and the primary and secondary users communicate to a common receiver, each secondary user can acquire its channel state by monitoring the transmission from the common receiver to the primary users. The problem is more complicated in other scenarios. A possible general solution is that the receiver for the secondary users will transmit a beacon signal whenever it senses that the primary users are not transmitting, which will be used by the secondary users for channel estimation.

A. Optimal Support and Throughput

Recall from Theorem 1 that the support of the optimal power distribution (at both the transmitter and receiver ends) in non-fading channels is a subset of $\{E_0, E_1, E_2, \ldots\}$. This result can be extended to the fading case as follows.

Corollary 1: Given g, the support of the optimal conditional transmitted power distribution $f_T(e_T | g)$ is a subset of $\{E_0/g, E_1/g, E_2/g, \ldots\}$, with $\{E_0, E_1, E_2, \ldots\}$ defined in (9).

To prove Corollary 1, the key is to show that for any transmitted power level e_T that leads to a received power level $e_R = e_T g$ with $E_n < e_R < E_{n+1}$, we can always merge e_T to E_n/g without reducing the system throughput. This is similar to Remark 3 in the proof of Theorem 1. The details are omitted due to space limitation.

Based on Corollary 1, the optimal $f_T(e_T|g)$ can be expressed in the following form

$$
f_T(e_T|g) = \sum_{n\geq 0} p_n(g) \cdot \delta(e_T - E_n/g) \tag{16}
$$

where $p_n(q)$ is the probability that each user transmits with power level E_n/g at the channel state g. Let $\Psi(g)$ be the PDF of g. The received power distribution $f_R(e_R)$ is given by

$$
f_R(e_R) = \sum_{n\geq 0} p_n \cdot \delta(e_R - E_n)
$$
 (17a)

where

$$
p_n \equiv \int_g p_n(g) \cdot \Psi(g) dg. \tag{17b}
$$

Given $f_R(e_R)$, the throughput of the system in (14) can be computed using (11).

The above discussion is for perfect channel state information at the transmitter (CSIT). If the available CSIT contains error, a simple (but sub-optimal) solution is to adopt power levels $\{E_n + \Delta_n\}$ where Δ_n are tolerance margins. This will result in loss in power efficiency that increases directly with CSIT error. Such loss becomes significant when CSIT is very unreliable (including no CSIT). We expect that better solutions are possible based on other power distribution optimization techniques and we are still working on this issue.

B. Throughput Optimization

From the above discussions, we need to optimize the distribution $\{p_n(g)|n = 0, 1, 2, \ldots\}$ in (16) for each channel state q . The exact solution is complicated when q is continuously distributed. To overcome the problem, we consider an approximate approach. We divide the range $[0, \infty)$ into M intervals according to $M+1$ thresholds $\{g^{(m)}|m=0,\ldots,M\}$ (with $g^{(0)} = 0$ and $g^{(M)} = \infty$) and assume that the received power distributions in each individual interval are the same, i.e.,

$$
p_n(g) = p_n^{(m)}
$$
 for $g \in [g^{(m-1)}, g^{(m)}), m = 1, ..., M.$ (18)

Here $p_n^{(m)}$ is the probability that the received power is E_n
when abangl gain $g \in [g(m-1) - g(m))$. Then (17b) on happy when channel gain $g \in [g^{(m-1)}, g^{(m)}]$. Then (17b) can be rewritten into the following form:

$$
p_n = \sum_{m=1}^{M} p_n^{(m)} q^{(m)} \tag{19a}
$$

where

$$
q^{(m)} = \int_{g \in [g^{(m-1)}, g^{(m)})} \Psi(g) dg.
$$
 (19b)

When the received power is E_n and the channel gain is $g \in [g^{(m-1)}, g^{(m)})$, the related transmitted power is $E_n g^{-1}$ with probability $p_n^{(m)} \Psi(g) dg$. The average transmitted power is given by

$$
\sum_{m=1}^{M} \sum_{n=1}^{N} \int_{g \in [g^{(m-1)}, g^{(m)})} (E_n g^{-1}) (p_n^{(m)} \Psi(g) dg)
$$

=
$$
\sum_{m=1}^{M} \sum_{n=1}^{N} p_n^{(m)} E_n / \bar{g}^{(m)}
$$
(20a)

where N is the number of received power levels and

$$
\bar{g}^{(m)} = \left(\int_{g \in [g^{(m-1)}, g^{(m)})} g^{-1} \Psi(g) dg \right)^{-1}.
$$
 (20b)

We can formulate the throughput optimization problem as follows.

$$
\max_{\{p_n^{(m)}\}} \quad T = 2\left(1 - p_0 - \sum_{n=1}^N p_n^2\right) \tag{21a}
$$

s.t.
$$
0 \le p_n^{(m)} \le 1, n = 0, ..., N, m = 1, ..., M
$$
 (21b)

$$
p_n = \sum_{m=1}^{M} p_n^{(m)} q^{(m)}, n = 0, 1, \dots, N
$$
 (21c)

$$
\sum_{n=0}^{N} p_n^{(m)} = 1, m = 1, ..., M
$$
 (21d)

$$
\sum_{m=1}^{M} \sum_{n=1}^{N} p_n^{(m)} E_n / \bar{g}^{(m)} \le \bar{e}.
$$
 (21e)

It can be seen that (21) is a convex problem with $M(N + 1)$ optimization variables and can be solved by standard convex optimization tools [21].

V. GENERAL SYSTEMS WITH MORE THAN TWO USERS

To optimize the power distribution for a cognitive radio system with more than two secondary users, we need to analyze the general K -user feasible region, which is a tedious issue and we will not pursuit it further. Instead, we will discuss a simple and sub-optimal solution below.

A. Sub-optimal Solution

We refer to a collision involving k $(2 \leq k \leq K)$ users as a type-k collision. In this subsection, we design the power distribution by only considering type-2 collisions. We will see that this simple scheme can still provide noticeable performance improvement.

Since only type-2 collisions are considered, we still confine the support of the received power distribution f_R within the set $\{E_0, E_1, \ldots, E_n, \ldots\}$ in (9) based on Theorem 1. With the received power distribution $\{p_n | n = 0, 1, \ldots\}$ defined in (19), the system packet throughput can be represented by

$$
T = T_1 + T_2. \tag{22a}
$$

In (22a), T_1 is the packet throughput contributed by the event when only one user transmits, i.e.,

$$
T_1 = K(1 - p_0)p_0^{K-1}
$$
 (22b)

and T_2 is that contributed by resolving type-2 collisions, i.e.,

$$
T_2 = 2\binom{K}{2}p_0^{K-2}\left((1-p_0)^2 - \sum_{n=1}^N p_n^2\right). \tag{22c}
$$

The function in (22a) is in general non-convex with respect to ${p_n}_{n \geq 0}$. However, for a given p_0 , maximizing (22a) is equivalent to minimizing $\sum_{n=1}^{N} p_n^2$, which is convex with respect to ${n \geq 1}$. to $\{p_n\}_{n\geq 1}$. Hence the throughput maximization problem can be solved through the following two steps. First we solve the following convex optimization problem for any given p_0 .

$$
\min_{\{p_n^{(m)}\}} \quad \sum_{n=1}^N p_n^2 \tag{23a}
$$

s.t.
$$
0 \le p_n^{(m)} \le 1, n = 1, ..., N, m = 1, ..., M
$$
 (23b)

$$
p_n = \sum_{m=1}^{M} p_n^{(m)} q^{(m)}, n = 1, \dots, N
$$
 (23c)

$$
\sum_{n=1}^{N} p_n^{(m)} \le 1, m = 1, ..., M
$$
\n(23d)

$$
\sum_{n=1}^{N} \sum_{m=1}^{M} p_n^{(m)} q^{(m)} = 1 - p_0
$$
 (23e)

$$
\sum_{n=1}^{N} \sum_{m=1}^{M} p_n^{(m)} E_n / \bar{g}^{(m)} \le \bar{e}
$$
 (23f)

with $\{q^{(m)}\}$ and $\{\bar{g}^{(m)}\}$ given in (19b) and (20b) respectively. Then the optimal p_0 is obtained by a full search.

B. User Density Consideration

In the above, we assumed a constant number (i.e., K) of active secondary users sharing the same spectrum hole. In a practical cognitive radio system, the number of active secondary users may vary from slot to slot. In general, the power distribution can be optimized based on the statistical mean of the throughput with respect to the distribution of K . The detailed discussion is similar to that in Subsection V-A and omitted here.

C. Feasible Region for Practical Systems

The discussions so far are based on ideal coding. We now briefly outline the treatments for practical systems. For example, consider a system based on a (3, 6) regular lowdensity parity-check (LDPC) code [22] with codeword length $10⁴$ followed by QPSK modulation. The data rate of each user is 1 bit/symbol. Interleave-division multiple-access (IDMA) transmission and iterative LMMSE detection with 30 iterations are assumed [23], [24]. We say that a packet transmission is successful if its bit error rate (BER) after iterative detection is no higher than 10^{-5} . The feasible region can be defined accordingly. Fig. 4 shows the feasible region of a twouser system without fading obtained by simulating the BER performance for all power pairs (e_1, e_2) . It can be seen that the feasible region S in Fig. 4 is bounded by four curves $e_1 = E_1$, $e_2 = \phi(e_1)$, $e_2 = E_1$ and $e_1 = \phi(e_2)$. Theorem 1 does not hold here as the boundary function $\phi(\cdot)$ is not monotonically

Fig. 4. The feasible region of a two-user IDMA system with (3, 6) regular LDPC encoding and iterative decoding. Number of iterations = 30.

increasing. $\frac{1}{1}$ To overcome the problem, we approximate the region S with an inner bound and an outer bound, as shown by the enlarged view in Fig. 4. Since the boundary functions for both bounds are monotonically increasing, following (9), we can obtain the discrete power levels for the inner bound as

$$
[E_0, \ldots, E_4, E_Q] = [0, 1.43, 3.11, 4.81, 6.16, 6.80], \quad (24a)
$$

and those for the outer bound as

$$
[E_0, \dots, E_5, E_Q] = [0, 1.43, 3.11, 4.81, 6.16, 6.82, 6.88].
$$
\n(24b)

Then a similar method as in Subsection V-A can be applied to the inner and outer bounds to obtain the corresponding system performance bounds.

Comparing Fig. 4 with Fig. 1, we can see that the feasible region for LDPC coding with iterative detection is not contained in that for ideal coding with SIC. This is possible since SIC is optimal for ideal coding only when the powers of the users are properly allocated [14]. Otherwise, non-ideal coding with iterative detection may perform better, especially when the received powers for the users are close.

VI. NUMERICAL RESULTS

A. Non-Fading Channels with Ideal Coding and SIC Decoding

Figure 5 provides the performance of various schemes in non-fading channels with different numbers of users K and $R = 1$ bit/symbol. For the conventional ALOHA scheme, the packet throughput with full load is calculated as [1]

$$
T = K p_1 (1 - p_1)^{K - 1} \tag{25}
$$

where p_1 is the transmission probability of each user. T in (25) is maximized at $p_1 = 1/K$ with $T = (1 - 1/K)^{K-1}$.

¹The system performance with iterative detection is determined by the residual interference after iteration instead of the initial value. A power increase for one user may decrease the residual interference to the other and so benefit the detection of the signal of the other. Thus it is possible that the boundary is not monotonically increasing.

Fig. 5. Performance comparison among various schemes in non-fading channels with ideal coding and different *K*. $R = 1$ bit/symbol.

The resultant average power consumption of each user is given by

$$
\bar{e}_{\text{ALOHA}} = E_1/K. \tag{26}
$$

For a fair comparison, we use \bar{e}_{ALOHA} in (26) as the power constraint \bar{e} in our proposed scheme. We select $N = 30$ in (23) for a maximum power constraint $E_{\text{max}} = 15 \text{dB}$. To calculate the throughput of the proposed scheme, we discretize the value of p_0 in (23) with step 0.01, and solve (23) for each discretized p_0 . We can see from Fig. 5 that compared with conventional ALOHA, the proposed scheme (denoted by ML-SIC for multiple-level transmission and SIC) improves throughput significantly. For example, at $K = 8$, a throughput gain of 28% is observed.

We also include in Fig. 5 the results obtained by optimizing the power distribution in systems with a SUD receiver (denoted by ML-SUD for multiple-level transmission and SUD). In this case, no interference cancellation is applied and so the signal from at most one user (with the highest received power) can be decoded successfully (for high-rate transmissions, i.e., $R \geq 1$ bit/symbol). Using a similar derivation procedure as in Section III, we can verify that the support of the optimal power distribution for the system in (1) with SUD is again a subset of $\mathcal E$ in (9). Then the throughput optimization problem is almost the same as in the ML-SIC scheme except that the term $\sum_{n=1}^{N} p_n^2$ in the objective function (23) is replaced by
 $\sum_{n=1}^{N} p_n^2$ (n) From Fig. 5, we can see that this scheme $\sum_{n=1}^{N} (p_n \sum_{l=n}^{N} p_l)$. From Fig. 5, we can see that this scheme performs better than conventional ALOHA, but worse than the proposed ML-SIC scheme.

Recall that for conventional ALOHA, collision probability increases as K increases. This can be compensated to a certain extent by using a smaller transmission probability, but throughput is still a decreasing function of K in general. Since throughput is non-negative, it will then converge to a limit when $K \to \infty$. This limit is $\lim_{K \to \infty} (1-1/K)^{K-1} = 1/e \approx$ 0.368 (using (25)) for conventional ALOHA. It is observed in Fig. 5 that the throughput of the proposed ML-SIC scheme also converges to a limit, but it is difficult to derive this limit analytically due to the optimization involved in solving (23).

Fig. 6. Performance comparison among various schemes in Rayleigh fading channels with ideal coding and different *K*. $R = 1$ bit/symbol.

B. Fading Channels with Ideal Coding and SIC Decoding

Figure 6 provides the performance of the proposed ML-SIC scheme in fading channels with ideal coding. In Fig. 6, we assume a Rayleigh fading channel with averaged power gain 1 for all users. We divide the whole range of q, i.e., $[0, \infty)$, into M intervals with equal probability $1/M$ and set $M = 25$. The channel-aware ALOHA protocol with SUD (CA-SUD) and $E_T = 5$ dB in (15) is considered as a reference [7]. For a fair comparison, we set the power constraint in (23) as

$$
\bar{e} = \bar{e}_{\text{CA-SUD}} = E_T \cdot \bar{p}_{\text{CA-SUD}} \tag{27a}
$$

where $\bar{p}_{\text{CA-SUD}}$ is the average transmission probability in [7] given by

$$
\bar{p}_{\text{CA-SUD}} = \int_{g} p(g)\Psi(g)dg.
$$
 (27b)

Figure 6 also includes the performance of conventional ALOHA and ML-SUD. Similar observations as in the nonfading case can be made. The proposed ML-SIC scheme improves the system performance significantly. For example, at $K = 8$, ML-SIC can obtain a throughput gain of 58% compared with CA-SUD. Note from Fig. 6 that when K is large, ML-SUD performs worse than CA-SUD although the former involves a joint power level and probability optimization. This is possible since the former is designed by considering type-2 collisions only.

In a fading channel, both the CA-SUD scheme and the proposed ML-SIC scheme can utilize the channel state information to optimize the transmission and so improve the system throughput. Fig. 7 further provides the performance comparison of these two schemes under different power constraints. The system setting is the same as that in Fig. 6 except that K is fixed to be 4. Clearly, to obtain the same throughput, the proposed ML-SIC scheme requires much less power than CA-SUD. As the power increase of secondary users may increase the interference to the primary users, the proposed scheme can be applied to alleviate this problem.

Figure 8 shows the received power distributions of the ML-SIC scheme in fading channels with two different average

Fig. 7. System throughput of a four-user Rayleigh fading channel with ideal coding under different power constraints. $R = 1$ bit/symbol.

Fig. 8. The received power distributions of the proposed ML-SIC scheme with different power constraints in Fig. 7.

power constraints. Clearly, the power distribution *shifts rightwards* when the average power constraint increases, which leads to a lower probability of unresolvable collisions involving two or more users with the same arrival power level.

Recall that the MPR model is originally proposed for lowrate CDMA applications where interference can be alleviated using SUD. The transmission rates considered in this section are for high-rate applications (symbol transmission rate $R \geq 1$) bit/symbol) where MUD is essential for interference suppression. The proposed ML-SIC scheme can efficiently utilize the potential benefit of MUD through multiple-level transmission and SIC receiver, and hence obtain significant performance improvement.

C. Fading Channels with LDPC Coding and Iterative Decoding

Figure 9 compares the performance of various schemes in an LDPC coded system. In the simulation, a (3, 6) regular LDPC code with codeword length 10^4 followed by OPSK

Fig. 9. Performance comparison among various schemes in fading channels with LDPC coding and different *K*.

modulation is adopted. IDMA transmission [23] is assumed with 30 iterations at the receiver. The two-user feasible region of this system has been shown in Fig. 4 and the bounding technique discussed in Subsection V-C is applied for the proposed scheme.

From Fig. 9, the curves for the inner and outer bounds of the proposed scheme almost coincide. This justifies the approximation used in Subsection V-C. The throughput gain, compared with conventional ALOHA, is even more impressive than that in the ideal coding case in Fig. 6. This confirms the discussion at the end of Section V that SIC is not optimal for ideally coded systems when the powers of the users are not properly allocated [14]. LDPC coded systems may perform better in a decentralized power control environment.

VII. CONCLUSIONS

In this paper, a random-power transmission scheme has been developed for random access systems with SIC receivers. A collision involving two packets is resolvable by SIC provided that these two packets have properly designed power levels. Based on this principle, we have studied optimal and suboptimal MPR strategies for cognitive radio systems with multiple secondary users transmitting in random access mode. We have limited our focus to ALOHA-type random access schemes. It is expected that the results can be extended to systems with more sophisticated random access protocols such as carrier sense multiple access (CSMA).

APPENDIX PROOF OF THEOREM 1

For a given distribution f, we define a new distribution $f^{[n]}$ constructed as follows.

$$
f^{[n]}(e) = \begin{cases} \sum_{l=0}^{n-1} \delta(e - E_l) \cdot \tau_l & e < E_n \\ f(e) & e \ge E_n \end{cases}
$$
 (28)

where $\tau_l = \int_{E_l \leq e \leq E_{l+1}} f(e)de, l = 0, 1, \ldots, n-1$. It can be verified that $f^{[0]} = f$.

Fig. 10. Illustration of Remark 3.

Remark 2:

$$
(f^{[n+1]}, f^{[n]}) \underline{b.e.}, (f^{[n]}, f^{[n]}), \forall n. \tag{29}
$$

Proof: From (28), a sample of $f^{[n+1]}$ can be equivalently obtained through the steps below:

Step 1: Draw a power value e_1 according to $f^{[n]}$;

Step 2: If $E_n < e_1 < E_{n+1}$, reduce e_1 to E_n ; otherwise, keep e_1 unchanged.

Clearly, we have $E(f^{[n+1]}, f^{[n]}) \le E(f^{[n]}, f^{[n]})$ from Step 2. In what follows, we will show that the power change in Step 2 does not decrease P_{2U} . Let us focus on the impact of power change from (e_1, e_2) to (E_n, e_2) when $E_n < e_1 < E_{n+1}$. This can be discussed case by case below. In Fig. 10, white circles $\{A_i\}$ represent power pairs drawn from $(f^{[n]}, f^{[n]})$ while black circles $\{A'_i\}$ represent those after the power change
in Step 2, which are also samples drawn from $(f^{[n+1]} \, f^{[n]})$ in Step 2, which are also samples drawn from $(f^{[n+1]}, f^{[n]})$. From Fig. 10, it can be seen that this power change leads to the following three possibilities.

- a) A_1 fails while A'_1 succeeds. Such events result in increased P_{2U} ;
- b) A_2 succeeds while A'_2 fails. Such events cannot happen as $f^{[n]}(e_2) = 0, \forall e_2 \in (E_{n-1}, E_n);$
In all other situations, both nower is
- c) In all other situations, both power pairs fail or succeed simultaneously.

Hence, $P_{2U}(f^{[n+1]}, f^{[n]}) \ge P_{2U}(f^{[n]}, f^{[n]})$. This leads to (29).

Following a similar discussion as that in Remark 2, we can further show $(f^{[n+1]}, f^{[n+1]})$ *b.e.* $(f^{[n+1]}, f^{[n]})$. From the transitive property of the relation $\underline{b.e.}$, we have the following. *Remark 3:*

$$
(f^{[n+1]}, f^{[n+1]}) \underline{b.e.} (f^{[n]}, f^{[n]}), \forall n. \tag{30}
$$

Based on Remark 3, we can prove Theorem 1 by showing that the optimal distribution f takes zero value in $(E_n, E_{n+1}), n \ge 0$. This can be seen by reduction to absurdity. Assume that the optimal power distribution f takes non-zero values in intervals (E_n, E_{n+1}) for some integers *n*. Denote by n^* the minimum integer for such intervals. We can always generate a new distribution f^* with reduced power by merging the power levels in $[E_{n^*}, E_{n^*+1}]$ to E_{n^*} . From Remark 3, (f^*, f^*) b.e. (f, f) . Furthermore, $E(f^*, f^*) < E(f, f)$, and so $(f^*, f^*) \underline{b}$, (f, f) . Hence f is not optimal.

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