# Spectrum Allocation Based on Game Theory in Cognitive Radio Networks

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Abstract—As a kind of intelligent communication technology, the characteristic of dynamic spectrum allocation of cognitive radio provides feasible scheme for sharing with the spectrum resources among the primary user and secondary users, which solves the current spectrum resource scarcity problem. In this paper, we comprehensively explored the cognitive radio spectrum allocation models based on game theory from cooperative game and non-cooperative game, which provide detailed overview and analysis on the state of the art of spectrum allocation based on game theory. In order to provide flexible and efficient spectrum allocation in wireless networks, this paper also provides the general framework model based on game theory for cognitive radio spectrum allocation.

*Index Terms*—Cognitive radio; Spectrum allocation; Game theory

# I. INTRODUCTION

In recent years, with the development of wireless communication technology [1], wireless spectrum resource gradually decrease. And the main reason of spectrum scarcity is the reasonable of the spectrum allocation. Wireless spectrum resource is allocated to different wireless communication system by the spectrum authorities of each country in a fixed spectrum allocation way [2, 3], and were authorized to obtain wireless spectrum communication system, which will be used to maintain the communication in a large-scale region for a long time, even if the authorized user in a certain place and a certain time without using its authorized spectrum, other unauthorized users can't use the frequency spectrum resource, which lead to the waste of time and space, the low spectrum utilization rate. The situation doesn't adapt to the rapid development of wireless communication [4].

In order to solve the deficient problem of spectrum resources, people put forward the concept of Cognitive Radio (Cognitive Radio, CR). Cognitive radio is an intelligent wireless communication system. It realizes spectrum sharing and dynamic spectrum allocation by the spectrum sensing and intelligent learning ability of the system. It is visible that spectrum allocation is an important content in the cognitive radio. Spectrum allocation refers to allocate spectrum to one or more given nodes based on the node number which needs to access system and its service requirements. The choice of spectrum allocation strategy directly determines the system capacity, spectrum utilization rate and whether it will meet users' continuous changing needs because of different business. The current spectrum allocation technologies are classified as below (as shown in Figure 1):

In the cognitive radio, spectrum allocation algorithm design choose spectrum allocation strategies that adapt to the time change characteristics of the wireless environment, which based on detecting available spectrums and controlling the transmit power, therefore, the cognitive radio spectrum allocation focuses on dynamic spectrum allocation [1]. At present, the cognitive radio spectrum allocation model mainly has six kinds: graph theory, the interference temperature, price auction, Partially Observable Markov Decision Process (POMDP), and the game theory.

Game theory logic spreads over the whole economics, and is widely used in politics, science, psychology, evolutionary biology and other social and behavioral sciences. Hence using the game theory to analyze the cognitive radio is a good method. Reference [2] analyzed the convergence of various kinds of game models in cognitive radio network, which combined with interference control and spectrum allocation. Reference [3] used game theory to analysis the power control of the cognitive radio, interference avoidance and Call Admission Control (CAC) and so on.



Figure 1. Classification of spectrum allocation schemes

Reference [4] applied the game framework to analysis the cognitive radio users' behavior in distributed adaptive spectrum allocation, and put forward a kind of adaptive spectrum allocation algorithm, and to minimize the interference to the authorized users. Cognitive users made decisions according to their respective interference benefit function and adaptively choose the minimum interference channel. Reference [5] proposed a kind of non-convex game, and used the optimization theory to study it, which comprehensively analysis the existence and uniqueness of a standard Nash equilibrium. The proposed algorithm is suitable for cooperative or noncooperative cognitive radio scene. Reference [6] used game theory model to solve the security problems in cognitive radio network, and discussed the attacking problems of different protocol layers.

Although cognitive radio can provide feasible scheme for sharing with the spectrum resources among the primary user and secondary users, we still are lack of comprehensive understanding on current spectrum resource scarcity problem. In this paper, we explored the cognitive radio spectrum allocation models based on game theory from cooperative game and noncooperative game, which provide detailed overview and analysis on the state of the art of spectrum allocation based on game theory. In order to provide flexible and efficient spectrum allocation in wireless networks, this paper provides the general framework model based on game theory for cognitive radio spectrum allocationalso proposes a cognitive radio spectrum allocation framework model based on game theory.

The remainder of this paper is organized as follows. Section II introduces Spectrum Allocation general Models. Section III explores the Spectrum Allocation Special Models in details. At last, section 6 draws the conclusion.

#### II. SPECTRUM ALLOCATION GENERAL MODELS

The spectrum allocation problems in cognitive radio include the secondary user game process, the primary user game process and the primary and the secondary users united game process. In the game of the secondary user system, cognitive radio users are game participants, their action strategy is to select the demand spectrum, decision making process is that secondary users select primary users' spectrum for communication and determine the number of required spectrums. During the process of the primary user game, the primary user with spectrum authorization is the participants in the game. Their action strategy is to choose the number of loan spectrums, the decision-making process is that the primary user chooses how much spectrums to loan to the secondary user for communication. And in the primary and secondary users united game, the primary and the secondary users are participants, primary users' strategy is the amount that they what to loan, secondary users' action is to choose the spectrum demand. Decision making process is the primary user determines the number of the loan spectrums. Secondary users decide the spectrum demand.

Reference [7] considered the problem that a primary user and multiple cognitive users share with spectrums. The paper considered the problem as a oligarchic market competition. Spectrum allocation between secondary users was the cooperation game. Firstly the author made a static game. All of the secondary users had the current adopted strategies and the mutual payment information, and then the secondary user gradually adjusted their strategies iteratively according to the last stage they observed. Reference [8] analyzed the problem of wireless spectrum allocation problem that multiple primary users sold spectrum opportunities to multiple cognitive users. Multiple primary users' competition could be decomposed into the problem of every two primary users' competition. This paper considered the wireless spectrum allocation model of two primary users and multiple (N = 100) cognitive users. Two primary users had two independent spectrum shared pool: H and L. Primary users had the right to decide which part of the channel of the spectrum shared pool to rent. The channel quality of the spectrum shared pool H and L also had certain differences, each cognitive user had different preference in using the channel. Secondary users adjusted their spectrum purchasing behavior through the observation of the quality and price which provided by primary users. Primary users adjusted their behavior to achieve the highest utility when they sold spectrum opportunities to secondary users; the utility of the target was the maximum of the channel utilization. Reference [9] analyzed the scene of the oligopoly market that multiple

primary users competed each other for providing spectrums for a secondary user in the cognitive radio network. Through the use of a balanced pricing scheme, each primary user's first goal was to maximize its profits under the condition that the quality of service (QoS) was limited. The decline of the primary user's quality of service was the cost that providing the spectrum access opportunities to secondary users. Through establishing collusion between primary users to lead the primary user could obtain higher utility than that of the Nash equilibrium. Reference [10] considered the scene that multiple secondary users and a primary user disturb each other. Firstly, constructing a potential game model to allocate power and spectrum at the same time. Then, forming the Stackelberg Game model based on this structure. They analyzed how the authorized users spectrums cognitive allocated to users for communication under the condition that their QoS were satisfied. The simulation results showed that the designed model had good convergence.

The general model based on game theory of the spectrum allocation problem in the cognitive radio is as below:

$$G = \{N, S_1, \cdots, S_N; u_1, \cdots, u_N\}$$
(1)

The expression (1) indicates that there are *N* game participants.  $\{S_1, \dots, S_N\}$  is the strategy space or strategy set of all participants. For any one of the game participants i,  $S_i$  is its strategy space. Any of a particular strategy uses  $s_i$  to express, and  $s_i \in S_i . \{s_1, \dots, s_N\}$  is a strategy combination which is constituted by the strategy every participant choose one,  $u_i$  indicates the revenue function of the game participant i,  $u_i\{s_1, \dots, s_N\}$  is the revenue of the game participant i when he chooses the strategy  $\{s_1, \dots, s_N\}$ .

The System model that considers all the secondary users as a whole can be shown like the following:



Figure 2. SYSTEM MODEL

The performance of the spectrum allocation algorithm based on game theory in cognitive radio largely depends on the selection of the utility function. Cognitive radio network under different application situations would select utility function based on different targets, such as setting the goal to maximize the system throughput, or to maximize spectrum utilization rate, to minimize system interference level, to ensure users fairness and so on.

#### **III. SPECTRUM ALLOCATION SPECIAL MODELS**

Game theory can be divided into the cooperative game and the non-cooperative game according to the method of cooperative. The difference between cooperative game and non-cooperative game is whether the behavior among participants have a binding agreement. If any, it is the cooperative game, else it is a cooperative game.

#### A. Cooperative Game

The definition of cooperative game is given in the form of characteristic function, like (N, v). We make  $N = \{1, 2, 3, \dots, n\}$  indicates the set of participants. N is an integer. It shows the number of participants. S is a subset of N. It shows the coalition among participants,  $S \subseteq N$ . Given a limited set of participants. The characteristic of cooperative game is ordered pair (N, v). Thereinto eigenfunction v is the mapping from  $2^N = \{S \mid S \subseteq N\}$  to the set of real numbers  $\mathbb{R}^N$ , namely  $v: 2^N \to R^N$ , and  $v(\varphi) = 0$ . v is the corresponding eigenfunction with every coalition S in N. V(s) is the utility that participants in a coalition S cooperate with each other. There have been a lot of researches which using the cooperative game theory to analyze the resource allocation problem in cognitive radio. Reference [11] described the problem of spectrum allocation process with the cooperative game theory in cognitive radio, the procedure is as follows:

They signed a spectrum use agreement before cognitive users accessing spectrums. The agreement ensured that the user could get more revenues through the cooperation than that of acting alone. The core could be used to test whether the cooperation was stable. When considering the principle of average and fairness, we can use shapely value to allocate cooperative income of cognitive users. When considering maximizing the minimum fair principle, we can use the kernel to allocate the benefit of cooperation of cognitive users. The spectrum allocation problem of the cognitive radio combined auction theory with cooperative game theory is analyzed in [12]. In the cognitive radio secondary users (SUs) cooperatively sensed spectrums to identify and obtain free spectrums and share them. The sensing and sharing scene of spectrum is modeled as a transferable utility (TU) cooperative game in the paper. They used Vickrey-Clarke-Groves (VCG) auction mechanism to allocate spectrum resources for each secondary user fairly. The secondary users formed alliances to sense spectrum together. Each secondary user's value could be calculated according to the activity information of the primary user which was obtained from spectrum sensing in the joint. The resulting game was balance and super additive. Every secondary user got a sum of income according to their value in the league. According to the secondary users' demand of spectrum, they used the income to bid for free spectrums through the VCG auction. VCG auction mechanism made secondary users bid honestly according

to their needs.

Reference [13] considered that the resource allocation network always cooperated and coexisted in cognitive network. They used the cooperation game theory to analyze the resource allocation problem in cognitive radio network. In the real cases, partners only had part of information related or didn't have to make a decision in advance. However, current game theory models always think partners have enough information in the cognitive radio network. In this paper, they considered incomplete information and put forward a algorithm based on the cooperative game theory was considered to allocate resources. Simulation results show that, with the traditional algorithm, the algorithm could improve resource allocation efficiency of the cognitive radio network. Reference [14] constructed an Asymmetric Nash Bargain Solution Based (ANBS) utility function based on OFDM cognitive radio environment, which achieved a new spectrum sharing algorithm through two users' bargaining based on sensing contribution weighted proportional fairness. The simulation results showed that using the cooperative game theory, the method in this paper not only realized the fairness and validity of spectrum resources allocation, but also helped to maximize the spectrum sensing.

Matching Game (Matching Game) is a kind of cooperation game which is widely researched and used. In the matching game models of spectrum allocation, the user and channel match as bilateral market. Cognitive users obtain the available channel information through spectrum sensing in cognitive radio network and send the channel information to the base station. Base station calculates the preference of the available channel according to channel information and allocates channels to the cognitive users according to the matching game algorithm.

There are also related researches of cognitive radio spectrum allocation with match game model. In order to manage spectrums in cognitive radio system effectively, reference [15] used reinforcement learning method based on POMDP (Partially Observable Markov Decision Processes) model to analyze time-varying characteristics of secondary users and channel states and puts forward Matching Game model (Matching Game) of spectrum allocation. Cognitive users adjusted their matching strategies through observing payoffs of maximum system of history information statistics. The simulation results showed that this method could realize efficient allocation of spectrum resources.

Primary users rent spectrums to secondary users to get some rewards, so there is a competitive relationship among primary users. Whereas it's competitive relationship is among secondary users for leasing spectrums. Therefore, it's generally selfish noncooperation to obtain the greatest utility among primary users and secondary users respectively. So it is a very effective method if we use non-cooperative game to study spectrum allocation in cognitive radio network.

# B. Non- cooperative Game

The main non-cooperative game models are: cournot

game model, bertrand game model, stackelberg games model, repeated games model, supermodular game model, potential game model, evolutionary game model, auction game model.

# (1) Cournot Games Model

Cournot game model belongs to the complete information static game, game participants compete for output. Reference [16] used this model to study selfish noncooperative spectrum allocation behavior among primary users. The price of spectrum of the primary user was the same, but the sale quantity was not identical. The primary user always knew other primary users' spectrum history strategies and determined the current strategy according to the historical information. The number of spectrum which was sold by primary user achieves stable equilibrium state after many games. Therefore, the purpose of the cournot game model was to maximize spectrum number which was sold by primary users system, the method was to maximize the utility function of primary users. Reference [17] studied cognitive radio dynamic spectrum allocation based on the game theory, considering the difference of spectrums; this paper used a cournot game model and added the spectrum similarity matrix to original pricing function. They put forward a new utility function to make the spectrum allocation closer to the real network environment. The simulation analysis showed that the allocation algorithm was more diversified than the original algorithm in considering the differences of spectrums. It was applicable to the actual network allocation. Reference [18] used game theory to analyse primary users' leasing spectrum behavior in the cognitive radio. First of all, the system model of spectrum allocation was established. Secondly, the cournot algorithm was designed based on the system model. Finally, the simulation completed the situation that the total number and the price of the lending spectrum vary with the increasing of the number of the primary users. The simulation results showed that the lease spectrum quantity increased a lot. Compared with static spectrum allocation algorithm, using cournot algorithm reduced the price of the spectrum.

#### (2) Bertrand Game Model

Bertrand game model also belongs to the category of complete information static game, game participants compete for the price. Reference [9] used bertrand game model to study the selfish noncooperative spectrum allocation behavior among primary between users. The primary user always knew other primary users' history sale price of the spectrum, and determined their own current price according to this historical information. After many games, the primary user sell spectrum price to reach the Nash equilibrium state. Therefore, the purpose of bertrand game model was to optimize the sale price of the primary user' system. The method was to make to maximize the utility function of the primary user. Reference [19] constructed the behavior of the primary user in spectrum allocation as bertrand game model. This paper put forward a oligopoly pricing framework for dynamic spectrum allocation. In this model, the primary user sold excessive spectrums to secondary users to get rewards. The paper put forward strict constraint and QoS punishment two kinds of methods to simulate the primary user whose ability was limited in the actual situation. In the oligopoly model which had strict constraints, the author proposed a low complexity searching method to get Nash equilibrium and proves its uniqueness. When reducing to a duopoly game, the analysis showed interesting gap in the leadership-subordinate pricing strategy. In the QoS punishment based on oligopoly model, this paper proposed a novel variable transformation method and deduced the unique Nash equilibrium. When the market information was limited, this paper provided three "StrictBEST", short-sighted optimal algorithm "StrictBR" and "QoSBEST", to make the price adjust principles based on the best response function (BRF) and bounded rationality (BR) for duopoly primary users.Numerical results demonstrated the validity of the analysis and proved that the "StrictBEST" and "QoSBEST" converged to the Nash equilibrium. "StrictBR" algorithm revealed the chaotic behavior of the dynamic price adaptation in response to the learning rate.

## (3) Stackelberg Game Model

Stackelberg game model belongs to the category of complete information dynamic game. Game participants compete for production. This model was used to study the primary user's selfish noncooperative spectrum allocation behavior in reference [20]. The price of the spectrum which were lended by primary users was the same, but the number of every primary user' sale spectrum each was not identical. Different from the game process of cournot game model, primary users didn't take the strategy at the same time during the game.

Part of the primary users took strategies first, and the other part took strategies later. The primary user who took the strategy later knew the sale spectrum strategy information which took the strategy first, and determined their own current strategies according to this information. After many game, the rental spectrum number of authorized user achieved stable equilibrium state. The goal of stackelberg game model was to maximize the amount of the spectrum rented by the primary user system. The method was to maximize the primary users and secondary users' utility function.

In order to understand the interaction between the secondary user and the primary user, Reference [21] first constructed a game of union form to study secondary users' subband assignment problem, and then combined joint form game with layered structure based on stackelberg game. This paper put forward a simple distributed algorithm in order that secondary user would find optimal bandwidth, which proved that the transmission power and the secondary user's subband allocation and the price of the primary user were associated through the price function of the primary user. This made the joint optimization possible. The paper also proved that if the primary user's subband allocation would be very stable. Stackelberg game

equilibrium of stratified game architecture was unique and optimal.

Reference [22] used the stackelberg game model to analyze the primary user allocated spectrum resources to the secondary user's problem in the cognitive radio technology. The primary user was the seller and the secondary user was the buyer in this spectrum sale game. Modeling the spectrum users is selfish, rational players. Considering that the primary user always knew more about the information of "the market" and the market price should always be purchased in advance. This paper used stackelberg game to analysis the spectrum pricing and allocation process under the condition of the asymmetric information, and which introduces parameter I to measure the negative effect from the secondary user to the primary user. If given a predefined value I, you could find a feasible pricing area to guarantee primary service. At last, it put forward a contract between the primary user and the secondary user as a asymmetric information matching, Having observed the increase of the secondary user's utility when the contract is effective.

The primary user and secondary user's utility function of stackelberg game model and the transmission system modulation model the same with the cournot game model. The first-mover advantages of stackelberg game model in spectrum allocation algorithm were also presented. The number that the primary user who adopt the strategy first sell spectrum number stable state is greater than those who adopt the strategy after. According to the process of the game, it is known that stackelberg game model is more suitable for authorized user to take strategies in sequential, and the cournot game model is suitable for the scene that authorized users take strategies at the same time.

#### (4) Repeated Game Model

Repeated game model belongs to the category of the dynamic game. It can be the complete information dynamic game, and also can be the incomplete information dynamic game. Repeated game is composed of multiple game phases, the game form of each phase is the same. It's different from one game that cooperative behavior would appear in the repeated game.

Reference [23] showed that players would estimate their future values of  $u_i$ . After many repeated game phases, they modified the original objective functions by discounting the expected returns in future stages by  $\delta$ , where  $0 < \delta < 1$ , so the anticipated value player *i* in stage *k* is as below:

$$u_{i,k(a)} = \delta^k u_i(a) \tag{2}$$

Reference [24] showed that when there is no communication between the players, memory of past events, or speculation of future events, the repeated game is called the myopic game. There are two convergence dynamics possible in a myopic game. They are the *best response dynamic* and the *better response dynamic*. The paper also gave the definition of the two *dynamics*:

Best response dynamic:

At every stage, one player  $i \in N$  is permitted to deviate from  $a_i$  to some randomly selected action  $b_i \in A_i$  if  $u_i(b_i, a_{-i}) \ge u_i(c_i, a_{-i}) \forall c_i \ne b_i \in A_i$  and  $u_i(b_i, a_{-i}) > u_i(a)$ 

Better response dynamic:

At every stage, one player  $i \in N$  is permitted to deviate from  $a_i$  to some randomly selected action  $b_i \in A_i$ , if  $u_i(b_i, a_{-i}) \ge u_i(a_i, a_{-i})$ 

Repeated game has been fully applied. Reference [25] put forward a utility function based on the primary user free probability which used the game theory model in the distributed cognitive radio network which had multiple primary users and a secondary user. The primary users could realize Nash equilibrium through adjusting learning rate with the repeated game. Simulation showed that this method makes the primary user's free probability bigger and the system utility higher. The system had the largest utility when the spectrum provided by the primary user was completely free. In addition, the Nash equilibrium was invalid when the total profit of the primary user's didn't reach the maximization. At last, the cooperation optimal solutions could obtain the highest system profit. Reference [26] researched the spectrum sharing problem that multiple systems coexisted and interfered mutually in a unauthorized band. They put forward the spectrum allocation algorithm based on repeated game theory to maximize the system throughput in non-cooperative scene. In the repeated game, the game participants may be active in establishing a "good" reputation to sacrifice immediate interests for long-term benefits.

#### (5) Supermodular Game Model

To all  $i \in N$ , the supermodular game restricts  $\{u_j\}$  like the following expressions [27]:

$$\frac{\partial^2 u_i(a)}{\partial a_i \partial a_j} \ge 0 \forall j \neq i \in N$$
(3)

$$\frac{\partial^2 u_i(a)}{\partial a_i \partial a_i} \le 0 \forall j \neq i \in N$$
(4)

When meets expression (3), the game is said to be supermodular; when expression (4) is satisfied, the game is said to be submodular.

Reference [28] mentioned that the supermodular game has weak FIP property, such as, it started from the initial action vector, there are a series of "selfish" strategy to change the way to make the game converges to the Nash equilibrium. In particular, in the supermodular game a best response sequence could also make a game process converges to the Nash equilibrium [28]. By the Topkis fixed point theorem [29], it is known that all supermodular games exist one Nash equilibrium at least. Furthermore, if radio only produces limit errors, or the radio makes the best response according to the average weight of the past behavior of their own observation. The entire process will be convergence [28] [30]. Reference [31] used the method of the supermodular game theory to study the spectrum sharing in the cognitive radio network. In this paper, they considered a bertrand competition game model, the main service providers competed for selling their free spectrums to maximize their profits. Then proving that bertrand competition was a smooth supermodular game, and using the method of the cyclic optimization to obtain the optimal price solutions. The simulation results verified the algorithm approximately converged to an equilibrium point, and analyzed the influence of equilibrium point of external variables.In the reference [32], Nie Nie etc. proposed a spectrum allocation algorithm based on exact potential game, and at the same time, in order to make up for the deficiency of the algorithm, they put forward a kind of spectrum allocation algorithm of no regret learning. Reference [33] analyzed the convergence of all kinds of game models in the cognitive radio. The paper also separately discussed the spectrum allocation problem of the supermodular game, exact potential game and other special game model in detail, and gave the corresponding spectrum allocation algorithm.

## (6) Potential Game

To the potential game, meeting the condition of exact potential game in  $U_i(s_i, s_{-i}) - U_i(s_i, s_{-i}) = P(s_i, s_{-i}) - P(s_i, s_{-i})$  that P get to the points of maximum are all the Nash equilibrium point of potential game, potential game has FIP attribute, so when the node take the selection of selfish strategy, it will converge to a Nash equilibrium.

Reference [32] used potential game theory framework to the analysis distributed adaptive channel allocation behavior of cognitive radio. The allocation model was set like this: in a cognitive radio network, N sending and receiving couples of cognitive users distributed in the scene equably, they were viewed as stationary, or slow moving. There were K available spectrums in the scene, satisfying K < N. The paper also defined two different objective functions for spectrum sharing game, it respectively captured the utility of the selfish users and cooperation users. In this paper, they proved that the channel allocation problem of the utility definition of the cooperation user could be constructed as a potential game, so it converged to a Nash equilibrium point of deterministic channel allocation. The utility function proposed by reference [32] can be proved with the

expression 
$$\frac{\partial^2 u_i(a)}{\partial a_i \partial a_i} = \frac{\partial^2 u_i(a)}{\partial a_i \partial a_i} \forall i, j \in N, a \in A$$
, that the

utility function has a exact potential function P, meeting  $U_i(s_i, s_{-i}) - U_i(s'_i, s_{-i}) = P(s_i, s_{-i}) - P(s'_i, s_{-i})$ .

Reference [33] would be a reference about the solution of the Nash equilibrium. Assuming that every player could clearly know the strategy information of their rivals. Game participants made the goal that maximizing the utility function of the next game process. Through observing the strategy of the opponent to determine their optimal strategies, and achieving Nash equilibrium ultimately through continuous repeated game process.

Reference [34] proposed a non-cooperative game theory framework to allocate powers and spectrums for

cognitive users of the cognitive radio network together. The proposed game was proved to be a exact potential game. The game converged to Nash equilibrium ultimately by following the best response dynamics. The paper also put forward the certain optimal response strategy based on the opponent of observation of each cognitive radio. From the simulation results, we could see that the resources are shared fairly in all the cognitive users.

Reference [35] analyzed the cognitive radio model based on the game theory: a potential game and no regret learning game. And putting forward a mathematical model based on dynamic spectrum allocation algorithm of the potential game. Finally, the simulations of these algorithm proved that advanced dynamic spectrum allocation algorithm based on potential game theory that had the best capacity performance. However, how to reduce the allocation cost is also a very big problem in the process of spectrum allocation.

# (7) Evolutionary Game

The evolutionary game theory is the new development of game theory in recent years. The basic thought of it is the evolution and genetic theories [36]. The evolutionary game theory assumes that the behavior subjects have "limited rationality", knowing the local information and need to adapt to environmental changes gradually through the test, the imitation and study and other dynamic adjustment process, and this is more close to the real life.

Reference [37] put forward a kind of spectrum allocation algorithm based on evolutionary game in the cognitive radio network. The primary users leased their free spectrums to secondary users. In the process of the evolutionary game, the secondary users in different groups competed for the limited spectrums resource. At the same time they also needed to complete their own evolutionary adjustment of spectrum selection strategy. i.e., if a user observed and found the current frequency band that it selected to access in the period was lower than the average incomes of all the users of the group, the user would have their own strategy adjustment (i.e., select to access to other usable frequency band), and tried to increase their incomes in the next period through imitating other users' good spectrum selecting strategy of the group. At the same time, the strategy equilibrium among the cognitive users was the result of learning adjustment of repeated game. When the spectrum selection process of the users reached to evolutionary equilibrium state, each user of the group would get the same benefits. At the same time they could realize the equilibrium stable spectrum choice under the evolutionary stable strategy. The primary users got their best utility through mutual competitive price. The simulation showed that the proposed algorithm was better than the equilibrium price and utility of the primary user. Reference [38] considered a dynamic spectrum leased problem of the spectrum secondary market of the cognitive radio network that the secondary service provider leased spectrum from the spectrum

brokers and then provided services to the secondary users. The optimal decision of the secondary providers and the secondary users decided dynamically under the competition. Since the secondary users could adapt to the service selection strategy according to the received service qualities and prices. Modeling the dynamic service selection to a evolutionary game in a lower level. Applying the replicator dynamics to simulate service choice adaption and evolutionary balance, using dynamic service choice, competitive secondary providers could dynamically lease spectrums to provide services to secondary users. In a higher layer making a spectrum leased differential game to simulate the competition. The evolutionary game service chose distributed to describe the state of differential game. Open loop and closed loop Nash equilibrium both had solutions as differential dynamic control game. Numerical comparison showed that it was better than the static control in the profit and convergence speed. Reference [39] used the method of the evolutionary game theory to study cooperative spectrum allocation problem in the cognitive radio. This paper pointed out that the cooperative spectrum sharing among primary users (PU) and multiple secondary users (SUs) helped to improve the whole system's throughput. They proposed a kind of two-tier game in the sharing of spectrum in which secondary users decided whether to cooperate under replicator dynamics and the primary user adjusted its strategy to allocate time slots for the cooperative secondary users' transmission. In addition, the author also designed a distributed algorithm to describe the secondary user's learning process, proved that the dynamics could effectively converge to the evolutionary stable strategy (ESS), this was also the optimal strategy of the primary and the secondary users. The simulation results showed that the proposed mechanism converged to the ESS automatically, at this time all the secondary users would keep their strategies. In addition, it showed that the mechanism could help secondary users to share information and obtained higher transmission rate than that in fully cooperative or non-cooperative scene.

# (8) Auction Game

In cognitive radio network, secondary users requests the primary user to rent spectrums, the primary user received the request from the secondary users and to loan the free spectrums to the secondary users in appropriate prices. Because the secondary users can hire multiple primary users' spectrums, different primary users bid mutually in order to attract secondary users. They also have to consider their own benefits at the same time, so as to form the game process that multiple primary users bid. Through the auction game, the primary users get the additional utility; the secondary users can also use the transmission information of spectrum of the primary users. It realizes the spectrum sharing and improves the spectrum utilization.

Reference [40] studied the multimedia flow problems of the cognitive radio network. There were a primary user and N secondary users in the network. This paper looked the spectrum allocation problem as a auction game and put forward three spectrum allocation scheme based on auction. Spectrum allocation separately used single object pay-as-bid ascending clock auction (ACA - S), the traditional ascending clock auction (ACA - T), alternative ascending clock auction (ACA - A) three methods in the three schemes. The author proved that the three algorithms converged to a limited number of clocks. The paper also proved ACA-T and ACA-A was cheat-proof, but ACA-T was not. In addition, this paper showed that the ACA-T and ACA-A could maximize the social welfare, but the ACA-S may not. Therefore, ACA-A was a very good way to solve the cognitive radio network of multimedia, because it could maximize the social welfare in a cheat-proof way. Finally, the simulation results proved the efficiency of the proposed algorithm.

Reference [41] put forward a new method to encourage the primary users to lease their spectrums: the secondary users bided to show that they were willing to spend how much power to transmit the primary signal to the destination. Due to the asymmetric cooperation, the primary users achieved power saving. In the centralized structure, a secondary system decision center (SSDC) selected a bid for each of the primary channel based on the best channel allocation. In the distributed cognitive network architecture, the author made a agreement based on the auction game. Each secondary user bided independently for each primary channel in the agreement. The recipients of each primary link selected the bid of the largest power saving. This simple, robust distributed strengthening learning mechanism allowed the user to modify their bid and increased their rewards. Results showed that the major effect of the strengthening learning was to improve the utilization efficiency of spectrum and met the performance requirement of individual secondary user.

#### (9) Performance Analysis

But it has to choose whether to use cooperative game model or non-cooperative game model according to the different applied scene of the cognitive radio network, and decide which one of the non-cooperative game model to choose. Non-cooperative game emphasizes individual rational and individual optimal decision; the participants of the game are selfish, rational. Their purpose is to maximize their income to take the optimal decision. The result may be efficient and also may be inefficient. As the Nash equilibrium of the solution of the non-cooperative game, although it can ensure the maximization of personal utility, but it do not have group's optimality generally, namely it's not pareto optimal. And cooperative game emphasizes collective rationality (including efficiency, justice, fairness, etc.) and overall optimal decision, the solution (such as nash bargaining solution) generally has the Pareto optimality and social optimality, namely the outcome of game can guarantee the interests of the other participants are not damaging, Under the precondition, at least one of the participants' interest is increased, so that the Collective interests increase. The participants of the cooperative game play the game through reaching binding cooperation agreements, so they can obtain higher earnings than the non-cooperative game. That is to say, cooperation can improve efficiency and realize the results which can't achieve in the noncooperative case.

For the nine models of the cooperative game and non-cooperative game, they respectively have the applicable range and advantages and disadvantages. Summarizing as below:

Game model	Advantages	Disadvantages
Matching game model [15]	Efficiency, fairness, pareto optimum	Maximization of collective benefit, but not the personal interests; limited application scope.
Cournot game model [16],[17],[18]	Simple model of two oligarchs, solve the spectrum allocation problem with two authorized user and multiple cognitive users.	Many constraints. static game model, poor flexibility, limited application scope.
Bertrand game model [9],[19]	Solve the spectrum allocation problem with two authorized user and multiple cognitive users; Suitable for the game among the authorized users.	Static game model. poor flexibility; limited application scope; Nonoptimal equilibrium;
Stackelberg game model [20],[21],[22]	Improving the tenemental amount of spectrum, reducing the tenemental prices of spectrum, high utilization.	Too much constrained condition
Repeated game [25],[26]	Maximization of the gross income; minimize total interference level.	High complexity
Supermodular game model [31],[32],[33]	Maximization throughput, Reducing the interference of cognitive users to primary users.	Rigorous application
Potential game model [34],[35]	Maximizing throughput, reducing the interference problem of cognitive users to primary users. Suitable for cooperation users.	Convergence is easier for the simple model.
Evolutionary game model [37],[38],[39]	Accurately predict the dynamic behavior.	High complexity
Auction game model [40],[41]	Influence to the game with different band utilization rate.	Limited application scope

TABLE I SUMMARY OF GAME MODELS

# IV. CONCLUSION

This paper illustrates that introducing the game theory into spectrum allocation of the cognitive radio and describes the analytical scheme of game theory of spectrum allocation in cognitive radio. The paper divided the spectrum allocation models based on game theory into cooperative game model and non-cooperative game model. And making related investigative summary and description about the matching game model of the cooperative game model of and eight kinds of non-cooperative game models: the cournot game model, the bertrand game model, the stackelberg game model, the repeated game, the supermodular game model, the potential game model, the evolutionary game model, the auction game model. And the problems that each model is suitable for researching are different. We should choose the right model according to of the problem you analysis. Game theory provides a good theoretical tool for the research of spectrum allocation in cognitive radio, but this method is still in its beginning stage and doesn't form a complete theoretical system, and also the game models which can apply is limited and the conditions are very strict. The spectrum allocation based on the game theory in the cognitive radio network still has a lot of problems to be solved. The future work will focus on optimized spectrum allocation algorithm based on game theory to achieve better performance and spectrum utilization.

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