

Dynamic Output Feedback Control Synthesis for Continuous-Time T–S Fuzzy Systems via a Switched Fuzzy Control Scheme

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Abstract—This correspondence paper is concerned with the problem of designing switched dynamic output feedback H_∞ controllers for continuous-time Takagi–Sugeno (T–S) fuzzy systems. A new type of dynamic output feedback controllers, namely, switched dynamic parallel distributed compensation (SDPDC) controllers, is proposed, which are switched by basing on the values of membership functions. A new method for designing SDPDC controllers for guaranteeing stabilities and H_∞ performances of closed-loop nonlinear systems is presented, where the design conditions are given in terms of the solvability of a set of linear matrix inequalities. It is shown that the new method provides better or at least the same results of the existing design methods via a pure DPDC scheme. A numerical example is given to illustrate the effectiveness of the proposed method.

Index Terms—Dynamic output feedback, H_∞ control, fuzzy control, linear matrix inequalities (LMIs), switched control, Takagi–Sugeno (T–S) fuzzy systems.

I. INTRODUCTION

In recent years, Takagi–Sugeno (T–S) fuzzy models of nonlinear systems [1] have attracted great interests from scientists and engineers. It can approximate a large class of nonlinear systems by fuzzy “blending” of some local linear models. As a result, the conventional linear system theory can be applied to the analysis and synthesis of the class of nonlinear control systems. In recent years, T–S fuzzy systems have been studied extensively by many researchers (see for instance, [2]–[12]). The advantage of these results is that the stability analysis and controller gain design can be converted into convex optimization problems in terms of linear matrix inequalities (LMIs) [13], which can be solved efficiently [14]. Among these results, quadratic Lyapunov function approaches are extensively applied (see [2], [5], and [7]). Because a common quadratic Lyapunov function is independent of fuzzy membership functions, the techniques based on a single Lyapunov functions might give conservative results. For obtaining more relaxed conditions, parameter-dependent Lyapunov functions (or fuzzy Lyapunov functions) [8], [10], [11] and piecewise Lyapunov functions [3], [6] have been exploited for designing H_∞ controllers for T–S fuzzy systems. However, most of the aforementioned research works of fuzzy control systems are based on the assumption that the states are available for controller implementation, which is not true

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in many practical cases. Therefore, the output feedback control of fuzzy systems is very important, and some results have been obtained in [15]–[20].

Recently, there have appeared a number of approaches for designing static output feedback controllers for fuzzy control systems, see [21]–[24]. Although dynamic output feedback problems can be transformed into static output feedback problems, dynamic output feedback problems are different from the static output feedback problems. It is well known that static output feedback control still is an open problem for linear time-invariant (LTI) systems; however, an LMI-based sufficient and necessary condition for the dynamic output feedback problem of LTI systems has been presented in [25]. Thus, for T–S fuzzy systems, it is necessary to study dynamic output feedback problems without using static output feedback approaches. By using dynamic parallel distributed compensation (DPDC) scheme, Li *et al.* [26] present a systematic framework for designing dynamic output feedback controllers for continuous-time T–S fuzzy systems. The corresponding discrete-time version is developed and successfully applied to a vehicle with triple trailers in [27]. By considering the relations of H_∞ disturbance attenuation γ of a general uncertain fuzzy system and the stability with unitary H_∞ disturbance attenuation of a fuzzy system without uncertainty, Yoneyama [28] gives a robust H_∞ output feedback controller design approach for continuous-time fuzzy systems. By combining a fuzzy-basis-dependent Lyapunov function and a transformation on the controller parameters, Lam and Zhou [29] studied the H_∞ dynamic output feedback control for discrete-time fuzzy systems. Moreover, based on a fuzzy linear fractional transformation model, Tuan *et al.* [30] provide an efficient and tractable way to handle the output feedback parallel distributed compensation problem. In [31], a robust H_∞ output feedback controller design method for a class of fuzzy uncertain dynamic systems with pole placement constraints is given. Moreover, based on fuzzy observers, a robust stabilization technique is proposed to override the effect of approximation error in the fuzzy approximation procedure in [32], and the technique is further extended to a mixed H_2/H_∞ fuzzy output feedback control design in [33]. By assigning both state and estimation error poles to a desired LMI region, the problem of observer-based fuzzy control is also studied in [34].

In the aforementioned dynamic output feedback control synthesis, those LMI-based conditions in [26], [27], [29], and [31] are convex, which can be solved efficiently [14], and the DPDC scheme plays an important role for developing controller design methods. However, when some subsystem of fuzzy models plays more important role than the other ones, the same DPDC control scheme is used in these approaches, which might result in conservative designs. Motivated by this, a new scheme, which will switch different DPDC controllers for dominant subsystems, is introduced in this correspondence paper. In this correspondence paper, the new control scheme is called switched DPDC (SDPDC) control scheme. Specifically, this correspondence paper is concerned with the problem of designing switched dynamic output feedback H_∞ controllers for continuous-time T–S fuzzy systems. A new type of dynamic output feedback controllers, namely, SDPDC controllers, is proposed, which are switched by basing on the values of membership functions. The control scheme is an extension of the DPDC control scheme [26]. Sufficient conditions for designing SDPDC controllers for guaranteeing the stability and H_∞ performances of closed-loop nonlinear systems are presented, where the design conditions are given in terms of solvability of a set of LMIs. It is shown that the new method provides better or at least the same results of the corresponding design methods via the pure DPDC scheme. This correspondence paper is organized as follows. Section II presents the

T–S fuzzy model, which is the new type of dynamic output feedback control scheme. Section III provides a technique for designing an H_∞ full order dynamic output feedback controller for continuous-time T–S fuzzy systems. An example is given to illustrate the effectiveness of the new proposed method in Section IV. Concluding remarks are given in Section V.

Notation: The superscript T stands for matrix transposition and the notation M^{-T} denotes the transpose of the inverse matrix of M . The symbol * within a square matrix represents the symmetric entries. A square matrix $E > (\geq) 0$ implies that $E + E^T > (\geq) 0$

$$[H_{ijl}]_{r \times g} =: \begin{bmatrix} H_{11l} & H_{12l} & \cdots & H_{1gl} \\ H_{21l} & H_{22l} & \cdots & H_{2gl} \\ \vdots & \vdots & \ddots & \vdots \\ H_{r1l} & H_{r2l} & \cdots & H_{rgl} \end{bmatrix}.$$

II. SYSTEM DESCRIPTION AND PROBLEM STATEMENT

The nonlinear systems under consideration are described by the following fuzzy system model:

Plant Rule 1 :

$$\begin{aligned} \text{IF } v_1(t) \text{ is } M_{i1}, v_2(t) \text{ is } M_{i2}, \dots, v_p(t) \text{ is } M_{ip} \\ \text{THEN } \dot{x}(t) = A_i x(t) + B_{1i} w(t) + B_{2i} u(t) \\ z(t) = C_{1i} x(t) + D_{11i} w(t) + D_{12i} u(t) \\ y(t) = C_{2i} x(t) + D_{21i} w(t) \end{aligned} \quad (1)$$

where $x(t) \in \mathbb{R}^{n_x}$ is the state vector, $u(t) \in \mathbb{R}^{n_u}$ is the control input vector, $z(t) \in \mathbb{R}^{n_z}$ is the controlled output vector, $y(t) \in \mathbb{R}^{n_y}$ is the measurable output vector, and $w(t) \in \mathbb{R}^{n_w}$ is the exterior disturbance. r is the number of IF–THEN rules, $v(t) \in \mathbb{R}^{p \times 1}$ denotes the premise variables and assumed to be measurable, and M_{ij} denotes the fuzzy sets. Denote

$$\mathbf{w}_i(v(t)) = \prod_{j=1}^p M_{ij}(v_j(t))$$

where $M_{ij}(v_j(t))$ is the grade of membership of $v_j(t)$ in M_{ij} , where it is assumed that

$$\sum_{i=1}^r \mathbf{w}_i(v(t)) > 0, \quad \mathbf{w}_i(v(t)) \geq 0; \quad i = 1, 2, \dots, r.$$

Let

$$\alpha_i(v(t)) = \frac{\mathbf{w}_i(v(t))}{\sum_{i=1}^r \mathbf{w}_i(v(t))}.$$

Then

$$0 \leq \alpha_i(v(t)) \leq 1 \quad \sum_{i=1}^r \alpha_i(v(t)) = 1 \quad (2)$$

where $\alpha_i(v(t))$ denotes the normalized membership functions. For the convenience of notations, $\alpha_i(v(t))$ is denoted as α_i and the vector $\alpha(v(t)) = [\alpha_1(v(t)), \dots, \alpha_r(v(t))]^T$ as α . By using the fuzzy inference method with a singleton fuzzifier, product inference, and center average defuzzifiers, the final output of T–S fuzzy model is obtained as

$$\begin{aligned} \dot{x}(t) &= A(\alpha)x(t) + B_1(\alpha)w(t) + B_2(\alpha)u(t) \\ z(t) &= C_1(\alpha)x(t) + D_{11}(\alpha)w(t) + D_{12}(\alpha)u(t) \\ y(t) &= C_2(\alpha)x(t) + D_{21}(\alpha)w(t) \end{aligned} \quad (3)$$

where

$$\begin{aligned} A(\alpha) &= \sum_{i=1}^r \alpha_i A_i & B_1(\alpha) &= \sum_{i=1}^r \alpha_i B_{1i} \\ B_2(\alpha) &= \sum_{i=1}^r \alpha_i B_{2i} & C_1(\alpha) &= \sum_{i=1}^r \alpha_i C_{1i} \\ D_{11}(\alpha) &= \sum_{i=1}^r \alpha_i D_{11i} & D_{12}(\alpha) &= \sum_{i=1}^r \alpha_i D_{12i} \\ C_2(\alpha) &= \sum_{i=1}^r \alpha_i C_{2i} & D_{21}(\alpha) &= \sum_{i=1}^r \alpha_i D_{21i}. \end{aligned} \quad (4)$$

A. Switched PDC Scheme

Denote

$$\Omega = \left\{ \alpha : 0 \leq \alpha_i \leq 1, \quad 1 \leq i \leq r, \quad \sum_{i=1}^r \alpha_i = 1 \right\} \quad (5a)$$

$$\Omega_l = \left\{ \alpha : 0 \leq \alpha_i \leq \alpha_l, \quad 1 \leq i \leq r, \quad \sum_{i=1}^r \alpha_i = 1, \quad \alpha \in \Omega \right\} \quad (5b)$$

$$\partial\Omega_l = \{ \alpha : \exists i \neq l \text{ such that } \alpha_i = \alpha_l \text{ and } \alpha \in \Omega_l \} \quad (5c)$$

$$\partial\Omega = \bigcup_{l=1}^r \partial\Omega_l \quad (5d)$$

where $1 \leq l \leq r$.

Remark 1: Ω is the set of all the vectors $\alpha = [\alpha_1, \alpha_2, \dots, \alpha_r]^T$, where α_i ($i = 1, \dots, r$) takes all possible values of membership functions. Ω_l is the set of all the vectors α with α_i ($i = 1, \dots, r$) satisfying $0 \leq \alpha_i \leq \alpha_l$, which describes the case where the l th rule plays a more important or at least the same role than other rules. Obviously, $\Omega = \bigcup_{l=1}^r \Omega_l$.

At any time or moment t , we have the vector $\alpha(v(t)) = [\alpha_1(v(t)), \alpha_2(v(t)), \dots, \alpha_r(v(t))]^T \in \Omega$. Then, there exists one l , where $l \in \{1, \dots, r\}$, such that the vector $\alpha(v(t)) \in \Omega_l$, which implies that the l th subsystem plays a more important or at least the same role than other subsystems. For this case, a specific controller gain $\mathcal{K}_l(\alpha)$ is applied for achieving better control effect. Thus, the following switched dynamic output feedback controller for continuous-time T–S fuzzy models is exploited in this correspondence paper:

$$\begin{bmatrix} \dot{\xi}(t) \\ u(t) \end{bmatrix} = \begin{cases} \mathcal{K}_1(\alpha) \begin{bmatrix} \xi(t) \\ y(t) \end{bmatrix}, & \text{for } \alpha \in \Omega_1 \\ \vdots \\ \mathcal{K}_r(\alpha) \begin{bmatrix} \xi(t) \\ y(t) \end{bmatrix}, & \text{for } \alpha \in \Omega_r \end{cases} \quad (6a)$$

where

$$\mathcal{K}_l(\alpha) = \begin{bmatrix} A_{Kl}(\alpha) & B_{Kl}(\alpha) \\ C_{Kl}(\alpha) & D_{Kl}(\alpha) \end{bmatrix}, \quad 1 \leq l \leq r. \quad (6b)$$

For the design of controller gains, a concept of DPDC scheme [26], [27] is used, i.e.,

$$\begin{aligned} A_{Kl}(\alpha) &= \sum_{i=1}^r \sum_{j=1}^r \alpha_i \alpha_j A_{Kijl} & B_{Kl}(\alpha) &= \sum_{i=1}^r \alpha_i B_{Kil} \\ C_{Kl}(\alpha) &= \sum_{i=1}^r \alpha_i C_{Kil} & D_{Kl}(\alpha) &= D_{Kl}. \end{aligned} \quad (6c)$$

The premise variables $v_i(t)$, where $1 \leq i \leq r$, are measurable; then, $\alpha_i(v(t))$, where $1 \leq i \leq r$, can be obtained online. For the design of controller gains $A_{Kl}(\alpha)$, $B_{Kl}(\alpha)$, $C_{Kl}(\alpha)$, and D_{Kl} , only the controller parameters A_{Kijl} , B_{Kijl} , C_{Kijl} , and D_{Kl} are to be designed.

Because the premise variables $\alpha_i(v(t))$, where $1 \leq i \leq r$, are available online, the switching can be perfectly determined. It is performed by following the steps.

Step 1) If $\alpha(v(t)) \in \partial\Omega$, then the controller gains do not switch.

Step 2) If $\alpha(v(t)) \in \Omega_l$, (because $\Omega = \bigcup_{l=1}^r \Omega_l$, there must exist the l), then the switch controller gain is denoted as $\mathcal{K}_l(\alpha)$.

Note that, if $\alpha \in \Omega_l$, where $1 \leq l \leq r$, the controller gain is then $\mathcal{K}_l(\alpha)$, which implies that the dynamic output feedback controller is the following concrete form:

$$\begin{aligned}\dot{\xi}(t) &= A_{Kl}(\alpha)\xi(t) + B_{Kl}(\alpha)y(t) \\ u(t) &= C_{Kl}(\alpha)\xi(t) + D_{Kl}(\alpha)y(t).\end{aligned}$$

Combining it and (4), the closed-loop system can be obtained as

$$\begin{aligned}\dot{\hat{x}}(t) &= \mathbf{A}_l \hat{x}(t) + \mathbf{B}_l w(t) \\ z(t) &= \mathbf{C}_l \hat{x}(t) + \mathbf{D}_l w(t),\end{aligned} \quad \alpha \in \Omega_l; \quad 1 \leq l \leq r \quad (7)$$

where

$$\hat{x}(t) = \begin{bmatrix} x(t) \\ \xi(t) \end{bmatrix}$$

and

$$\begin{aligned}\mathbf{A}_l &= \begin{bmatrix} A(\alpha) + B_2(\alpha)D_{Kl}C_2(\alpha) & B_2(\alpha)C_{Kl}(\alpha) \\ B_{Kl}(\alpha)C_2(\alpha) & A_{Kl}(\alpha) \end{bmatrix} \\ \mathbf{B}_l &= \begin{bmatrix} B_1(\alpha) + B_2(\alpha)D_{Kl}D_{21}(\alpha) \\ B_{Kl}(\alpha)D_{21}(\alpha) \end{bmatrix} \\ \mathbf{C}_l &= [C_1(\alpha) + D_{12}(\alpha)D_{Kl}C_2(\alpha) \quad D_{12}(\alpha)C_{Kl}(\alpha)] \\ \mathbf{D}_l &= [D_{11}(\alpha) + D_{12}(\alpha)D_{Kl}D_{21}(\alpha)], \quad 1 \leq l \leq r.\end{aligned} \quad (8)$$

Then, the problem considered in this correspondence paper is formulated as follows.

Given a prescribed H_∞ performance $\gamma > 0$, design a fuzzy dynamic output feedback controller of the form (6) such that the following are satisfied.

- 1) The closed-loop system (7) is asymptotically stable.
- 2) Under zero initial condition: $x(0) = 0$

$$\int_0^\infty z^T(t)z(t) \leq \gamma^2 \int_0^\infty w^T(t)w(t). \quad (9)$$

Note that the zero initial condition $x(0) = 0$ is required in the H_∞ performance; the definition of H_∞ performance can also be found in [20] and [21].

III. MAIN RESULT

In this section, two lemmas first are given. In Lemma 1, a nonconvex sufficient condition for designing H_∞ controllers for continuous-time fuzzy systems is exploited. Moreover, some matrix properties are studied in Lemma 2. Last, based on Lemmas 1 and 2, an LMI-based method of designing H_∞ SDPDC controllers via full order dynamic output feedback for continuous-time fuzzy systems is proposed.

Lemma 1: Given a prescribed H_∞ performance γ , if there exists matrix $P = P^T \in \mathbb{R}^{2n_x \times 2n_x}$ satisfying

$$P > 0 \quad (10)$$

$$\begin{bmatrix} P\mathbf{A}_l + \mathbf{A}_l^T P & * & * \\ \mathbf{B}_l^T P & -\gamma^2 I & * \\ \mathbf{C}_l & \mathbf{D}_l & -I \end{bmatrix} < 0, \quad 1 \leq l \leq r; \quad \alpha \in \Omega_l \quad (11)$$

where \mathbf{A}_l , \mathbf{B}_l , \mathbf{C}_l , and \mathbf{D}_l are the same as in (8). Then, the dynamic output feedback controller (6) such that the closed-loop system (7) is asymptotically stable with an H_∞ performance bounded by γ .

Proof: Consider a candidate Lyapunov function $V(t)$

$$V(t) = \hat{x}^T(t)P\hat{x}(t)$$

where

$$\hat{x}(t) = \begin{bmatrix} x(t) \\ \xi(t) \end{bmatrix} \quad P \in \mathbb{R}^{2n_x \times 2n_x} \quad P > 0.$$

Then

$$\begin{aligned}\dot{V}(t) + z^T(t)z(t) - \gamma^2 w^T(t)w(t) &= 2\hat{x}^T(t)P(\mathbf{A}_l \hat{x}(t) + \mathbf{B}_l w(t)) + (\mathbf{C}_l \hat{x}(t) + \mathbf{D}_l w(t))^T \\ &\quad \times (\mathbf{C}_l \hat{x}(t) + \mathbf{D}_l w(t)) - \gamma^2 w^T(t)w(t) \\ &= \hat{x}^T(t) (P\mathbf{A}_l + \mathbf{A}_l^T P + \mathbf{C}_l^T \mathbf{C}_l) \hat{x}(t) \\ &\quad + 2\hat{x}^T(t) (P\mathbf{B}_l + \mathbf{C}_l^T \mathbf{D}_l) w(t) \\ &\quad + w^T(t) (-\gamma^2 I + \mathbf{D}_l^T \mathbf{D}_l) w(t) \\ &= \begin{bmatrix} \hat{x}(t) \\ w(t) \end{bmatrix}^T \begin{bmatrix} P\mathbf{A}_l + \mathbf{A}_l^T P + \mathbf{C}_l^T \mathbf{C}_l & * \\ \mathbf{B}_l^T P + \mathbf{D}_l^T \mathbf{C}_l & -\gamma^2 I + \mathbf{D}_l^T \mathbf{D}_l \end{bmatrix} \begin{bmatrix} \hat{x}(t) \\ w(t) \end{bmatrix}, \\ &1 \leq l \leq r; \quad \alpha \in \Omega_l.\end{aligned} \quad (12)$$

On the other hand, applying Schur complement to (11), we have the following:

$$\begin{bmatrix} P\mathbf{A}_l + \mathbf{A}_l^T P + \mathbf{C}_l^T \mathbf{C}_l & * \\ \mathbf{B}_l^T P + \mathbf{D}_l^T \mathbf{C}_l & -\gamma^2 I + \mathbf{D}_l^T \mathbf{D}_l \end{bmatrix} < 0 \quad 1 \leq l \leq r; \quad \alpha \in \Omega_l.$$

Pre- and postmultiplying the aforementioned inequality by $\begin{bmatrix} \hat{x}^T(t) & w^T(t) \end{bmatrix}$ and its transpose, then it follows that

$$\begin{bmatrix} \hat{x}(t) \\ w(t) \end{bmatrix}^T \begin{bmatrix} P\mathbf{A}_l + \mathbf{A}_l^T P + \mathbf{C}_l^T \mathbf{C}_l & * \\ \mathbf{B}_l^T P + \mathbf{D}_l^T \mathbf{C}_l & -\gamma^2 I + \mathbf{D}_l^T \mathbf{D}_l \end{bmatrix} \begin{bmatrix} \hat{x}(t) \\ w(t) \end{bmatrix} \leq 0, \quad 1 \leq l \leq r; \quad \alpha \in \Omega_l.$$

Combining it and (12) yields the following:

$$\dot{V}(t) + z^T(t)z(t) - \gamma^2 w^T(t)w(t) \leq 0. \quad (13)$$

Integrating both sides of (13) yields the following:

$$\begin{aligned} & \int_0^{\infty} \dot{V}(t)dt + \int_0^{\infty} z^T(t)z(t)dt - \gamma^2 \int_0^{\infty} w^T(t)w(t)dt \\ & = V(\infty) - V(0) + \int_0^{\infty} z^T(t)z(t)dt - \gamma^2 \int_0^{\infty} w^T(t)w(t)dt \leq 0. \end{aligned}$$

Using the fact that $x(0) = 0$ and $V(\infty) \geq 0$, we obtain the following:

$$\int_0^{\infty} z^T(t)z(t)dt \leq \gamma^2 \int_0^{\infty} w^T(t)w(t)dt.$$

Hence, (11) holds; then, the H_{∞} performance is fulfilled.

If the disturbance $w(t) = 0$, then from (13), we have $\dot{V}(t) < 0$. Hence, the closed-loop system (7) is asymptotically stable. Thus, the proof is complete. ■

Remark 2: Note that the condition in Lemma 1 is not convex with respect to P , $A_{Kl}(\alpha)$, $B_{Kl}(\alpha)$, $C_{Kl}(\alpha)$, and D_{Kl} ; therefore, the condition cannot directly be used for designing controllers. What follow are some properties of matrix are studied in Lemma 2 and based on Lemmas 1 and 2 and an LMI-based sufficient condition for designing H_{∞} controller is proposed in Theorem 1.

Lemma 2: If $X = X^T$, $Y = Y^T$, and M and N are nonsingular, satisfy the following:

$$MN^T + XY = I$$

then

$$\begin{bmatrix} X & M \\ M^T & -N^{-1}YM \end{bmatrix} \begin{bmatrix} Y & N \\ N^T & -M^{-1}XN \end{bmatrix}$$

are symmetrical and satisfy

$$\begin{bmatrix} X & M \\ M^T & -N^{-1}YM \end{bmatrix} \begin{bmatrix} Y & N \\ N^T & -M^{-1}XN \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}.$$

Proof: From $MN^T + XY = I$, we have the following:

$$\begin{aligned} NM^T &= I - YX \\ N^T &= M^{-1}(I - XY). \end{aligned}$$

Then

$$\begin{aligned} N^T X M^{-T} &= M^{-1}(I - XY)X M^{-T} \\ &= M^{-1}X(I - YX)M^{-T} \\ &= M^{-1}X N M^T M^{-T} = M^{-1}XN \\ M^T Y N^{-T} &= M^T Y (I - XY)^{-1} M = M^T (I - YX)^{-1} Y M \\ &= M^T (N M^T)^{-1} Y M = M^T M^{-T} N^{-1} Y M \\ &= N^{-1} Y M \end{aligned}$$

which imply that both $M^{-1}XN$ and $N^{-1}YM$ are symmetrical. Furthermore, both

$$\begin{bmatrix} X & M \\ M^T & -N^{-1}YM \end{bmatrix} \begin{bmatrix} Y & N \\ N^T & -M^{-1}XN \end{bmatrix}$$

are symmetrical.

Now, consider

$$\begin{aligned} & \begin{bmatrix} X & M \\ M^T & -N^{-1}YM \end{bmatrix} \begin{bmatrix} Y & N \\ N^T & -M^{-1}XN \end{bmatrix} \\ &= \begin{bmatrix} XY + MN^T & XN - MM^{-1}XN \\ M^T Y - N^{-1}YMN^T & M^T N + N^{-1}YMM^{-1}XN \end{bmatrix} \\ &= \begin{bmatrix} XY + MN^T & 0 \\ M^T Y - N^{-1}YMN^T & M^T N + N^{-1}YXN \end{bmatrix}. \end{aligned}$$

From $MN^T + XY = I$ and $M^T Y N^{-T} = N^{-1} Y M$, it follows that

$$\begin{aligned} & \begin{bmatrix} XY + MN^T & 0 \\ M^T Y - N^{-1}YMN^T & M^T N + N^{-1}YXN \end{bmatrix} \\ &= \begin{bmatrix} I & 0 \\ M^T Y - M^T Y N^{-T} N^T & M^T N + N^{-1}(I - N M^T)N \end{bmatrix} \\ &= \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}. \end{aligned}$$

Then, from the aforementioned proof, we have the following:

$$\begin{bmatrix} X & M \\ M^T & -N^{-1}YM \end{bmatrix} \begin{bmatrix} Y & N \\ N^T & -M^{-1}XN \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}.$$

Thus, the proof is complete. ■

Based on Lemmas 1 and 2, a convex method of designing H_{∞} SDPDC controllers via full order dynamic output feedback for continuous-time fuzzy systems is proposed in the following theorem.

Theorem 1: Given a prescribed H_{∞} performance bound γ , if there exist matrices $X = X^T \in \mathbb{R}^{n_x \times n_x}$, $Y = Y^T \in \mathbb{R}^{n_x \times n_x}$, $\hat{A}_{Kijl} \in \mathbb{R}^{n_x \times n_x}$, $\hat{B}_{Kil} \in \mathbb{R}^{n_x \times n_y}$, $\hat{C}_{Kil} \in \mathbb{R}^{n_u \times n_x}$, $\hat{D}_{Kl} \in \mathbb{R}^{n_u \times n_y}$, $J_{ijl} = J_{jil}^T \in \mathbb{R}^{e \times e}$, and $0 \leq R_{ijl} \in \mathbb{R}^{e \times e}$, where $1 \leq i, j, l \leq r$, satisfying the LMIs given by (14)–(17) shown at the bottom of the page, where

$$\begin{bmatrix} X & I_{n_x \times n_x} \\ I_{n_x \times n_x} & Y \end{bmatrix} > 0 \quad (14)$$

$$\Lambda_{iil} < J_{iil}, \quad 1 \leq i \leq r; \quad 1 \leq l \leq r \quad (15)$$

$$\Lambda_{ijl} + \Lambda_{jil} < J_{ijl} + J_{jil}^T, \quad 1 \leq i < j \leq r; \quad 1 \leq l \leq r \quad (16)$$

$$[J_{ijl}]_{r \times r} + \mathbf{He}(E_l^T [R_{ijl}]_{(r-1) \times r}) < 0, \quad 1 \leq l \leq r \quad (17)$$

the expressions for Λ_{ijl} and E_l , shown at the bottom of the page, then there exists a dynamic output feedback controller (6) with gains (18) such that the closed-loop system (7) is asymptotically stable with an H_∞ performance bounded by γ

$$\begin{aligned} D_{Kl} &= \hat{D}_{Kl} \\ C_{Kil} &= (\hat{C}_{Kil} - D_{Kl}C_{2i}X)M^{-T} \\ B_{Kil} &= N^{-1}(\hat{B}_{Kil} - YB_{2i}D_{Kl}) \\ A_{Kijl} &= N^{-1}(\hat{A}_{Kijl} - NB_{Kjl}C_{2i}X - YB_{2i}C_{Kj}M^T \\ &\quad - Y(A_i + B_{2i}D_{Kl}C_{2j})X)M^{-T} \end{aligned} \quad (18)$$

where M and N satisfy the following:

$$MN^T = I - XY. \quad (19)$$

Proof: Define

$$P = \begin{bmatrix} Y & N \\ N^T & -M^{-1}XN \end{bmatrix}.$$

Then, from Lemma 2, we have the following:

$$P^{-1} = \begin{bmatrix} X & M \\ M^T & -N^{-1}YM \end{bmatrix}$$

where X , Y , M , and N satisfy the conditions of Theorem 1.

Because M is nonsingular, the following nonsingular matrix can be constructed:

$$T = \begin{bmatrix} X & I_{n_x \times n_x} \\ M^T & 0 \end{bmatrix}.$$

Then

$$\begin{aligned} T^T P T &= \begin{bmatrix} X & I_{n_x \times n_x} \\ M^T & 0 \end{bmatrix}^T \begin{bmatrix} Y & N \\ N^T & -M^{-1}XN \end{bmatrix} \begin{bmatrix} X & I_{n_x \times n_x} \\ M^T & 0 \end{bmatrix} \\ &= \begin{bmatrix} I_{n_x \times n_x} & 0 \\ Y & N \end{bmatrix} \begin{bmatrix} X & I_{n_x \times n_x} \\ M^T & 0 \end{bmatrix} \\ &= \begin{bmatrix} X & I_{n_x \times n_x} \\ I_{n_x \times n_x} & Y \end{bmatrix}. \end{aligned}$$

Combining it and (14), it follows that $P > 0$.

Consider

$$\begin{aligned} &\begin{bmatrix} \alpha_1 I \\ \vdots \\ \alpha_r I \end{bmatrix}^T \text{He} \left(E_l^T [R_{ijl}]_{(r-1) \times r} \begin{bmatrix} \alpha_1 I \\ \vdots \\ \alpha_r I \end{bmatrix} \right) \\ &= \text{He} \left(\begin{bmatrix} (\alpha_l - \alpha_1)I \\ \vdots \\ (\alpha_l - \alpha_{l-1})I \\ (\alpha_l - \alpha_{l+1})I \\ \vdots \\ (\alpha_l - \alpha_r)I \end{bmatrix}^T [R_{ijl}]_{(r-1) \times r} \begin{bmatrix} \alpha_1 I \\ \vdots \\ \alpha_r I \end{bmatrix} \right). \end{aligned} \quad (20)$$

If $\alpha = [\alpha_1 \ \alpha_2, \dots, \alpha_r]^T \in \Omega_l$, then we have the following:

$$\alpha_l - \alpha_i \geq 0, \quad 1 \leq i \neq l \leq r.$$

Combining it with (20) and $R_{ijl} \geq 0$, then it follows

$$\begin{aligned} &\begin{bmatrix} \alpha_1 I \\ \vdots \\ \alpha_r I \end{bmatrix}^T \text{He} \left(E_l^T [R_{ijl}]_{(r-1) \times r} \begin{bmatrix} \alpha_1 I \\ \vdots \\ \alpha_r I \end{bmatrix} \right) \geq 0, \\ &\alpha \in \Omega_l, \quad 1 \leq l \leq r. \end{aligned} \quad (21)$$

Moreover, pre- and postmultiplying (17) by $[\alpha_1 I, \dots, \alpha_r I]$ and its transpose yield the following:

$$\begin{aligned} &\begin{bmatrix} \alpha_1 I \\ \vdots \\ \alpha_r I \end{bmatrix}^T [J_{ijl}]_{r \times r} \begin{bmatrix} \alpha_1 I \\ \vdots \\ \alpha_r I \end{bmatrix} \\ &+ \begin{bmatrix} \alpha_1 I \\ \vdots \\ \alpha_r I \end{bmatrix}^T \text{He} \left(E_l^T [R_{ijl}]_{(r-1) \times r} \begin{bmatrix} \alpha_1 I \\ \vdots \\ \alpha_r I \end{bmatrix} \right) < 0, \quad 1 \leq l \leq r. \end{aligned}$$

Considering it and (21), then we can obtain the following:

$$\begin{bmatrix} \alpha_1 I \\ \vdots \\ \alpha_r I \end{bmatrix}^T [J_{ijl}]_{r \times r} \begin{bmatrix} \alpha_1 I \\ \vdots \\ \alpha_r I \end{bmatrix} < 0, \quad \alpha \in \Omega_l; \quad 1 \leq l \leq r$$

$$\begin{aligned} \Lambda_{ijl} &= \begin{bmatrix} A_i X + X A_i^T + B_{2i} \hat{C}_{Kjl} + (B_{2i} \hat{C}_{Kjl})^T & * & * & * \\ \hat{A}_{Kijl} + (A_i + B_{2i} \hat{D}_{Kl} C_{2j})^T & Y A_i + A_i^T Y + \hat{B}_{Kjl} C_{2i} + (\hat{B}_{Kjl} C_{2i})^T & * & * \\ (B_{1i} + B_{2i} \hat{D}_{Kl} D_{21j})^T & (Y B_{1i} + \hat{B}_{Kjl} D_{21i})^T & -\gamma^2 I & * \\ C_{1i} X + D_{12i} \hat{C}_{Kjl} & C_{1i} + D_{12i} \hat{D}_{Kl} C_{2j} & D_{11i} + D_{12i} \hat{D}_{Kl} D_{21j} & -I \end{bmatrix} \\ E_l &= \begin{bmatrix} -\tilde{I} & 0 & \cdots & 0 & \tilde{I} & 0 & \cdots & 0 \\ 0 & -\tilde{I} & \cdots & 0 & \tilde{I} & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & -\tilde{I} & \tilde{I} & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & \tilde{I} & -\tilde{I} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & \tilde{I} & 0 & \cdots & -\tilde{I} \end{bmatrix}_{(r-1) \times r} \rightarrow l-1, \quad \tilde{I} \in R^{e \times e}; \quad e = 2n_x + n_z + n_w \\ &\downarrow \\ &l \end{aligned}$$

i.e.,

$$\sum_{i=1}^r \sum_{j=1}^r \alpha_i \alpha_j J_{ijl} < 0, \quad \alpha \in \Omega_l; \quad 1 \leq l \leq r. \quad (22)$$

Multiplying (15) and (16) by α_i^2 , where $1 \leq i \leq r$, and $\alpha_i \alpha_j$, where $1 \leq i < j \leq r$, respectively, and summing them, then we have the following:

$$\sum_{i=1}^r \sum_{j=1}^r \alpha_i \alpha_j \Lambda_{ijl} < \sum_{i=1}^r \sum_{j=1}^r \alpha_i \alpha_j J_{ijl}, \quad \alpha \in \Omega_l; \quad 1 \leq l \leq r.$$

Combining it and (22) then yields the following:

$$\sum_{i=1}^r \sum_{j=1}^r \alpha_i \alpha_j \Lambda_{ijl} < 0, \quad \alpha \in \Omega_l; \quad 1 \leq l \leq r$$

which can be rewritten as

$$\begin{bmatrix} H_{11l} & * & * & * \\ H_{21l} & H_{22l} & * & * \\ H_{31l} & H_{32l} & H_{33l} & * \\ H_{41l} & H_{42l} & H_{43l} & H_{44l} \end{bmatrix} < 0, \quad \alpha \in \Omega_l; \quad 1 \leq l \leq r \quad (23)$$

where

$$\begin{aligned} H_{11l} &= A(\alpha)X + XA^T(\alpha) + B_2(\alpha)\hat{C}_{Kl}(\alpha) \\ &\quad + (B_2(\alpha)\hat{C}_{Kl}(\alpha))^T \\ H_{21l} &= \hat{A}_{Kl}(\alpha) + (A(\alpha) + B_2(\alpha)\hat{D}_{Kl}C_2(\alpha))^T \\ H_{22l} &= YA(\alpha) + A^T(\alpha)Y + \hat{B}_{Kl}(\alpha)C_2(\alpha) \\ &\quad + (\hat{B}_{Kl}(\alpha)C_2(\alpha))^T \\ H_{31l} &= (B_1(\alpha) + B_2(\alpha)\hat{D}_{Kl}D_{21}(\alpha))^T \\ H_{32l} &= (YB_1(\alpha) + \hat{B}_{Kl}(\alpha)D_{21}(\alpha))^T \\ H_{33l} &= -\gamma^2 I \\ H_{41l} &= C_1(\alpha)X + D_{12}(\alpha)\hat{C}_{Kl}(\alpha) \\ H_{42l} &= C_1(\alpha) + D_{12}(\alpha)\hat{D}_{Kl}C_2(\alpha) \\ H_{43l} &= D_{11}(\alpha) + D_{12}(\alpha)\hat{D}_{Kl}D_{21}(\alpha) \\ H_{44l} &= -I. \end{aligned}$$

From (18), we can obtain the following:

$$\begin{aligned} \hat{D}_{Kl} &= D_{Kl} \\ \hat{C}_{Kl}(\alpha) &= C_{Kl}(\alpha)M^T + D_{Kl}C_2(\alpha)X \\ \hat{B}_{Kl}(\alpha) &= NB_{Kl}(\alpha) + YB_2(\alpha)D_{Kl} \\ \hat{A}_{Kl}(\alpha) &= NA_{Kl}(\alpha)M^T + NB_{Kl}(\alpha)C_2(\alpha)X \\ &\quad + YB_2(\alpha)C_{Kl}(\alpha)M^T \\ &\quad + Y(A(\alpha) + B_2(\alpha)D_{Kl}C_2(\alpha))X. \end{aligned}$$

Combining it and (23), it follows that

$$\begin{bmatrix} \Phi_{11l} & * & * & * \\ \Phi_{21l} & \Phi_{22l} & * & * \\ \Phi_{31l} & \Phi_{32l} & \Phi_{33l} & * \\ \Phi_{41l} & \Phi_{42l} & \Phi_{43l} & \Phi_{44l} \end{bmatrix} < 0, \quad \alpha \in \Omega_l; \quad 1 \leq l \leq r \quad (24)$$

where

$$\begin{aligned} \Phi_{11l} &= \text{He} (A(\alpha)X + B_2(\alpha) (C_{Kl}(\alpha)M^T + D_{Kl}C_2(\alpha)X)) \\ \Phi_{21l} &= NA_{Kl}(\alpha)M^T + NB_{Kl}(\alpha)C_2(\alpha)X \\ &\quad + YB_2(\alpha)C_{Kl}(\alpha)M^T \\ &\quad + Y(A(\alpha) + B_2(\alpha)D_{Kl}C_2(\alpha))X \\ &\quad + (A(\alpha) + B_2(\alpha)D_{Kl}C_2(\alpha))^T \\ \Phi_{22l} &= \text{He} (YA(\alpha) + (NB_{Kl}(\alpha) + YB_2(\alpha)D_{Kl})C_2(\alpha)) \\ \Phi_{31l} &= (B_1(\alpha) + B_2(\alpha)D_{Kl}D_{21}(\alpha))^T \\ \Phi_{32l} &= (YB_1(\alpha) + (NB_{Kl}(\alpha) + YB_2(\alpha)D_{Kl})D_{21}(\alpha))^T \\ \Phi_{33l} &= -\gamma^2 I \\ \Phi_{41l} &= C_1(\alpha)X + D_{12}(\alpha) (C_{Kl}(\alpha)M^T + D_{Kl}C_2(\alpha)X) \\ \Phi_{42l} &= C_1(\alpha) + D_{12}(\alpha)D_{Kl}C_2(\alpha) \\ \Phi_{43l} &= D_{11}(\alpha) + D_{12}(\alpha)D_{Kl}D_{21}(\alpha) \\ \Phi_{44l} &= -I. \end{aligned}$$

Moreover, pre- and postmultiplying matrix

$$\begin{bmatrix} PA_l + A_l^T P & * & * \\ B_l^T P & -\gamma^2 I & * \\ C_l & D_l & -I \end{bmatrix} \quad (25)$$

by $\text{diag} [T^T \quad I \quad I]$ and its transpose then yield the following:

$$\begin{bmatrix} T^T & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} PA_l + A_l^T P & * & * \\ B_l^T P & -\gamma^2 I & * \\ C_l & D_l & -I \end{bmatrix} \begin{bmatrix} T & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} = \begin{bmatrix} \Phi_{11l} & * & * & * \\ \Phi_{21l} & \Phi_{22l} & * & * \\ \Phi_{31l} & \Phi_{32l} & \Phi_{33l} & * \\ \Phi_{41l} & \Phi_{42l} & \Phi_{43l} & \Phi_{44l} \end{bmatrix}.$$

From the aforementioned equality and (24), then we can obtain the following:

$$\begin{bmatrix} T^T & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} PA_l + A_l^T P & * & * \\ B_l^T P & -\gamma^2 I & * \\ C_l & D_l & -I \end{bmatrix} \begin{bmatrix} T & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} < 0, \quad \alpha \in \Omega_l; \quad 1 \leq l \leq r.$$

Because $\text{diag} [T^T \quad I \quad I]$ is nonsingular

$$\begin{bmatrix} PA_l + A_l^T P & * & * \\ B_l^T P & -\gamma^2 I & * \\ C_l & D_l & -I \end{bmatrix} < 0, \quad \alpha \in \Omega_l; \quad 1 \leq l \leq r. \quad (26)$$

TABLE I
NUMBERS OF LMIS AND VARIABLES

	The method of [26] (DPDC scheme)	Theorem 1 (switched DPDC scheme)
LMIs	$2 + r + \frac{1}{2}r(r - 1)$	$1 + r + r^2 + \frac{1}{2}r^2(r - 1)$
Variables	$n_x + (1 + r^2)n_x^2 + n_u n_y + r n_x(n_y + n_u) + [1 + r(2n_x + n_w + n_z)](2n_x + n_w + n_z)$	$n_x + (1 + r^3)n_x^2 + r n_u n_y + r^2 n_x(n_y + n_u) + r[1 + r(2n_x + n_w + n_z)](2n_x + n_w + n_z) + r^2(r - 1)(2n_x + n_w + n_z)^2$

Applying Lemma 1 to (26) and $P > 0$, then we have the closed-loop system (7) that is asymptotically stable with an H_∞ performance bounded by γ . Thus, the proof is complete. ■

Remark 3:

- 1) Theorem 1 presents an LMI-based approach for designing switched dynamic output feedback H_∞ controllers, which can be solved efficiently via LMI control toolbox [14]. The new proposed method can provide less conservative results than the existing methods via the DPDC scheme [26]. In fact, if $R_{ijml} = 0$, $\hat{A}_{Kijl} = \hat{A}_{Fij}$, $\hat{B}_{Kil} = \hat{B}_{Fi}$, $\hat{C}_{Kil} = \hat{C}_{Fi}$, and $\hat{D}_{Kil} = \hat{D}_F$ in the conditions of Theorem 1, then the design method given in Theorem 1 reduces to the corresponding control design method based on the DPDC scheme [26]. However, more decision variables are involved in the condition given by Theorem 1. The comparisons of the complexity are give in Table I.
- 2) By introducing an SDPDC control scheme with a quadratic Lyapunov function, the convex approach in Theorem 1 can give less conservative results for designing fuzzy controllers than the existing approaches [26]. On the other hand, some convex techniques based on a weighting dependent Lyapunov function (fuzzy Lyapunov function) [8], [11] are proposed to replace the single quadratic one for discrete-time fuzzy systems for obtaining relaxed design conditions. For continuous fuzzy systems, a fuzzy Lyapunov approach is given in [10] by assuming the time derivatives to be computable. However, some membership function cannot satisfy the assumption, for example, the triangular and trapezoidal membership functions. Therefore, the quadratic Lyapunov function method is used in this correspondence paper.
- 3) By (14), we can obtain $Y > 0$ and $X - Y^{-1} > 0$, which imply that $I - XY$ is nonsingular. Therefore, we always find nonsingular M and N satisfying (19). In particular, by orthogonal-triangular decomposition of $I - XY$ (the qr function in Matlab can perform the orthogonal-triangular decomposition of a matrix), we can obtain a solution of N and M (more details can be found in [35]).
- 4) Notice that a congruent transformation is applied to (25), which shows that (26) is equivalent to (24). Moreover, it is easily seen that (23) and (24) are equivalent. Thus, the nonlinear matrix inequality (26) with respect to $X, Y, M, N, A_{Kijl}, B_{Kil}, C_{Kil}$, and D_{Kil} is equivalent to an LMI (23) with respect to $X, Y, \hat{A}_{Kijl}, \hat{B}_{Kil}, \hat{C}_{Kil}$, and \hat{D}_{Kil} . On the other hand, both (26) and (23) are nonlinear with respect to α_i , where $1 \leq i \leq r$; however, the nonlinearity can be removed by some existing techniques, which can be found in many references, such as [2], [5], and [10]. The technique in [5], which can give less conservative results than [2] and [10], is used in this correspondence paper.

IV. EXAMPLE

Consider a nonlinear two-degree-of-freedom helicopter system [9], which is described by the following T-S fuzzy model:

- Rule 1:** IF $p_1(t)$ is 4°
 THEN $\dot{x}(t) = A_1x(t) + B_{11}w(t) + B_{21}u(t)$
 $z(t) = C_{11}x(t) + D_{111}w(t) + D_{121}u(t)$
 $y(t) = C_{21}x(t) + D_{211}w(t)$
- Rule 2:** IF $p_1(t)$ is 40°
 THEN $\dot{x}(t) = A_2x(t) + B_{12}w(t) + B_{22}u(t)$
 $z(t) = C_{12}x(t) + D_{112}w(t) + D_{122}u(t)$
 $y(t) = C_{22}x(t) + D_{212}w(t)$
- Rule 3:** IF $p_1(t)$ is 76°
 THEN $\dot{x}(t) = A_3x(t) + B_{13}w(t) + B_{23}u(t)$
 $z(t) = C_{13}x(t) + D_{113}w(t) + D_{123}u(t)$
 $y(t) = C_{23}x(t) + D_{213}w(t)$

where $p_1(t)$ is a pitch angle, and

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -0.0798 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad B_{11} = \begin{bmatrix} -0.03 \\ -0.02 \\ -0.02 \\ -0.02 \end{bmatrix}$$

$$B_{21} = \begin{bmatrix} 0 & 0 \\ 5.1404 & 0 \\ 0 & 0 \\ 0 & 3.4338 \end{bmatrix} \quad C_{11} = [0.8 \quad -0.2 \quad 0.1 \quad -0.6]$$

$$D_{111} = -0.6 \quad D_{121} = [0.9 \quad 0.5]$$

$$C_{21} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad D_{211} = \begin{bmatrix} -0.005 \\ -0.008 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -0.5648 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad B_{12} = \begin{bmatrix} 0.01 \\ 0.01 \\ 0.01 \\ -0.01 \end{bmatrix}$$

$$B_{22} = \begin{bmatrix} 0 & 0 \\ 2.1714 & 0 \\ 0 & 0 \\ 0 & 4.4716 \end{bmatrix} \quad C_{12} = [0.6 \quad 0.2 \quad 0.2 \quad -0.1]$$

$$D_{112} = 0.1 \quad D_{122} = [1 \quad 1]$$

$$C_{22} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad D_{212} = \begin{bmatrix} 0.001 \\ 0.005 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -0.2692 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad B_{13} = \begin{bmatrix} 0.04 \\ 0.03 \\ 0.03 \\ 0.02 \end{bmatrix}$$

$$B_{23} = \begin{bmatrix} 0 & 0 \\ 0.2554 & 0 \\ 0 & 0 \\ 0 & 14.1593 \end{bmatrix} \quad C_{13} = [1 \quad 0.6 \quad 0.3 \quad 0.5]$$

$$D_{113} = 0.8 \quad D_{123} = [0.3 \quad 0.8]$$

$$C_{23} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad D_{213} = [0.004 \quad 0.008]$$

the membership functions for Rules 1 to 3 are shown in Fig. 1.

The DPDC controller design method [26] (Theorem 1 with $R_{ijml} = 0$, $\hat{A}_{Kijl} = \hat{A}_{Fij}$, $\hat{B}_{Kil} = \hat{B}_{Fi}$, $\hat{C}_{Kil} = \hat{C}_{Fi}$, and $\hat{D}_{Kil} = \hat{D}_F$)

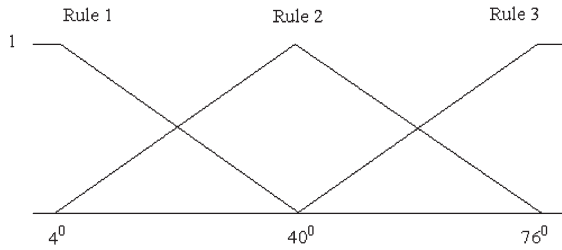


Fig. 1. Membership functions.

TABLE II
 H_∞ PERFORMANCE BOUND

	Theorem 1 (switched DPDC scheme)	The method of [26] (DPDC scheme)
$\sqrt{\gamma}$	0.4932	0.6204
cputime (sec)	3137.1026	278.6250

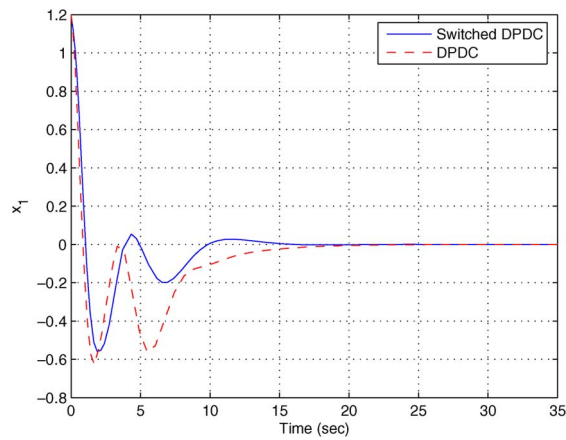


Fig. 2. Response of the state $x_1(t)$.

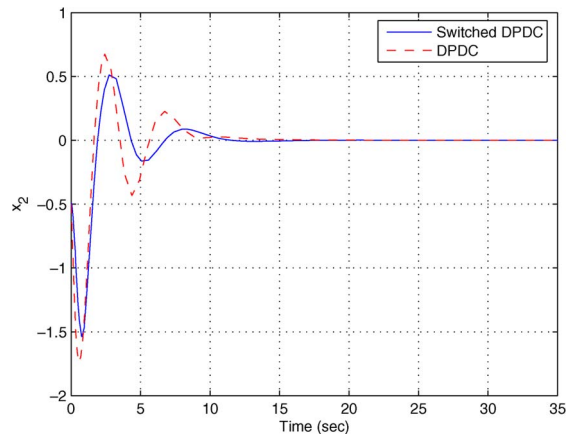


Fig. 3. Response of the state $x_2(t)$.

and the SDPDC controller method (Theorem 1) are applicable. The obtained optimal H_∞ performance indices are shown in Table II, where “cputime” means how long the process uses the CPU.

In Table II, it can be seen that the obtained optimal H_∞ performance bounded by Theorem 1 (the SDPDC control scheme) is smaller than the method in [26] (the DPDC control scheme); because more variables are introduced, Theorem 1 needs more computational time (cputime) than the method in [26]. Now, the obtained controllers by Theorem 1 and the method in [26] will be used to do simulations under

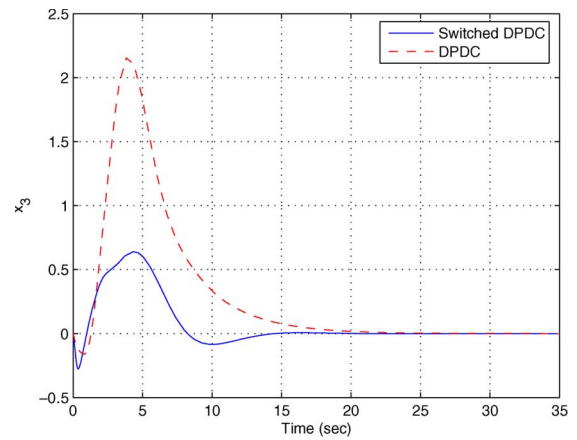


Fig. 4. Response of the state $x_3(t)$.

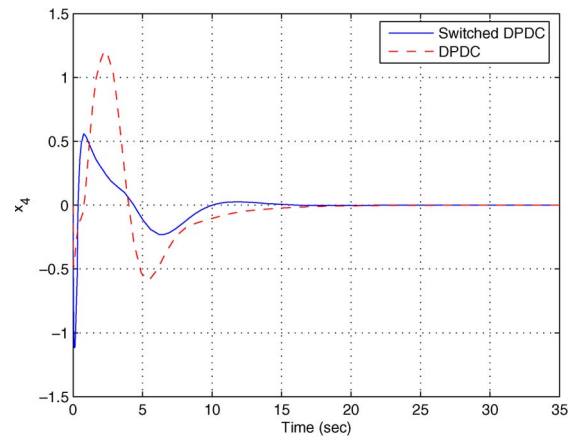


Fig. 5. Response of the state $x_4(t)$.

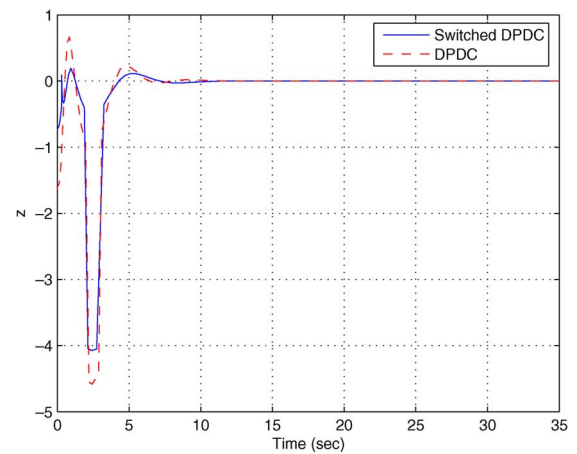


Fig. 6. Controlled output $z(t)$.

the assumption that the initial condition $x(0) = [1.2 \ -0.5 \ 0 \ 0]^T$ and the exogenous disturbance input

$$w(t) = \begin{cases} 6, & 2 \leq t \leq 3 \\ 0, & \text{others.} \end{cases}$$

The responses of $x(t)$, $z(t)$, and $u(t)$ are given in Figs. 2–7. Moreover, the switching of the gains of SDPDC controller is shown in Fig. 8, during the simulation. Figs. 2–7 show that the SDPDC controller can achieve better H_∞ performance than the DPDC controller.

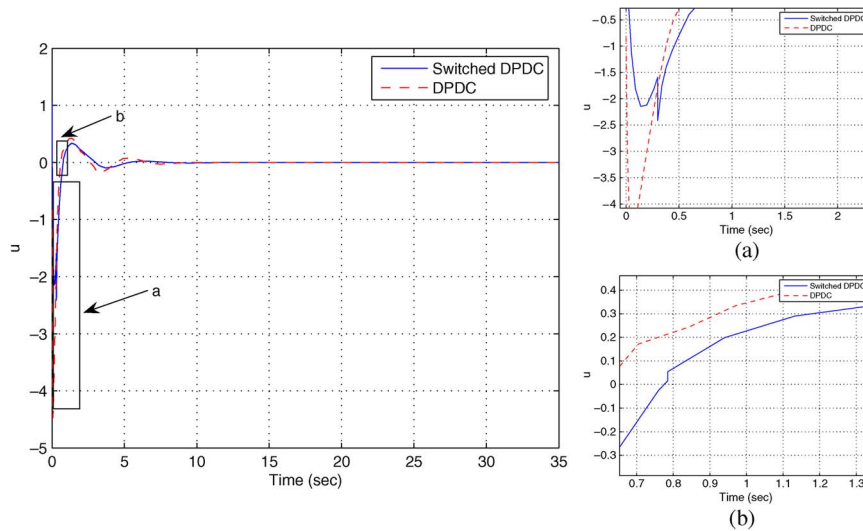


Fig. 7. Control input $u(t)$. (a) $t \in [0, 2.3]$. (b) $t \in [0.65, 1.35]$.

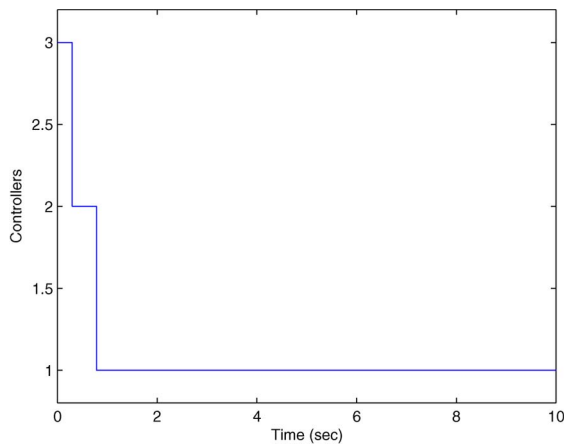


Fig. 8. Switching between the gains of controller during the simulation.

V. CONCLUSION

In this correspondence paper, the problem of designing switched H_∞ controllers with pole placement constraints via the dynamic output feedback for continuous-time T–S fuzzy systems has been studied. A new control scheme, namely, SDPDC control scheme, is proposed, which is an extension of the DPDC scheme. A new method for designing SDPDC controllers for guaranteeing the stability and H_∞ performances of closed-loop nonlinear systems is presented, where the design conditions are given in terms of the solvability of a set of LMIs. It is shown that the new method is less conservative than the corresponding design methods via the pure DPDC scheme. A numerical example has been given to illustrate the effectiveness of the proposed method.

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