Optimization Criteria for Optimal Placement of Piezoelectric Sensors and Actuators on a Smart Structure: A Technical Review

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ABSTRACT: This article presents in a unified way, the various optimization criteria used by researchers for optimal placement of piezoelectric sensors and actuators on a smart structure. The article discusses optimal placement of piezoelectric sensors and actuators based upon six criteria: (i) maximizing modal forces/moments applied by piezoelectric actuators, (ii) maximizing deflection of the host structure, (iii) minimizing control effort/maximizing energy dissipated, (iv) maximizing degree of controllability, (v) maximizing degree of observability, and (vi) minimizing spill-over effects. Optimal piezoelectric sensor and actuator locations on beam and plate structures for each criterion and modes of interest are presented in a tabular form. This technical review has two objectives: (i) practicing engineers can pick the most suitable philosophy for their end application and (ii) researchers can come to know about potential gaps in this area.

Key Words: smart structure, active vibration control, optimal placement, sensors, actuators, piezoelectric materials, optimization criteria.

Nomenclature

Y Young's modulus of elasticity
σ stress
μ mass per unit length
I moment of inertia
ψ normalized modal function
Q_j modal control force
ω_j modal frequency
λ_j j-th modal eigenvalue of controllability Grammian
ζ_j modal damping ratio
m_i modal mass of i-th eigenmode
T time
w transverse deflection of the host structure
d_{31} piezoelectric strain constant
e_{31} piezoelectric stress constant
M moments applied by actuator
M_0 maximum value of moments applied by actuator
x_p position of sensor/actuator
a, b, h length, breadth, thickness
α_1, α_2 normalized length coordinates of sensor/actuator
β_1, β_2 normalized breadth coordinates of sensor/actuator

Ω driving frequency (dimensionless)
E_z applied electric field in z-direction
d distance from piezoelectric sensor/actuator centerline to host structure centerline
$\eta_i(t)$ time response of the i-th eigenmode
X state vector
Y system output
A system influence matrix
B position matrix of the actuator
C position matrix of the sensor
B_a control influence vector
B_v measurement influence vector
K feedback control gain
V_i voltage applied on i-th piezoelectric actuator
V_{act} external control voltage applied on the actuator.
G transfer function between actuator voltage and plate deflection
G_c controllability Gramian matrix
G_o observability Gramian matrix
a_0 amplitude of external excitation
x x-coordinate
D modal damping matrix
Λ diagonal eigenvalue matrix

Subscripts

s host structure
p piezoelectric
j j-th mode
x, y in the x- or y-direction

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*Superscripts*

T transpose

INTRODUCTION

SINCE TIME IMMEMORIAL, man has been controlling structural vibrations by modifying mass, stiffness, and damping of the structure. This may increase overall mass of the structure and is found to be unsuitable for controlling low frequency vibrations (Fahy and Walker, 1998; Brennan and Ferguson, 2004). This method does not suit applications where weight restrictions are present and low frequency vibrations are encountered. For such applications, smart structures are being developed, which are lightweight and attenuate low frequency vibrations (Fuller et al., 1997). A structure in which external source of energy is used to control structural vibrations is called a 'smart structure' and the technique is called 'active vibration control (AVC)'. A smart structure essentially consists of sensors to capture the dynamics of the structure, a processor to manipulate the sensor signal, actuators to obey the order of processor, and a source of energy to actuate the actuators (Figure 1).

This field has recently gained lot of interest due to three main reasons: (i) increased interest of man in space exploration, nano-positioning, micro-sensing, etc. (Loewy, 1997; Moheimani and Fleming, 2006), (ii) advent of fast processors, real-time operating systems, etc. (Piersol and Paez, 2010), and (iii) development of stable and high performance sensors and actuators (Barlas and van Kuik, 2010; Rao and Sunar, 1994). Piezoelectric sensors/actuators are being used extensively for AVC because piezoelectric materials have excellent electromechanical properties: fast response, easy fabrication, design flexibility, low weight, low cost, large operating bandwidth, low power consumption, generation of no magnetic field while converting electrical energy into mechanical energy, etc. Piezoelectric materials generate strains when an electric signal is applied on them and vice versa. So, they can be used as sensors and actuators for structural vibrations (Fabunni, 1980). This effect occurs naturally in quartz but can be induced in other materials such as specially formulated ceramics consisting mainly of lead, zirconium, and titanium (PZT). Piezoelectric materials can be used as sensors and actuators in the form of distributed layers (Bailey and Hubbard, 1985; Hanagud et al., 1985; Tzou and Hollkamp, 1994), surface bonded patches (Crawley and de Luis, 1987; Sharma et al., 2007), embedded patches (Crawley and de Luis, 1987; Raja et al., 2002; Elsoufi et al., 2007), cylindrical stacks (Li et al., 2008), screen printed piezoelectric layer (Glynne-Jones et al., 2001), active fiber composite patches (Raja et al., 2004), functionally graded piezoelectric material patches (Yang and Xiang, 2007), etc. Surface mounted or embedded piezoelectric patches can control a structure better than a distributed one because the influence of each patch on the structural response can be individually controlled (Tzou and Fu, 1994).

Performance of AVC not only depends upon the control law but also on the placement of piezoelectric sensors and actuators (Crawley and de Luis, 1987). When a designer of a smart structure has to place a limited number of sensor/actuator patches over the structure, a lot of options are available. Unwise placement of even collocated sensor–actuator pairs over a smart
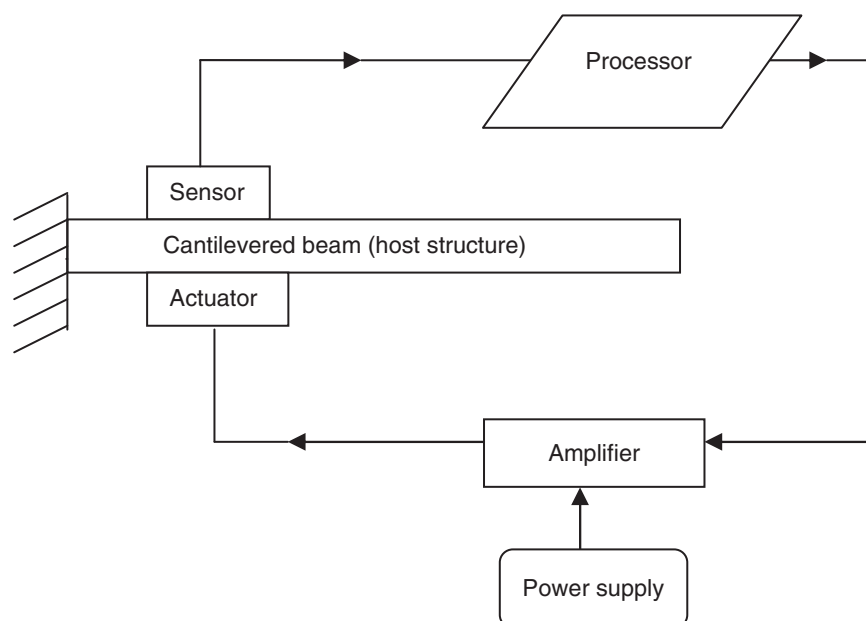
Figure 1. *Schematic of a smart structure.*

structure controlled by negative velocity feedback can make it unstable (Liu and Yang, 1993), and a wise placement of even non-collocated sensor–actuator pairs over a structure can make it stable (Yang and Lee, 1993a). It is therefore important that the option for placement so selected, should not make the structure unstable. Also, it would be better if sensors and actuators are so placed that it best suits the end application of the smart structure. Keeping in mind the end application of the smart structure, a criterion can be fixed for sensor/actuator locations to maximize the performance of AVC. Such a criterion is referred to as 'optimization criterion'. Once 'optimization criterion' is fixed, desired sensor/actuator locations can be found using a suitable search algorithm called 'optimization technique'. Such a placement of sensors and actuators which is done by optimizing some fixed criterion is called 'optimal placement'. Optimization techniques like univariate search method (Reklaitis et al., 1983), modified method of feasible direction (Vanderplaasts, 1984), simulated annealing (Kirkpatrick et al., 1983; Cerny, 1985), tabu search (Glover and Laguna, 1997), genetic algorithms (Goldberg, 1989; Mitchell, 1996), sensitivity analysis (Conte et al., 2003; Saltelli et al., 2004), gradient algorithm (Haftka and Gurdal, 1993), invasive weed optimization (Mehrabian and Lucas, 2006), etc. are used in AVC to find the optimal sensor/actuator locations. Optimal placement of sensors and actuators over a structure can be different for different criteria. An optimization criterion can be based upon: maximization of modal forces/moments applied by the actuator, maximization of deflection of the host structure, minimal change in host structural dynamics, desired host structural dynamics, minimization of control effort, minimization of host vibrations, maximization of degree of controllability/observability of modes of interest, etc.

Much of the work in AVC is concentrated on the modeling and control of a smart structure (Crawley and de Luis, 1987; Baz and Poh, 1988; Anderson et al.,1992; Dosch et al., 1992; Tzou and Hollkamp, 1994; Varadan et al., 1996; Smittakorn and Heyliger, 2000; Sunar et al., 2001; Raja et al., 2002; Sharma et al., 2007). These developments have been well documented in various topical reviews (Rao and Sunar, 1994; Benjeddou, 2000; Tauchert et al., 2000; Fripp and Atalla, 2001; Alkhatib and Golnaraghi, 2003; Garg and Anderson, 2003; Bars et al., 2006; Kandagal and Venkatraman, 2006; Song et al., 2006). Relatively, only few works are focused on finding optimal locations of sensors/actuators on a smart structure for AVC. In these works, researchers have used many 'optimization criteria' and 'optimization techniques' to find optimal locations. Optimization techniques used to find optimal piezoelectric sensor/actuator locations on a smart structure are well documented (Padula and Kincaid, 1999).

In a similar review, 'optimization criteria' used by researchers for placement of piezoelectric, shape memory alloys, and magnetostrictive actuators on a smart structure have been mentioned (Frecker, 2003). However to the best knowledge of authors, a review article which presents: (i) 'state-of-the-art' of each 'optimization criterion' used for placement of piezoelectric sensors and actuators, and (ii) the optimal placements so obtained, is absent in the literature. Presentation of such 'most used' optimization criteria to find optimal sensor/actuator locations in a unified way would be of immense use to the practicing engineers and scientists working in this field. So, the authors are motivated to write a 'technical review' in this relatively new area of research.

In this 'technical review', a well-knitted knowledge bank of various criteria used by researchers for optimal placement of piezoelectric sensors/actuators on a smart structure is presented. 'State-of-the-art' of optimal placement of sensors/actuators over a smart structure based upon six optimizing criteria: (i) maximizing modal forces/moments applied by piezoelectric actuators, (ii) maximizing deflection of the host structure, (iii) minimizing control effort/maximizing energy dissipated, (iv) maximizing degree of controllability, (v) maximizing degree of observability, and (vi) minimizing spill-over effects, are discussed one by one. Optimal placements so obtained are presented in a tabular form for beam and plate smart structures.

OPTIMAL PLACEMENT OF PIEZOELECTRIC SENSORS AND ACTUATORS

In following sections, 'state-of-the-art' of optimal placement of piezoelectric sensors and actuators based upon the above-mentioned six criteria are presented one by one.

Maximizing Modal Forces/Moments Applied by Piezoelectric Actuators

Piezoelectric actuators are desired to strain the host structure in a direction opposite to the strains developing in the host structure. So, it can be reasoned that piezoelectric actuators should be placed in the regions of high average strains and away from the areas of zero strain (strain nodes). If an electric field is applied across piezoelectric actuators in the same direction, as shown in Figure 2(a), the host structure will be deformed in extension mode. If the field is applied across piezoelectric actuators in the opposite directions as shown in Figure 2(b), the host structure will be deformed in bending mode (Crawley and de Luis, 1987). For control of first mode of a cantilevered beam, collocated actuator pair should be placed near the root. For second mode control, actuators should not be placed at a distance 0.216 of beam
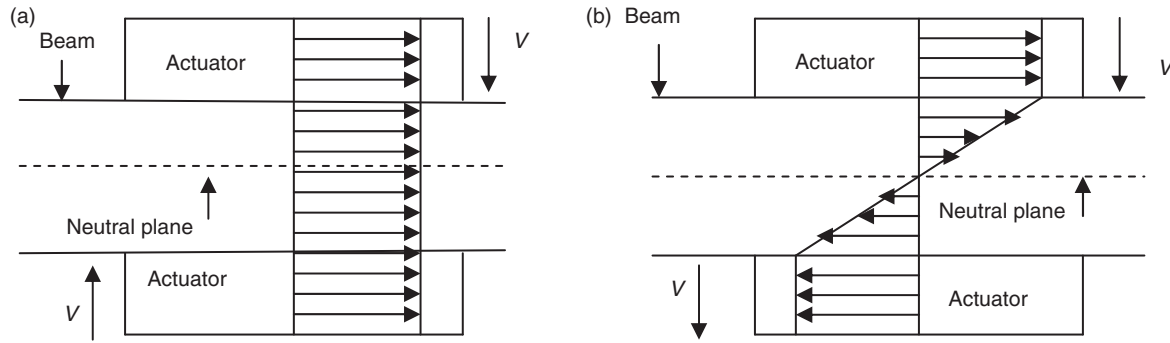
Figure 2. *A sketch of the surface bonded piezoelectric-substrate strain distribution during: (a) extension and (b) bending mode of the collo-cated piezoelectric actuator pair.*

length from the root as this is a location of zero strain node. Segmented actuators at the opposite sides of zero strain node and driven 180° out of phase would suit to control such modes (Crawley and de Luis, 1987).

Actuators can be so placed such that modal force available to modes of interest is maximized. To achieve this goal, second-order differential equations of motion are written in modal domain. When a plate is controlled using independent modal space control, modal force applied by an actuator to excite j-th mode depends upon the location of the actuator and is given by (Bin et al., 2000):

$$Q_j(t) = -a_p b_p \left(\frac{h_s + h_p}{2} \right) L[\psi_j] V_j(t), \quad (2.1)$$

where operator $L = e_{31}\{(\partial^2/\partial x^2) + (\partial^2/\partial y^2)\}$ for isotropic piezoelectric material. The vector, which gets multiplied by actuator control voltages, that is $L[\psi_j]$, is maximized to achieve maximum modal force objective. Optimal location thus obtained is where the sum of modal strains in x- and y-direction is maximum. Optimal placements of two actuators on a cantilevered plate are adjacent to each other at mid-point of the cantilevered edge for first mode. For control of second mode, both actuators are placed at extreme corners of the cantilevered edge. For control of third mode, one actuator is placed at a distance of 0.25 times length of cantilevered edge on the cantilevered edge and second actuator is placed at a distance of 0.75 times length of cantilevered edge on the cantilevered edge (Bin et al., 2000). Piezoelectric actuators apply moments and thus strains on the structure (as shown in Figure 2). Moments applied by piezoelectric actuator on the structure are function of actuator placement and thickness and are calculated as (Main et al., 1994):

$$M = -2b_s Y_p d_{31} \int_{(h_s-h_p)/2}^{(h_s+h_p)/2} E_z y \, dy = -2b_s \int_0^d \sigma_x y \, dy, \quad (2.2)$$

where 'y' is the distance from beam centerline in the transverse direction. Thus, moments applied can be optimized so as to get optimal thickness and placement of actuator. Such optimal thickness and placement of actuator would apply maximum moments on the structure and thus result in maximum curvature of the host structure. Therefore, curvature of the host structure (2.3) which is a function of modulus ratio of piezoelectric and host structure can be optimized (Main et al., 1994):

$$\frac{d^2 y}{dx^2} = \left(\frac{1}{d} \right) \frac{d_{31} E_z (6\rho_a \rho_b)}{\rho_Y (1 - \rho_a \rho_c^2 - 2\rho_a^3) + (6\rho_a \rho_c^2 + 2\rho_a^3)}, \quad (2.3)$$

where $\rho_Y = \frac{Y_s}{Y_p}$, $\rho_a = \frac{h_p}{2d}$, and $\rho_c = \frac{h_s}{2d}$. Plots showing optimal thickness versus modulus ratio for a given actuator location and optimal actuator location versus modulus ratio for given actuator thickness can be obtained for an embedded piezoelectric actuator. Also, a plot showing optimal thickness versus modulus ratio of a surface bounded piezoelectric actuator can be obtained. So for a given modulus ratio, optimal thickness as well as location of actuator with respect to neutral plane of the host structure can be obtained (Main et al., 1994).

Maximizing Deflection of the Host Structure

When an external voltage is applied on the surface bonded piezoelectric actuator, it produces transverse deflections in the host structure. Transverse deflection of the host structure is a function of actuator placement. So, transverse deflection of the host structure can be used as criterion for optimal placement of actuators (Bruch et al., 2000; Correia et al., 2000; Correia et al., 2001; Sunar et al., 2001; Moita et al., 2006). Using assumed mode shapes method, the dynamic transverse deflection of the beam with surface bonded piezoelectric patches is given by (Zhang et al., 2008):

$$w(x, t) = \sum_{i=1}^{\infty} \psi_i(x)\eta_i(t). \quad (2.4)$$

Also, output sensor voltage is given by (Zhang et al., 2008):

$$V_{\text{sensor}} = e_{31} b_p d \int_{x_1}^{x_2} \frac{\partial^2 w(x,t)}{\partial x^2} \, dx, \qquad (2.5)$$

where x_1 and x_2 are the sensor co-ordinates along x-axis. Optimal position of actuator is where the system's strain value is highest. Highest strain value corresponds to the position where beam's curvature is highest. Optimal position of sensor is where V_{sensor} maximizes. Then, the optimal position of collocated sensor–actuator pair is ascertained by the following equation (Zhang et al., 2008):

$$\frac{\partial^3 w(x,t)}{\partial x^3} = 0. \qquad (2.6)$$

Maximum deflection of host structure can be written as a sum of average angular displacements of adjacent edges of the actuator relative to opposite edges (Ip and Tse, 2001):

$$w_{\text{max},j} = \frac{1}{\omega_j^2} \frac{a_0}{2\zeta_j m_j} \left| \left(\theta_1 a_p + \theta_2 b_p \right)_j \right|, \qquad (2.7)$$

where θ_1 and θ_2 are average angular displacements of length and breadth vector about x- and y-axis, respectively. This summation can be optimized to find optimal position as well as orientation of the actuator. For simply supported plates, optimal actuator position is exactly at the anti-node of mode of interest. For cantilevered plate optimal actuator positions are close to the mid-span of cantilevered edge for the fundamental mode and near the corner of the cantilevered edge for second and third modes. For fourth and fifth modes, it is recommended to place actuators at anti-nodes (Ip and Tse, 2001). Similarly, in case of cantilevered beam, dynamic deflection can be taken as criterion to decide optimal position and length of the piezoelectric actuator pair. Beam dynamic deflection (2.8) is a function of partial derivative of beam mode shapes with respect to the length coordinate of the beam (Barboni et al., 2000):

$$w = \sum_{j=1}^{\infty} \eta_j \frac{\partial \eta_j}{\partial \tilde{x}} \bigg|_{\alpha_1}^{\alpha_2} M_0 \frac{\left(\Omega \sin Z_j t - Z_j \sin \Omega t \right)}{Z_j \left(\Omega^2 - Z_j^2 \right)}, \qquad (2.8)$$

where $\tilde{x} = \frac{x}{a_s}$ and $Z_j = \frac{\omega_j^2 \mu}{Y_s I_s} a_s^4$. Effects of position and size of the actuator takes place in the term $\frac{\partial \eta_j}{\partial X}$. Therefore, this derivative can be partially differentiated and optimized to find optimal position of the actuator when its length is fixed. Optimal position thus obtained is located on the beam where opposite edges of actuators correspond to points of equal curvature of beam mode. Similarly,

optimal length of the actuator (when position is fixed) corresponds to beam positions where opposite edges of the actuator have opposite curvatures. Optimal position as well as length of the actuator is achieved when it is placed between two consecutive points where the curvatures become zero. This location is anti-node of the mode and is strained the highest (Barboni et al., 2000). Maximum plate deflection at a particular mode (j,k) can be expressed as a function of product of two sine functions of actuator position coordinates (Yang and Zhang, 2006):

$$|w_{\text{max}}| = \sum_{j=1}^{\infty} \sum_{k=1}^{\infty} |A_{jk}| \sin^2(j\pi\alpha_0) \sin^2(k\pi\beta_0), \qquad (2.9)$$

where function A_{jk} depends upon plate and piezoelectric actuator material properties, $\alpha_0 = \frac{\alpha_1 + \alpha_2}{2}$ and $\beta_0 = \frac{\beta_1 + \beta_2}{2}$. The product function (2.10) is named as 'position mode function' (PMF) and can be partially differentiated to find optimal actuator location (Yang and Zhang, 2006):

$$\chi_{jk} = \sin^2(j\pi\alpha_0) \sin^2(k\pi\beta_0). \qquad (2.10)$$

Optimal locations thus obtained are at the anti-nodes of respective vibration modes (Yang and Zhang, 2006). Optimal actuator locations to excite several modes simultaneously are obtained by taking combined position mode function (CPMF), which is summation of position mode functions of participating modes (Yang and Zhang, 2006). Modal displacement of a structure is a function of actuator placement and therefore, an influence matrix of actuators can be constructed. To excite only some modes, rows of influence matrix corresponding to modes of interest can be maximized and remaining rows minimized. The sensor–actuator pair must be located near the centerline along the fixed edge of cantilevered plate to damp the first mode. To damp the second mode, optimal location is when one edge of the sensor–actuator pair is on the free edge of the plate and the other edge is on the cantilevered edge of the plate. To control the first two modes simultaneously, one sensor–actuator pair is placed at the centerline along the cantilevered edge and other is placed adjacent to it along the cantilevered edge (Quek et al., 2003).

Minimizing Control Effort/Maximizing Energy Dissipated

In active vibration control, external source of energy is utilized to cause deflection of the structure. The dynamic equations of motion of smart structure in modal domain are written as (Zhang et al., 2008):

$$\ddot{\eta}(t) + D\dot{\eta}(t) + \Lambda \eta(t) = B_a V_{\text{act}}. \qquad (2.11)$$

Modal amplitude 'η' can be controlled by external voltage V_{act}. In state-space form, system (2.11) can be written as (Yang and Lee, 1993a):

$$\dot{X}(t) = AX(t) + BV_{act}(t)$$
$$Y(t) = CX(t), \qquad (2.12)$$

where $B = \begin{bmatrix} 0 \\ B_a \end{bmatrix}$ and $C = \begin{bmatrix} 0 & B_v \end{bmatrix}$. The active damping control law to increase energy dissipation through negative velocity feedback control law is (Yang and Lee, 1993a):

$$V_{act} = -KY = -KB_v\dot{\eta}. \qquad (2.13)$$

Electrical energy spent in structural vibration suppression is a function of actuator placement and is given by (Kim and Kim, 2005; Yang et al., 2005):

$$J_e = \int_0^\infty V_{act}^T R V_{act}\, dt, \qquad (2.14)$$

where R is the weighing matrix and is real symmetric positive definite. It is desired that minimum energy is spent in structural vibration suppression. Therefore, minimization of J_e can be used as a criterion for optimal placement with constraint of minimum vibration suppression level. Using Equation (2.14), optimal value of J_e is (Yang et al., 2005):

$$J_e = -x^T(0)Px(0), \qquad (2.15)$$

where matrix P is the solution of Lyapunov equation and '$x(0)$' is the initial condition. Using the above criterion, minimum energy is used to control the transverse vibrations of a cantilevered beam when the actuator is placed near the root (Baz and Poh, 1988). Optimal distribution of piezoelectric layer over cantilevered plate coincides with areas of high strain for first and third modes. Second and fourth modes which are anti-symmetric, have asymmetric distribution of piezoelectric layer. There is a critical coverage ratio over which additional treatment of active layer is not profitable. For single (second and fourth) as well as for multiple modes, split distribution of piezoelectric layer is the optimal distribution (Kim and Kim, 2005). Equation (2.13), when substituted in (2.11) reduces the dynamic equation of motion of smart structure to:

$$\ddot{\eta}(t) + (D + B_a K B_v)\dot{\eta}(t) + \Lambda\eta(t) = 0. \qquad (2.16)$$

The closed-loop system is stable if the location of collocated sensor–actuator is selected so that generalized damping matrix $(D + B_a K B_v)$ is positive definite. However, in case of non-collocated sensor–actuator

pair this condition may not be necessarily achieved (Yang and Lee, 1993a). In such situation, asymmetric generalized damping matrix $(D + B_a K B_v)$ can be decomposed as a summation of a symmetric (C_d) and a skew-symmetric matrix (C_g) such that (Yang and Lee, 1993a):

$$C_d = D + D_c; \quad D_c = \frac{1}{2}\big[B_a K B_v + (B_a K B_v)^T\big], \quad (2.17)$$

$$C_g = \frac{1}{2}\big[B_a K B_v - (B_a K B_v)^T\big]. \qquad (2.18)$$

The closed-loop system would remain asymptotically stable with infinite gain margin provided that the symmetric part 'C_d' of the generalized damping matrix is positive definite. Skew-symmetric matrix 'C_g' does not influence the system stability at all. Hence to find the optimal sensor/actuator locations for structural vibration control, one should only ensure the positive definiteness of symmetric matrix 'C_d' (Yang and Lee, 1993a). It is further desired that smart structure dissipates energy as fast as possible. Energy dissipated by active vibration control of a smart structure is (Yang and Lee, 1993a.b; Yang et al., 2005):

$$J_d = -\int_0^\infty \dot{x}^T Q x\, dt, \qquad (2.19)$$

where $Q = \begin{bmatrix} 0 & 0 \\ 0 & D_c \end{bmatrix}$. Energy dissipated as given by Equation (2.19), depends upon piezoelectric placement as well as on feedback gain. So, piezoelectric placements as well as control gains need to be optimized simultaneously so that energy dissipated by the system is maximized. With velocity feedback, 'D_c' should always be positive definite and with state feedback, the real eigenvalues of closed-loop system matrix should be negative (Yang and Lee, 1993a). Using Equation (2.19), optimal value of J_d can be obtained (Yang and Lee, 1993b; Yang et al., 2005) as:

$$J_d = -x^T(0)Px(0), \qquad (2.20)$$

where matrix P is the solution of Lyapunov equation. For control of the first four modes using Equation (2.20), optimal location of collocated sensor–actuator pair comes out near the beam root for velocity feedback control and at the root for state feedback control (Yang and Lee, 1993b). Optimal placement of one collocated sensor–actuator pair is at 0.153 times beam length from beam root for velocity feedback control. One collocated sensor–actuator pair is at 0.15 times beam length and the second one at 0.71 times beam length from beam root for velocity feedback control when two collocated sensor–actuator pairs are used. If three collocated sensor–actuator pairs are used,

then optimal placements are: (i) first pair at 0.158, (ii) second pair at 0.359, and (iii) third pair at 0.828 times beam length from beam root for velocity feedback control (Yang et al., 2005). A criterion for the simultaneous minimization of energy spent by the actuators and maximization of energy dissipated by the structure can be obtained by using performance index of LQR optimal control as (Yang and Lee, 1993a,b; Bruant et al., 2001; Gaudiller and Hagopian, 1996):

$$J_{ed} = \int_0^\infty (\dot{x}^T Q x + V_{\mathrm{act}}^T R V_{\mathrm{act}}) dt. \qquad (2.21)$$

Equation (2.21) can be minimized to find the optimal location of the actuator patch. Using (2.21), optimal value of J_{ed} can be written as (Yang and Lee, 1993a,b):

$$J_{ed} = x^T(0) P x(0), \qquad (2.22)$$

where matrix P is the solution of algebraic Riccati equation (ARE). Using criterion (2.22) to control the first four modes of a cantilevered beam simultaneously, actuator is placed at 0.059 times beam length and sensor at 0.067 times beam length from beam root for velocity feedback control. For state feedback control, actuator is placed at 0.042 times beam length and sensor at 0.223 times beam length. Optimal placement of collocated sensor–actuator pair is at 0.125 times beam length for velocity feedback control (Yang and Lee, 1993a). The optimal solutions given by Equations (2.15), (2.20), and (2.22) depend upon actuator locations and the initial states. These initial states may not be known. Therefore a procedure that minimizes the trace of matrix P instead of solving Equations (2.15), (2.20), and (2.22), can be used. The optimization criterion thus becomes (Livine and Athans, 1970):

$$\text{Minimize } J = \text{tr}[P], \qquad (2.23)$$

where tr[.] denotes the trace of matrix. The trace is also the sum of eigenvalues of the matrix. Placement of sensor–actuator which minimizes the trace of matrix [P], would be optimal from a possible set of locations for controlling the structural vibrations (Nam and Weisshaar, 1996; Fahroo and Wang, 1997; Demetriou, 2000; Li et al., 2002; Gao et al., 2003; Si et al., 2003; Kim and Kim, 2005; Yang et al., 2005; Kumar and Narayanan, 2007). When LQR performance index is optimized using this method, the optimal locations of 10 sensor–actuator pairs over a cantilevered plate to control first six natural modes are at regions of high modal strain energies. Five sensor–actuator pairs are placed at maximum modal strain energies of first two modes, two at maximum modal strain energies of fourth

mode, and three around the center of the plate (Kumar and Narayanan, 2007). Optimal locations of four collocated piezoelectric actuator pairs on a plate shaped wing are: two at the leading and the trailing edge of the root, one at the edge of the tip, and one at the mid cord of the out board region. Actuators placed at the tip, leading edge, and at the trailing edge of the root are also thicker than the remaining two (Nam et al., 1996). For control of first mode of a cantilevered beam, optimal actuator patch location is at root. One sensor is placed at a distance of 0.37 times beam length and second at 0.66 times beam length from the root (Demetriou, 2000). Minimum control energy can also be ensured by minimizing (Lammering et al., 1994):

$$Z = \text{tr}[F^{-T} F^{-1}], \qquad (2.24)$$

where $F = m^{-1} \psi^T$ and m is modal mass matrix.

Maximizing Degree of Controllability

Regardless of the control algorithm being applied, necessary condition for effective active vibration control is that the smart structure should be controllable. A closed-loop system (2.12), is completely controllable if every state variable can be affected in such a way so as to cause it to reach a particular value within a finite amount of time by some unconstrained control. If one state variable cannot be affected in this way, the system is said to be uncontrollable (Inman, 2006). Controllability is a function of both the system dynamics and location and number of actuators. Control influence matrix 'B' is determined by actuator locations on the smart structure. A standard check for the controllability of a system is a 'rank test' of a matrix 'R' such that (Inman, 2006):

$$R = \begin{bmatrix} B & AB & A^2 B & A^3 B & \ldots\ldots & A^{2n-1} B \end{bmatrix}_{1 \times 2n} \qquad (2.25)$$

Closed-loop system (2.12) is completely state controllable if and only if $2n \times 2n$ matrix R is full rank (Inman, 2006). Rank of a matrix is the number of independent rows (or columns) of the matrix when the rows (columns) are treated like vectors. Matrix R is called the 'controllability matrix' for the matrix pair [A B]. Controllability of a system only tells us whether the system is controllable or not. Degree of controllability of a system can be increased with proper placement of actuators using various techniques. In a smart structure, optimal actuator locations are where electrical energy consumed is smallest and modal forces generated are the largest. Criterion (2.14) ensures actuator locations where energy required to control structural vibrations is

minimum. Using this criterion, the optimal control energy is obtained as (Hac and Liu, 1993):

$$J_e = \left[e^{At_1}\{x_0\} - \{x_{t_1}\}\right]^T G_C^{-1}(t)\left[e^{At_1}\{x_0\} - \{x_{t_1}\}\right], \quad (2.26)$$

where the aim is to bring the modal system to a desired state $\{x_{t_1}\}$ from initial state $\{x_0\}$ after some time 't_1' and, 'G_c' is the controllability Gramian matrix defined as (Hac and Liu, 1993):

$$G_c(t) = \int_0^{t_1} e^{At} B B^T e^{A^T t} \, dt. \quad (2.27)$$

Controllability Grammian matrix (2.27) is a measure of the degree of controllability of a system. Effects of actuator are contained in G_c by way of matrix 'B'. Minimization of control energy (2.26) means minimization of G_c^{-1}. In other words, minimum control energy would be used in structural vibration control if determinant of controllability Gramian matrix is maximized. Eigenvalues of controllability Grammian matrix are also a measure of the degree of controllability. Higher the eigenvalues of controllability Grammian matrix, higher is the controllability. If any eigenvalue of controllability Grammian is very less, then the corresponding mode is difficult to control and would require a huge amount of control energy for attenuation. Therefore, configuration of actuators, which maximizes the performance index (Peng et al., 2005):

$$J_e = \left(\sum_{j=1}^{2n} \lambda_j\right) \sqrt[2n]{\prod_{j=1}^{2n} (\lambda_j)} \quad (2.28)$$

would require minimal control effort to suppress structural vibrations. Here, n is the number of first modes to be controlled and λ_j is eigenvalue of controllability Grammian. Criterion (2.28) is equivalent to maximizing the performance index (Bruant and Proslier, 2005):

$$J_e = \mathrm{tr}[G_c] * (\det G_c)^{1/2n}. \quad (2.29)$$

Based on criterion (2.28), optimal location of four actuators to control five modes of cantilevered plate is: two actuators at extreme corners of cantilevered edge and other two actuators placed adjacent to each other near center of the plate (Peng et al., 2005). Criterion (2.28) ensures global controllability of the system for the first n eigenmodes. To control each mode individually by applying minimal control effort, each diagonal term of G_c can be maximized, that is, performance index should be (Bruant and Proslier, 2005):

$$J_e = \max\{(G_c)_{11}, (G_c)_{22}, \ldots, (G_c)_{nn}\}, \quad (2.30)$$

where $(G_c)_{ii}$ are the diagonal elements of G_c. Number of uncontrollable modes is equal to the number of very small singular values of controllability Grammian matrix (Grace et al., 1990). So, minimal singular value of controllability Grammian can also be maximized to search for optimal location of actuators (Sadri et al., 1999; Kermani et al., 2004). Optimal actuator locations for i-th mode can also be found using a measure of 'modal controllability' defined as (Hamden and Nayfeh, 1989; Sadri et al., 1998; Sadri et al., 1999; Aldraihem et al., 2000; Zhang et al., 2009):

$$\delta = \|f_i\|, \quad (2.31)$$

where $f_i^T = \frac{q_i^T B}{\|q_i\|}$ and 'q_i' is the normalized eigenvector of the i-th mode. Simultaneous maximization of minimal singular value of controllability Grammian and 'modal controllability' of the i-th mode, gives optimal actuator locations between nodal lines for simply supported plate (Sadri et al., 1999).

A square matrix can always be decomposed in the form of $U_1 W U_2$, where U_1 and U_2 are unitary matrices and W is a diagonal matrix with singular values as the diagonal elements (Gopal, 2008). This is called singular value decomposition (SVD). Singular values of control influence matrix 'B', determines the magnitude of control forces. These singular values are found by performing SVD of 'B'. So, to achieve maximum control forces, product of singular values can be taken as controllability index (Wang and Wang, 2001):

$$\Gamma = \prod_{i=1}^{n} \kappa_i, \quad (2.32)$$

where 'κ_i' are the singular values of matrix 'B'. Higher the controllability index 'Γ', smaller will be the required actuator voltages for vibration suppression. Therefore, maximization of controllability index gives optimal location of the actuator (Wang and Wang, 2000; Dhuri and Sheshu, 2006; Dhuri and Sheshu, 2009). For control of first mode of simply supported beam, optimal location is at the mid-span (anti-node) of the beam. For simultaneous control of first two modes, there are two optimal locations at distance of approximately 0.3 times 'beam length' from both ends. Similarly, there are three optimal locations when the first three modes are controlled simultaneously at distance of approximately 0.27, 0.50, 0.73 times 'beam length' from the left end. Similarly for cantilevered beam, optimal location is at the root for first mode. For simultaneous control of first two modes, optimal location is at 0.56 times 'beam length' from the root. For simultaneous control of first three modes, optimal location is at distance of 0.70 times 'beam length' from the root. If the free end of the

cantilevered beam is propped then the optimal location for control of first mode is at 0.3 times 'beam length' from the root. For simultaneous control of first two modes, optimal location is at 0.73 times 'beam length' from the root. For simultaneous control of three modes, optimal location is at 0.53 times 'beam length' from the root (Wang and Wang, 2000).

Transfer function (G) between actuator voltage and plate deflection gives the response characteristics of the system. Degree of controllability can be quantitatively measured using H_2 norm of the transfer function. H_2 norm of the transfer function is defined as the expected root-mean-square value of the output when the input is a unit variance white noise. Square of H_2 norm of the transfer function in modal form is (Moheimani and Ryall, 1999):

$$\|G_i\|_2^2 = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \mathrm{tr}\{G(r_1)G*(r_1)\}\mathrm{d}w, \qquad (2.33)$$

where $\|G_i\|_2$ is H_2 norm of the transfer function for i-th mode, r_1 is the location of piezoelectric actuator, and superscript (*) denotes the complex conjugate. Modal controllability is a measure of controller authority over i-th mode and can be defined as (Moheimani and Ryall, 1999):

$$M_i(r_1) = \left(\frac{f_i(r_1)}{\alpha_i} \times 100\right) \%, \qquad (2.34)$$

where $f_i(r_1) = \|G_i\|_2$ and $\alpha_i = \max f_i(r_1)$. Square of H_2 norm of the transfer function in spatial form is (Moheimani and Ryall, 1999):

$$\langle\langle G \rangle\rangle_2^2 = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_R \mathrm{tr}\{G(r_1)G*(r_1)\}\mathrm{d}r\,\mathrm{d}w, \qquad (2.35)$$

where $\langle\langle G \rangle\rangle_2^2$ is H_2 norm of the transfer function in spatial form and 'R' is the spatial domain of the smart structure. Spatial controllability is a measure of controller authority over the entire structural modes in an average sense and is defined as (Moheimani and Ryall, 1999):

$$S(r_1) = \left(\frac{1}{\beta}\sqrt{\sum_{i=1}^{n} f_i(r_1)}\right) \times 100 \%, \qquad (2.36)$$

where $\beta = \max \sum_{i=1}^{n} f_i(r_1)$ and n is the number of low frequency modes considered. Spatial H_2 norm is different than modal H_2 norm in the sense that it introduces an additional average operation over the spatial domain R. Spatial H_2 norm is related to modal H_2 norm as (Moheimani and Ryall, 1999):

$$\langle\langle G \rangle\rangle_2^2 = \sum_{i=1}^{J} \|G_i\|_2^2. \qquad (2.37)$$

Optimal piezoelectric actuator locations can be found by either maximizing modal controllability (2.34) or spatial controllability (2.36) (Moheimani and Fu, 1998; Moheimani and Ryall, 1999; Halim and Moheimani, 2003; Liu et al., 2006; Dhuri and Sheshu 2007; Qui et al., 2007; Guney and Eskinat, 2008). Using criterion (2.34), first and second modes are completely controllable if actuator is placed at the root of the beam, third mode is completely controllable if actuator is placed at 0.735 times beam length from the root of the beam and fourth mode is completely controllable if actuator is placed at 0.829 times beam length from the root of the beam (Moheimani and Ryall, 1999). For control of first mode of a simply supported plate, optimal placement of actuator is in the middle of the plate (Halim and Moheimani, 2003). Optimal piezo actuator location can also be obtained where the spatial controllability is maximized and modal controllability is guaranteed for each mode of interest (Demetriou and Armaou, 2005).

Maximizing Degree of Observability

Every state variable in the system has some effect on the output of the system. A closed-loop system (2.12) is said to be completely observable if, examination of the system output determines information about each of the state variables. If one state variable cannot be observed in this way, the system is said to be unobservable (Inman, 2006). Observability is a function of both system dynamics and location and number of sensors. The output influence matrix 'C' is determined by the position of sensors on the smart structure. The standard check for the observability of a system is a rank test of a matrix 'O' where (Inman, 2006):

$$O = \begin{bmatrix} C & CA & CA^2 & CA^3 & \ldots\ldots & CA^{2n-1} \end{bmatrix}_{1 \times 2n}^T. \qquad (2.38)$$

The system is completely state controllable if, and only if, $2n \times 2n$ 'matrix O' is full rank. Matrix O is called the 'observability matrix' for the matrix pair [A C]. Observability of a system only tells us weather the system is observable or not. Degree of observability depends upon the location of sensors and can be increased with proper placement of sensors using various methods. Optimal location of sensors is determined using the same methodology as is used for actuators (Bruant and Proslier, 2005). In a smart structure, optimal sensor locations are where vibration amplitudes or the changes in vibration mode shapes of host structure is relatively large (Li et al., 2004), that is, system output index (2.39) is as large as possible (Hac and Liu, 1993):

$$J_o = \int_0^{\infty} \{Y\}^T\{Y\}\mathrm{d}t. \qquad (2.39)$$

Measure of observability is the observability Gramian matrix 'G_o' defined as (Hac and Liu, 1993):

$$G_o(t) = \int_0^\infty e^{At} C C^T e^{A^T t}\, \mathrm{d}t. \qquad (2.40)$$

Effects of sensor are contained in G_o by way of matrix 'C'. Information about observability is hidden in eigenvalues of the observability Grammian matrix. If the i-th eigenvalue of G_o is small, it means that the i-th mode will not be well observed. Therefore, sensor location is so selected that eigenvalues of observability Grammian matrix corresponding to desired modes are maximized (Bruant and Proslier, 2005). To maximize these eigenvalues, maximization of following measure can be done (Hac and Liu, 1993; Baruh, 1992):

$$J_0 = \mathrm{tr}[G_0] * \left| (G_0) \right|^{1/2n}, \qquad (2.41)$$

where n is the number of first modes to be observed. Criterion (2.41) ensures global observability of the system for the first n eigenmodes. To observe each mode individually, diagonal terms of G_o can be maximized, that is, performance index should be (Bruant and Proslier, 2005):

$$J_o = \max\left\{ (G_o)_{11}, (G_o)_{22}, \ldots\ldots\ldots, (G_o)_{nn} \right\}, \qquad (2.42)$$

where, $(G_o)_{ii}$ are the diagonal elements of G_o. Optimal sensor locations can also be found using a measure of 'modal observability' of the i-th mode (Kim and Junking, 1991) which is defined as (Hamden and Nayfeh, 1989):

$$\delta = \| f_i \|, \qquad (2.43)$$

where $f_i^T = \dfrac{q_i^T C}{\| q_i \|}$ and 'q_i' is the normalized eigenvector of the i-th mode. For a collocated sensor–actuator system (i) measure of modal observability is equivalent to the measure of modal controllability (Kim and Junking, 1991; Aldraihem et al., 1997), (ii) optimal sensor locations are same as optimal actuator locations when we maximize the degree of controllability/observability using H_2 norm (Qui et al., 2007), and (iii) measures of modal and spatial observability using H_2 norm are equivalent to the measure of modal and spatial controllability using H_2 norm (Halim and Moheimani, 2003).

To implement state feedback control law on a smart structure, modal displacements, and modal velocities of the modes to be controlled are estimated from output of the sensor. The error vector between estimated and actual modal displacement is given by (Sun and Tong, 2001):

$$\{ e(t) \} = [B_n]^{-1} [B_r] \{ \eta_r(t) \}, \qquad (2.44)$$

where $B_n \,\&\, B_r$ are the measurement influence vectors corresponding to first n observed modes and r residue modes, respectively, and $\eta_r(t)$ are residue modal vectors. Square of the norm of error vector (2.44) is given by (Sun and Tong, 2001):

$$\| e(t) \|^2 = \{ e(t) \}^T \{ e(t) \} = \{ \eta_r(t) \}^T [B_e] \{ \eta_r(t) \}. \qquad (2.45)$$

$[B_e] = [B_r]^T \left([B_n]^{-1} \right)^T [B_n]^{-1} [B_r]$ is a symmetric positive definite matrix. Then, degree of observability can be increased by finding optimal sensor locations using the criterion (Sun and Tong, 2001):

$$\| e(t) \|^2 \leq \lambda_{\max}([B_e]) \{ \eta_r(t) \}^T \{ \eta_r(t) \}, \qquad (2.46)$$

where $\lambda_{\max}([B_e])$ is the maximum eigenvalue of $[B_e]$.

Minimizing Spillover Effects

Many times, a smart flexible structure is discretized into finite number of elements for vibration analysis and control. It is sufficient to account for low frequency dynamical behavior in most practical situations. While implementing the control law, the model is reduced to include only first few low frequency modes of interest. Only first few low frequency modes are considered in state observer. However, state feedback control law based on a reduced model may excite the residual modes. These residual modes appear in sensor output but are not included in the control design as shown in Figure 3. This closed-loop interaction with low damping of the residual modes, results into spillover instability (Preumont, 2002; Han et al., 1997). If subscript 'c' refers to controlled modes, and subscript 'r' refers to residual modes then the open-loop system is described as (Preumont, 2002):

$$\begin{aligned} \dot{X}_c &= A_c X_c + B_c V_{\mathrm{act}} \\ \dot{X}_r &= A_r X_r + B_r V_{\mathrm{act}}. \\ Y &= C_c X_c + C_r X_r \end{aligned} \qquad (2.47)$$

Assuming a perfect knowledge of the controlled modes, full state observer is (Preumont, 2002):

$$\dot{\hat{X}}_c = A_c \hat{X}_c + B_c V_{\mathrm{act}} + L_c \left(Y - C_c \hat{X}_c \right), \qquad (2.48)$$
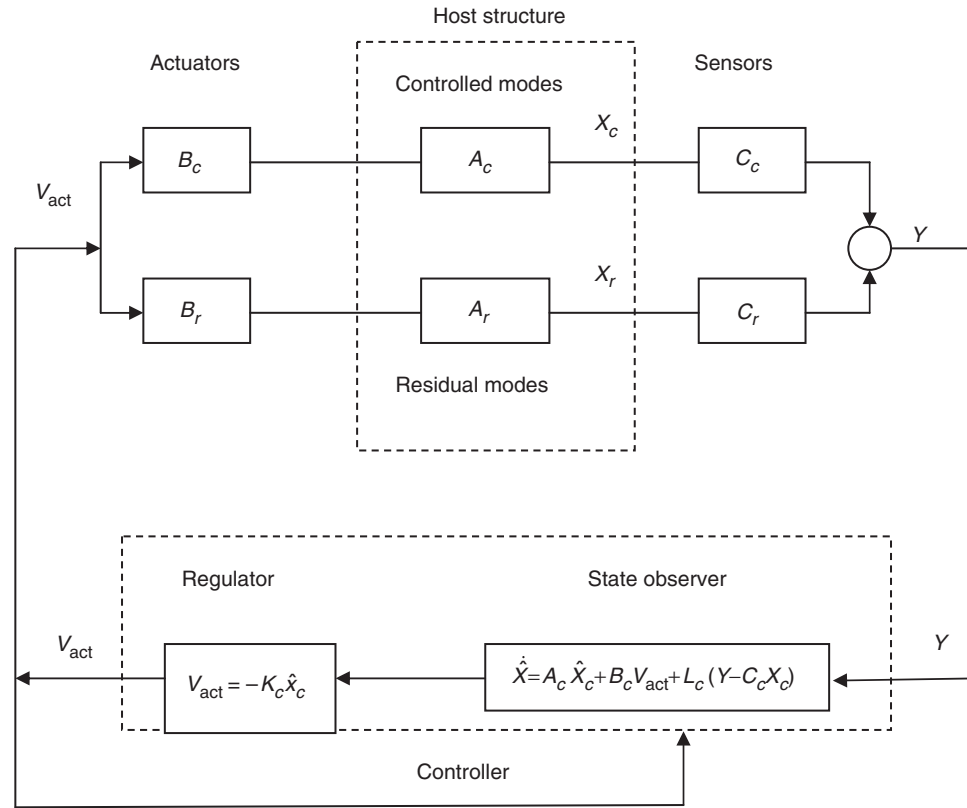
Figure 3. *Schematic illustration of spill-over effects.*

where $\hat{X}_c$ is the estimated value of X_c and L_c is the observer gain matrix. The state feedback control law is (Preumont, 2002):

$$V_{\text{act}} = -K_c\hat{X}_c. \qquad (2.49)$$

If X_c and X_r are the state variables and $e_c = X_c - \hat{X}_c$ then the interaction between control system and the residual modes is given by (Preumont, 2002):

$$\begin{bmatrix} \dot{X}_c \\ \dot{e}_c \\ \dot{X}_r \end{bmatrix} = \begin{bmatrix} A_c - B_cK_c & B_cK_c & 0 \\ 0 & A_c - L_cC_c & -L_cC_r \\ -B_rK_c & B_rK_c & A_r \end{bmatrix} \begin{bmatrix} X_c \\ e_c \\ X_r \end{bmatrix}. \qquad (2.50)$$

B_rK_c and L_cC_r terms are the result of sensor output being contaminated by residual modes via the term C_rX_r (observation spillover), and the feedback control exciting the residual modes via the term B_rV_{act} (control spillover). As a result, the eigenvalues of the system shift away from their decoupled locations as assumed in control law. If the stability margin of the residual modes is small, even a small shift in eigenvalues would make them unstable. This is known as spillover instability. These spillover effects can reduce the performance and stability of the controller (Joshi, 1989). However, spillover effects can be reduced if sensors and actuators are placed over a smart structure

in such a way that effects of residual modes are minimal. Actuators so placed would control desired modes with minimal control spillover and sensors so placed would sense desired modes with minimal observation spillover. To minimize the spillover effects, performance index (2.28) can be modified as (Hac and Liu, 1993):

$$J = \left(\sum_{j=1}^{2n} \lambda_j\right)^{2n}\sqrt{\prod_{j=1}^{2n}(\lambda_j)} - \gamma\left(\sum_{j=2n_c+1}^{2(n_c+n_r)} \lambda_j\right)^{2n_r}\sqrt{\prod_{j=2n_c+1}^{2(n_c+n_r)}(\lambda_j)} \qquad (2.51)$$

where n_c and n_r are the number of the controlled modes and residual modes, respectively, and γ is the weighting constant, which can be selected by the designer. Maximization of this performance index would maximize eigenvalues corresponding to the modes to be controlled and minimize eigenvalues corresponding to the modes to be ignored. This results in reduction of spillover effects (Han and Lee, 1999). When this criterion is used to control first three modes of a composite cantilevered plate instrumented with two piezoelectric sensors and two piezoelectric actuators, optimal locations are: (i) one sensor at one end of the cantilevered edge, (ii) both actuators exactly below first sensor and placed adjacent to each other, and (iii) second sensor at second end of the cantilevered edge (Han and Lee, 1999).

Table 1. Optimal locations of surface bonded piezoelectric sensor and actuator patches on a smart beam structure.

Criterion	Boundary condition	Modes to be controlled	Sensor and actuator locations	Source
Maximizing modal forces/moments	Cantilevered	First	Actuators must be placed near root	(Crawley and de Luis, 1987)
		Second	One actuator at distance less than 0.216 times beam length driven 180° out of phase with second actuator at distance greater than 0.216 times beam length bonded to opposite side	
Maximizing deflection of the host structure	Cantilevered	First	Optimal length of collocated actuator pair is equal to the length of the beam	(Barboni et al., 2000)
		Second or higher	Collocated actuator pair is where opposite edges of actuator correspond to points of equal curvature of beam mode	
Minimizing the control efforts	Cantilevered	First	Actuators must be placed near root	(Baz and Poh, 1988)
Optimal value of LQR performance index	Cantilevered	First	Actuator is placed at root, one sensor is placed at a distance of 0.37 times beam length and second at 0.66 times beam length from the root	(Demetriou, 2000)
Maximizing energy dissipated	Cantilevered	First four	Actuator at 0.059 times beam length and sensor at 0.067 times beam length for velocity feedback control Actuator at 0.042 times beam length and sensor at 0.223 times beam length for state feedback control Collocated sensor–actuator pair at 0.125 times beam length for velocity feedback control	(Yang and Lee, 1993a)
	Cantilevered	First four	Collocated sensor–actuator pair near beam root for velocity feedback control Collocated sensor–actuator pair at beam root for state feedback control	(Yang and Lee, 1993b)
	Cantilevered	First four	Collocated sensor–actuator pair at 0.153 times beam length from beam root for velocity feedback control (when one pair is used) First collocated sensor–actuator pair at 0.15 times beam length and second at 0.71 times beam length from beam root for velocity feedback control (when two pairs are used) First collocated sensor–actuator pair at 0.158, second at 0.359, and third at 0.828 times beam length from beam root for velocity feedback control (when three pairs are used)	(Yang et al., 2005)
Maximizing degree of controllability	Cantilevered	First	Collocated actuator pair at root	(Wang and Wang, 2000)
		First two	Collocated actuator pair at 0.56 times beam length from root	
		First three	Collocated actuator pair at 0.7 times beam length from root	
	Cantilevered at one end and propped at another	First	Collocated actuator pair at 0.3 times beam length from root	
		First two	Collocated actuator pair at 0.73 times beam length from root	
		First three	Collocated actuator pair at 0.53 times beam length from root	
	Simply Supported	First	Collocated actuator pair at mid-span	
		First two	Collocated actuator pair at 0.3 times beam length from any end	
		First three	Collocated actuator pair at 0.27 or 0.5 or 0.73 times beam length from left end	
Maximizing degree of modal controllability	Cantilevered	First	Actuator at root	(Moheimani and Ryall, 1999)
		Second	Actuator at root	
		Third	Actuator at 0.735 times beam length from beam root	
		Fourth	Actuator at 0.829 times beam length from beam root	

Table 2. Optimal locations of surface bonded piezoelectric sensor and actuator patches on a smart plate structure.

Criterion	Boundary condition	Modes to be controlled	Sensor and actuator locations	Source
Maximizing modal forces/ moments applied by actuator	Cantilevered	First	Two actuators adjacent to each other at mid-point of the cantilevered edge	(Bin et al., 2000)
		Second	Two actuators at extreme corners of the cantilevered edge	
		Third	One actuator at 0.25 times length of cantilevered edge on the cantilevered edge and second actuator at 0.75 times length of cantilevered edge on the cantilevered edge	
Maximizing deflection of the host structure	Cantilevered	First	Actuator close to the middle of cantilevered edge	(Ip and Tse, 2001)
		Second and Third	Actuators near the corner of the cantilevered edge	
		Fourth and Fifth	Actuators at anti-nodes	
	Cantilevered	First	Collocated sensor–actuator pair must be located near the centerline along the cantilevered edge	(Quek et al., 2003)
		Second	One edge of the collocated sensor–actuator pair is on the free edge of the plate and the other edge is on the cantilevered edge of the plate	
		First and Second	One collocated sensor–actuator pair is placed at the centerline along the cantilevered edge and other is placed adjacent to it along the cantilevered edge	
	Simply Supported	First five	Actuators at anti-nodes of modes of interest	(Ip and Tse, 2001; Yang and Zhang, 2006)
Minimizing the control efforts	Cantilevered	First and Third	Actuators should cover areas of high strain	(Kim and Kim, 2005)
		Second and Fourth	Optimal distribution of piezoelectric actuator layer is asymmetric	
Optimal value of LQR performance index	Cantilevered	First six	Five collocated sensor–actuator pairs at maximum modal strain energies of first two modes, two at maximum modal strain energies of fourth mode, and three around the center of the plate	(Kumar and Narayanan, 2007)
Expected value of LQR performance index	Cantilevered (Plate shaped wing)	First torsion mode	Two actuators at leading and trailing edge of the root, one at the edge of the tip, and one at the mid cord of the out board region	(Nam et al., 1996)
Maximizing degree of controllability	Cantilevered	First five	Two actuators at extreme corners of cantilevered edge and two adjacent to each other near center of the plate	(Peng et al., 2005)
	Simply Supported	First	Actuator at middle of the plate	(Halim and Moheimani, 2003)
		First four	Actuators between nodal lines	(Sadri et al., 1999)
Maximizing degree of observability and minimizing spillover effects	Cantilevered	First three	One sensor at one end of cantilevered edge, two actuators exactly below this sensor placed adjacent to each other, and another sensor at other cantilevered end	(Han and Lee, 1999)

To ensure maximum utilization of control energy with minimum spillover effects, criterion (2.24) can be modified as (Lammering et al., 1994):

$$Z = \mathrm{tr}\left[\left(F_r F^{-1}\right)^T \left(F_r F^{-1}\right)\right]. \qquad (2.52)$$

In this way, significant vibration control can be achieved with only little spillover effect. When criterion (2.46) is used to maximize the degree of observability, spillover effects can be greatly minimized if we minimize the maximum eigenvalue of $[B_e]$ (Sun and Tong, 2001). Spillover effects can also be addressed by imposing a constraint in criterion (2.36). High frequency residual modes can be considered for which spatial controllability can be defined as (Halim and Moheimani, 2003):

$$S_r(r_1) = \left(\frac{1}{\beta_r}\sqrt{\sum_{i=J+1}^{\bar{J}} f_i(r_1)}\right) \times 100\ \% \qquad (2.53)$$

where $\bar{J}$ corresponds to the highest frequency mode that is considered for the spillover reduction, and $\beta_r = \max \sum_{i=J+1}^{\bar{J}} f_i(r_1)$. Therefore, spillover effects can be reduced by imposing the constraint (Halim and Moheimani, 2003; Demetriou and Armaou, 2005):

$$S_r(r_1) \leq c, \qquad (2.54)$$

along with the criterion (2.36) where 'c' is the upper allowable level for spatial controllability for spillover reduction. Criteria (2.30) and (2.42) increase the degree of 'modal controllability' and 'modal observability', respectively, but without considering residual modes. These criteria can be modified to include residual modes. New criterion thus obtained for good controllability with minimal spillover effects is (Bruant and Proslier, 2005):

$$J = \max\left\{(G_c)_{11}, (G_c)_{22}, \ldots, (G_c)_{n_c n_c}\right\} \quad \text{and}$$
$$\min\left\{(G_c^r)_{11}, (G_c^r)_{22}, \ldots, (G_c^r)_{n_r n_r}\right\} \qquad (2.55)$$

and for good observability with minimal spill-over effects is (Bruant and Proslier, 2005):

$$J = \max\left\{(G_o)_{11}, (G_o)_{22}, \ldots, (G_o)_{n_c n_c}\right\} \quad \text{and}$$
$$\min\left\{(G_o^r)_{11}, (G_o^r)_{22}, \ldots, (G_o^r)_{n_r n_r}\right\} \qquad (2.56)$$

where $(G_c^r)_{ii}$ and $(G_o^r)_{ii}$ are the eigenvalues corresponding to residual modes.

In all sections of 'Optimal Placement of Piezoelectric Sensors and Actuators', 'state-of-the-art' of optimal placement of piezoelectric sensors and actuators based upon six optimization criteria are discussed one by one. Optimal placements so obtained for surface bonded piezoelectric sensors and actuators are presented criterion wise for smart beam structures in Table 1, and for smart plate structures in Table 2. According to all criteria, as can be observed from the results presented in the Table 1: (i) to control the first mode of a cantilevered beam, optimal placement of actuators is near the root of the beam, and (ii) to control the higher modes of a cantilevered beam, optimal placement of actuators is away from the root of the beam. According to all criteria, results presented in Table 2 reflect that in case of a cantilevered plate: (i) optimal placement of actuator is close to the mid-point of cantilevered edge for control of the first mode, (ii) optimal placement of actuators is at the corners of cantilevered edge for control of second mode, and (iii) optimal placement of actuators is in between the mid-point and corners of the cantilevered edge for control of higher modes. According to all criteria in case of simply supported plates, the optimal actuator placements are either: (i) at locations where average strains in x- and y-directions are maximum, or (ii) at anti-nodes. Actuators should not be placed at nodes. It is also observed from Tables 1 and 2 that: (i) optimal placements of piezoelectric sensors and actuators on a smart structure depend upon the optimization criterion and boundary conditions, and (ii) optimal locations obtained for each criterion cannot be predicted using intuitive method. Placement of sensors and actuators based upon intuitive method can result in poor performance or even failure of AVC scheme (Yang and Lee, 1993a). It is therefore recommended that the placement of sensors and actuators on a smart structure should be according to a suitable criterion. Out of six criteria discussed in the sections of 'Optimal Placement of Piezoelectric Sensors and Actuators', the first four give the optimal placement of piezoelectric actuators. Criterion 2.5 gives the optimal placement of piezoelectric sensor. Optimal placement of piezoelectric sensors as well as actuators can be ascertained by applying criterion 2.6.

CONCLUSIONS

In this 'technical review', the authors have presented the 'most used' optimization criteria by researchers for placement of piezoelectric sensors and actuators on a smart structure. Attempt is made to discuss 'state-of-the-art' for each criterion. Optimal locations so obtained, are presented for beam and plate structures in a tabular form. A practicing engineer or a researcher can pick the most suitable placement for his application using information provided in the table and avoid intuitive methods. The following conclusions are drawn from this 'technical review':

1. Optimal placements of piezoelectric sensors and actuators on a smart structure depend upon the optimization criterion.

2. If actuators are placed in the region where average modal strains are highest, then such placement of actuators would apply maximum modal forces/moments on the smart structure.

3. If actuators are placed at the antinodes of modes of interest, then such placement of given actuators would result in maximum deflection of the host structure.

4. Most of the researchers have used 'minimizing control effort/maximizing energy dissipation' as criterion for optimal placement of actuators.

5. Degree of controllability can be maximized to place actuators at locations where the modes of interest of smart structure are best controlled. Similarly, degree of observability can be maximized to place sensors at locations where the modes of interest are best observed. However, criterion 2.6 can be used for optimal placements of both piezoelectric sensors and actuators on a smart structure and to ensure minimum spillover effects as well.

6. Actuators used in smart structures are more massive than sensors. Therefore, optimal placement of actuators has greater significance than that of sensors. Out of the six criteria discussed above, four (2.1–2.4) are exclusively dedicated for optimal placement of actuators, and one criteria (2.6) is dedicated for optimal placement of both sensors and actuators. Criterion 2.5 is the only criterion exclusively dedicated for optimal placement of sensors.

7. Optimal placement of piezoelectric sensors and actuators depends upon boundary conditions. While searching for optimal locations the boundary conditions must be properly understood.

8. Optimal locations of piezoelectric sensors and actuators can be found by either selecting a single criterion or a combination of more than one criterion depending upon the end application.

9. Most of the work is targeted on simple beam and plate structures. Studies on optimal piezoelectric sensor and actuator placement on real-life complex structures is absent in the literature. Therefore, future research should be targeted on optimal placement of piezoelectric patches on real-life structures.

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