TWO-STAGE SPECTRUM SENSING FOR COGNITIVE RADIOS

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ABSTRACT

We consider a two-stage sensing scheme for cognitive radios where coarse sensing based on energy detection is performed in the first stage and, if required, fine sensing based on cyclostationary detection in the second stage. We design the detection threshold parameters in the two sensing stages so as to maximize the probability of detection, given constraints on the probability of false alarm. We compare this scheme with ones where only energy detection or cyclostationary detection is performed. The performance comparison is made based on the probability of detection, probability of false alarm and mean detection time.

***Index Terms*—** Two-stage spectrum sensing, cognitive radios, energy detection, cyclostationary detection

1. INTRODUCTION

Cognitive radios have emerged as a promising solution to improving spectrum utilization. It has been widely recognized that while there is a perceived scarcity of radio spectrum, large portions of licensed spectrum remain under-utilized [4]. Cognitive radios determine empty portions of licensed spectrum, and utilize such portions for secondary use in order to meet regulatory constraints of limiting harmful interference to licensed wireless systems. The determination of empty spectrum is typically done by spectrum sensing and is a critical challenge in cognitive radios. In particular, (i) spectrum sensing has to reliably determine the presence or absence of ongoing licensed transmissions, and (ii) sensing of multiple radio channels (possibly spanning several hundreds of MHz) has to be done as fast as possible.

Two sensing techniques that have been commonly considered in cognitive radios are energy detection [1], [2], [9] and cyclostationary detection [6], [8]. While energy detection is a simple detection technique, its performance is not robust to noise and is known to be poor at low SNRs. Cyclostationary detection on the other hand provides better detection but is computationally more complex and needs a much higher sensing time.

In this paper, we consider a two-stage sensing approach based on energy detection and cyclostationary detection. For a given channel, in the first stage, energy detection is performed. If the energy is above a certain threshold λ, the channel is declared to be occupied. Else, cyclostationary detection is performed in the second stage. If the decision metric in this stage exceeds a certain threshold γ, the channel is declared to be occupied. Else, it is declared to be empty and available for secondary use. We analyze the performance of such a two-stage sensing approach in terms of the probabilities of detection and false-alarm and the mean detection time to determine occupancy of a channel. Thresholds λ and γ that maximize the probability of detection under this approach, with the probability of false alarm being constrained, are also determined.

A two-stage sensing based on energy detection with different sensing bandwidths in the two stages was proposed in [7]. Our work differs from [7] in two ways. We consider two-stage sensing over the channel bandwidth, whereas [7] first searches over a larger bandwidth in the coarse sensing stage and then over the channel bandwidth in the fine sensing stage. Moreover, in [7], both stages are based on energy detection. Further, our approach in this paper is to design the detection thresholds so as to optimize detection performance and study the performance trade-offs of the two-stage sensing scheme with the resulting thresholds.

The remainder of the paper is organized as follows. In Section 2, we first present the two-stage sensing scheme. Section 3 analyzes the scheme from a detection performance and mean detection time viewpoint. In Section 4, we present simulation results of two-stage sensing for an OFDM signal and finally in Section 5, we draw conclusions.

2. TWO-STAGE SPECTRUM SENSING

The two-stage spectrum sensing that we propose is shown in figure 1. We assume that there are L channels to be sensed and that channels are sensed serially. In the coarse sensing stage, the channel is sensed using energy detection. If the decision metric is greater than a threshold λ, the channel is declared to be occupied. Else, the received signal is analyzed by fine sensing consisting of cyclostationary detection. If the constituent detection metric is greater than a threshold γ, the channel is declared occupied, else it is declared to be empty. In the following we shall discuss the two stages of energy detection [2], [9] and cyclostationary detection [3], [6] in the context of two-stage sensing.

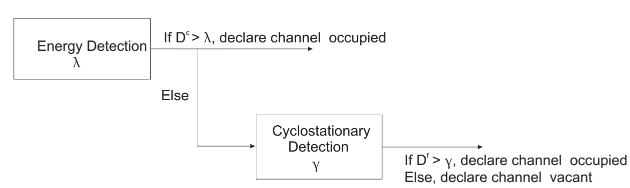

Fig. 1. Two-stage spectrum sensing scheme.

2.1. Coarse Sensing

An energy detector which serially searches every channel within the band is used as coarse sensing stage. The energy detector accumulates the energy of M^c samples and then compares it with a threshold λ to decide whether the primary user is present or not.

Denoting H_1 and H_0 as the respective probabilities of primary user presence and absence, the energy detector makes its decision based on $M^{\rm c}$ observations $x_k,\ k=1,\ldots,M^{\rm c}$, given by

$$x_k = \begin{cases} n_k & \text{under } \ H_0 \\ s_k + n_k & \text{under } \ H_1 \end{cases} \tag{1}$$

with the primary user's signal and receiver noise denoted by s_k and n_k, respectively. The noise is assumed to be an i.i.d. random Gaussian process with zero mean and variance σ_n^2, while the signal is assumed to be an i.i.d. random process of zero mean and variance σ_s^2.

The decision rule used by the energy detector is given by

$$D^{\rm c} = \sum_{k=1}^{M^{\rm c}} x_k^2 \underset{H_0}{\overset{H_1}{\gtrless}} \lambda. \tag{2}$$

The test statistic $D^{\rm c}$ for large $M^{\rm c}$ can be modeled by a Gaussian distribution as follows [2],

$$D^{\rm c} \sim \begin{cases} \mathcal{N}(M^{\rm c}\sigma_n^2, 2M^{\rm c}\sigma_n^4) & \text{under } \ H_0 \\ \mathcal{N}(M^{\rm c}(\sigma_n^2+\sigma_s^2), 2M^{\rm c}(\sigma_n^2+\sigma_s^2)^2) & \text{under } \ H_1. \end{cases} \tag{3}$$

Then, the probability of false alarm, $P_f^{\rm c}$, and probability of detection, $P_d^{\rm c}$, for the energy detector stage are

$$P_f^{\rm c} = Q\left(\frac{\lambda - M^{\rm c}\sigma_n^2}{\sqrt{2M^{\rm c}\sigma_n^4}}\right), \tag{4}$$

$$P_d^{\rm c} = Q\left(\frac{\lambda - M^{\rm c}(\sigma_n^2+\sigma_s^2)}{\sqrt{2M^{\rm c}(\sigma_n^2+\sigma_s{}^2)^2}}\right), \tag{5}$$

where $Q(a)$ is the Q-function.

2.2. Fine Sensing

Upon detection of the possible empty channels in the coarse sensing stage, a final decision about the vacancy of the channel is made after the fine sensing stage. For the fine sensing stage, a cyclostationary detector, which has better performance than the energy detector, particularly for low SNR, is employed.

Cyclostationary processes are random processes for which the statistical properties such as the mean and autocorrelation change periodically as a function of time [5]. Many of the signals used in wireless communications and radar systems possess this property. Cyclostationarity may be caused by modulation and coding [5], or it may be intentionally produced to help channel estimation, equalization or synchronization such as the use of the cyclic prefix (CP) in an OFDM signal [8]. In our work, we use the second-order time domain cyclostationary detector presented in [3].

A random process $x_k,\ k=1,\ldots,M^{\rm f}$ is wide-sense second-order cyclostationary if there exists a $K>0$ such that

$$\mu_x(k) = \mu_x(k+K),\ \forall k$$

and

$$R_x(k,\kappa) = R_x(k+K,\kappa),\ \forall(k,\kappa)$$

where $\mu_x(k) = E[x_k]$ is the mean value of the random process x_k, $R_x(k,\kappa) = E[x_k x_{k+\kappa}^*]$ is the autocorrelation function, and K is called the cyclic period.

Due to the periodicity of the autocorrelation $R_x(k,\kappa)$, it has a Fourier-series representation as follows [3]:

$$R_x(k,\kappa) = \sum_\alpha R_x^\alpha(\kappa)e^{j\alpha k},$$

where the Fourier coefficients are

$$R_x^\alpha(\kappa) = \lim_{M^{\rm f}\to\infty} \frac{1}{M^{\rm f}} \sum_{k=0}^{M^{\rm f}-1} R_x(k,\kappa)e^{-j\alpha k}$$

with α called the cyclic frequency and $R_x^\alpha(\kappa)$ called the cyclic autocorrelation function.

To check if $R_x^\alpha(\kappa)$ is null for a given candidate cycle, consider the following estimator of $R_x^\alpha(\kappa)$

$$\begin{aligned}\hat{R}_x^\alpha(\kappa) &= \frac{1}{M^{\rm f}} \sum_{k=0}^{M^{\rm f}-1} x_k x_{k+\kappa}^* e^{-j\alpha k} \\ &= R_x^\alpha(\kappa) + \epsilon_x^\alpha(\kappa)\end{aligned} \tag{6}$$

where $\epsilon_x^\alpha(\kappa)$ represents the estimation error which vanishes as $M^{\rm f}\to\infty$. Due to the error $\epsilon_x^\alpha(\kappa)$, the estimator $\hat{R}_x^\alpha(\kappa)$ is seldom exactly zero in practice, even when α is not a cyclic frequency. This raises an important issue about deciding whether a given value of $\hat{R}_x^\alpha(\kappa)$ is "zero" or not. To answer this question statistically, we use the decision-making approach of [3].

In general, we consider a vector of $\hat{R}_x^\alpha(\kappa)$ values rather than a single value in order to check simultaneously for the presence of cycles in a set of lags κ.

Let $\kappa_1,\ldots,\kappa_N$ be a fixed set of lags, α be a candidate cyclic frequency, and

$$\begin{aligned}\hat{\mathbf{R}}_x = \big[&\Re\{\hat{R}_x^\alpha(\kappa_1)\},\ldots,\Re\{\hat{R}_x^\alpha(\kappa_N)\},\\ &\Im\{\hat{R}_x^\alpha(\kappa_1)\},\ldots,\Im\{\hat{R}_x^\alpha(\kappa_N)\}\big]\end{aligned}$$

represent a $1\times 2N$ row vector consisting of cyclic correlation estimators from (6) with $\Re$ and $\Im$ representing the real and imaginary parts, respectively. If the asymptotic value of $\hat{\mathbf{R}}_x$ is given as $\mathbf{R}_x$ where

$$\begin{aligned}\mathbf{R}_x = \big[&\Re\{R_x^\alpha(\kappa_1)\},\ldots,\Re\{R_x^\alpha(\kappa_N)\},\\ &\Im\{R_x^\alpha(\kappa_1)\},\ldots,\Im\{R_x^\alpha(\kappa_N)\}\big].\end{aligned}$$

Then, we can write $\hat{\mathbf{R}}_x = \mathbf{R}_x + \boldsymbol{\epsilon}_x$ where

$$\begin{aligned}\boldsymbol{\epsilon}_x = \big[&\Re\{\epsilon_x^\alpha(\kappa_1)\},\ldots,\Re\{\epsilon_x^\alpha(\kappa_N)\},\\ &\Im\{\epsilon_x^\alpha(\kappa_1)\},\ldots,\Im\{\epsilon_x^\alpha(\kappa_N)\}\big]\end{aligned}$$

is the estimation error vector.

In [3], the test statistic related to the cyclostationary detector has been derived as follows

$$D^{\rm f} = M^{\rm f}\hat{\mathbf{R}}_x\hat{\boldsymbol{\Sigma}}^{-1}\hat{\mathbf{R}}_x^H \tag{7}$$

where $\hat{\boldsymbol{\Sigma}}$ is the covariance matrix of $\hat{\mathbf{R}}_x$. In [3], it is shown that the test statistic $D^{\rm f}$ under the hypothesis H_0, asymptotically has a central chi-squared distribution, while under the hypothesis H_1 follows a Gaussian distribution. Hence, for a large $M^{\rm f}$ we can write

$$D^{\rm f} \sim \begin{cases} \chi^2_{2N} & \text{under } \ H_0 \\ \mathcal{N}(M^{\rm f}\hat{\mathbf{R}}_x\hat{\boldsymbol{\Sigma}}^{-1}\hat{\mathbf{R}}_x^H, 4M^{\rm f}\hat{\mathbf{R}}_x\hat{\boldsymbol{\Sigma}}^{-1}\hat{\mathbf{R}}_x^H) & \text{under } \ H_1 \end{cases} \tag{8}$$

Having the asymptotic distribution of the test statistic $D^{\rm f}$, we say that if $D^{\rm f}\geq\gamma$ we can declare α is a cyclic frequency for some κ_n and therefore the primary user is present. Else, we declare α is not a cyclic frequency and thus the primary user is absent, which means that this band is empty and can be used by the cognitive radio.

The probability of detection, $P_d^{\rm f}$, and the probability of false alarm, $P_f^{\rm f}$, can be obtained as

$$P_f^{\rm f} = P(D_f \geq \gamma | H_0) = \frac{\Gamma(\gamma/2, N)}{\Gamma(N)}, \tag{9}$$

$$P_d^{\rm f} = P(D_f \geq \gamma | H_1) = Q\left(\frac{\gamma - M^{\rm f}\hat{\mathbf{R}}_x\hat{\boldsymbol{\Sigma}}^{-1}\hat{\mathbf{R}}_x^H}{\sqrt{(4M^{\rm f}\hat{\mathbf{R}}_x\hat{\boldsymbol{\Sigma}}^{-1}\hat{\mathbf{R}}_x^H)}}\right), \tag{10}$$

where $\Gamma(a)$ is the gamma function and $\Gamma(a,x)$ is the incomplete gamma function ($\Gamma(a,x) = \int_x^\infty t^{a-1}e^{-t}dt$).

3. PROBLEM FORMULATION AND ANALYSIS

In the two-stage sensing scheme, a false alarm occurs if, under H_0, (i) $D^{\rm c} > \lambda$, or (ii) $D^{\rm f} > \gamma$ given that $D^{\rm c} \leq \lambda$. Similarly, a correct detection occurs if, under H_1, (i) $D^{\rm c} > \lambda$, or (ii) $D^{\rm f} > \gamma$ given that $D^{\rm c} \leq \lambda$. Hence, the overall probabilities of false alarm and detection for a single channel are given by

$$P_f = P_f^{\rm c} + (1 - P_f^{\rm c})P_f^{\rm f}, \quad (11)$$

$$P_d = P_d^{\rm c} + (1 - P_d^{\rm c})P_d^{\rm f}. \quad (12)$$

The goal is to design a decision strategy (determination of λ and γ) in order to maximize the probability of detection of each channel subject to a false alarm rate constraint (equivalent to minimizing the interference to the primary user subject to an information rate constraint). Therefore the corresponding problem is given by

$$\max_{(\lambda,\gamma)} P_d(\lambda, \gamma) \quad \text{s.t. } P_f \leq \beta. \quad (13)$$

The inequality constraint in the problem (13) can be reduced to an equality constraint by the following theorem.

Theorem 4.1: The optimal value of the probability of detection in (13) is attained by $P_f = \beta$.

Proof: P_d is a differentiable and decreasing function of the thresholds λ and γ. Thus, the first derivative of P_d with respect to λ and γ is negative. Hence, the maximum P_d is attained for the lowest possible λ and γ. Furthermore, the first derivative of P_f with respect to λ and γ is also negative, using a similar explanation as before. Assume (λ^*, γ^*) to be the optimal solution of (13) corresponding to $P_f < \beta$. Suppose we keep λ^* to be constant but decrease γ until we get to $P_f = \beta$. In this case, a higher probability of detection is attained for $\gamma < \gamma^*$. Therefore (λ^*, γ^*) can not be the optimal solution of the problem. The same explanation holds if we keep γ^* constant and decrease λ until we get to $P_f = \beta$. Thus, the optimal P_d is attained by $P_f = \beta$. □

Hence, we can rewrite the problem (13) as

$$\max_{(\lambda,\gamma)} P_d(\lambda, \gamma) \quad \text{s.t. } P_f = \beta. \quad (14)$$

Furthermore, for a given false alarm rate constraint β, we have the following relation between λ and γ

$$\lambda = f(\gamma) = Q^{-1}\left(\frac{\beta - \frac{\Gamma(\gamma/2,N)}{\Gamma(N)}}{1 - \frac{\Gamma(\gamma/2,N)}{\Gamma(N)}}\right)\sqrt{2M^{\rm c}\sigma_n^4} + M^{\rm c}\sigma_n^2. \quad (15)$$

Therefore, the problem (14) can be simplified to an unconstrained problem as follows

$$\max_{\gamma} P_d(f(\gamma), \gamma). \quad (16)$$

The optimal γ and $\lambda = f(\gamma)$ can then be obtained from (16) and (15). We can show that the problem is unimodal in γ and therefore can be solved by an unconstrained optimization algorithm such as the gradient descent algorithm.

3.1. Mean Detection Time Analysis

In order to compare the agility of the two-stage sensing scheme with energy detection and cyclostationary detection, we need to compare their mean detection time. The mean detection time of the two-stage sensing has two terms as follows

$$\bar{T} = \bar{T}_{\rm c} + \bar{T}_{\rm f}$$

where $\bar{T}_{\rm c}$ is the coarse sensing time which is equal to LT_1, with $T_1 = \frac{M^{\rm c}}{2W}$ (W is the channel bandwidth) the sensing time in each channel for the coarse sensing stage and $\bar{T}_{\rm f}$ is the fine sensing stage mean detection time. $\bar{T}_{\rm f}$ can be derived as follows

$$\bar{T}_{\rm f} = E[K]T_2 \quad (17)$$

where $E[K]$ is the mean number of reported channels for the fine sensing stage and $T_2 = \frac{M^{\rm f}}{2W}$ is the sensing time of each channel. K is a random variable which follows a binomial distribution, with parameters L and P_{rep}, where P_{rep} is the probability that a channel would be reported to the fine sensing stage and is given by

$$P_{rep} = Pr(H_0)(1 - P_f^{\rm c}) + Pr(H_1)(1 - P_d^{\rm c}). \quad (18)$$

Hence, the mean detection time of the fine sensing stage is

$$\bar{T}_{\rm f} = LP_{rep}T_2, \quad (19)$$

and the total mean detection time is

$$\bar{T} = L(T_1 + P_{rep}T_2). \quad (20)$$

4. SIMULATION RESULTS

In this section, we compare the detection performance of the proposed two-stage sensing scheme with energy detection and cyclostationary detection. A DVB OFDM signal with 10 channels and a channel bandwidth of 8 MHz is employed. Each OFDM signal has 8192 carriers with a CP of length 1024. In these simulations, we have used an OFDM signal consisting of 18 OFDM symbols. Furthermore, a Kaiser window of length 61 is applied in the simulations. Denoting the OFDM symbol length by T_s, the described OFDM signal exhibits cyclostationarity with cyclic frequencies of $\alpha = \frac{2\pi m}{T_s}$, $m = \pm1, \pm2, ...$ [6]. Here, we only use $\alpha = \frac{2\pi}{T_s}$ as the cyclic frequency which has to be detected by the cyclostationary detector.

Fig. 2 shows the probability of detection variation with γ for different values of β for two-stage sensing with sensing times $T_1 = 2\ ms$ and $T_2 = 18\ ms$ at SNR$= -17$ dB. From the figure, it becomes clear that the maximum probability of detection is attained when the probability of false alarm satisfies constraint (13) with equality as shown in Theorem 4.1.

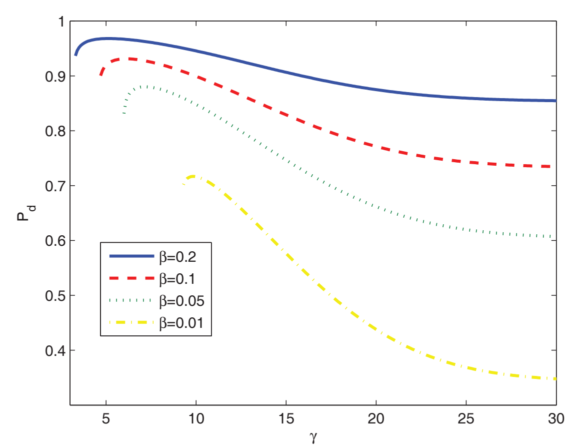

Fig. 2. Probability of Detection versus γ

In the following scenarios, we assume that all the channels experience the same SNR and have the same probability of false alarm constraint, $\beta = 0.1$. Therefore, they will all have the same probability of detection. The same probability of false alarm constraint is imposed on all three sensing schemes. Fig. 3 shows the detection performance versus SNR for the three sensing schemes for sensing times $T_1 = 2\,ms$ and $T_2 = 18\,ms$. As we can see, for an SNR that is less than $-12\,dB$, the two-stage sensing scheme performs better than either energy detection or cyclostationary detection.

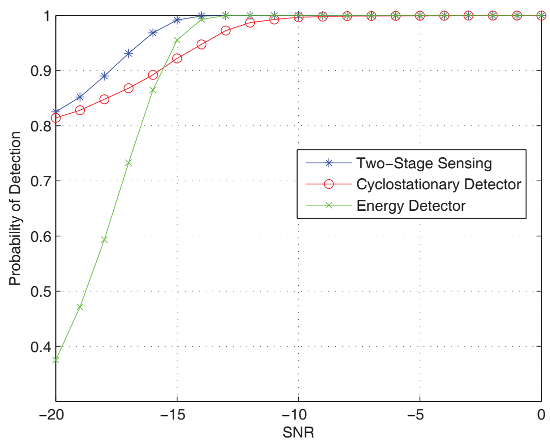

Fig. 3. Detection Performance Comparison

In order to see how the detection time looks like for the two-stage sensing compared to the energy and cyclostationary detectors, we present the mean detection time of the two-stage sensing for different SNRs. In Fig. 4 we consider $Pr(H_0) = 0.2$. As we can see, in the range where the two-stage sensing performs better than energy detection, (SNR less than $-12\,dB$), two-stage sensing outperforms the cyclostationary detector in terms of mean detection time as well as detection performance. In Fig. 5, where $Pr(H_0) = 0.8$, and thus P_{rep} is higher, two-stage sensing does not always have a smaller mean detection time than the cyclostationary detector.

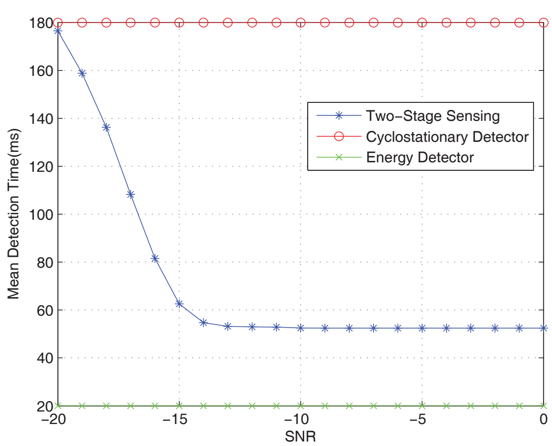

Fig. 4. Mean Detection Time Comparison for $Pr(H_0) = 0.2$

5. CONCLUSIONS

We analyzed a two-stage sensing scheme in terms of its detection performance and mean detection time. In particular, we designed

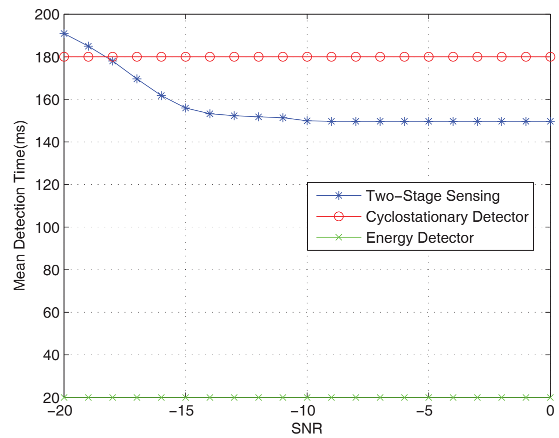

Fig. 5. Mean Detection Time Comparison for $Pr(H_0) = 0.8$

optimum thresholds for the energy detection and cyclostationary detection stages so as to maximize the probability of detection given constraints on the probability of false alarm. Using a DVB OFDM signal, we showed the performance trade-offs for the proposed sensing scheme. We observed that at low SNR, where the energy detector is not reliable, the two-stage sensing scheme provides improved detection. Furthermore, we show that the mean detection time of the two-stage sensing scheme is much lower than the cyclostaionarity detection scheme for most of the SNR range.

6. REFERENCES

[1] D. Cabric, S. M. Mishra and R. W. Brodersen, "Implementation issues in spectrum sensing for cognitive radios", *Asilomar Conference on Signals, Systems and Computers*, pp. 772-776, Nov. 2004.

[2] D. Cabric, A. Tkachenko and R. W. Brodersen, "Spectrum sensing measurements of pilot, energy, and collaborative detection", *IEEE MILCOM*, Oct. 2007.

[3] A. Dandawate and G. B. Giannakis, "Statistical tests for presence of cyclostationarity", *IEEE Transactions on Signal Processing*, pp. 2355-2369, Sept. 1994.

[4] Federal Communications Commission - Second Report and Order and Memorandum Opinion and Order, "Unlicensed operation in the TV broadcast bands," *FCC 08-260*, Nov 2008.

[5] W. Gardner, "Signal interception: a unifying theoretical framework for feature detection", *IEEE Transactions on Communications*, pp. 897-906 , Aug. 1988.

[6] J. Lunden, V. Koivunen, A. Huttunen and H. V. Poor, "Spectrum sensing in cognitive radios based on multiple cyclic frequencies", *IEEE Proc. CrownCom*, Aug. 2007.

[7] L. Luo, N. M. Neihart, S. Roy and D. J. Allstot, "A two-stage sensing technique for dynamic spectrum access", *IEEE Transactions on Wireless Communications*, pp. 3028-3037, June 2009.

[8] P. D. Sutton, K. E. Nolan and L. E. Doyle, "Cyclostationary signatures in practical cognitive radio applications", *IEEE Journal on Selected Areas in Communications*, pp. 13-24, Jan. 2008.

[9] R. Tandra and A. Sahai, "SNR walls for signal detection", *IEEE Journal of Selected Topics in Signal Processing*, pp. 4-17, Feb. 2008.