A Multi-Sensor Based Occupancy Estimation Model forSupporting Demand Driven HVAC Operations

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Abstract

Heating, ventilation, and air conditioning (HVAC) is a major energy consumer in buildings, and. implementing demand driven HVAC operations is a way to reduce HVAC related energy consumption. This relies on the availability of occupancy information, which determines peak/off-hour modes that impact cooling/heating loads of HVAC systems. This research proposes an occupancy estimation model that is built on a combination of nonintrusive sensors that can detect indoor temperature, humidity, $CO₂$ concentration, light, sound and motion. Sensor data is processed in real time using a radial basis function (RBF) neural network to estimate the number of occupants. Field tests carried out in two shared lab spaces for 20 consecutive days report an overall detection rate of 87.62% for self-estimation and 64.83% for crossestimation. The results indicate the ability of the proposed system to monitor the occupancy information of multioccupancy spaces in real time, supporting demand driven HVAC operations.

1. INTRODUCTION

Due to the rising energy demand and diminishing energy resources, sustainability and energy conservation is becoming an increasingly important topic. In the U.S., buildings account for 40% of total energy consumption, 48% of which is consumed by heating, ventilation, and air conditioning (HVAC) systems (DOE 2011). Given the fact that in the U.S., new construction represents only less than three percent of the existing building stock in any

given year (Shelley and Roessner 2004) and that buildings are generally in operation for 30 to 50 years, there is great potential of energy savings through improving the operations of HVAC systems in existing buildings. This has attracted considerable attention in the academia and given rise to active research on this topic. The Building Level Energy Management System (BLEMS) project is such a research effort. The objective of this DOE sponsored project is to study the behavior of buildings and that of building occupants, and to proactively and reactively optimize the building energy consumption while responding to comfort preferences of the occupants. The study reported in this paper was completed within the scope of the BLEMS project as a reactive energy-saving measure, which has a focus on estimating the occupancy to support the implementation of demand driven HVAC operations.

In traditional HVAC operations, the ventilation and conditioning demand is assumed to be at the peak based on maximum occupancy during operational hours, and the temperature and humidity are used as the sole inputs in adjusting the operations, which often results in waste of HVAC related energy consumption (Agarwal et al. 2010). Even with improved HVAC systems that run at different capacities at different times of the day, e.g. minimum capacity at off hours, energy can still be wasted e.g. by over cooling unoccupied spaces. The idea of demand driven HVAC operations is therefore proposed and researched in the academia as a way to address such waste of energy. Demand driven HVAC operations replace the assumption that the ventilation and conditioning demand is at the peak with the actual demand based on real-time sensing of the environment. Previous research has proven that the application of demand driven HVAC operations

could save up to 56% of HVAC related energy consumption (Sun et al. 2011). The actual ventilation and conditioning demands depend on various factors that should be input into HVAC systems for energy-efficient operations, among which fine-grained occupancy information is a key input (Agarwal et al. 2010). Occupancy information enables timely reaction to changing ventilation and conditioning demands, and minimizes energy consumption without compromising the occupant comfort.

Due to the importance of the occupancy information, a number of occupancy detection systems have been proposed in previous research, which reported consequent HVAC energy savings between 10% and 56% based on simulations (Erickson et al. 2009; Erickson and Cerpa 2010; Sun et al. 2011; Tachwali et al. 2007; Warren and Harper 1991). However, these occupancy detection systems have certain limitations with respect to their accuracy, cost, intrusiveness, and privacy, and therefore bear potentials for improvement. This paper proposes an occupancy estimation model that has the following features: (1) affordable. This study uses a number of offthe-shelf low-cost sensors; (2) high-resolution. The proposed model can count the number of occupants at the room level with a sample rate of one reading per minute; (3) accurate,. The proposed model can achieve an accuracy of around 85%; and (4) non-intrusive. The system causes little intrusion to either the buildings or the occupants.

2. PREVIOUS STUDIES

Potential benefits of energy savings by implementing demand driven HVAC operations have motivated research efforts in providing effective occupancy detection solutions. $CO₂$ sensors have been widely used for this purpose (Leephakpreeda et al. 2001; Nielsen and Drivsholm 2010; Sun et al. 2011), as a larger occupancy in a space usually results in higher $CO₂$ concentrations. However, it usually takes some time for the $CO₂$ concentration to build up, and the $CO₂$ concentration is affected by not only occupancy but also other factors such as passive ventilation (e.g. through open windows). Such limitations indicate that the $CO₂$ sensor based systems are unable to provide accurate and real-time occupancy information by themselves. Researchers have also proposed various video based systems (Benezeth et al. 2011; Erickson et al. 2009; Wang et al. 2010), which detect the occupancy in a monitored space by using image-processing techniques. These video based systems generally suffer from the requirement for line of sight in the monitored spaces, which compromises the applicability of these systems especially in heavilypartitioned spaces. Moreover, the use of video cameras usually requires large image storage space, and can cause privacy concerns among users.

To overcome these limitations, researchers have proposed to use a combination of various ambient sensors. Agarwal et al. (2010) used a magnetic reed switch door sensor and a passive infrared (PIR) sensor for occupancy detection, which could report the actual occupancy most of the time. Their occupancy detection algorithm was only applicable for single-occupancy offices, and was built on an assumption that occupants always keep their doors open when they are in the offices or being somewhere nearby. Meyn et al. (2009) used measurements from cameras, PIRs, and $CO₂$ sensors, as well as historical data of building utilization, to estimate the building occupancy level. The estimation was done by solving a receding-horizon convex optimization problem. The reported accuracy was 89%. The system was not able to estimate the number of occupants at the room level, and the error tended to accrue over time. Henze et al. (2006) proposed an occupancy detection system that comprised of three PIRs and one telephone sensor for each room and relied on the belief networks algorithm. The system could detect if any occupant was present with an accuracy of 76%, but was not able to count the number of occupants. Dong et al. (2010) proposed a system that estimated the occupancy of a space by sensing the $CO₂$ concentration, acoustics, and motion in the space. Field tests were carried out in two rooms, with three algorithms including supporting vector machine, artificial neural network, and hidden Markov model. All algorithms yielded an accuracy of around 75%. The authors indicated that the reported accuracy can be further improved. Hailemariam et al. (2011) built an occupancy detection system that used light sensors, motion sensors, $CO₂$ sensors, and sound sensors. Decision trees algorithm was used to estimate the occupancy of cubicles in an office. An accuracy of 98.4% was achieved using the motion sensor alone, and a decline in the accuracy was reported when other sensors were integrated. The system was not configured to count the number of occupants. Melfi et al.

(2011) proposed a novel occupancy detection system that used existing IT infrastructure. Occupants' MAC and IP addresses, and mouse and keyboard activities are monitored for occupancy detection. Accuracies reported in field tests done in two buildings were around 80% at the building level and 40% at the floor level. The system was not able to detect the occupants that do not use a computer. Hutchins et al. (2007) proposed an approach that could recover missing or corrupted sensor data in occupancy estimation. The proposed approach consisted of an inhomogeneous Poison process and a hidden Markov process. The system was not validated with field tests, and was only applicable at the building level.

3. METHODOLOGY

Radial basis function (RBF) neural network is a multidimensional spatial interpolation approach in a neural network, which is based on the local response feature of biological neurons. RBF neural network has a simple and direct training process, as well as rapid learning convergence rates. It has an efficient uniform approximation property for arbitrary, non-linear functions that make the RBF neural network desirable for the application in this study.

The study is conducted for fitting the estimated occupancy and the ground truth. The data is dispersed, and groups of such data $(x_{ii}, y_i)(i = 1, 2, \dots m; j = 1, 2, \dots n)$ are obtained from the BLEMS sensors. Then a suitable and appropriate analytic expression $y = f(x_i, c)$ ($j = 1, 2, ... n$) is used to reflect the relationship between x_i ($j = 1, 2, ... n$) and *y*, which is used to "optimally" approximate the sensor data or fit the ground truth data. The most common way to solve the analytic expression is utilizing parameter selection. For this study, the relationship between the sensor data and occupancy is considered as non-linear, which makes it applicable and necessary to apply the RBF neural network to solve the relationship expression $y = f(x_i, c)$ ($i = 1, 2, ...n$) between sensor data and occupancy ground truth,

RBF neural network is a type of feedforward neural network. The network structure is similar to the multilayer feedforward network, consisting of three layers: the input layer composed of source sensor nodes, the

hidden layer with local responding function, and the output layer for response to input. The RBF neural network uses the radial basis function as the basis for hidden units to establish the hidden layers, which are used to convert low-dimensional inputs to high-dimensional inputs. In this way, a linear inseparable problem in lowdimensional space can be made separable in highdimensional space.

The neuron model of RBF neural network is shown in Figure 1. In this model, the node activation function applies radial basis function, which is always defined as function of Euclidean distance from one arbitrary point to another. Here *x* is the input vector, with the *w as the weight vector; while y is the output vector.*

Figure 1. Neuron model of the RBF neural network.

It can be seen in figure 1 that the activation function of the RBF neural network considers the distances $\left\Vert dist\right\Vert$ between the input vector and the weight vector as independent variables. The general expression of the RBF neural network is:

$$
R\big(||\, dist\, ||\big) = e^{-||\, dist\, ||^2}
$$

In figure 1, b is a threshold used for adjusting the sensitivity of neurons. As the distances between the weight vector and the input vector reduce, the output of the network will increase. When the weight vector and the input vector converge, the output will equal to 1. Therefore, applying the radial basis neuron and the linear neuron can establish a generalized regression neural network for function approximation.

As aforementioned, the general structure of the RBF neural network contains an input layer, a hidden layer, and an output layer, as shown in Figure 2. The input layer is responsible for transmitting signals. The hidden layer and the output layer have different roles in network, so their learning strategies differ from each other. The hidden layer adjusts the parameters of the activation function at a relatively low speed for non-linear optimization, while the output layer adjusts the linear weights at a high learning speed for linear optimization.

Figure 2. General structure of a RBF neural network.

The RBF neural network has three types of parameters to solve: the basis function centers, the variances, and the weights between the hidden layer and the output layer. The basis function centers are always calculated based by the K-means clustering approach: first of all, *h* training samples are randomly selected as the clustering centers c_i $(i = 1, 2, 3, \ldots, h)$. Then group these samples according to the nearest neighbor rule, applying the Euclidean distance between x_p and c_i to assign x_p to different groups $\mathcal{P}_p(p=1,2,...,P)$. The last step is to readjust the clustering centers, by calculating the mean of training samples in each clustering group \mathcal{G}_p . If the new clustering center c_i stays constant, it can be considered as the final basis function center. The second type of parameter is the Variances. As the Gaussian function is utilized as the basis function of the RBF neural network, variances can be generated by the following equation:

 $\mu_i = \frac{c_{\text{max}}}{\sqrt{2h}}, i = 1, 2, ...,$ $\frac{c_{\text{max}}}{\sqrt{n}}$, *i* = 1, 2, ..., *h h* $\sigma_i = \frac{c_{\text{max}}}{\sqrt{2i}}$, $i = 1, 2, ..., h$, in which c_{max} is the

maximum distance between the clustering centers. The third type of parameter is the Weights between the hidden layer and the output layer, reachable by the least square

method:
$$
w = \exp(\frac{h}{c_{\max}^2} ||x_p - c_i||^2)
$$

where $i = 1, 2, \dots h$; $p = 1, 2, 3, \dots P$

In this study, Matlab was utilized to realize the RBF neural network. Three steps were followed The first step was to design an approximate radial basis function network, this is a trial process of adding the number of neurons in a hidden layer until the output error satisfies the preset value; The next step is to develop an exact radial basis function network based on the input vectors and expansion velocity, compared to the first step, this step requires a quick and error free generation of radial basis function; and the last step is to calculate the returned matrix from the input matrix handled by the radial basis function, compare it with ground truth and finally acquire the error rate.

4. TEST SETUP

BLEMS sensor nodes are built and used in the tests (Figure 3). Each sensor node consists of an Arduino Black Widow stand-alone single-board microcontroller computer with integrated support for 802.11 WiFi. Mounted close to the door at a height of about 1.5 m, each sensor node includes the following sensors: a light sensor, a sound sensor, a motion sensor, a $CO₂$ sensor, a temperature sensor, a relative humidity sensor, and a PIR sensor that detects objects as they pass through the door. A script is written and uploaded to the sensor node using Arduino to configure the microcontroller to process the raw data. The processed data reported by the sensor node include 11 variables, which can be categorized into three types: instant variables that show the instant output of a sensor at the time the data is queried, including *lighting, sound, motion, CO2 concentration*, *temperature, relative humidity,* and *reflector* (infrared); count variables that sum the number of times a sensor's output changes in the last minute, including *motion count* and *reflector count*; average variables that show the average value of a sensor's output over a certain period of time, including *sound average* (5 seconds) and *long sound average* (5 minutes). The data is automatically queried every one minute, time stamped, and stored in an SQL database.

Figure 3. BLEMS sensor node.

Figure 4. Mobile device used to collect occupancy ground truth.

Figure 5. BLEMS test bed layout.

The BLEMS project uses a three-story educational building at the University of Southern California as the test bed. A total of 50 BLEMS sensors are planned to be deployed in the building. Figure 5 shows the deployment on the third floor. In order to train the occupancy estimation model to be used in the test bed building, this study installed two sensor nodes in two multi-occupancy labs and collected sensor data and occupancy data for a period of 20 days. Lab 1 has an area of about 40 m^2 , and is shared by 5 PhD students. The lab hosts meetings at times, which can involve up to 10 attendees. Lab 2 has a similar size and is shared by up to 8 PhD students. A touch-screen mobile device is mounted close to the door

in both labs to collect the ground truth of the occupancy (Figure 4). During the test period, all occupants are told to log in or out when they enter or leave the lab using the mobile device. The ground truth data is then sent to and stored in the same database where the sensor data resides. In lab 2, a camera is also installed to validate the collected ground truth by manual random spot-checking.

The sensor data was collected for 20 consecutive days, starting from 00:00 AM, Sep. 12th to 00:00 AM, Oct. 1st. At a one-minute sampling rate, after excluding all corrupted data points due to wireless connection breaks, a total of 25,898 data points were collected in both labs. The data is divided into four groups (G1, G2, G3, G4) as shown in Table 1.

Four tests are carried out using different groups of sensor data (Table 2). In order to implement selfestimation, in tests 1 and 2, the model is trained, validated and tested using sensor data from the same lab. In order to implement cross-estimation, in tests 3 and 4, the model is trained and validated using sensor data from one lab, and tested using the sensor data from the other lab.

	Training $+$ Validation	Testing
Test 1	G1	G ₂
Test 2	G3	G ₄
Test 3	G1.G2	G3.G4
Test 4	G3.G4	G1.G2

Table 1: Group of sensor data.

Table 2: Test design.

5. TEST RESULTS

Two parameters are defined to evaluate the results. The first one is the root mean square error (RMSE) of the results, which measures the deviation of the estimated occupancy from the actual occupancy. The second parameter is the error rate, which shows the accuracy of all validated data. The concept of tolerance is also introduced. Tolerance measures the tolerated error between the estimated and the actual occupancy. Tolerance is necessary in that for the purpose of driving HVAC systems, a small error can be acceptable, and the HVAC systems do not need to be adjusted every time the occupancy slightly changes. Due to the initialization random feature, for each step, five experiments were carried out and the best one was chosen for analysis. The following results are all based on tolerance =1.

5.1. Self-estimation

Test 1 adopted all the data from lab 1 for training, validation, and testing. The test yielded an RMSE of 1.202 and error rate of 11.26%, or an accuracy of 88.74%. Test 2 used the data from lab 2 and resulted in an accuracy of 86.50%, with an RMSE of 1.499. To better compare the estimated output and the ground truth to visualize the differences between them, both the estimated occupancy (rounded) data and the ground truth occupancy data are depicted in Figure 6 (test 1) and Figure 7 (test 2). The test results also show that the events when the space switched from occupied to unoccupied or vice versa could be detected by the model 82.35% of the time and 70.13% of the time for test 1 and test 2, respectively.

Figure 6. Estimation result for test 1.

Figure 7. Estimation result for test 2.

5.2. Cross Estimation

Tests 3 and 4 are cross estimation results, where data from one lab is applied for training and validation, and data from the other lab is used for testing. Test 3 utilized the model, which was trained using the data from lab 1 to estimate the occupancy in lab 2. The RMSE was 2.310 and the error rate was 33.57%, or an accuracy of 66.43%. Test 4 utilized the model which was trained using the data from lab 2 to estimate the occupancy in lab 1. The error rate was 36.77%, and the RMSE was 2.743. To better compare the estimated output and the ground truth data to visualize the differences between them, both estimated occupancy (rounded) data and ground truth occupancy data are shown on Figure 8 (test 3) and Figure 9 (test 4). The test results also show that the events when the space switched from occupied to unoccupied or vice versa could be detected by the model 71.21% of the time and 77.04% of the time for test 3 and test 4, respectively.

Figure 8. Estimation result for test 3.

Figure 9. Estimation result for test 4.

6. DISCUSSION AND CONCLUSIONS

The self-estimation test results show that the proposed model can yield accurate estimates of the number of occupants (tolerance=1) 88.74% and 86.50% of the time for lab 1 and lab 2, respectively. The tolerance is necessary because the estimated occupancy is given by the model in a decimal format, which needs to be rounded to compare with the ground truth. The rounding process causes additional errors and need to be offset by the tolerance. In addition, when used for demand driven HVAC operations, a certain level of error is fairly acceptable, as the HVAC systems do not need to be so sensitive that they respond to any slight changes in occupancy. Instead, adding or subtracting one occupant in a room shouldn't cause significant changes in HVAC operations, unless the room switches form unoccupied to occupied or vice versa.

The cross-estimation tests yield an accuracy of 66.43% when the model is trained and validated in lab 1 and tested in lab 2, or 63.23% the other way round. Compared with the self-estimation results, a decline in the accuracies indicates that possibility of having a universal occupancy estimation model, which is trained in one space and used in other spaces, is limited by certain constraints. One such constraint is the differences in environmental settings. For example, there is no window hence no natural lighting in lab 1, and the artificial lighting is always off whenever the lab is unoccupied. Therefore, that lighting sensor reading is zero always indicates the lab is unoccupied. However, the door in lab 2 is always open, and the lighting sensor reading is always positive even late at night, due to the lighting in the corridor. Another constraint lies in the fact that the temperature sensors, humidity sensors and $CO₂$ sensors used in this study are not calibrated. Therefore, a consistent $CO₂$ sensor reading of 0.20 may be associated with 3 occupants in lab 1 but 6 occupants in lab 2. For the building level estimation, the authors plan to classify test bed spaces based on their characteristics and implement a cross-estimation model for each category of space. Lastly, the chosen test bed differs from regular office spaces in terms of the behaviors of occupants (PhD students vs. office workers). Therefore the model obtained from this study needs to be calibrated or re-measured before it is applied to office spaces, though the approach used in this study will remain valid.

The proposed system is low-cost and high-resolution. The sensor node prototype cost about \$230 USD, and will be even lower if mass-produced. As respect to resolution, the proposed system can provide the occupancy information at the room level, and indicate the exact number of occupants, which can all be done instantly upon users' request.

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