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Relay Selection in Dual-Hop Transmission Systems: Selection Strategies and Performance Results

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***Abstract*— We address the problem of selecting an appropriate relay station for forwarding data from a source to a destination node in dual-hop communication systems. In this regard, we focus on regenerative relays, which try to fully decode the messages received from the source before forwarding them to the actual destination. Several different scenarios are considered with different kinds of channel state information (CSI) available at the selecting entity. For all cases, we present the optimal selection strategies aiming at either maximizing the mean mutual information or minimizing the outage probability. The performance of all schemes is evaluated by means of numerical and simulation results and it turns out that with minimal CSI often almost the same performance as with perfect CSI can be achieved.**

I. Introduction

Relayed transmission, where one or multiple relay stations are actively involved in transmitting data from a source to a destination node, is likely to play an important role in future cellular communication systems as well as wireless ad hoc and sensor networks. This is because it entails several different benefits, such as the potential to increase the system capacity, to extend the radio range or to reduce network infrastructure costs and hardware requirements of mobile devices [1], [2]. In recent years, in particular the concept of so-called cooperative communication has gained a lot of research attention. The fundamental idea of this approach is that multiple users cooperate by acting as relay stations for each other, thus establishing a virtual multiple antenna system suitable for greatly improving the effective channel conditions, see for example [3]–[6].

In most previous works on relayed transmission, it was assumed that a certain relay station is available and for this relay the transmission then has been optimized. However, a crucial factor for the performance of such systems first of all is the selection of an appropriate relay station out of a set of potential candidates, which might be either fixed relays being part of a certain network infrastructure or—in case of cooperative communication—simply other users nearby. Though it is in principle possible to use multiple relays in parallel, this does not necessarily improve the performance compared to single-relay transmission since generally additional resources are required for that purpose and it would come along with an increased signal processing complexity. Recently, the relay selection problem has attracted an increasing amount of research attention, see for example [7]–[9], but the main focus so far mainly was either on rather unrealistic scenarios with perfect channel state information (CSI) of all hops or on suboptimal distributed algorithms with relatively limited performance.

In contrast to previous works, we develop several different centralized relay selection schemes in this paper, where the selection of an appropriate relay station is performed based on different kinds of channel knowledge, including perfect, quantized and statistical CSI. For all considered cases, we present the optimal selection strategies aiming at either maximizing the mean mutual information between the source and the destination or minimizing the outage probability. In this regard, we always focus on dual-hop transmission with regenerative relays, which try to fully decode the messages received from the source before forwarding them to the actual destination.

The remainder of this paper is structured as follows: In Section II, we introduce our system and channel model whereas the actual relay selection strategies are presented and analyzed in Section III. Afterwards, some performance results are given in Section IV, followed by our main conclusions in Section V.

II. System and Channel Model

We consider a dual-hop communication system, where the data transmission from the source to the destination is subdivided into two different phases of equal length. During the first phase, the source encodes a certain number of information bits and transmits the corresponding codeword to an intermediate relay station, which first of all tries to decode the message before re-encoding and forwarding it to the actual destination node. The relay station involved in this system is always selected prior to a new transmission out of a set of N potential candidate nodes. This selection might be done at the source, the destination, or a centralized control entity, taking perfect or partial CSI of all source-to-relay and relay-to-destination links into account. In this regard, we assume that this CSI is signaled to the selecting entity using an error-free zero-delay control channel. For simplicity, we do not consider the case where the destination also evaluates the signal directly received from the source in our further analysis, but it can easily be checked that the selection strategies that we will present are still optimal if the direct path is taken into account as well.

Below, the instantaneous signal-to-noise ratio (SNR) of the link between the source and the i-th potential relay node will be denoted by $\gamma_{R,i}$ and similarly the SNR of the link between the i-th potential relay node and the destination by $\gamma_{D,i}$. All links are assumed to undergo independent fading and to be constant during the transmission of one frame while consecutive channel realizations are independent of each other. In general, the probability density functions (pdf) of $\gamma_{R,i}$ and

$\gamma_{D,i}$ will be denoted by $p_{\gamma_{R,i}}(\gamma)$ and $p_{\gamma_{D,i}}(\gamma)$, respectively, and similarly the corresponding cumulative distribution functions (cdf) by $F_{\gamma_{R,i}}(\gamma)$ and $F_{\gamma_{D,i}}(\gamma)$. In the following, the optimal selection strategies will always be given in a generic way based on these general pdfs and cdfs, but in addition we will explicitly consider the important case that all hops are subject to not necessarily identically distributed Nakagami-m fading with integer fading parameters, for which we have

$$p_{\gamma_{R,i}}(\gamma) = \frac{\epsilon_{R,i}^{m_{R,i}}}{\Gamma(m_{R,i})}\,\gamma^{m_{R,i}-1}\exp\left(-\epsilon_{R,i}\,\gamma\right) \quad (1)$$

$$p_{\gamma_{D,i}}(\gamma) = \frac{\epsilon_{D,i}^{m_{D,i}}}{\Gamma(m_{D,i})}\,\gamma^{m_{D,i}-1}\exp\left(-\epsilon_{D,i}\,\gamma\right), \quad (2)$$

where we have introduced for brevity the short-hand notations $\epsilon_{R,i} = \frac{m_{R,i}}{\bar{\gamma}_{R,i}}$ and $\epsilon_{D,i} = \frac{m_{D,i}}{\bar{\gamma}_{D,i}}$ and where $m_{R,i}$ and $\bar{\gamma}_{R,i}$ denote, respectively, the fading parameter and average SNR of the i-th source-to-relay link and $m_{D,i}$ and $\bar{\gamma}_{D,i}$ the corresponding parameters of the i-th relay-to-destination link. Using [10, eqs. (3.381,1) and (8.352,1)], we get for the cdfs in that case

$$F_{\gamma_{R,i}}(\gamma) = 1-\exp\left(-\epsilon_{R,i}\,\gamma\right)\sum_{k=0}^{m_{R,i}-1}\frac{1}{k!}\left(\epsilon_{R,i}\,\gamma\right)^k \quad (3)$$

$$F_{\gamma_{D,i}}(\gamma) = 1-\exp\left(-\epsilon_{D,i}\,\gamma\right)\sum_{k=0}^{m_{D,i}-1}\frac{1}{k!}\left(\epsilon_{D,i}\,\gamma\right)^k. \quad (4)$$

All relay nodes are assumed to have perfect channel knowledge of the corresponding source-to-relay links and the destination is assumed to have perfect channel knowledge of all relay-to-destination links. Finally, we assume that all transmit signals are circularly symmetric complex Gaussian distributed.

III. Relay Selection Strategies

We consider two different approaches for selecting an appropriate relay station: In the first case, we aim at maximizing the mean mutual information between the source and the destination whereas in the second case our goal is to minimize outage probability, i.e., the probability that the instantaneous mutual information falls below a given information rate R. In general, if we select the i-th relay node, the mutual information between source and destination is given by [5]

$$I_i = \frac{1}{2}\min\left\{\log_2\left(1+\gamma_{R,i}\right),\log_2\left(1+\gamma_{D,i}\right)\right\} \quad (5)$$

$$= \frac{1}{2}\log_2\left(1+X_i\right), \quad (6)$$

where we have introduced for brevity the short-hand notation

$$X_i = \min\left\{\gamma_{R,i},\gamma_{D,i}\right\}. \quad (7)$$

Clearly, if our goal is to maximize the *mean* mutual information, we always have to select the relay station for which the expected mutual information conditioned on the CSI available at the selecting entity is maximal, i.e., the relay node to be selected in this case corresponds to

$$k_{\text{mmi}} = \arg\max_i \frac{1}{2}\,\mathbb{E}_{X_i|\text{CSI}}\left[\log_2\left(1+X_i\right)|\text{CSI}\right] \quad (8)$$

$$= \arg\max_i \frac{1}{2}\int_0^\infty \log_2(1+\gamma)\,p_{X_i|\text{CSI}}(\gamma|\text{CSI})\,d\gamma, \quad (9)$$

where $p_{X_i|\text{CSI}}(\gamma|\text{CSI})$ denotes the pdf of X_i conditioned on the CSI that is available at the selecting entity. If, in contrast, we want to minimize the outage probability, we always have to select the relay for which the conditional probability that the mutual information falls below R given the available CSI is minimized, i.e., in this case we have

$$k_{\text{out}} = \arg\min_i \text{Prob}\left[I_i \le R|\text{CSI}\right] \quad (10)$$

$$= \arg\min_i F_{X_i|\text{CSI}}\left(2^{2R}-1|\text{CSI}\right), \quad (11)$$

with $F_{X_i|\text{CSI}}(\gamma|\text{CSI})$ as the cdf of X_i conditioned on the available CSI. Having closed-form expressions for the conditional pdf and cdf of X_i, it is hence straightforward to determine the optimal relay station to be selected in either case. If the selection aims at maximizing the mean mutual information, this still requires the evaluation of a single integral, but this generally at least can be easily done numerically for virtually arbitrary fading distributions. In the following paragraphs, we first of all express the conditional pdf and cdf of X_i for various kinds of CSI based on the general SNR distributions of the individual links and then we consider the important case that all links are Nakagami-m fading as a concrete example, for which we solve the integral in (9) analytically in closed-form.

A. Selection Based on Perfect CSI of all Hops

Ideally, the selecting entity has perfect CSI of all links, i.e., all $\gamma_{R,i}$ and $\gamma_{D,i}$ are perfectly known. Hence, we have

$$p_{X_i|\text{CSI}}(\gamma|\gamma_{R,i},\gamma_{D,i}) = \delta(\gamma-\min\{\gamma_{R,i},\gamma_{D,i}\}) \quad (12)$$

where $\delta(\cdot)$ denotes Dirac's delta function and likewise

$$F_{X_i|\text{CSI}}(\gamma|\gamma_{R,i},\gamma_{D,i}) = H(\gamma-\min\{\gamma_{R,i},\gamma_{D,i}\}), \quad (13)$$

with $H(\cdot)$ as the Heaviside step-function. Clearly, for both strategies that we consider, we should always select the relay for which $\min\{\gamma_{R,i},\gamma_{D,i}\}$ is maximal and the instantaneous mutual information is then simply given by

$$I_{\text{inst}} = \frac{1}{2}\log_2\left(1+\max_i\min\{\gamma_{R,i},\gamma_{D,i}\}\right). \quad (14)$$

For determining the actual mean mutual information that can be achieved this way, we first of all consider the distribution of $X_i = \min\{\gamma_{R,i},\gamma_{D,i}\}$, whose cdf can easily be shown by exploiting the independence of $\gamma_{R,i}$ and $\gamma_{D,i}$ to be given by

$$F_{X_i}(\gamma) = F_{\gamma_{R,i}}(\gamma)+F_{\gamma_{D,i}}(\gamma)-F_{\gamma_{R,i}}(\gamma)\,F_{\gamma_{D,i}}(\gamma). \quad (15)$$

Deriving (15) with respect to γ yields the corresponding pdf

$$p_{X_i}(\gamma) = p_{\gamma_{R,i}}(\gamma)\left[1-F_{\gamma_{D,i}}(\gamma)\right]+p_{\gamma_{D,i}}(\gamma)\left[1-F_{\gamma_{R,i}}(\gamma)\right]. \quad (16)$$

Hence, we get for the cdf of $Y = \max_i X_i$

$$F_Y(\gamma) = \prod_{i=1}^N F_{X_i}(\gamma) \quad (17)$$

and consequently we obtain for the corresponding pdf

$$p_Y(\gamma) = \frac{\partial}{\partial\gamma}F_Y(\gamma) = \sum_{i=1}^N p_{X_i}(\gamma)\prod_{\substack{j=1\\ j\ne i}}^N F_{X_j}(\gamma). \quad (18)$$

The mean mutual information $\bar{I}$ is then generally given by $\bar{I} = \frac{1}{2}\int_0^\infty \log_2(1+\gamma)p_Y(\gamma)d\gamma$. For the important case of Nakagami-m fading on all hops, we can combine (1) and (2) with (15), (16) and (18) and then calculate $\bar{I}$ analytically in closed-form for an arbitrary number of relay stations and arbitrary distribution parameters by making use of the integration result derived in [11, appendix B]. However, since the general expression is very lengthy, it is not explicitly presented here. Instead, we rather consider only the special case of Rayleigh fading on all hops ($m_{R,i} = m_{D,i} = 1 \;\forall\; i$), for which we can derive a more compact form. In this case, we have

$$p_{X_i}(\gamma) = \left(\frac{1}{\bar{\gamma}_{R,i}} + \frac{1}{\bar{\gamma}_{D,i}}\right) e^{-\gamma\left(\frac{1}{\bar{\gamma}_{R,i}}+\frac{1}{\bar{\gamma}_{D,i}}\right)} \quad (19)$$

$$F_{X_i}(\gamma) = 1 - e^{-\gamma\left(\frac{1}{\bar{\gamma}_{R,i}}+\frac{1}{\bar{\gamma}_{D,i}}\right)}. \quad (20)$$

Hence, it can easily be shown that (18) reduces to

$$p_Y(\gamma) = \sum_{i=1}^{N} \sum_{\{k_1,\ldots,k_N\}\in\mathbb{X}_i} (-1)^{\sum_{j=1}^N k_j - 1} \times \left(\frac{1}{\bar{\gamma}_{R,i}} + \frac{1}{\bar{\gamma}_{D,i}}\right) e^{-\gamma \sum_{j=1}^N k_j \left(\frac{1}{\bar{\gamma}_{R,j}}+\frac{1}{\bar{\gamma}_{D,j}}\right)}, \quad (21)$$

where $\mathbb{X}_i$ denotes the set of all index tuples $\{k_1,\ldots,k_N\}$ with $k_j \in \{0;1\}$ for $j \neq i$ and $k_i = 1$. After performing a simple substitution, we then finally obtain with [10, eq. (4.331,2)]

$$\bar{I} = \frac{1}{2\ln 2}\sum_{i=1}^{N} \sum_{\{k_1,\ldots,k_N\}\in\mathbb{X}_i} (-1)^{\sum_{j=1}^N k_j - 1}\left(\frac{1}{\bar{\gamma}_{R,i}} + \frac{1}{\bar{\gamma}_{D,i}}\right) \frac{e^{\sum_{j=1}^N k_j\left(\frac{1}{\bar{\gamma}_{R,j}}+\frac{1}{\bar{\gamma}_{D,j}}\right)}}{\sum_{j=1}^N k_j\left(\frac{1}{\bar{\gamma}_{R,j}}+\frac{1}{\bar{\gamma}_{D,j}}\right)} E_1\left(\sum_{j=1}^N k_j\left(\frac{1}{\bar{\gamma}_{R,j}}+\frac{1}{\bar{\gamma}_{D,j}}\right)\right), \quad (22)$$

where $E_1(\cdot)$ denotes the exponential integral function [10].

The average outage probability in case of perfect CSI of all hops is simply the probability that the best relay channel cannot support the desired information rate R, i.e., we have

$$P_{\text{out}}(R) = \text{Prob}\left[\max_i X_i \leq 2^{2R}-1\right] = F_Y\left(2^{2R}-1\right) \quad (23)$$

with $F_Y(\gamma)$ according to (17).

B. Selection Based on Statistical CSI Only

In practical systems, having perfect CSI of all hops available at the selecting entity seems to be rather unrealistic. If the relay selection is done at the source, for example, this would require significant feedback from both all relay stations as well as the actual destination node. Besides, a selection based on perfect CSI might lead to frequent changes of the utilized relay node and hence result in a considerable protocol overhead. A more viable approach requiring only very low-rate feedback and infrequent changes of the utilized relay therefore is to select a relay only based on knowledge of the SNR distributions of the individual links. In this case, the pdf and cdf of X_i conditioned on the available CSI simply correspond to the unconditional functions according to (15) and (16), respectively. Hence, for maximizing the mean mutual information, we always have to use the relay with maximal $\mathbb{E}[I_i]$ whereas for outage minimization we have to select the relay for which $F_{X_i}(2^{2R}-1)$ is minimal. The mean mutual information and outage probability that can be achieved this way are then given by

$$\bar{I} = \max_i \mathbb{E}[I_i] = \max_i \frac{1}{2}\int_0^\infty \log_2(1+\gamma)p_{X_i}(\gamma)d\gamma \quad (24)$$

$$P_{\text{out}}(R) = \min_i F_{X_i}\left(2^{2R}-1\right). \quad (25)$$

Please note that for Nakagami-m fading on all hops, we can calculate $\mathbb{E}[I_i] = \frac{1}{2}\int_0^\infty \log_2(1+\gamma)p_{X_i}(\gamma)d\gamma$ analytically in closed-form again by using (1), (2), and (16) as well as the integration result from [11, appendix B], yielding to

$$\mathbb{E}[I_i] = \frac{1}{2\ln 2}\left[\sum_{k=0}^{m_{D,i}-1}\sum_{\nu=1}^{k+m_{R,i}} \frac{1}{k!}\frac{\Gamma(k+m_{R,i})}{\Gamma(m_{R,i})} e^{\epsilon_{R,i}+\epsilon_{D,i}} \times\Gamma(\nu-k-m_{R,i},\epsilon_{R,i}+\epsilon_{D,i})\frac{\epsilon_{R,i}^{m_{R,i}}\epsilon_{D,i}^{k}}{(\epsilon_{R,i}+\epsilon_{D,i})^\nu} + \sum_{k=0}^{m_{R,i}-1}\sum_{\nu=1}^{k+m_{D,i}} \frac{1}{k!}\frac{\Gamma(k+m_{D,i})}{\Gamma(m_{D,i})} e^{\epsilon_{R,i}+\epsilon_{D,i}} \times\Gamma(\nu-k-m_{D,i},\epsilon_{R,i}+\epsilon_{D,i})\frac{\epsilon_{D,i}^{m_{D,i}}\epsilon_{R,i}^{k}}{(\epsilon_{R,i}+\epsilon_{D,i})^\nu}\right], \quad (26)$$

with $\Gamma(\cdot,\cdot)$ as the upper incomplete gamma function [10].

C. Selection Based on Perfect and Statistical CSI

As a combination of the previously considered approaches, the relay selection might be done based on perfect CSI of one hop but only statistical CSI of the other one. In this regard, we assume for notational convenience that the relay-to-destination links, i.e., the SNRs $\gamma_{D,i}$, are perfectly known whereas for the source-to-relay links only statistical CSI is available, but the complementary case might be treated in exactly the same way. In general, the conditional cdf of X_i is given in this case by

$$F_{X_i|\text{CSI}}(\gamma|\gamma_{D,i}) = \begin{cases} F_{\gamma_{R,i}}(\gamma), & \text{for } \gamma < \gamma_{D,i} \\ 1, & \text{for } \gamma \geq \gamma_{D,i} \end{cases} \quad (27)$$

and hence we obtain for the corresponding pdf

$$p_{X_i|\text{CSI}}(\gamma|\gamma_{D,i}) = p_{\gamma_{R,i}}(\gamma)\,H(\gamma_{D,i}-\gamma) + \left(1-F_{\gamma_{R,i}}(\gamma_{D,i})\right)\delta\left(\gamma-\gamma_{D,i}\right), \quad (28)$$

where $H(\cdot)$ denotes the Heaviside-function and $\delta(\cdot)$ Dirac's delta function. As before, the optimal relay can then easily be determined by means of the generic expressions according to (9) and (11), respectively. For Nakagami-m fading, we again can calculate the conditional expectation in (9) analytically as

$$\mathbb{E}_{X_i|\text{CSI}}[I_i] = \frac{1}{2\ln 2}\frac{\epsilon_{R,i}^{m_{R,i}}}{\Gamma(m_{R,i})}\mathcal{I}\left(\gamma_{D,i}, m_{R,i}, \epsilon_{R,i}\right) + \frac{1}{2}\left(1-F_{\gamma_{R,i}}(\gamma_{D,i})\right)\log_2(1+\gamma_{D,i}), \quad (29)$$

where we have introduced for brevity the short-hand notation

$$\mathcal{I}(a,b,c) = \int_0^a \ln(1+x)\,x^{b-1}\,e^{-c\,x}dx, \quad (30)$$

with $a, c \in \mathbb{R}^+$, $b \in \mathbb{N}$. This integral can be solved in closed-form by means of repeated partial integration and with the help of [10, eqs. (2.325,1), (3.381,1) and (8.352,1)] (a detailed derivation is omitted here due to space constraints), yielding

$$\mathcal{I}(a,b,c) = \sum_{k=0}^{b-1} \frac{(b-1)!}{k!\, c^{b-k}} \Big[-e^{-ac} \ln(1+a)\, a^k + (-1)^k e^c \times [E_1(c) - E_1(c(1+a))] + \sum_{\nu=1}^{k} \binom{k}{\nu} (-1)^{k-\nu} \times \Gamma(\nu) \sum_{\kappa=0}^{\nu-1} \frac{c^{\kappa-\nu}}{\kappa!} \left(1 - e^{-ca}(1+a)^\kappa\right) \Big], \quad (31)$$

where $E_1(\cdot)$ denotes the exponential integral function again. The calculation of the actual mean mutual information that can be achieved this way seems to be mathematically involved, wherefore it will only be investigated by means of Monte-Carlo simulations in Section IV. For determining the average outage probability that we get by selecting the relay according to (11), we first of all note that if all $\gamma_{D,i}$ are known, we can immediately reduce the potential candidate set by not further considering all relays with $\gamma_{D,i} < 2^{2R} - 1$ and out of the remaining ones, we then simply have to select the one for which $F_{\gamma_{R,i}}\left(2^{2R}-1\right)$ is minimal. Hence, it can easily be seen that we can calculate the average outage probability as

$$P_{\text{out}}(R) = \sum_{\mathcal{A} \cup \bar{\mathcal{A}} = \{1,\ldots,N\}} \prod_{i \in \mathcal{A}} \left[1 - F_{\gamma_{D,i}}\left(2^{2R}-1\right)\right] \times \prod_{j \in \bar{\mathcal{A}}} F_{\gamma_{D,j}}\left(2^{2R}-1\right) \min_{i \in \mathcal{A}} F_{\gamma_{R,i}}\left(2^{2R}-1\right) \quad (32)$$

where the summation has to be taken over all possibilities for partitioning the set of relay indices $\{1,\ldots,N\}$ into two disjoint subsets $\mathcal{A}$ and $\bar{\mathcal{A}}$, where $\mathcal{A}$ always corresponds to the potential candidate set of relays with $\gamma_{D,i} \geq 2^{2R} - 1$ and $\bar{\mathcal{A}}$ is simply the complementary set of $\mathcal{A}$ in $\{1,\ldots,N\}$.

D. Selection Based on Quantized and Perfect CSI

As a tradeoff between the previously considered cases with perfect CSI of all hops and perfect CSI of only the relay-to-destination links, we might design a system where the relay selection is done based on perfect CSI of one hop while for the other one aside from statistical CSI also quantized values of the instantaneous SNR are available. In this case, the signaling load can be directly adjusted by appropriately adjusting the quantizer resolution. For notational convenience, we assume in the following again that the relay-to-destination links are perfectly known whereas for the source-to-relay links only quantized and statistical CSI is available, but as before the complementary case might be treated in exactly the same way.

Generally, we consider an arbitrary scalar P-bit quantizer with $K = 2^P$ different quantization levels. The relays (which themselves are assumed to have perfect knowledge of the associated source-to-relay links) subdivide the complete SNR range into K disjoint sections and always signal only the index of the section containing the instantaneous SNR value on that link to the selecting entity. If we denote this section for the i-th relay by $\Delta_{R,i}$ and the corresponding upper and lower bounds by ζ_i^{up} and ζ_i^{low}, respectively, the conditional cdf of X_i given $\Delta_{R,i}$ and $\gamma_{D,i}$ can easily be shown to be given by

$$F_{X_i|\text{CSI}}(\gamma|\Delta_{R,i},\gamma_{D,i}) = \begin{cases} \frac{F_{\gamma_{R,i}}\left(\max\{\min\{\gamma,\ \zeta_i^{\text{up}}\},\ \zeta_i^{\text{low}}\}\right) - F_{\gamma_{R,i}}(\zeta_i^{\text{low}})}{F_{\gamma_{R,i}}(\zeta_i^{\text{up}}) - F_{\gamma_{R,i}}(\zeta_i^{\text{low}})}, & \gamma < \gamma_{D,i} \\ 1, & \gamma \geq \gamma_{D,i} \end{cases} \quad (33)$$

Deriving (33) with respect to γ, we obtain the conditional pdf

$$p_{X_i|\text{CSI}}(\gamma|\Delta_{R,i},\gamma_{D,i}) = H\left(\gamma - \chi_i^{\text{low}}\right) H\left(\chi_i^{\text{up}} - \gamma\right) \times \frac{p_{\gamma_{R,i}}(\gamma) + \left(F_{\gamma_{R,i}}(\zeta_i^{\text{up}}) - F_{\gamma_{R,i}}(\chi_i^{\text{up}})\right) \delta\left(\gamma - \chi_i^{\text{up}}\right)}{F_{\gamma_{R,i}}(\zeta_i^{\text{up}}) - F_{\gamma_{R,i}}(\chi_i^{\text{low}})}, \quad (34)$$

where we have introduced the short-hand notations

$$\chi_i^{\text{up}} = \min\{\zeta_i^{\text{up}}, \gamma_{D,i}\} \quad (35)$$

$$\chi_i^{\text{low}} = \min\{\zeta_i^{\text{low}}, \gamma_{D,i}\} \quad (36)$$

and with $H(\cdot)$ as the Heaviside step function with $H(0) = 1$.

In case of Nakagami-m fading on all hops, the conditional expectation in (9) can be expressed analytically in closed-form again by combining (34) with (1) and (3) as

$$\mathbb{E}_{X_i|\text{CSI}}[I_i] = \frac{F_{\gamma_{R,i}}(\zeta_i^{\text{up}}) - F_{\gamma_{R,i}}\left(\chi_i^{\text{up}}\right)}{2\left[F_{\gamma_{R,i}}(\zeta_i^{\text{up}}) - F_{\gamma_{R,i}}(\chi_i^{\text{low}})\right]} \log_2(1+\chi_i^{\text{up}}) + \frac{\epsilon_{R,i}^{m_{R,i}}\left[\mathcal{I}\left(\chi_i^{\text{up}}, m_{R,i}, \epsilon_{R,i}\right) - \mathcal{I}\left(\chi_i^{\text{low}}, m_{R,i}, \epsilon_{R,i}\right)\right]}{2\ln 2\,\Gamma(m_{R,i})\left[F_{\gamma_{R,i}}(\zeta_i^{\text{up}}) - F_{\gamma_{R,i}}\left(\chi_i^{\text{low}}\right)\right]} \quad (37)$$

with $\mathcal{I}(\cdot,\cdot,\cdot)$ according to (31). As before, it seems to be hard to calculate the corresponding mean mutual information analytically in closed-form, wherefore it will be investigated only by means of Monte Carlo simulations in Section IV.

An interesting question that arises in this context is how to partition the SNR range in an optimal way. If we want to maximize the mean mutual information, the optimal partitioning depends not only on the SNR distribution of the corresponding source-to-relay link, but also on the distribution of the SNR of the associated relay-to-destination link. Consider for example the special case where $\gamma_{D,i}$ never exceeds a certain upper bound b. Then, it would not make any sense to have more than one quantization section for SNRs $\gamma_{R,i}$ above b since the minimum would be in any case at most b and consequently a finer granularity in this SNR region would not improve the performance. Similar considerations can be made for the more realistic case where $\gamma_{D,i}$ might take on any real value, but the probability that a certain value b is exceeded is very small.

The optimal quantizer for outage minimization actually requires only one bit granularity, because we only have to know whether the SNRs on the source-to-relay links are larger or smaller than $2^{2R}-1$. If we design the quantizer in that way, we always can select a relay which can support the desired rate, provided that there is one such relay. Therefore, the average outage probability with an optimal 1-bit quantizer is exactly the same as for the case of perfect CSI of all hops according to (23). Hence, the feedback load might be drastically reduced without any impact on the outage performance.

E. Selection Based on Quantized and Statistical CSI

Finally, we consider the case where the relay selection is done based on statistical CSI of all relay-to-destination links as well as both statistical and quantized CSI of the corresponding source-to-relay links. This might correspond to a scenario where the relay selection is performed at the source, with a limited feedback channel from the relays and a very low-rate feedback channel from the destination. However, please note that the complementary case can be treated in exactly the same way again, which is therefore not explicitly considered here.

The conditional cdf of X_i given $\Delta_{R,i} = [\zeta_i^{\text{low}}, \zeta_i^{\text{up}})$ can easily be shown to be given in this case by

$$F_{X_i|\text{CSI}}(\gamma|\Delta_{R,i}) = F_{\gamma_{D,i}}(\gamma) + \left(1 - F_{\gamma_{D,i}}(\gamma)\right) \times \frac{F_{\gamma_{R,i}}\left(\max\left\{\zeta_i^{\text{low}}, \min\left\{\gamma, \zeta_i^{\text{up}}\right\}\right\}\right) - F_{\gamma_{R,i}}(\zeta_i^{\text{low}})}{F_{\gamma_{R,i}}(\zeta_i^{\text{up}}) - F_{\gamma_{R,i}}(\zeta_i^{\text{low}})} \quad (38)$$

and hence we get for the corresponding pdf

$$p_{X_i|\text{CSI}}(\gamma|\Delta_{R,i}) = H(\zeta_i^{\text{up}} - \gamma)\left[\frac{H(\gamma - \zeta_i^{\text{low}})}{F_{\gamma_{R,i}}(\zeta_i^{\text{up}}) - F_{\gamma_{R,i}}(\zeta_i^{\text{low}})} \times \left[p_{\gamma_{R,i}}(\gamma)(1 - F_{\gamma_{D,i}}(\gamma)) - p_{\gamma_{D,i}}(\gamma)(F_{\gamma_{R,i}}(\gamma) - F_{\gamma_{R,i}}(\zeta_i^{\text{low}}))\right] + p_{\gamma_{D,i}}(\gamma)\right]. \quad (39)$$

For Nakagami-m fading, we can derive an exact analytical closed-form solution for the conditional expectation in (9) again, which can easily be shown with (39) as well as (1) – (4) to be given by (40) shown at the bottom of this page.

If we want to minimize the outage probability, the optimal quantizer requires only one bit again, since we only have to know whether the SNR on the source-to-relay links is above or below $2^{2R}-1$. Then, we can immediately neglect all relays which would definitely cause an outage due to a poor source-to-relay link and out of the remaining ones, we simply have to select the one with minimal $F_{\gamma_{D,i}}(2^{2R}-1)$. Hence, we have exactly the same situation as with perfect/statistical CSI and therefore the average outage probability corresponds to (32).

IV. Performance Results

The performance of our schemes has been extensively investigated by means of Monte-Carlo simulations and—where available—analytical results. Fig. 1 depicts the mean mutual information over the number of available relay stations for Rayleigh fading links with 20 dB average SNR. For the schemes based on quantized CSI, simple one bit quantizers have been used with equiprobable SNR partitions. Obviously, a single feedback bit is sufficient to achieve almost the same performance as in case that we have perfect CSI of the corresponding hops. Besides, with perfect or quantized/perfect CSI, the mean mutual information can be steadily increased by increasing the number of relay nodes whereas in case of perfect/statistical and quantized/statistical CSI the curves saturate. This is because even if we can always find a relay with a good source-to-relay link, there is an inevitable uncertainty about the quality of the associated relay-to-destination link.

Fig. 2 shows the mean mutual information for a system with two imbalanced relays, where both hops of the first relay have a fixed average SNR of 20 dB whereas the average SNR of both hops of the second relay takes on several different values. Obviously, if one relay has much better hops on average, the performance of all selection strategies is approximately the same, which is reasonable because then virtually always only the better relay is selected in all cases. However, if the average SNRs are comparable, we obtain considerable differences, where the mean mutual information naturally is generally increasing with the quality of the CSI that is available.

The impact of the quantization granularity on the schemes based on quantized CSI is illustrated in Fig. 3. In this regard, the quantizers always split the SNR range into equiprobable partitions and we assume that we have five potential relay nodes. Obviously, if the source-to-relay links have a lower SNR than the relay-to-destination links, a considerable gain

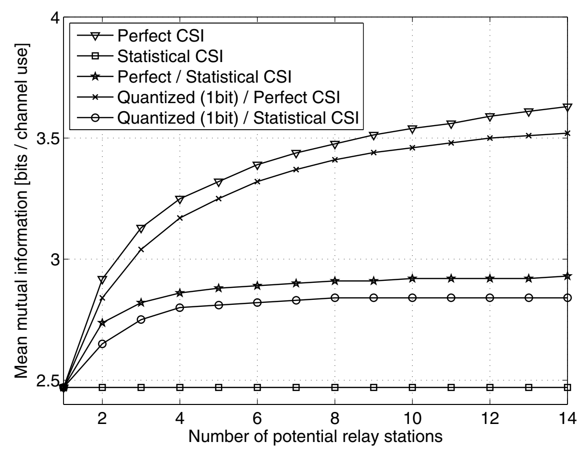

Fig. 1. Impact of the number of potential relay stations on the mean mutual information for $\bar{\gamma}_{R,i} = \bar{\gamma}_{D,i} = 20\,\text{dB}\ \forall\ i$ and Rayleigh-fading on all hops.

$$\mathbb{E}_{X_i|\text{CSI}}[I_i] = \frac{1}{2\ln 2}\left[\sum_{k=0}^{m_{R,i}-1}\left[\frac{\epsilon_{R,i}^{m_{R,i}}\epsilon_{D,i}^{k}}{k!\,\Gamma(m_{R,i})}\left(\mathcal{I}(m_{R,i}+k, \epsilon_{R,i}+\epsilon_{D,i}, \zeta_i^{\text{high}}) - \mathcal{I}(m_{R,i}+k, \epsilon_{R,i}+\epsilon_{D,i}, \zeta_i^{\text{low}})\right)\right.\right.$$
$$+\frac{\epsilon_{D,i}^{m_{D,i}}\epsilon_{R,i}^{k}}{\Gamma(m_{D,i})k!}\left(\mathcal{I}(m_{D,i}+k, \epsilon_{R,i}+\epsilon_{D,i}, \zeta_i^{\text{high}}) - \mathcal{I}(m_{D,i}+k, \epsilon_{R,i}+\epsilon_{D,i}, \zeta_i^{\text{low}}) - \exp\left(-\epsilon_{R,i}\zeta_i^{\text{low}}\right)(\zeta_i^{\text{low}})^k\right.$$
$$\left.\left.\times\left(\mathcal{I}(m_{D,i}, \epsilon_{D,i}, \zeta_i^{\text{high}}) - \mathcal{I}(m_{D,i}, \epsilon_{D,i}, \zeta_i^{\text{low}})\right)\right)\right]\frac{1}{F_{\gamma_{R,i}}(\zeta_i^{\text{up}}) - F_{\gamma_{R,i}}(\zeta_i^{\text{low}})} + \left.\frac{\epsilon_{D,i}^{m_{D,i}}\mathcal{I}\left(m_{D,i}, \epsilon_{D,i}, \zeta_i^{\text{up}}\right)}{\Gamma(m_{D,i})}\right] \quad (40)$$

can be obtained by using 1-4 feedback bits whereas otherwise the performance is virtually independent of the quantizer resolution. This is because then the relay-to-destination links represent the bottleneck, which are not quantized here.

Fig. 4 shows the outage probability for a system with five relays and Rayleigh fading links. Since with optimal 1-bit quantization the outage probabilities for selection based on perfect/quantized as well as statistical/quantized CSI correspond to the ones for selection based on perfect and perfect/statistical CSI, respectively, these curves are not explicitly included here. As can be seen, especially the probability that only very low rates can be achieved can be significantly reduced with better CSI and selection based on perfect/statistical CSI represents a good tradeoff between the other two approaches.

V. Conclusion

We have presented the optimal relay selection strategies for dual-hop transmission with regenerative relays for various different kinds of CSI at the selecting entity. We have focused on two different selection criteria, aiming at either maximizing the mean mutual information or minimizing the outage probability. The selection rules have been given in a generic form based on the fading distributions of the individual links and—where feasible—closed-form expressions for the mean mutual information and average outage probability have been derived for the important case of Nakagami-m fading on all hops. Finally, we have compared the performance of the different schemes by means of numerical as well as simulation results.

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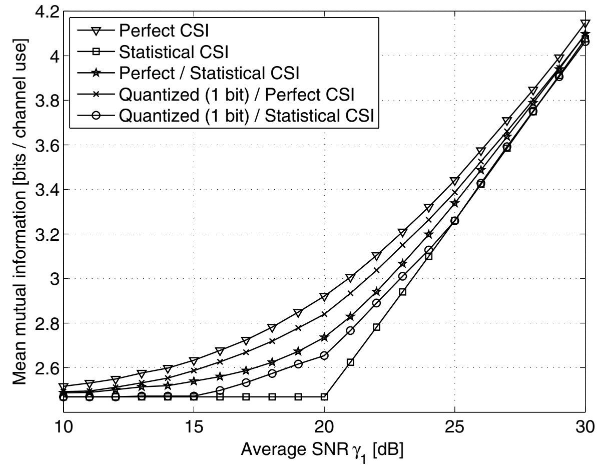

Fig. 2. Mean mutual information for a system with two relays with $\bar{\gamma}_{R,1} = \bar{\gamma}_{D,1} = \gamma_1$, $\bar{\gamma}_{R,2} = \bar{\gamma}_{D,2} = 20\,\text{dB}$, and Rayleigh-fading on all hops.

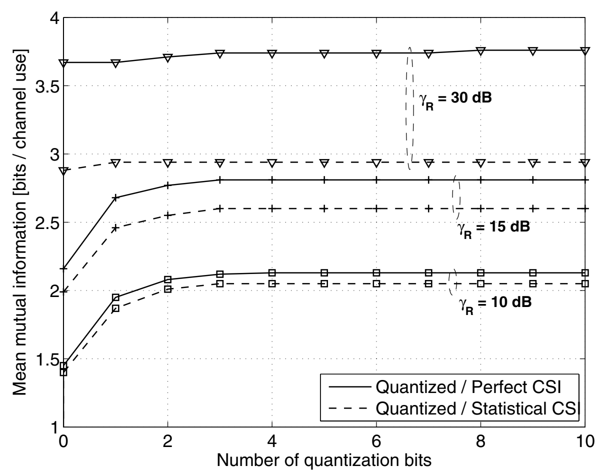

Fig. 3. Mean mutual information vs. the number of quantization bits for $N = 5$, $\gamma_{\bar{D},i} = 20\,\text{dB}\ \forall\ i$, $\gamma_{\bar{R},i} = \bar{\gamma}_R\ \forall\ i$, and Rayleigh-fading on all hops.

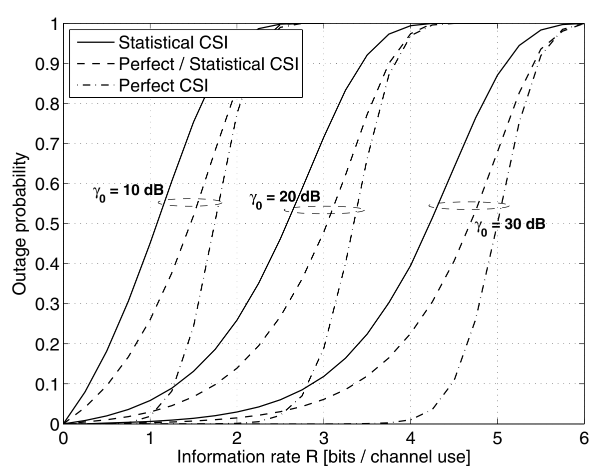

Fig. 4. Information outage probability as a function of the information rate for $N = 5$, $\bar{\gamma}_{R,i} = \bar{\gamma}_{D,i} = \gamma_0\ \forall\ i$ and Rayleigh-fading on all hops.