Inter-Cell Interference in Noncooperative TDD Large Scale Antenna Systems

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***Abstract*—In this paper we study the performance of cellular networks when their base stations have an unlimited number of antennas. In previous work, the asymptotic behavior of the signal to interference plus nose ratio (SINR) was obtained. We revisit these results by deriving the rigorous expression for the SINR of both downlink and uplink in the scenario of infinite number of antennas.**

We show that the contamination of the channel estimates happens whenever a pilot sequence is received at a base station simultaneously with non-orthogonal signals coming from other users. We propose a method to avoid such simultaneous transmissions from adjacent cells, thus significantly decreasing interference. We also investigate the effects of power allocation in this interference-limited scenario, and show that it results in gains of over 15dB in the signal to interference ratio for the scenario simulated here. The combination of these two techniques results in rate gains of about 18 times in our simulations.

***Index Terms*—Antenna arrays, cellular networks, mobile communication, multiple access interference**

I. Introduction

THE EXPONENTIAL increase in demand for high data rates, as well as the higher user density in cellular networks require new ways of mitigating interference allowing a larger number of users to share bandwidth. This, along with the Green Touch initiative to decrease the power consumption in communications networks, motivates the analysis of cellular systems with a very large number of base station (BTS) antennas. Such systems have been studied extensively (see [2]- [5] and references therein).

In [6], the author derived estimates for SINR values in a non-cooperative cellular network in which the number of BTS antennas tends to infinity. It is shown in [6] that not all interference vanishes, and therefore, SINR does not grow indefinitely. The reason is that the channel estimates made at the BTS contain not only the desired channel vector and additive white noise, but also components directed towards users in other cells who are assigned non-orthogonal pilot sequences. The numerical results obtained in [6] show that this type of non-cooperative cellular network may provide breakthrough data transmission rates.

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We present here an extension of the results in [1], which was written by the authors of this paper. In [1], we investigate the performance of the downlink of a cellular system with a large number of antennas, analyze the impact of power allocation in this setting and devise a method for mitigating interference. Here, we extend these results to the uplink of such systems. The main aspect of this extension is the proposal and analysis of the performance of a scheme that decreases the residual interference, including both uplink and downlink communications in this analysis. The techniques proposed in this paper are shown to increase rates overall, with the focus on guaranteeing higher quality of service to even poorly located users.

This paper is organized as follows. First, in Section II, we describe our system model and assumptions. Then, in Section III, we derive a rigorous expression for the asymptotic SINR for both the downlink and uplink of cellular networks as the number of BTS antennas tends to infinity for the case when all users transmit their pilots simultaneously. Later in this section, we propose a scheme where the time overlap of pilots is decreased, resulting in less contamination of the channel estimates and consequently a substantial reduction in interference. Noting that the asymptotic expressions for the SINR are not affected by additive noise, we investigate the impact of distributed power allocation algorithms in Section IV. In Section V, we present the results of numerical simulations and show that the proposed interference reduction algorithms and power allocation increase data transmission rates approximately by a factor of 20.

II. System Model

We consider a cellular network composed of L *hexagonal cells*, each consisting of a central M*-antenna* base station (BTS) and N_u *single-antenna users* that share the same bandwidth.

We assume that Orthogonal Frequency-Division Multiplexing (OFDM) is used. Consequently, we consider a flat-fading channel model for each OFDM subcarrier. For a given subcarrier we denote by $\mathbf{g}_{ikl} = \sqrt{\beta_{ikl}}\,\mathbf{h}_{ikl}$ the *channel vector* between the i-th BTS and the k-th user of the l-th cell. We assume that the *small-scale fading vectors*, $\mathbf{h}_{ikl} \sim \mathbb{C}\mathcal{N}(\mathbf{0}, \mathbf{I})$, are statistically independent across users. By β_{ikl} we denote the *large-scale fading coefficients* (log normal distribution and geometric decay). These coefficients are constant with respect to frequency and BTS antenna index.

We further assume frequency block fading model, in which the channel vector $\mathbf{g}_{ikl}$ is constant across N_{smooth} subcarriers. Thus each user would need to send a pilot only in one

Fig. 1. TDD transmission protocol for $U = 2$, $K = 5$, $N = 1$ and $D = 3$.

subcarrier for every N_{smooth} subcarriers. So the *maximum number of users per cell* is $N_u = K\,N_{smooth}$, where K is the number of *available pilot sequences* at each cell. By ρ_{kl} and P_{kl} we denote the *pilot power* and *BTS transmit power* respectively.

The block fading model in the frequency domain is a rough approximation to the actual behavior of the channel gains. However, sending only one pilot every coherence bandwidth can be used in real situations. It provides the base stations with evenly spaced samples allowing it to interpolate to find the gains for the other subcarriers.

We also assume a time block fading model. Thus channel vectors $\mathbf{h}_{ikl}$ stay constant during coherence blocks of T OFDM symbols. The channel vectors in different coherence blocks are assumed to be independent. The large-scale fading coefficients are assumed to remain constant, as they change more slowly by some orders of magnitude.

We assume reciprocity between uplink and downlink channels, i.e., β_{ikl} and $\mathbf{h}_{ikl}$ are equal for both directions.

Let us consider here a Time-Division Duplexing (TDD) scheme. For every user, each coherence interval is organized in four phases:

- first, each user sends *uplink data* to its BTS for U *symbol periods*.
- then, the user sends a pilot sequence of length K to its BTS;
- the BTS then uses this pilot to estimate the corresponding channel vector, with which it processes the data received at the uplink phase ;
- the BTS then transmits *downlink* data for D *symbol periods* to its mobile units using the channel estimates as beamforming vectors;

We assume that the estimation process takes N OFDM symbols. Therefore, each coherence interval has length $T = U + K + N + D$, as depicted in Figure 1.

III. Asymptotic Behavior of SINR

In this section, we analyze the asymptotic behavior of the SINR as the number of BTS antennas M tends to infinity while the number of users per cell N_u remains finite, constant and equal to the length of the pilot sequence K. Here we revisit the results obtained in [6], deriving more general expressions for the SINR of downlink and uplink taking into account all pertaining variables, such as pilot powers, base station transmit powers and normalization constants.

The analysis for $M = \infty$ facilitates the presentation of the concept of time-shifted pilots, and it constitutes a useful limiting case. Certainly a finite-M analysis is needed to provide first-order performance estimates of practical systems, but that is beyond the scope of this paper. Practically speaking one would like to have several-times as many service-antennas as terminals. In turn the number of terminals that can be serviced simultaneously is limited by their mobility: if half of the coherence interval is used for reverse-link pilots, then the maximum number of terminals is equal to the coherence-time divided by twice the channel delay-spread. If high radiated energy-efficiency (bits/Joule) is a priority then even greater number of service-antennas may be desirable, since every doubling of M permits a reduction in total radiated power by a factor of two.

In this asymptotic scenario, we first analyze the case where all users send the pilots simultaneously. Next, we propose and analyze a scheme with time-shifted pilots and show that this scheme causes a significant reduction of the interference in the asymptotic regime.

In both cases, we assume that in all cells the same set of K orthogonal pilots of length K is used. The k - th users in all cells use the same pilot sequence $\psi_k = (\psi_{k1}, \ldots, \psi_{kK}), |\psi_{kj}| = 1$. Since the pilots are orthogonal we have $|\psi_{k'}^{\dagger} \psi_k| = K\delta_{k,k'}$.

A. Aligned Pilots

Here, we focus on the case where pilots are sent simultaneously by all users in the system. At the pilot-transmission stage, the i-th base station receives the signal

$$\mathbf{y}_{B_i} = \sum_{l=1}^{L}\sum_{k=1}^{K}\sqrt{\rho_{kl}\,\beta_{ikl}}\mathbf{h}_{ikl}\psi_k + \mathbf{z}_i, \tag{1}$$

where $\mathbf{z}_i \in \mathbb{C}^{M\times K}$ is the additive noise. Without loss of generality, we assume that the entries of $\mathbf{z}_i$ are i.i.d. $\mathcal{CN}(0,1)$ random variables and that all gains are scaled accordingly.

The i-th base station estimates the vectors $\mathbf{g}_{ik'i}$ for users located in the same cell as $\hat{\mathbf{g}}_{ik'i} = \frac{\mathbf{y}_{B_i}\,\psi_{k'}^{\dagger}}{K}$, which results in

$$\hat{\mathbf{g}}_{ik'i} = \sqrt{\rho_{k'i}\beta_{ik'i}}\,\mathbf{h}_{ik'i} + \sum_{l=1,l\neq i}^{L}\sqrt{\rho_{k'l}\beta_{ik'l}}\,\mathbf{h}_{ik'l} + \mathbf{z}'_i, \tag{2}$$

where $\mathbf{z}'_i = \frac{\mathbf{z}_i\,\psi_k^{\dagger}}{K} \sim \mathcal{CN}(0, \frac{1}{K}I_M)$.

The base station computes the beamforming vector to its k'-th user as the normalized version of (2):

$$\mathbf{w}_{k'i} = \frac{\hat{\mathbf{g}}_{ik'i}}{\|\hat{\mathbf{g}}_{ik'i}\|} = \frac{\hat{\mathbf{g}}_{ik'i}}{\alpha_{k'i}\sqrt{M}}. \tag{3}$$

The scalar $\alpha_{k'i} = \frac{\|\hat{\mathbf{g}}_{ik'i}\|}{\sqrt{M}}$ is a normalization factor. To compute its value as $M \to \infty$, we use the following well known lemma.

Lemma 1. *Let $\mathbf{x}, \mathbf{y} \in \mathbb{C}^{M\times 1}$ be two independent vectors with distribution $\mathcal{CN}(\mathbf{0}, c\,\mathbf{I})$. Then*

$$\lim_{M\to\infty}\frac{\mathbf{x}^{\dagger}\,\mathbf{y}}{M} \overset{a.s.}{=} 0 \text{ and } \lim_{M\to\infty}\frac{\mathbf{x}^{\dagger}\,\mathbf{x}}{M} \overset{a.s.}{=} c. \tag{4}$$

Using the fact that the channel vectors of different users are independent, and applying the above lemma, we can derive the asymptotic behavior of $\alpha^2_{k'i}$:

$$\lim_{M\to\infty}\alpha^2_{k'i}=\lim_{M\to\infty}\frac{1}{M}\left(\sum_{l=1}^{L}\rho_{k'l}\beta_{ik'l}\|\mathbf{h}_{ik'l}\|^2+\right.$$
$$+\sum_{l_1=1}^{L}\sum_{l_2=1,l_2\neq l_1}^{L}\sqrt{\rho_{k'l_1}\rho_{k'l_2}\beta_{ik'l_1}\beta_{ik'l_2}}\,\mathbf{h}^{\dagger}_{ik'l_1}\mathbf{h}_{ik'l_2}+$$
$$\left.+\|\mathbf{z}'_i\|^2+\sum_{l=1}^{L}\sqrt{\rho_{k'l}\beta_{ik'l}}\,\mathbf{h}^{\dagger}_{ik'l}\mathbf{z}'_i\right)\overset{\text{a.s.}}{=}\sum_{l=1}^{L}\rho_{k'l}\,\beta_{ik'l}+\frac{1}{K}.$$

Once the pilot sequences are received and the channel vectors are estimated, each BTS transmit the downlink data to its respective users. The k'-th user of the i-th cell receives

$$y_{U_{k'i}}=\sum_{l=1}^{L}\sum_{k=1}^{K}\sqrt{P_{kl}\beta_{lk'i}}\mathbf{h}^{\dagger}_{lk'i}\mathbf{w}_{kl}s_{kl}+v_{k'i},\qquad(5)$$

where s_{kl} is the signal intended to the k-th user in the l-th cell and $v_{k'i}$ is the unit variance additive white Gaussian noise.

Denote each term in the double sum of (5) by Q_{kl} and let $S_{kl}=\mathbb{E}\left[|Q_{kl}|^2\right]$ be the variance of the received signal s_{kl}. Then

$$S_{kl}=\mathbb{E}\left[|Q_{kl}|^2\right]=\mathbb{E}\left[P_{kl}\beta_{lk'i}|\mathbf{h}^{\dagger}_{lk'i}\mathbf{w}_{kl}|^2|s_{kl}|^2\right]$$
$$=\frac{P_{kl}\,\beta_{lk'i}}{\alpha^2_{kl}M}\left|\sum_{l_1=1}^{L}\sqrt{\rho_{kl_1}\beta_{lkl_1}}\mathbf{h}^{\dagger}_{lk'i}\mathbf{h}_{lkl_1}+\mathbf{h}^{\dagger}_{lk'i}\mathbf{z}'_i\right|^2$$

Consider first the case $k=k'$. Applying Lemma 1 to the terms inside the absolute value, we get

$$\lim_{M\to\infty}\frac{1}{M}\left(\sum_{l_1=1}^{L}\sqrt{\rho_{k'l_1}\beta_{lk'l_1}}\mathbf{h}^{\dagger}_{lk'i}\mathbf{h}_{lk'l_1}+\mathbf{h}^{\dagger}_{lk'i}\mathbf{z}'_i\right)$$
$$\overset{\text{a.s.}}{=}\sqrt{\rho_{k'i}\beta_{lk'i}}.\qquad(6)$$

Note that $S_{k'l}$ is a continuous function of $\left(\sum_{l_1=1}^{L}\sqrt{\rho_{k'l_1}\beta_{lk'l_1}}\mathbf{h}^{\dagger}_{lk'i}\mathbf{h}_{lk'l_1}+\mathbf{h}^{\dagger}_{lk'i}\mathbf{z}'_i\right)$. Thus, from the Continuous Mapping Theorem [7, p.27], we have

$$\lim_{M\to\infty}\frac{S_{k'l}}{M}\overset{\text{a.s.}}{=}\frac{P_{k'l}\,\rho_{k'i}\beta^2_{lk'i}}{\alpha^2_{k'l}}$$

Now let $k\neq k'$. According to Lemma 1, S_{kl} vanishes as $M\to\infty$ since $\mathbf{h}_{lk'i}$, $\mathbf{h}_{lkl_1}$ are independent for any l_1.

In (5), the received power of the desired signal is $S_{k'i}$ and all other values S_{kl} contribute to interference. One can see that only the users in the neighboring cells, the k'-th users, who use the same pilot sequence, create interference that does not vanish as $M\to\infty$. The reason for this is that the beamforming vectors of these users contain components directed towards the k'-th user in the i-th cell, generating directed interference that does not vanish as $M\to\infty$. The variance of the additive noise, however, is unitary regardless of the number of BTS antennas, thus rendering the effect of the noise null in the asymptotic region.

We obtained the asymptotic behavior of the numerator and denominator of the SINR, which are formed of independent variables. Therefore, we proved the following theorem.

Theorem 1. *The downlink SINR of the k'-th user in the i-th cell is*

$$\varsigma^D_{ik'}=\frac{P_{k'i}\beta^2_{ik'i}/\alpha^2_{k'i}}{\sum_{l=1,l\neq i}^{L}P_{k'l}\beta^2_{lk'i}/\alpha^2_{k'l}},\qquad(7)$$

with $\alpha_{k'l}=\sum_{j=1}^{L}\rho_{k'j}\,\beta_{lk'j}+\frac{1}{K}$.

Note that additive noise impacts only the normalization constants $\alpha_{k'l}$. Therefore base station transmit powers P_{kl} are scalable. This allows for the use of lower power levels, resulting in a more power-efficient system.

One may think possible SINR gains can be obtained by choosing optimal pilot powers $\rho_{k'l}$, since they appear in expression (7). The following theorem shows that in fact this is not the case.

Theorem 2. *If a set of SINRs can be achieved for a particular choice of pilot powers ρ_{kl}, it can also be achieved for any other choice of ρ_{kl}.*

Proof: Defining the variables $P'_{kl}=P_{kl}/\alpha^2_{kl}$ we get

$$\varsigma^D_{ik'}=\frac{P'_{k'l}\beta^2_{ik'i}}{\sum_{l=1}^{L}P'_{k'l}\beta^2_{lk'i}}.\qquad(8)$$

This implies that any set of downlink SINR values obtained for one set of normalization factors can be obtained if they are changed by appropriately scaling the BTS transmit powers. ■

Uplink communication suffers similar interference generated by the contamination of the channel estimates. In this phase, users send data to their respective BTSs, which then multiply the received signal by the channel estimates to obtain the desired signal as well as interference.

The i-th BTS receives the signal

$$y_{B_i}=\sum_{l=1}^{L}\sum_{k=1}^{K}\sqrt{P^U_{kl}\,\beta_{ikl}}\,\mathbf{h}_{ikl}q_{kl}+\nu_i$$

where P^U_{kl} is the uplink transmit power and q_{kl} is the uplink signal sent by the k-th user of the l-th cell and ν_i is additive white Gaussian noise. To decode the signal sent by its k'-th user, the BTS applies Maximal Ratio Combining (MRC) by multiplying the received signal by the channel estimate obtained in (2):

$$\hat{q}_{k'i}=\hat{\mathbf{g}}^{\dagger}_{ik'i}y_{B_i}$$
$$=\sum_{l_1}^{L}\sum_{l_2=1}^{L}\sum_{k=1}^{K}\sqrt{\rho_{k'l_1}\,\beta_{ik'l_1}\,P^U_{kl_2}\,\beta_{ikl_2}}\,\mathbf{h}^{\dagger}_{ik'l_1}\mathbf{h}_{ikl_2}\,q_{kl_2}$$
$$+\sum_{l=1}^{L}\sqrt{P^U_{kl}\,\beta_{ikl}}\mathbf{z}'^{\dagger}\mathbf{h}_{ikl}+\sum_{l=1}^{L}\sqrt{\rho_{k'l}\,\beta_{ik'l}}\,\mathbf{h}^{\dagger}_{ik'l}\nu_i$$
$$+\ \mathbf{z}'^{\dagger}_i\,\nu_i\qquad(9)$$

Since multiplication on the receiver does not modify transmit power, normalization is not required in the uplink.

The asymptotic behavior of the uplink SINR is similar to that of the downlink. In fact, by using Lemma 1 we can rewrite

(9) as

$$\hat{q}_{k'i} = \underbrace{\sqrt{\rho_{k'i} P^U_{k'i}} \beta_{ik'i} \|\mathbf{h}_{ik'i}\|^2 q_{k'i}}_{\text{signal}} + \underbrace{\sum_{l=1, l\neq i}^{L} \sqrt{\rho_{k'l} P^U_{k'l}} \beta_{ik'l} \|\mathbf{h}_{ik'l}\|^2 q_{k'l}}_{\text{interference}} + o(M) \quad (10)$$

Consequently, the uplink SINR for the k'-th user of the i-th cell is

$$\varsigma^U_{ik'} = \frac{\rho_{k'i} P^U_{k'i} \beta^2_{ik'i}}{\sum_{l\neq i}^{L} \rho_{k'l} P^U_{k'l} \beta^2_{ik'l}} \quad (11)$$

Note that the dependence of the SINR on the pilot powers is different for the downlink and uplink. In the case of the downlink SINR, as seen in expression (7), the pilot powers appear in the normalization constants, whereas in the uplink SINR (expression (11)), the pilot powers appear multiplying the transmit powers. As a result, any attempt to assign different pilot powers to users would impact uplink and downlink differently, thus making it impossible to optimize pilot powers for both directions simultaneously. Moreover, in both cases, any effect of pilot power allocation can be achieved through transmit power allocation. Therefore, no loss is incurred in assigning a constant pilot power ρ to all users. Consequently, we henceforth assume that $\rho_{kl} = \rho$ for all k, l, resulting in an uplink SINR of

$$\varsigma^U_{ik} = \frac{P^U_{k'i} \beta^2_{ik'i}}{\sum_{l\neq i}^{L} P^U_{k'l} \beta^2_{ik'l}} \quad (12)$$

We also observe that the interference terms are also different for the two directions of communication. In the downlink, each user receives interference from neighboring BTSs because of the contaminated channel estimate used as beamforming. Thus, the long-term fading coefficients that form the SINR expression depend on the distance between each mobile user and the fixed neighboring BTSs. In the uplink, interference is due to users in other cells who use the same pilot sequence, and hence the coefficients depend on the distances between each BTS and the users from other cells. As a result, the statistical properties of the interference are different and so are the SINR values. Nonetheless, as shown in the numerical results of Section V, these dissimilarities amount to little and the distribution of SINR values is very similar for uplink and downlink, a fact previously noted in [6].

B. Time-Shifted Pilots

In Section III-A, we show that, as the number of BTS antennas M tends to infinity, the only cause of interference is the superposition of non-orthogonal pilot sequences coming from different users, which contaminates the channel estimates used for spatial filtering. As a result, we devise a scheme that avoids such superposition.

In fact, when a mobile unit sends a pilot sequence to its BTS, any non-orthogonal signal sent simultaneously from a mobile station will cause the channel estimate to be contaminated. In order to illustrate this point, let us consider a scenario with only two cells with K users each. Let us assume that, in the first phase, all users in cell 1 are sending pilot sequences while all users in cell 2 are sending uplink data to their BTS. As a result, BTS 1 receives

$$\mathbf{y}_{P_1} = \sum_{k=1}^{K} \sqrt{\rho\, \beta_{1k1}} \mathbf{h}_{1k1} \psi_k + \sqrt{P_{k2}\, \beta_{1k2}} \mathbf{h}_{1k2} q_{k2} + z_1 \quad (13)$$

and the estimate of the k'-th user channel is

$$\hat{\mathbf{g}}_{1k'1} = \frac{\mathbf{y}_{B_1} \psi_{k'}}{K} = \sqrt{\rho\, \beta_{1k'1}} \mathbf{h}_{1k'1} + \frac{z_1 \psi_{k'}}{K} + \frac{1}{K} \sum_{k=1}^{K} \sqrt{P_{k2}\, \beta_{1k2}} \mathbf{h}_{1k2} \left(q_{k2} \psi^\dagger_{k'} \right). \quad (14)$$

Note that each term $\left(q_{k2} \psi^\dagger_{k'} \right)$ is a random scalar, and thus the channel estimate is a linear combination of the desired channel vector and the channel vectors between the BTS and the users in cell 2.

Let us assume now that, in the second phase, users in cell 1 send uplink data and users in cell 2 send pilot sequences. Now, BTS 1 receives

$$\mathbf{y}_{U_1} = \sum_{k=1}^{K} \sqrt{P_{k1}\, \beta_{1k1}} \mathbf{h}_{1k1} q_{k1} + \sqrt{\rho\, \beta_{1k2}} \mathbf{h}_{1k2} \psi_k + \nu_1 \quad (15)$$

and performs MRC to obtain

$$\hat{\mathbf{g}}^\dagger_{1k'1} \mathbf{y}_{U_1} = \sqrt{\rho\, P_{k'1}} \beta_{1k'1} \|\mathbf{h}_{1k'1}\|^2 q_{k'1} + \sum_{k=1}^{K} \sqrt{\rho\, P_{k2}} \beta_{1k2} \|\mathbf{h}_{1k2}\|^2 \left(q_{k2} \psi^\dagger_{k'} \right) \psi_k + o(M). \quad (16)$$

We can see that the pilot sequences sent by the users in cell 2 do not vanish as $M \to \infty$, since the channels between them and BTS 1 are present in the estimates, as shown in (14).

The example above shows that the simultaneous transmission of pilot sequences in one cell and uplink signals in neighboring cells does not solve the problem of interference due to the contamination of channel estimates.

In the remainder of this section, we propose a scheme in which pilot sequences are sent simultaneously with downlink data from other cells, and we show that this superposition does not create interference as $M \to \infty$.

This scheme is based on partitioning the L cells into groups of cells $A_1, A_2, \ldots, A_\Gamma$. Communication during each coherence interval is divided in two stages. In the first stage, users from cells in the group A_γ transmit their pilot sequences simultaneously, while users from al other groups receive downlink data. Once users in A_γ finish their pilot sequence, they start receiving downlink data while a different group starts sending pilots. This protocol is repeated until users in all groups transmit their pilots.Then, at this point, all users switch to transmitting uplink data to their BTSs.

For both the downlink and uplink, the BTSs use the channel estimates computed in the first stage. Figure 2 depicts this scheme for when cells are divided into three groups. Note that, for any given coherence interval, the allocation of time slots to pilot, uplink and downlink data must now follow a certain structure. Note that this scheme does not require a longer coherence interval than the scheme presented in the last

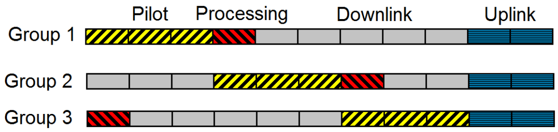

Fig. 2. Time- shifted pilot scheme with $K = 3, N = 1, D = 5$ and $U = 2$.

section, but rather it establishes a different tradeoff between rates, number of users in the system, and coherence interval.

Here we show that, under the proposed scheme, all the interference coming from cells in different groups vanishes. We also discuss important issues generated by the fact that pilots are now transmitted simultaneously with downlink data.

Let the i-th cell be in the group A_γ. Then, in the first phase, this BTS receives

$$\mathbf{y}_{B_i} = \sum_{j \in A_\gamma} \sum_{k=1}^{K} \sqrt{\rho \beta_{ikj}} \mathbf{h}_{ikj} \psi_k + \sum_{l \notin A_\gamma} \sum_{k=1}^{K} \sqrt{P_{kl}\, c_{il}} \mathbf{h}_{il} \mathbf{w}_{kl} \mathbf{s}_{kl} + \mathbf{z}_i, \tag{17}$$

where $\mathbf{s}_{kl}$ is a $K \times 1$ vector of the intended signals to the k-th user in the l-th cell, $c_{il} \in \mathbb{C}$ is the fading coefficient (constant with respect to frequency and antenna indices), and $\mathbf{h}_{il} \in \mathbb{C}^{M \times M}$ is the channel matrix between antennas of the i-th and the l-th base stations. The quantities c_{il} and $\mathbf{h}_{il}$ almost do not change in time, since base stations do not move. So we assume that c_{il} and $\mathbf{h}_{il}$ are constant in time.

The results presented below are based on the assumption that for randomly chosen base stations i and l the quantities c_{il} and $\mathbf{h}_{il}$ are generated as random variable with log-normal distribution and i.i.d. Gaussian distribution respectively. We obtained similar results for a Ricing fading model that accounts for a line-of-sight components, which we discuss later in the section.

The i-th BTS uses $\mathbf{y}_{B_i}$ to get the estimate

$$\hat{\mathbf{g}}_{ik'i} = \frac{\mathbf{y}_{B_i} \psi_{k'}^{\dagger}}{K} = \sum_{j \in A_\gamma} \sqrt{\rho \beta_{ik'j}} \mathbf{h}_{ik'j} + \frac{1}{K} \sum_{l \notin A_\gamma} \sum_{k=1}^{K} \sqrt{P_{kl} c_{il}} \mathbf{h}_{il} \mathbf{w}_{kl} \mathbf{s}_{kl} \psi_{k'}^{\dagger} + z_i' \tag{18}$$

Note that the signal received by the base station receives not only pilot signals but also downlink signals transmitted by BTS from different groups. These downlink signals are typically more powerful than pilots. Thus it is a priori unclear whether the BTS can obtain an accurate estimate of the channel vector in the presence of these strong downlink signals.

The beamforming vector is defined as it was in (3):

$$\mathbf{w}_{k'i} = \frac{\hat{\mathbf{g}}_{ik'i}}{\|\hat{\mathbf{g}}_{ik'i}\|} = \frac{\hat{\mathbf{g}}_{ik'i}}{\alpha_{k'i}\sqrt{M}}, \text{ i.e. } \quad \alpha_{k'i} = \frac{\|\hat{\mathbf{g}}_{ik'i}\|}{\sqrt{M}} \tag{19}$$

The asymptotic behavior of the normalization factor $\alpha_{k'i}$ in this case is obtained by applying Lemma 1

$$\lim_{M\to\infty} \alpha_{k'i}^2 = \lim_{M\to\infty} \frac{1}{M} \left(\sum_{j \in A_\gamma} \rho \beta_{ik'j} \|\mathbf{h}_{ik'j}\|^2 + \frac{1}{K} \sum_{l \notin A_\gamma} \sum_{k=1}^{K} P_{kl} c_{il} \|\mathbf{h}_{il} \mathbf{w}_{kl}\|^2 \left| \mathbf{s}_{kl} \psi_k'^H \right|^2 + \|\mathbf{z}_i'\|^2 \right) \tag{20}$$

The above expression is obtained by considering that the cross-terms vanish since they are inner products of independent random vectors. In order to compute the limit in (20), we focus on the terms of the form $P_{kl} c_{il} \|\mathbf{h}_{il}\mathbf{w}_{kl}\|^2 \left|\mathbf{s}_{kl}\psi_k^H\right|^2$.

We see that α_{ki} in this case fluctuates depending on the signals, the beamforming vectors and the transmit powers of neighboring BTSs. However, its exact value is known to the BTS once it computes $\hat{\mathbf{g}}_{iki}$. Below we derive lower and upper bounds on α_{ki}. These bounds are important for the design of a power allocation scheme, discussed in section IV.

First, note that the inner product between data $\mathbf{s}_{kl}$ and pilot sequence ψ_k is a random variable, independent of channel coefficients. We assume that the signal is composed of PSK symbols, and therefore bound it from above by $\left|\mathbf{s}_{kl}\psi_k^H\right|^2 \leq K$ (similar bounds can be obtained for other types of modulation).

The product between channel matrix and beamforming can also be bounded as $\|\mathbf{h}_{il}\mathbf{w}_{kl}\|^2 \leq |\lambda_{\max}(\mathbf{h}_{il})|^2$, where $\lambda_{\max}(\mathbf{h}_{il})$ is the largest singular value of $\mathbf{h}_{il}$. For the case where there is no line of sight between the BTSs, we assume that the channel matrices have independent entries with a Gaussian distribution. As a result, the study of the distribution of eigenvalues in random matrices in [8] show that

$$\lim_{M\to\infty} \frac{|\lambda_{\max}(\mathbf{h}_{il})|^2}{M} = 4. \tag{21}$$

Therefore we have the following result.

Theorem 3. *As $M \to \infty$, we can bound α_{ki}^2 by*

$$\sum_{j \in A_\gamma} \rho \beta_{ik'j} + \frac{1}{K} \leq \lim_{M\to\infty} \alpha_{ki}^2 \leq \sum_{j \in A_\gamma} \rho \beta_{ik'j} + \sum_{l \notin A_\gamma} 4 P_{kl} c_{il} + \frac{1}{K}. \tag{22}$$

The upper bound in expression (21) is required due to the intricate and dynamic dependence between channel matrix and beamforming vectors. The bound assumes that the beamforming vector $\mathbf{w}_{kl}$ coincides with the eigenvector associated with the largest eigenvalue of $\mathbf{h}_{il}^{\dagger}\mathbf{h}_{il}$, and is only valid when the entries of the channel matrix between BTSs are assumed to be i.i.d. Gaussian, which corresponds to a Rayleigh fading model. A Rician-like fading model that accounts for a line-of-sight component, which may be realistic for the propagation between BTSs, generates channel matrices dominated by one or a few eigenvectors corresponding to the line of sight. In such cases, the largest eigenvalue of $\mathbf{h}_{il}^{\dagger}\mathbf{h}_{il}$ would grow as M^2, not M. The worst case for such scenario, with the beamforming vector aligned with the dominating eigenvector, would result in a very poor channel estimate, since the interference coming from neighboring BTSs would be stronger than the pilot sequences by a factor of M.

For scenarios with line of sight between neighboring BTSs, a small modification in the beamforming vectors avoids the potential issue described above while keeping the asymptotic behavior of the system unchanged. To describe this modification, let us assume the extreme case where the matrix $\mathbf{h}_{il}$ has rank 1, assuming it is formed exclusively by the line-of-sight component. In order to eliminate interference, the BTS projects all of its beamforming vectors onto the null space of $\mathbf{h}_{il}$, thus forcing $\mathbf{h}_{il}\mathbf{w}_{kl} = 0$. The effect of this projection is minimal: beamforming vectors would no longer have a component perfectly aligned with the channel vector in all M dimensions; it would instead be aligned in $M-1$ dimensions. For very large M, this effect is negligible.

In the general case, let us assume that the channel matrix between the i-th and the l-th BTSs is of the form

$$\mathbf{h}_{il} = \mathbf{h}_{il}^{\text{LOS}} + \mathbf{h}_{il}^{\text{NLOS}},$$

where $\mathbf{h}_{il}^{\text{LOS}} \in \mathbb{C}^{M\times M}$ is a deterministic matrix whose rank m does not grow with M and $\mathbf{h}_{il}^{\text{NLOS}}$ is a complex Gaussian full-rank matrix. Then, we compute the beamforming vectors as

$$\mathbf{w}_{kl} = \frac{\bar{\mathbf{h}}_{il}^{\text{LOS}}\,\hat{\mathbf{g}}_{lkl}}{\|\bar{\mathbf{h}}_{il}^{\text{LOS}}\,\hat{\mathbf{g}}_{lkl}\|},$$

where $\bar{\mathbf{h}}_{il}^{\text{LOS}} = \mathbf{I} - (\mathbf{h}_{il}^{\text{LOS}})^{\ddagger}\mathbf{h}_{il}^{\text{LOS}}$. Here, $(\mathbf{h}_{il}^{\text{LOS}})^{\ddagger}$ denotes its Moore-Penrose pseudo-inverse. The resulting beamforming vector is the projection of $\hat{\mathbf{g}}_{lkl}$ onto the null space of $\mathbf{h}_{il}^{\text{LOS}}$. As a consequence, the line of sight component is cancelled, removing its interference on the channel estimates. Moreover, this procedure does not change the asymptotic performance of the technique, since the only effect is that the power at the receiver grows with $(M-m)^2$ instead of M^2 for a constant m. Therefore, this procedure allows us to consider only the non-line-of-sight component of the channel matrices as interference during the channel estimation process, thus validating the uuper bound in (22).

After estimating corresponding channel vectors the BTSs in group A_γ transmit data to their users. Simultaneously, all but one of the remaining groups of BTSs ($A_{\gamma'}$) are also transmitting data to their respective users. At this stage, the k'-th user of the i-th BTS receives

$$\begin{aligned} y_{U_{k'i}} &= \sum_{j\in A_\gamma}\sum_{k=1}^{K}\sqrt{P_{kj}\,\beta_{jk'i}}\mathbf{h}_{jk'i}^{\dagger}\,\mathbf{w}_{kj}\,s_{kj} \\ &+ \sum_{l\in A_{\gamma'}}\sum_{k=1}^{K}\sqrt{\rho\,c'_{klk'i}}\,h'_{klk'i}\psi_k + v_{k'i} \\ &+ \sum_{l\notin A_\gamma\cup A_{\gamma'}}\sum_{k=1}^{K}\sqrt{P_{kl}\,\beta_{lki}}\,\mathbf{h}_{lki}^{\dagger}\,\mathbf{w}_{lk}\,s_{lk}, \end{aligned} \tag{23}$$

where $c'_{klk'i}$ and $h'_{klk'i}$ are slow and fast fading coefficients between the k-th user of the l-th cell and the k'-th user of the i-th cell, and $v_{k'i}$ is the additive noise term.

The received power of the desired signal $s_{k'i}$ is

$$S_{k'i} = \left|\sqrt{P_{k'i}\,\beta_{ik'i}}\mathbf{h}_{ik'i}^{\dagger}\,\mathbf{w}_{k'i}\right|^2 = \frac{P_{k'i}\,\beta_{ik'i}}{\alpha_{ik'}^2 M}\left|\mathbf{h}_{ik'i}^{\dagger}\hat{\mathbf{g}}_{ik'i}\right|^2.$$

Below we find an expanded expression of the product $\mathbf{h}_{ik'i}^{\dagger}\hat{\mathbf{g}}_{ik'i}$ and analyze the asymptotic behavior of each term:

$$\begin{aligned} \mathbf{h}_{ik'i}^{\dagger}\hat{\mathbf{g}}_{ik'i} &= \sum_{j\in A_\gamma}\sqrt{\rho\beta_{ik'j}}\,\underbrace{\mathbf{h}_{ik'i}^{\dagger}\,\mathbf{h}_{ik'j}}_{ⓐ} + \\ &+ \frac{1}{K}\sum_{l\notin A_\gamma}\sum_{k=1}^{K}\sqrt{P_{kl}c_{il}}\,\underbrace{\mathbf{h}_{ik'i}^{\dagger}\,\mathbf{h}_{ikl}\mathbf{w}_{kl}\mathbf{s}_{kl}\psi_{k'}^{\dagger}}_{ⓑ} + \underbrace{\mathbf{h}_{ik'i}^{\dagger}\,\mathbf{z}'_i}_{ⓒ}. \end{aligned} \tag{24}$$

From Lemma 1 it follows that

$$\lim_{M\to\infty}\frac{ⓐ}{M} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j. \end{cases}$$

For the analysis of the second term, we first note that $\mathbf{s}_{kl}\psi_{k'}^{\dagger}$ is a scalar whose value does not grow with M. In fact, since we assume PSK signals, we have that $\mathbf{s}_{kl}\psi_{k'}^{\dagger} \leq K$.

To establish the behavior of the interaction between channel and beamforming, consider the vector $\mathbf{q}_{ikl} = \frac{\mathbf{h}_{il}\mathbf{w}_{kl}}{\|\mathbf{h}_{il}\mathbf{w}_{kl}\|}$. From the model adopted here, the channel vectors $\mathbf{g}_{ik'i}$ are independent of $\mathbf{q}_{ikl}$, for $i \neq l$. Then, we have

$$\begin{aligned} \mathbf{h}_{ik'i}^{\dagger}\,\mathbf{h}_{il}\mathbf{w}_{kl} &= \|\mathbf{h}_{il}\mathbf{w}_{kl}\|\,\mathbf{h}_{ik'i}^{\dagger}\,\mathbf{q}_{ikl}, \text{ and} \\ \|\mathbf{h}_{il}\mathbf{w}_{kl}\||\mathbf{h}_{ik'i}^{\dagger}\,\mathbf{q}_{ikl}| &\leq |\lambda_{\max}(\mathbf{h}_{il})||\mathbf{h}_{ik'i}^{\dagger}\,\mathbf{q}_{ikl}|. \end{aligned}$$

According to (21) $\lambda_{\max}(\mathbf{h}_{il})$ grows proportionally to $\sqrt{M}$. Denote by f the inner product between $\mathbf{h}_{ik'i} \sim \mathcal{CN}(\mathbf{0},\mathbf{I})$ and an independent unit-norm vector $\mathbf{q}_{ikl}$. It is not difficult to show that $f \sim \mathcal{CN}(0,1)$, irrespective of the vector dimensions and the distribution of $\mathbf{q}_{ikl}$. Therefore we have

$$\lim_{M\to\infty}\frac{|ⓑ|}{M} \leq \lim_{M\to\infty}\frac{K\,|\lambda_{\max}(\mathbf{h}_{il})|\,|\mathbf{h}_{ik'i}^{\dagger}\,\mathbf{q}_{ikl}|}{M} \overset{\text{a.s.}}{=} 0. \tag{25}$$

The term ⓒ allows immediate application of Lemma 1:

$$\lim_{M\to\infty}\frac{\mathbf{h}_{ik'i}^{\dagger}\,\mathbf{z}'_i}{M} \overset{\text{a.s.}}{=} 0.$$

Collecting the above results, we have that the received power of the desired signal behaves as

$$\lim_{M\to\infty}\frac{S_{k'i}}{M} = P_{k'i}\,\rho\,\beta_{ik'i}^2\,/\alpha_{k'i}^2.$$

Similarly, signals coming from other BTSs in the group A_γ shown in expression (23) cause directed interference, whose power behaves asymptotically as

$$\lim_{M\to\infty}\frac{\left|\sqrt{P_{kj}\,\beta_{jk'i}}\mathbf{h}_{jk'i}^{\dagger}\,\mathbf{w}_{jk}\,s_{jk}\right|^2}{M} \overset{\text{a.s.}}{=} P_{k'j}\,\rho\,\beta_{jk'i}^2\,/\alpha_{k'j}^2$$

The pilot signals coming from users in cells of group $A_{\gamma'}$ propagate through a single-input single-output (SISO) channel, with finite power and not transmitted by very large antenna arrays. Therefore, the received power of any such user does not grow with M, that is

$$\lim_{M\to\infty}\frac{\left\|\sqrt{\rho\,c'_{klk'i}}\,g'_{klk'i}\psi_k\right\|^2}{M} \overset{\text{a.s.}}{=} 0. \tag{26}$$

The BTSs from groups other than A_γ and $A_{\gamma'}$ transmit signals using beamforming vectors uncorrelated to the channel vectors between them and users in the group A_γ. Therefore, for any $i \in A_\gamma$ and $l \notin A_\gamma \cup A_{\gamma'}$, irrespective of k, k', we have

$$\lim_{M\to\infty} \frac{\left|\mathbf{h}_{lk'i}^\dagger \mathbf{w}_{lk}\right|^2}{M} \overset{\text{a.s.}}{=} 0.$$

Combining these results we obtain the following expression for the SINR at the downlink:

$$\varsigma_{k'i}^D = \frac{P_{k'i}\beta_{ik'i}^2/\alpha_{k'i}^2}{\sum_{l\in A_\gamma, l\neq i} P_{k'l}\beta_{lk'i}^2/\alpha_{k'l}^2}. \quad (27)$$

Expression (27) is different from (7) in that only base stations in the same group cause interference. This gives significant gains in data transmission rates.

The same can be accomplished for the uplink by using the channel estimates obtained in (18) at the receivers. In this scheme, users from all cells transmit to their BTSs simultaneously. The signal received by the i-th BTS can be written as

$$\mathbf{y}_{U_i} = \sum_{l=1}^{L}\sum_{k=1}^{K} \sqrt{P_{kl}^U\,\beta_{ikl}}\mathbf{h}_{ikl}\mathbf{q}_{kl} + \nu_i.$$

In order to obtain the signal sent by its k'-th user, the BTS performs MRC, resulting in

$$\hat{\mathbf{g}}_{ik'i}^\dagger \mathbf{y}_{U_i} = \sum_{\substack{l\in\{1,L\}\\ k_1\in\{1,K\}\\ j_1\in A_\gamma}} \sqrt{\rho P_{k_1 l}^U\,\beta_{ik'j_1}\,\beta_{ik_1 l}}\,\underbrace{\mathbf{h}_{ik'j_1}^\dagger \mathbf{h}_{ik_1 l}}_{ⓓ}\mathbf{q}_{kl}$$
$$+\frac{1}{K}\sum_{\substack{l\in\{1,L\}\\ k_1,k_2\in\{1,K\}\\ j_2\notin A_\gamma}} \sqrt{P_{k_2 j_2}^D P_{k_1 l}^U c_{ij_2}}\,\underbrace{\mathbf{s}_{k_2 j_2}^\dagger \psi_{k'} \mathbf{w}_{k_2 j_2}^\dagger \mathbf{h}_{ij_2}^\dagger \mathbf{h}_{ik_1 l}}_{ⓔ}\mathbf{q}_{k_1 l}$$
$$+\nu_i'$$

where ν_i' denotes the terms related to additive noise that vanish as $M \to \infty$. The remaining terms have the same stochastic properties as those in the downlink. From Lemma 1, we have that

$$\lim_{M\to\infty}\frac{ⓓ}{M} = \begin{cases} 1 & \text{if } j_1 = l \text{and } k_1 = k' \\ 0 & \text{if otherwise.}\end{cases}$$

The terms of the form ⓔ have exactly the same distribution as ⓑ in expression (24), and thus its asymptotic behavior is as obtained in expression (25). Hence, we can rewrite the above expression as

$$\hat{\mathbf{g}}_{ik'i}^\dagger \mathbf{y}_{U_i} = \sum_{j\in A_\gamma} \sqrt{\rho P_{k'j}^U}\beta_{ik'j}\|\mathbf{h}_{ik'j}\|^2 \mathbf{q}_{k'j} + o(M)$$

The resulting uplink SINR is thus

$$\varsigma_{k'i}^U = \frac{P_{k'i}^U\,\beta_{ik'i}^2}{\sum_{j\in A_\gamma, j\neq i} P_{k'j}^U\,\beta_{ik'j}^2}, \quad (28)$$

effectively showing that, like in the downlink, the proposed scheme guarantees that only cells in the same group interfere with each other.

C. Cell Grouping

Since grouping cells and shifting the pilot sequences in time has the result of completely avoiding interference, it can be thought of as being analogous to frequency reuse, with the advantage that the entire band can be used by all cells.

We suggest partitioning cells into groups $A_1, \ldots, A_\Gamma$ in exactly the same way as they would be partitioned in frequency reuse $r = \Gamma$ systems. (Usually frequency reuse systems are considered only for certain integers r; we skip those details here.) Then, in a particular OFDM subcarrier, a network with frequency reuse $r = \Gamma$ and aligned pilots will have the same SINR value as a network with time-shifted pilots with Γ groups and frequency reuse $r = 1$.

Using this cell grouping scheme, we can compare the rates obtained by the aligned pilot and the shifted pilot approaches.

Let us first assume that no uplink data is being transmitted. Denote by B the available bandwidth and remind ourselves that $N_u = KN_{smooth}$. The time-shifted pilot method requires that $K = T/\Gamma$. The aligned pilot method has the optimal value $K = \frac{T-N}{2}$. Hence the data transmission rates for aligned pilots and shifted pilots are

$$R_{ki}^{ap} = \epsilon\frac{B}{\Gamma}\frac{T-N}{2}N_{smooth}(T-N-\frac{T-N}{2})\log_2(1+\varsigma_{ik}^D)$$
$$R_{ki}^{sp} = \epsilon B\frac{T}{\Gamma}N_{smooth}(T-N-\frac{T}{\Gamma})\log_2(1+\varsigma_{ik}^D).$$

Here the factor ϵ accounts for the effect of cyclic prefix, guard intervals, and particular modulation constellation, equal for both schemes. The ratio of these two rates is

$$\frac{R_{ki}^{sp}}{R_{ki}^{ap}} = \frac{T(T-N-\frac{T}{\Gamma})}{\left(\frac{T-N)}{2}\right)^2}. \quad (29)$$

The expression above shows that the rate gain increases with Γ, as expected. However, the gain saturates and

$$\lim_{\Gamma\to\infty}\frac{R_{ik}^{sp}}{R_{ik}^{ap}} = 4\frac{T}{T-N}.$$

As an example, consider a network with $T = 9$ and $N = 1$. Choosing $\Gamma = 3$, we obtain $\frac{R_{ik}^{sp}}{R_{ik}^{ap}} = 2.8125$. This example shows that the time-shifted pilots allow achieving a very significant gain in data transmission rate even for small Γ in the case where no uplink data is sent.

The analysis above is no longer valid for cases including uplink data. However, a fair comparison can be made by considering scenarios with the same coherence interval and same number of users, via time-sharing. This is done in Section V, where it is explained in detail.

IV. Power Allocation

The power allocation problem for systems with SINR expressions of the form of (7) and (27) has been extensively studied [9]–[13]. Although the primary scenario of such studies is the uplink of CDMA systems, the algorithms can be applied to the scenario studied in this paper as well.

We focus on setting SINR targets $\hat{\varsigma}_{kl}$ for the users and allocating powers P_{kl} with the objective of maximizing the number of users who achieve their targets under constraints $P_{kl} \leq \overline{P}_{kl}$. The reason for this is that in real-world systems, there is a possibility that not all users are able to achieve their SINR targets. In such cases, optimal algorithms for when all targets are achievable become unstable and have very poor performance. We focus here on distributed algorithms when there are no communication between BTSs. The only assumption is that each user sends to its BTS the current SINR value once every coherence interval. This is a standard assumption since this information is also useful at the BTS for other reasons, such as rate adaptation.

In this section P_{kl} is used for both uplink and downlink in order to simplify notation. Note, however, that powers are allocated independently for downlink and uplink, as different sets of powers are optimal for the two directions of communication. In practice, downlink power allocation runs at the BTSs while uplink power allocation is run by users, both distributively.

A. Aligned Pilots

It is clear in expressions (7) and (12), corresponding to the SINR at downlink and uplink respectively, that only users assigned the same pilot sequence interfere with each other. As a consequence, we have one independent power allocation problem for each different pilot sequence.

We adopt here a distributed algorithm proposed in [10]. The algorithm attempts to minimize outage by using a soft removal criterion. The principle of soft removal is that users who cannot achieve their target SINRs will be allocated progressively lower power. At the 0-th iteration we assume equal powers $P_{kl} = P$. At the i-th iteration the algorithm updates powers according to

$$P_{kl}(i) = \begin{cases} \hat{\varsigma}_{kl} R_{kl}(i) & \text{if } R_{kl}(i) \leq \frac{\bar{P}_{kl}}{\hat{\varsigma}_{kl}} \\ \frac{\bar{P}_{kl}}{\hat{\varsigma}_{kl} R_{kl}(i)} & \text{otherwise,} \end{cases} \tag{30}$$

where $R_{kl}(i) = P_{kl}(i-1)/\varsigma_{kl}(i)$.

In [10] it is shown that the above algorithm converges. We present simulation results for this algorithm in the next section.

B. Time-Shifted Pilots

In the case of time-shifted pilots, the independence of the power allocation problem can only be claimed for the uplink, as expression (28) only contains terms pertaining to users assigned the k'-th pilot within group A_γ. Thus, for the uplink, we have one independent power allocation problem for each pilot sequence in each group.

In the downlink, however, such independence cannot be claimed directly. The normalization factors $\alpha_{ki}, i \in A_\gamma$, depend on the transmit powers of BTSs from other groups $A_{\gamma'}$, and therefore α_{ki} can vary significantly from one iteration to another. This may result in that transmit powers will be increasing at each iteration and significantly exceed the constraints $\overline{P}_{kl}$. To avoid this undesirable behavior we propose the following approach.

As a first step, the BTSs compute the components of the normalization that depend only on the large-scale fading coefficients, $\xi_{kl} = \sum_{j\in A_\gamma} \rho\beta_{lkj}, l \in A_\gamma$. Note that, asymptotically, this slowly varying component is proportional to the norm of the channel vector when only one group is active:

$$\xi_{kl} = \frac{\|\hat{\mathbf{g}}_{kl}\|}{M}. \tag{31}$$

Thus, for each group, one coherence interval is enough to compute ξ_{kl}. Hence the estimation of ξ_{kl} for all BTSs in all groups only requires Γ coherence intervals, which is negligible when compared to the time during which coefficients β_{lkj} stay almost constant. Thus, according to Theorem 3, we can upper bound α_{kl}^2 by

$$\bar{\alpha}_{kl}^2 = \xi_l + \sum_{j\notin A_\gamma} 4\bar{P}_{kj} c_{lj} + \frac{1}{K}. \tag{32}$$

Define the variables $P'_{kl} = P_{kl}/\alpha_{kl}^2$ and $\bar{P}'_{kl} = \bar{P}_{kl}/\bar{\alpha}_{kl}^2$. At the i-th iteration the l-th BTS computes

1) $R_{kl}(i) = \frac{P'_{kl}(i-1)}{\varsigma_{kl}(i)}$;

2)
$$P'_{kl}(i) = \begin{cases} \hat{\varsigma}_{kl} R_{kl}(i) & \text{if } R_{kl}(i) \leq \frac{\bar{P}'_{kl}}{\hat{\varsigma}_{kl}} \\ \frac{\bar{P}'_{kl}}{\hat{\varsigma}_{kl} R_{kl}(i)} & \text{otherwise,} \end{cases}$$

3) $\alpha_{kl}(i) = ||\hat{\mathbf{g}}_{lkl}||^2$ and $P_{kl}(i) = P'_{kl}(i)\alpha_{kl}(i)^2$.

Theorem 4. *The above algorithm converges and at any iteration $P_{kl}(i) \leq \bar{P}_{kl}$.*

Proof: Step 2 guarantees that $P'_{kl}(i) \leq \bar{P}'_{kl}$ at every iteration. This corresponds by definition to $P'_{kl}(i)\,\alpha_{kl}(i)^2 \leq \bar{P}_{kl}$. Once the power is updated at step 3, we see that $P_{kl}(i) \leq \bar{P}_{kl}$.

To show that it converges, we first note that this algorithm applies the power allocation method in [10] to the variables P'_{kl}. We can rewrite the downlink SINR expression in (27) in terms of such variables as

$$\varsigma_{k'i}^D = \frac{P'_{k'i}\beta_{ik'i}^2}{\sum_{l\in A_\gamma, l\neq i} P'_{k'l}\beta_{lk'i}^2}. \tag{33}$$

In [10], the convergence of algorithm is proven precisely for SINR expressions with this structure. ∎

V. Numerical Results

The scenarios simulated here consist of a cellular system organized in hexagonal cells with radius 1.6 km, with users uniformly distributed in each cell, with the exception of a circle of 100m around each BTS. The system uses a frequency band of $B = 20$ MHz and a carrier frequency of 1.9 GHz. We model slow fading coefficients β_{ikl} assuming an average decay of $38dB$/decade and log normal shadowing with a standard deviation of $8dB$. We assume $T = 11$ and $N = 1$. For aligned pilots we assume $K = 5$, $D, = 3$ and $U = 2$, and $K = 3$, $D = 5$ and $U = 2$ for time-shifted pilots .

First, we compare the SINR obtained by assuming equal power allocation and the distributed soft removal algorithm in (30) for both uplink and downlink. We assign equal target SINR to all users, and vary its value. We plot in Figure 3 the fraction of users who reach the target once the algorithm converges, for frequency reuse $r = 3$. Due to the "soft" characteristic of the algorithm, we allow a $0.2dB$ tolerance, as the SINR of the majority of the users is around (and very

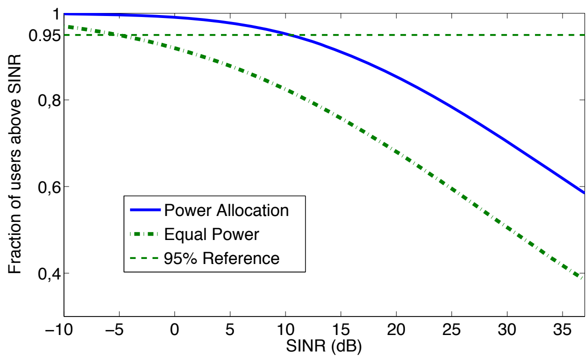

Fig. 3. Fraction of users above SINR for $r = 3$.

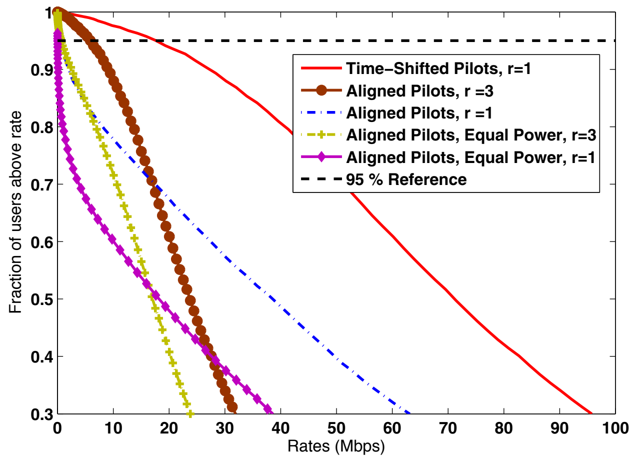

Fig. 4. Fraction of users above rate for aligned and time-shifted pilots for the downlink.

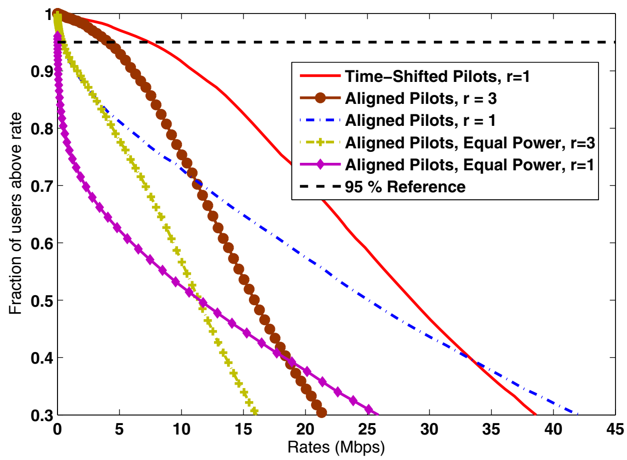

Fig. 5. Fraction of users above rate for aligned and time-shifted pilots for the uplink.

close) to the target. We also plot the fraction of users with SINRs larger than a given value for the case of equal power. In this scenario, we see that, for instance, 95% of the users have SINR above $10.5dB$ after power allocation, whereas the same fraction of users has SINR above $-5dB$ with equal power. A gain of over $10dB$ due to power allocation is seen for other percentages of users. Figure 3 also shows clearly that the SINR for both uplink and downlink are similar, a fact also noted in [6].

In Figures 4 and 5, we show the rate gains obtained by using the time-shifted pilot approach compared with the aligned pilot approach, both with power allocation, respectively for the downlink and uplink. In both figures, we plot the fraction of users that can achieve at least a certain rate.

A comparison between approaches with different lengths of pilot sequences ($K = 5$ and $K = 3$ for aligned and shifted approaches in our cases) must account for the fact that longest pilot sequences allow a larger number of users. In order to do so, we consider all systems with the same number of users, corresponding to $K = 5$, and assume that the time-shifted approach accommodates the users through time sharing. As a consequence, the rates for the time-shifted approach are decreased by a factor of $\frac{3}{5}$.

First, note in Figure 4 that the downlink rates corresponding to the time-shifted approach and $r = 1$ is roughly 3 times larger than the aligned pilot approach with $r = 3$. Also consider the curve for the aligned pilot approach with $r = 1$, which shows that time-shifted pilots have provided a rate increase for all users in this scenario, even though it required time-sharing to serve the same number of users.

We also see in Figure 4 that the rate gains of the time-shifted approach for the worst $1\% - 30\%$ and of the users is even more substantial (we see a rate gain of 32 times for the 95% level, from 548 Kbps to 17.6 Mbps). The gain drops when we consider higher SINR users, but is still very significant (1.8 times for the 50% level).

Finally, we also note that in the case of aligned pilots and equal powers, for the 95% level, we have rates 0.016 Mbps and 0.68 Kbps for $r = 1$ and $r = 3$ respectively. Thus the time-shifted approach together with power allocation gives us a 26-fold increase in data transmission rates at this level.

Figure 5 also shows significant gains for the uplink. In this case, the comparison between the shifted pilot approach with $r = 1$ and the aligned pilot approach with $r = 3$ results in a gain of 1.8 times.

Comparing the time-shifted approach to the aligned approach for $r = 1$, we also see substantial gains for the worst 60% of the users. In fact, at the 95% level, the gain in this case is of over 18 times(7.39 Mbps for the time-shifted approach versus 0.39 Mbps for the aligned pilot approach).

It is worth noting that although we see substantial gains for the 95% level, the best 40% of the users perform better in this specific scenario without pilot shifting. This behavior can be easily explained. The time-shifted approach guarantees a distance of at least one cell to the nearest interferer, which greatly increases the SINR of less favorably located users. Well-positioned users, however, do not get a proportional increase in rates even if their SINRs also increases, since their relationship is logarithmic. Thus, the decrease in rates due to time sharing (by a factor of $\frac{3}{5}$ in this scenario) is more significant to those users. Nonetheless, guaranteeing a much larger rate to the vast majority of the users is a substantial advantage of the proposed method.

VI. Conclusion

In this paper, we derive expressions for the asymptotic behavior of the SINR in both the downlink and the uplink of a cellular network as the number of base station antennas tends to infinity. We show that the fundamental limitation of such networks is the interference present in the channel estimates computed by the base stations, due to the overlapping of non-orthogonal pilot sequences from neighboring cells.

We analyze two different cases based on timing of pilot sequences: first, when all users transmit pilots to their base stations simultaneously; then, when the transmission of pilots is shifted in time from one cell to the next, avoiding overlap. We see, in the latter case, that it is possible to completely cancel interference from adjacent cells, as long as the pilots do not overlap in time. It is possible to harness SINR gains similar to those of frequency reuse while still sharing the same band, therefore resulting in a substantial increase in rates. In the simulations, we obtained a gain of 32 times for the downlink rate and 18 times for the uplink rate for the chosen scenario.

We also discuss the use of power allocation algorithms in this scenario, and show that such systems greatly benefit from it, as communication is limited not by additive noise, but by interference from adjacent terminals. Numerical simulations show that applying power allocation algorithms in this interference-limited setting provides significant gains, on the order of 15 dB for the scenario in question.

Finally, we show that the proposed techniques are especially beneficial to users in unfavorable locations that would otherwise suffer with low SINR. As a consequence, these techniques would result in significantly more users with high quality of service.

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