

An Opportunistic-Based Protocol for Bidirectional Cooperative Networks

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Abstract—In this paper, a new *opportunistic source selection (OSS)* protocol is studied in bidirectional cooperative networks. Unlike existing protocols, this protocol exploits multiuser nature of the bidirectional cooperative networks and it opportunistically supports two traffic flows based on instantaneous channel conditions. This makes the OSS protocol much more reliable than existing protocols. In order to show the performance improvement, we first derive a lower bound of the outage probability of the OSS protocol. Numerical results demonstrate that this lower bound is extremely tight and it indicates that the OSS protocol achieves full diversity order two in a bidirectional cooperative network with two sources and one relay. Then exact and approximate lower bounds of average bit error rates (BERs) at both sources in the OSS protocol are derived. Those lower bounds are very close to the exact average BERs as shown by numerical results. Lastly, an optimum power allocation scheme is developed for the OSS protocol. This scheme can optimize the outage probability, average BER, and data-rate of the OSS protocol at the same time.

Index Terms—Cooperative systems, multiuser systems, diversity methods, fading channels, space-time codes.

I. INTRODUCTION

IN traditional unidirectional cooperative networks, several relays assist in the communication between one source and one destination in order to achieve spatial diversity [1]. However, the half-duplex constraint of every terminal induces a severe loss of bandwidth efficiency as demonstrated by a pre-log factor $1/2$ in the data-rate expression. In order to overcome this difficulty, bidirectional cooperative networks were studied in many publications including [2], where two sources exchanged information with the help of several relays. As a result, there were two traffic flows in a bidirectional cooperative network and they were supported by the same physical channels concurrently. Although each traffic flow still had the pre-log factor $1/2$ in its data-rate expression, the total data-rate of the network, which was the summation of the data-rates of both traffic flows, no longer suffered from the pre-log factor $1/2$. Therefore, bidirectional cooperative networks might have higher bandwidth efficiency than unidirectional cooperative networks.

Manuscript received November 28, 2008; revised February 10, 2009 and May 7, 2009; accepted May 12, 2009. The associate editor coordinating the review of this paper and approving it for publication was D. Reynolds.

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This research is supported in part by the Ubiquitous Computing and Network(UCN) Project, Knowledge and Economy Frontier R&D Program of the Ministry of Knowledge Economy (MKE) in Korea as a result of UCN's subproject 09C1-C2-12T.

Digital Object Identifier 10.1109/TWC.2009.081580

Recently, many novel protocols were studied in the context of bidirectional cooperative networks, such as *physical layer network coding (PNC)* [3]–[5], *analog network coding (ANC)* [6]–[8], and *time division broadcast (TDBC)* [3], [8]–[11]¹. Those protocols successfully improved the bandwidth efficiency of the cooperative networks, because they concurrently support two traffic flows by the same physical channels. Due to the same reason, however, the reliability of the networks is contaminated. For example, the PNC and ANC protocols can not utilize the direct channel between the two sources. Thus, those two protocols can only achieve diversity order one and may not perform very well at high signal-to-noise ratio (SNR) range [8]. On the other hand, although the TDBC protocol can achieve full diversity order two [8]–[10], we notice that the average bit error rate (BER) of one source is much higher than that of the other one, when the relay is not located at the center between the two sources. This may limit the practical implementation of the TDBC protocol, because we intend to provide uniform Quality of Service (QoS) to every user in communication networks.

In this paper, we propose an *opportunistic source selection (OSS)* protocol for the bidirectional cooperative networks in order to improve the reliability of the networks. Unlike the PNC, ANC, and TDBC protocols, the proposed OSS protocol exploits the multiuser nature of the bidirectional cooperative networks and it supports two traffic flows in an opportunistic fashion based on instantaneous channel conditions. In order to show the reliability of the proposed OSS protocol, we analyze its outage probability and average BER, which are two widely-used performance metrics for cooperative networks [1], [12]. Specifically, we first derive a lower bound of the outage probability of the OSS protocol. Irrespective of the values of channel variances and average SNR, numerical results demonstrate that this lower bound is extremely tight to the exact outage probability. Based on this lower bound, the diversity order of the OSS protocol is also investigated. Then we derive the exact lower bounds of the average BERs at both sources in the OSS protocol. Those exact lower bounds are very tight to the exact average BERs as demonstrated by numerical results. Although they are not given in closed form, it is not hard to calculate them. Moreover, approximate lower bounds are also derived in truly closed form. Lastly, an optimum power allocation scheme is developed for the OSS protocol. This scheme simultaneously minimizes the outage probability and maximizes the data-rate of the OSS protocol. It also minimizes the average BERs at both sources.

¹Note that the TDBC protocol was called *straightforward network coding* scheme in [3].

The rest of this paper is organized as follows. Section II describes the system model of the OSS protocol. In Section III, we derive a lower bound of the outage probability of the OSS protocol and analyze the diversity order. In Section IV, the exact and approximate lower bounds of the average BERs at both sources are presented. Lastly, an optimum power allocation scheme is developed in Section V. Section VI presents some numerical results and Section VII concludes this paper.

II. SYSTEM MODEL

We consider a bidirectional cooperative network with two sources and one relay, where the sources intend to exchange information with the help of the relay. We use S_1 , S_2 , and R to denote the first source, the second source, and the relay, respectively. Every terminal has only one antenna and is half-duplex. The relay works in the amplify-and-forward mode. We assume that all wireless channels in this bidirectional cooperative network are reciprocal. Specifically, let g represent the fading coefficient of the channel between S_1 and S_2 , h the channel between S_1 and R , and f the channel between R and S_2 . Furthermore, we assume that g , h , and f are complex Gaussian random variables with zero mean and variances Ω_g , Ω_h , and Ω_f , respectively. The additive noise associated with every channel is a complex Gaussian random variable with zero mean and unit variance. Let γ_1 denote the instantaneous received SNR of the signals transmitted by S_2 , relayed by R , and received by S_1 . For example, if S_2 transmits with power E_s and R relays with power E_r as in a traditional unidirectional cooperative network, the instantaneous SNR γ_1 is given by [12], [19]

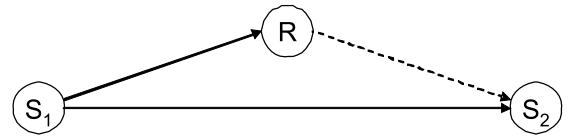
$$\gamma_1 = E_s |g|^2 + \frac{E_s E_r |h f|^2}{E_s |f|^2 + E_r |h|^2 + 1}. \quad (1)$$

Similarly, let γ_2 denote the instantaneous received SNR of the signals transmitted by S_1 , relayed by R , and received by S_2 . For example, if S_1 transmits with power E_s and R relays with power E_r as in a traditional unidirectional cooperative network, the instantaneous SNR γ_2 is given by

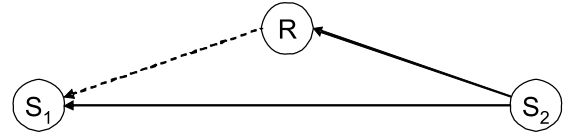
$$\gamma_2 = E_s |g|^2 + \frac{E_s E_r |h f|^2}{E_s |h|^2 + E_r |f|^2 + 1}. \quad (2)$$

Since the two sources intend to exchange information, there are two traffic flows in this bidirectional cooperative network. One is from S_1 via R to S_2 and the other is from S_2 via R to S_1 . Each traffic flow can be seen as a traditional unidirectional cooperative network. For example, in the first traffic flow, S_1 is the transmitter, and R and S_2 are the receivers. Hence, it is reasonable to assume that R knows h and S_2 knows h , f , and g as in the conventional unidirectional cooperative networks [1], [12]. Similarly, due to the existence of the second traffic flow, it is reasonable to assume that R knows f and S_1 knows h , f , and g . In all, we assume that the two sources know h , f , and g , and the relay knows h and f as in many previous publications [2], [13]–[15]. In order to achieve this channel state information (CSI) assumption, a possible pilot signalling scheme can be as follows. At first, every terminal broadcasts one pilot symbol to all other terminals. This makes S_1 know g and h , S_2 know g and f , and R know h and f .

If S_2 has a larger received SNR, i.e. $\gamma_2 > \gamma_1$



If S_1 has a larger received SNR, i.e. $\gamma_1 > \gamma_2$



————— Time slot 1 - - - - - Time slot 2

Fig. 1. System model of the OSS protocol.

Then R uses one pilot symbol to transmit f to S_1 and uses another pilot symbol to transmit h to S_2 . In all, five pilot symbols are needed to achieve our CSI assumption.

A bidirectional cooperative network can be seen as a multiuser network, because the two sources are actually two different users. We propose the OSS protocol which exploits the multiuser diversity [16] inherent in any multiuser systems.

OSS Protocol

- 1) At the start of every two time slots, the instantaneous SNR γ_1 is calculated at S_1 and S_2 by using (1). Also, the instantaneous SNR γ_2 is calculated at S_2 and S_1 by using (2).²
- 2) If $\gamma_2 > \gamma_1$, only S_1 transmits to S_2 with the help of R in the next two time slots. That is, an information-bearing symbol of S_1 is transmitted to S_2 as in a traditional unidirectional cooperative network. If $\gamma_1 > \gamma_2$, only S_2 transmits to S_1 with the help of R in the next two time slots. That is, an information-bearing symbol of S_2 is transmitted to S_1 as in a traditional unidirectional cooperative network.³

The proposed OSS protocol is depicted in Fig. 1. Unlike the PNC, ANC, and TDBC protocols in [3]–[11] where two traffic flows are supported concurrently, the OSS protocol supports two traffic flows opportunistically based on instantaneous channel conditions. This is a fundamental difference between the OSS protocol and the PNC, ANC, and TDBC protocols. This difference actually makes the OSS protocol have a much higher reliability than the other protocols.

Note that, although the OSS protocol is proposed for a bidirectional cooperative networks with two sources and one relay in this paper, it is very easy to extend it to more complicated cooperative networks. The essence of the OSS protocol is to choose only one source from all the sources which are intending to transmit information. The choice of this source should be based on instantaneous channel conditions and should generate the optimum performance. Therefore, as long as several sources intend to exchange information

²Since the two sources know the exact values of h , f , and g , both of them can calculate γ_1 and γ_2 .

³Certainly, there is a fairness problem between the two sources and it actually can be solved by the proportional fairness algorithm [17]. However, this topic is beyond the scope of this paper.

in a cooperative network, the OSS protocol can be applied. However, it may be hard to analyze the performance of the OSS protocol when it is used in more complicated cooperative networks. Furthermore, more pilot symbols are needed to implement the OSS protocol in such cooperative networks.

III. OUTAGE PROBABILITY AND DIVERSITY ORDER

When powerful channel coding is implemented, outage probability can be used to evaluate the performance of cooperative networks [1]. In this section, we first derive a lower bound of the outage probability of the OSS protocol. Based on this lower bound, we also show that the OSS protocol achieves the full diversity order two in a bidirectional cooperative network with two sources and one relay.

For the OSS protocol, the data-rates R_1 at S_1 and R_2 at S_2 are given by

$$R_1 = \frac{1}{2} \log_2(1 + \gamma_1), \quad (3)$$

$$R_2 = \frac{1}{2} \log_2(1 + \gamma_2). \quad (4)$$

The pre-log factor 1/2 is because the OSS protocol uses two time slots to accomplish the information exchange [2]. Since the two traffic flows happen opportunistically, the outage probability of the OSS protocol is defined by

$$P_{\text{outage}}(R) = \Pr(R_1 < R, \gamma_1 > \gamma_2) + \Pr(R_2 < R, \gamma_2 > \gamma_1) \quad (5)$$

$$= \Pr(R^{\text{OSS}} < R), \quad (6)$$

where R is a pre-determined target rate. Moreover, R^{OSS} can be seen as the data-rate of the OSS protocol and it is given by

$$R^{\text{OSS}} = \frac{1}{2} \log_2(1 + \max(\gamma_1, \gamma_2)). \quad (7)$$

It is very hard to obtain the exact outage probability $P_{\text{outage}}(R)$, because γ_1 and γ_2 both have complicated expressions as shown in (1) and (2). Thus, we try to find a lower bound of $P_{\text{outage}}(R)$ by using the following well-known inequality [12]:

$$\frac{xy}{x+y+1} < \frac{xy}{x+y} < \min(x, y). \quad (8)$$

A lower bound of the outage probability of the OSS protocol is derived in the following theorem.

Theorem 1: When $E_s \geq E_r$, the outage probability $P_{\text{outage}}(R)$ of the OSS protocol can be lower-bounded by $P_{\text{outage}}(R) > P_{\text{outage}}^{L,1}(R)$, where $P_{\text{outage}}^{L,1}(R)$ is given by:

$$\begin{aligned} & P_{\text{outage}}^{L,1}(R) \\ &= A_1 \left(\frac{E_s}{2E_r} \Omega_h, \Omega_f, \frac{E_s}{2E_r} \Omega_g, \frac{2^{2R}-1}{E_r} \right) \\ &+ A_1 \left(\frac{E_s}{2E_r} \Omega_f, \Omega_h, \frac{E_s}{2E_r} \Omega_g, \frac{2^{2R}-1}{E_r} \right) \\ &+ A_2 \left(\Omega_h, \Omega_f, \frac{E_s}{2E_r} \Omega_f, \frac{E_s}{2E_r} \Omega_g, \frac{2^{2R}-1}{E_r} \right) \\ &+ A_2 \left(\Omega_f, \Omega_h, \frac{E_s}{2E_r} \Omega_h, \frac{E_s}{2E_r} \Omega_g, \frac{2^{2R}-1}{E_r} \right). \quad (9) \end{aligned}$$

The functions $A_1(x, y, z, v)$ and $A_2(x, y_1, y_2, z, v)$ are defined as

$$\begin{aligned} & A_1(x, y, z, v) \\ &= \frac{y}{(y+2x)(2xz+yz-xy)} (2xz+yz-xy \\ &\quad - (2xz+yz)e^{-\frac{v}{2z}} + xye^{-v(\frac{1}{2x}+\frac{1}{y})}), \quad (10) \end{aligned}$$

$$\begin{aligned} & A_2(x, y_1, y_2, z, v) \\ &= \frac{xy_1}{(x+y_1)(xz+2y_2z-xy_2)(2xz+2y_1z-xy_1)} \\ &\quad \times \frac{1}{x+2y_2} ((xz+2y_2z-xy_2)(2xz+2y_1z-xy_1) \\ &\quad - 2(x+2y_2)(x+y_1)z^2e^{-\frac{v}{2z}} \\ &\quad - y_1(x+2y_2)(xz+2y_2z-xy_2)e^{-v(\frac{1}{x}+\frac{1}{y_1})} \\ &\quad + 2y_2(x+y_1)(2xz+2y_1z-xy_1)e^{-v(\frac{1}{x}+\frac{1}{2y_2})}). \quad (11) \end{aligned}$$

When $E_r \geq E_s$, the outage probability $P_{\text{outage}}(R)$ can be lower-bounded by $P_{\text{outage}}(R) > P_{\text{outage}}^{L,2}(R)$, where

$$\begin{aligned} & P_{\text{outage}}^{L,2}(R) \\ &= A_1 \left(\frac{E_r}{2E_s} \Omega_h, \Omega_f, \frac{\Omega_g}{2}, \frac{2^{2R}-1}{E_s} \right) \\ &+ A_1 \left(\frac{E_r}{2E_s} \Omega_f, \Omega_h, \frac{\Omega_g}{2}, \frac{2^{2R}-1}{E_s} \right) \\ &+ A_2 \left(\Omega_h, \Omega_f, \frac{E_r}{2E_s} \Omega_f, \frac{\Omega_g}{2}, \frac{2^{2R}-1}{E_s} \right) \\ &+ A_2 \left(\Omega_f, \Omega_h, \frac{E_r}{2E_s} \Omega_h, \frac{\Omega_g}{2}, \frac{2^{2R}-1}{E_s} \right). \quad (12) \end{aligned}$$

Proof: See Appendix A. ■

Numerical results will demonstrate that the lower bounds $P_{\text{outage}}^{L,1}(R)$ and $P_{\text{outage}}^{L,2}(R)$ are extremely tight to the exact outage probability $P_{\text{outage}}(R)$, irrespective of the values of channel variances and average SNR. Therefore, based on this lower bound, we derive the diversity order of the OSS protocol in the following corollary.

Corollary 1: The OSS protocol achieves the full diversity order two in a bidirectional cooperative network with two sources and one relay.

Proof: For simplicity, we assume that $E_s = E_r = E$ in this proof and note that this assumption does not affect the analysis of the diversity order [19]. When E is very large, we have the following result

$$\lim_{E \rightarrow \infty} P_{\text{outage}}^{L,1}(R) = \frac{C_1(R)}{E^2} + \mathcal{O}\left(\frac{1}{E^3}\right). \quad (13)$$

where $C_1(R)$ is a function of R and it is always positive. The second term $\mathcal{O}(1/E^3)$ means that it decreases with E as fast as $1/E^3$. Therefore, the diversity order of the OSS protocol is exactly two. ■

Corollary 1 indicates that the diversity order of the OSS protocol is two, which is because the direct channel g between S_1 and S_2 is fully utilized. In fact, the diversity order of the TDBC protocol is two as well. Unlike the TDBC protocol, however, the OSS protocol does not try to support two traffic

flows concurrently. Instead, the traffic flows are supported opportunistically depending on instantaneous channel conditions in the OSS protocol. Therefore, as will be demonstrated in Section VI, the outage probability of the OSS protocol is actually much lower than that of the TDBC protocol, although they have the same diversity order. This means the OSS protocol indeed improves the reliability of the bidirectional cooperative networks. Moreover, the PNC and ANC protocols can only achieve diversity order one [8]. Thus, they must have higher outage probabilities than the OSS protocol at moderate and high SNR range. This will be demonstrated in Section VI as well.

Although the OSS protocol has a very high reliability, its data-rate or throughput may not be as good as the TDBC, PNC, and ANC protocols. This is because the OSS does not try to support two traffic flows concurrently; while the TDBC, PNC, and ANC protocols always support two traffic flows concurrently. As a result, if one bidirectional cooperative network is designed to achieve very high data-rate or throughput of information exchange, the TDBC, PNC, and ANC protocols may be used; while if this network has a very strict requirement on QoS, the OSS protocol may be a better choice.

Furthermore, we notice that, in the OSS protocol, one source has a higher data-rate than the other source when the distances from the relay to the two sources are different. This is because the source with a better channel link has more opportunities to transmit its own information-bearing symbols. For example, in Fig. 9, we set $E_s = 2E_r$ and we see that the data-rate of one source becomes lower when the relay is closer to that user. In fact, this can be considered as a special property of the OSS protocol and one can actually make use of this property. It is practically possible that the two sources have different amount of data to exchange. For instance, it is possible that S_1 is a base station and S_2 is a mobile terminal. In this case, S_1 may have much more data to transmit, and hence, may need a much higher data-rate than S_2 . The OSS protocol can address this issue easily by choosing a particular relay node that is close to S_2 .

When the two sources have about the same amount of data to exchange, there are a few ways to address the fairness issue of the OSS protocol. One possibility is to properly choose a relay (out of multiple candidate relays) which has similar distances to the two sources. If such relay does not exist, then a proper scheduling scheme proposed for multiuser systems can be used. For example, we can easily implement the proportional fairness algorithm proposed in [17] to achieve fairness between the two sources. Another method to address the fairness issue when an equidistant relay node is not available is to use different sizes of constellations with different transmission powers at the two sources in order to make the two sources to have similar data-rates.

IV. LOWER BOUND OF AVERAGE BIT ERROR RATE AT EACH SOURCE

Another important and commonly-used performance metric for cooperative networks is average error rate [12]. In this section, we analyze the average BERs of both sources in

the OSS protocol. We first derive exact lower bounds of the average BERs. Numerical results demonstrate that those lower bounds are very tight, irrespective of the values of channel variances and average SNR. Moreover, each of them contains only one integration over a finite range, and hence, it is not hard to calculate. In order to further reduce the computational complexity, we also present approximations of those exact lower bounds. Those approximate lower bounds are given in closed form and they are also very close to the exact average BERs as demonstrated by numerical results.

For the OSS protocol, when M -QAM modulation is used, the exact conditional BER, conditioned on instantaneous channel coefficients, at S_1 is given by $P_b(\gamma_1)$, where the function $P_b(x)$ is defined as [20]⁴

$$P_b(x) = \frac{2}{\sqrt{M} \log_2 M} \sum_{j=1}^{\log_2 \sqrt{M}} \left[\sum_{i=0}^{(1-2^{-j})\sqrt{M}-1} A_{j,i}(M) \times Q \left((2i+1) \sqrt{\frac{3x}{M-1}} \right) \right]. \quad (14)$$

In (14), the coefficient $A_{j,i}(M)$ is defined as

$$A_{j,i}(M) = (-1)^{\lfloor i2^{j-1}/M \rfloor} \left(2^{j-1} - \left\lfloor 2^{j-1}i/\sqrt{M} + 1/2 \right\rfloor \right) \quad (15)$$

Due to the opportunistic selection in the OSS protocol, the average BER $P_{b,1}$ at S_1 is given by $P_{b,1} = \mathbb{E}[P_b(\gamma_1)|\gamma_1 > \gamma_2]$. Note that, although $P_{b,1}$ is measured at S_1 , it is the average BER of the signals transmitted by S_2 , relayed by R, and received by S_1 . The exact conditional BER at S_2 is given by $P_b(\gamma_2)$ and the average BER $P_{b,2}$ equals to $P_{b,2} = \mathbb{E}[P_b(\gamma_2)|\gamma_2 > \gamma_1]$. In order to perform the expectations, one needs the conditional moment generating functions (MGFs) of γ_1 and γ_2 . However, those MGFs are very hard to obtain due to the complicated expressions of γ_1 and γ_2 . Therefore, we try to find lower bounds of $P_{b,1}$ and $P_{b,2}$ by using (8). We first present the following lemma and it is crucial to analyze the average BERs.

Lemma 1: Assume X , Y , and Z are mutually independent exponential random variables with means Ω_X , Ω_Y , and Ω_Z , respectively. Let $U_1 = \min(C_s Y, C_r X)$, $U_2 = \min(C_s X, C_r Y)$, $W_1 = U_1 + C_s Z$ and $W_2 = U_2 + C_s Z$. When $C_s \geq C_r$, the conditional MGF $\text{MGF}_{W_1|W_1>W_2}(s)$ of W_1 , conditioned on $W_1 > W_2$, is given by $\text{MGF}_{W_1|W_1>W_2}(s) = T_1(s; \Omega_X, \Omega_Y, \Omega_Z, C_s, C_r)$, where the function $T_1(s; x, y, z, u, v)$ is defined as

$$T_1(s; x, y, z, u, v) = \left(\frac{x+y}{x} \frac{\frac{1}{vx} + \frac{1}{uy}}{\frac{1}{vx} + \frac{1}{uy} - s} - \frac{y}{x} \frac{\frac{1}{vx} + \frac{1}{vy}}{\frac{1}{vx} + \frac{1}{vy} - s} \right) \left(\frac{\frac{1}{uz}}{\frac{1}{uz} - s} \right). \quad (16)$$

When $C_r \geq C_s$, the conditional MGF $\text{MGF}_{W_1|W_1>W_2}(s)$ of W_1 , conditioned on $W_1 > W_2$, is given by

⁴If other modulation schemes are used, the conditional BER can be obtained by using [20] as well.

MGF $_{W_1|W_1>W_2}(s) = T_2(s; \Omega_X, \Omega_Y, \Omega_Z, C_s, C_r)$, where the function $T_2(s; x, y, z, u, v)$ is defined as

$$T_2(s; x, y, z, u, v) = \left(\frac{x+y}{y} \frac{\frac{1}{vx} + \frac{1}{uy}}{\frac{1}{vx} + \frac{1}{uy} - s} - \frac{x}{y} \frac{\frac{1}{ux} + \frac{1}{uy}}{\frac{1}{ux} + \frac{1}{uy} - s} \right) \left(\frac{\frac{1}{uz}}{\frac{1}{uz} - s} \right). \quad (17)$$

Proof: See Appendix B. ■

Lemma 1 helps us derive the exact lower bounds of $P_{b,1}$ and $P_{b,2}$ in the following theorem.

Theorem 2: When M -QAM is used as the modulation scheme and $E_s \geq E_r$, the average BER $P_{b,1}$ at S_1 in the OSS protocol is exactly lower-bounded by $P_{b,1}^{L,1}$, where $P_{b,1}^{L,1} = W_1(\Omega_h, \Omega_f, \Omega_g)$. The function $W_1(x, y, z)$ is defined as

$$W_1(x, y, z) = \frac{2}{\sqrt{M} \log_2 M} \sum_{j=1}^{\log_2 \sqrt{M} (1-2^{-j}) \sqrt{M}-1} \sum_{i=0}^{\log_2 \sqrt{M} (1-2^{-j}) \sqrt{M}-1} \frac{A_{j,i}(M)}{\pi} \times \int_0^{\frac{\pi}{2}} T_1 \left(-\frac{3(2i+1)^2}{2(M-1) \sin^2 \theta}; x, y, z, E_s, E_r \right) d\theta. \quad (18)$$

Similarly, the average BER $P_{b,2}$ at S_2 is exactly lower-bounded $P_{b,2}^{L,1}$, where $P_{b,2}^{L,1} = W_1(\Omega_f, \Omega_h, \Omega_g)$.

On the other hand, when M -QAM is used as the modulation scheme and $E_r \geq E_s$, the average BER $P_{b,1}$ at S_1 in the OSS protocol is exactly lower-bounded by $P_{b,1}^{L,2}$, where $P_{b,1}^{L,2} = W_2(\Omega_h, \Omega_f, \Omega_g)$. The function $W_2(x, y, z)$ is defined as

$$W_2(x, y, z) = \frac{2}{\sqrt{M} \log_2 M} \sum_{j=1}^{\log_2 \sqrt{M} (1-2^{-j}) \sqrt{M}-1} \sum_{i=0}^{\log_2 \sqrt{M} (1-2^{-j}) \sqrt{M}-1} \frac{A_{j,i}(M)}{\pi} \times \int_0^{\frac{\pi}{2}} T_2 \left(-\frac{3(2i+1)^2}{2(M-1) \sin^2 \theta}; x, y, z, E_s, E_r \right) d\theta. \quad (19)$$

Similarly, the average BER $P_{b,2}$ at S_2 is exactly lower-bounded $P_{b,2}^{L,2}$, where $P_{b,2}^{L,2} = W_2(\Omega_f, \Omega_h, \Omega_g)$.

Proof: See Appendix C. ■

Numerical results will demonstrate that the exact lower bounds $P_{b,1}^{L,1}$, $P_{b,1}^{L,2}$, $P_{b,2}^{L,1}$, and $P_{b,2}^{L,2}$ are very tight to the exact average BERs, irrespective of the values of channel variances and average SNR. Although each of those exact lower bounds contains one integration, this integration is taken over a finite range and it is not hard to calculate numerically. However, it is very hard to present the exact lower bounds in truly closed form, because the integrations in (18) and (19) are very difficult to solve. In order to further reduce the computational complexity, we derive approximations of the exact lower bounds in the following corollary. These approximate lower bounds are given in closed form and they are very accurate.

Corollary 2: The lower bounds $P_{b,1}^{L,1}$ and $P_{b,2}^{L,1}$ can be approximated by $\hat{P}_{b,1}^{L,1}$ and $\hat{P}_{b,2}^{L,1}$, respectively, where $\hat{P}_{b,1}^{L,1} =$

$V_1(\Omega_h, \Omega_f, \Omega_g)$ and $\hat{P}_{b,2}^{L,1} = V_1(\Omega_f, \Omega_h, \Omega_g)$. The function $V_1(x, y, z)$ is defined as

$$V_1(x, y, z) = \frac{2}{\sqrt{M} \log_2 M} \sum_{j=1}^{\log_2 \sqrt{M} (1-2^{-j}) \sqrt{M}-1} \sum_{i=0}^{\log_2 \sqrt{M} (1-2^{-j}) \sqrt{M}-1} A_{j,i}(M) \times \left[\frac{1}{12} T_1 \left(-\frac{3(2i+1)^2}{2(M-1)}; x, y, z, E_s, E_r \right) + \frac{1}{4} T_1 \left(-\frac{2(2i+1)^2}{M-1}; x, y, z, E_s, E_r \right) \right]. \quad (20)$$

On the other hand, the lower bounds $P_{b,1}^{L,2}$ and $P_{b,2}^{L,2}$ can be approximately by $\hat{P}_{b,1}^{L,2}$ and $\hat{P}_{b,2}^{L,2}$, respectively, where $\hat{P}_{b,1}^{L,2} = V_2(\Omega_h, \Omega_f, \Omega_g)$ and $\hat{P}_{b,2}^{L,2} = V_2(\Omega_f, \Omega_h, \Omega_g)$. The function $V_2(x, y, z)$ is defined as

$$V_2(x, y, z) = \frac{2}{\sqrt{M} \log_2 M} \sum_{j=1}^{\log_2 \sqrt{M} (1-2^{-j}) \sqrt{M}-1} \sum_{i=0}^{\log_2 \sqrt{M} (1-2^{-j}) \sqrt{M}-1} A_{j,i}(M) \times \left[\frac{1}{12} T_2 \left(-\frac{3(2i+1)^2}{2(M-1)}; x, y, z, E_s, E_r \right) + \frac{1}{4} T_2 \left(-\frac{2(2i+1)^2}{M-1}; x, y, z, E_s, E_r \right) \right]. \quad (21)$$

Proof: See Appendix D. ■

It is not surprising that the approximate lower bounds in Corollary 2 are not as tight as the exact lower bounds in Theorem 2. However, they are still very close to the exact average BERs as shown by numerical results and they are much easier to calculate. Furthermore, our numerical results demonstrate that the average BERs at both sources are very similar to each other in the OSS protocol, especially at high SNR range. This may make the OSS protocol more attractive than the ANC and TDBC protocols, because one source can have a much higher average BER than the other source in the latter two protocols.

V. OPTIMUM POWER ALLOCATION

In the previous sections, we analyzed the outage probability and the average BERs assuming that the transmission power at each terminal was fixed. In this section, we develop an optimum power allocation scheme for the OSS protocol. When it comes to optimum power allocation, several optimization goals are usually considered: minimization of the outage probability, minimization of the average BER, and maximization of the data-rate. Very ideally, our power allocation scheme can optimize the outage probability, average BER, and data-rate of the OSS protocol at the same time. This will further improve the reliability of the OSS protocol.

In order to design an optimum power allocation scheme, we now assume that the transmission powers of S_1 , S_2 , and R are E_1 , E_2 , and E_r , respectively. As a result, the received SNRs at S_1 and S_2 are rewritten as

$$\gamma_1 = E_2 |g|^2 + \frac{E_2 E_r |hf|^2}{E_2 |f|^2 + E_r |h|^2 + 1}, \quad (22)$$

$$\gamma_2 = E_1 |g|^2 + \frac{E_1 E_r |hf|^2}{E_1 |h|^2 + E_r |f|^2 + 1}. \quad (23)$$

Furthermore, we assume that the total transmission power of the bidirectional cooperative network is constrained to be E_{tot} , and hence, $E_1 + E_2 + E_r = E_{tot}$. First of all, it follows from (5)–(7) that the optimum power allocation scheme that minimizes the outage probability also maximizes the data-rate, i.e.⁵

$$\begin{aligned} (E_1, E_2, E_r) &= \arg \min_{E_1, E_2, E_r} P_{\text{outage}}(R) \\ &= \arg \max_{E_1, E_2, E_r} R^{\text{OSS}} \\ \text{subject to} \quad &E_1 + E_2 + E_r = E_{tot}. \end{aligned} \quad (24)$$

Moreover, by using (7) again, the optimization problem (24) is equivalent to the following two simpler optimization problems

$$(E_1, E_2, E_r) = \arg \max_{E_1, E_2, E_r} \gamma_1, \quad (25)$$

$$(E_1, E_2, E_r) = \arg \max_{E_1, E_2, E_r} \gamma_2, \quad (26)$$

where both of them suffer the constraint $E_1 + E_2 + E_r = E_{tot}$. Note that the conditional BERs $P_b(\gamma_1)$ and $P_b(\gamma_2)$ are determined by the instantaneous SNRs through (14). Thus, it is easy to see that the solutions to those two optimization problems in (25) and (26) actually minimize the conditional BER $P_b(\gamma_1)$ and $P_b(\gamma_2)$ for every channel realization, and hence, they minimize the average BERs $P_{b,1}$ and $P_{b,2}$ as well. Therefore, by solving the optimization problems in (25) and (26), we can develop an optimum power allocation scheme which minimizes the outage probability and average BERs, and maximizes the data-rate of the OSS protocol at the same time. Such optimum power allocation scheme is given in the following Theorem.

Theorem 3: When $E_1 + E_2 + E_r = E_{tot}$, the optimum power allocation scheme that minimizes the outage probability and average BERs, and maximizes the data-rate of the OSS protocol is described as follows. The sources first calculate γ_1 by assuming E_1, E_2 and E_r equal to

$$E_1 = 0, \quad (27)$$

$$E_2 = \frac{3E_{tot}|h|^2}{|h|^2 - |f|^2} \left(1 - \frac{|f|^2}{\sqrt{|gh|^2 - |gf|^2 + |hf|^2}} \right) \quad (28)$$

$$E_r = E_{tot} - E_2. \quad (29)$$

If E_2 is not a real number in (28), the sources calculate γ_1 by assuming $E_1 = E_r = 0$ and $E_2 = E_{tot}$.

Then the sources calculate γ_2 by assuming E_1, E_2 and E_r equal to

$$E_1 = \frac{3E_{tot}|f|^2}{|f|^2 - |h|^2} \left(1 - \frac{|h|^2}{\sqrt{|gf|^2 - |gh|^2 + |hf|^2}} \right) \quad (30)$$

$$E_2 = 0, \quad (31)$$

$$E_r = E_{tot} - E_1. \quad (32)$$

⁵In this paper, we adopt the sum-power constraint when developing the optimum power allocation scheme as in [23]–[26]. An alternative constraint is the individual-power constraint which limits the power at every terminal as shown in [27]. The latter constraint is more practical, but it usually leads to a power allocation scheme whose performance is not as good as the one based on the sum-power constraint. In fact, after obtaining the optimum power allocation scheme based on the sum-power constraint, it is very easy to extend it to the optimum power allocation scheme based on the individual-power constraint.

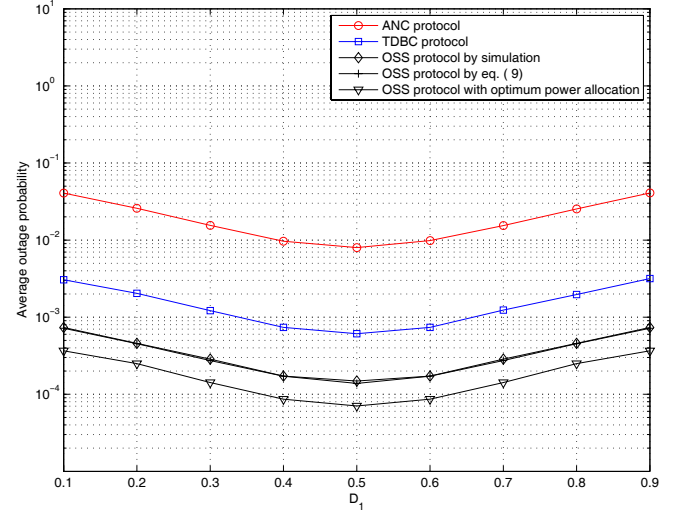


Fig. 2. Outage probabilities of the ANC, TDBC, and OSS protocols, $10 \log_{10} E = 15$ dB and $R = 1$ bps/Hz. For the ANC and TDBC protocols, $E_s = E_r = E$. For the OSS protocol, $E_s = 2E$ and $E_r = E$.

If E_1 is not a real number in (30), the sources calculate γ_2 by assuming $E_2 = E_r = 0$ and $E_1 = E_{tot}$.

If $\gamma_2 > \gamma_1$, S_1 and R allocate the powers as in (30)–(32). They cooperate as in a traditional unidirectional cooperative network over the next two time slots and transmit the signals of S_1 to S_2 . If $\gamma_1 > \gamma_2$, S_2 and R allocate the powers as in (27)–(29). They cooperate as in a traditional unidirectional cooperative network over the next two time slots and transmit the signals of S_2 to S_1 .

Proof: By using the Lagrange method, it can be easily shown that the solutions to the optimization problems in (25) and (26) are given in (27)–(29) and (30)–(32). Thus, the solution to (24) is described in Theorem 3. ■

Numerical results will demonstrate that the optimum power allocation scheme given in Theorem 3 considerably reduces the outage probability and average BER, and improves the data-rate of the OSS protocol, especially when the relay is close to one of the two sources. Note that the optimum power allocation scheme is based on the assumption that the two sources know h, f , and g , and the relay knows h and f . This assumption may require a slightly large amount of signalling overhead as discussed in Section II, and hence, it might not be suitable for fast fading environments. In such environments, suboptimum power allocations should be studied based on partial CSI at each terminal.

VI. NUMERICAL RESULTS

This section presents some numerical results to demonstrate the performance of the OSS protocol. We assume that all three terminals are located in a straight line and R is between S_1 and S_2 . We fix the distance between S_1 and S_2 as one and let D_1 denote the distance between S_1 and R. Furthermore, we set the path loss factor as four. As a result, the values of Ω_g, Ω_h and Ω_f equal to one, D_1^{-4} , and $(1 - D_1)^{-4}$, respectively, [28]. We assume that the total transmission power of the bidirectional cooperative network is $E_{tot} = 3E$.

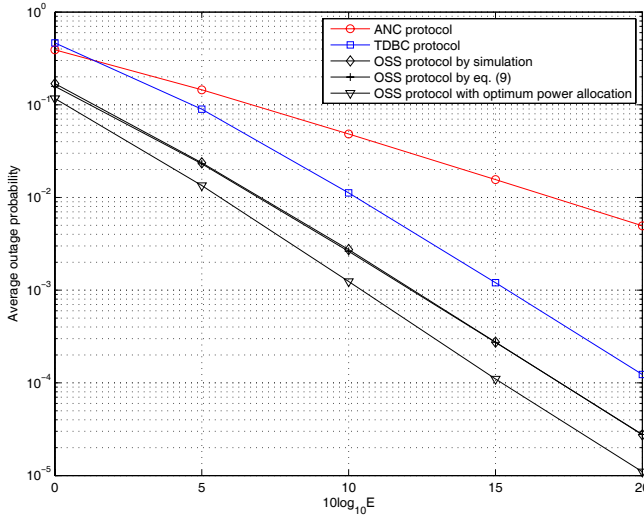


Fig. 3. Outage probabilities of the ANC, TDBC, and OSS protocols, $D_1 = 0.3$ and $R = 1$ bps/Hz. For the ANC and TDBC protocols, $E_s = E_r = E$. For the OSS protocol, $E_s = 2E$ and $E_r = E$.

In Figs. 2 and 3, we set $R = 1$ bps/Hz and compare the exact outage probability $P_{\text{outage}}(R)$ of the OSS protocol with the lower bound $P_{\text{outage}}^{L,1}(R)$ when $E_s = 2E$ and $E_r = E$. Our lower bound is extremely tight to the exact outage probability, irrespective of the values of channel variances and average SNR. Moreover, we present the outage probabilities of the ANC and TDBC protocols as well.⁶ In our simulations, we assume that capacity-achieving random coding is used at the sources. The data-rate and outage probability expressions for the TDBC and ANC protocols can be found in [8]. In order to satisfy the total transmission constraint $E_{\text{tot}} = 3E$, we set $E_1 = E_2 = E_r = E$ for the ANC and TDBC protocols.⁷ The outage probability of the OSS protocol is much lower than those of the ANC and TDBC protocols, which means that the OSS is indeed more reliable. Fig. 3 also shows that the outage probability of the OSS protocol is parallel with that of the TDBC protocol. Since the TDBC protocol achieves the full diversity order two [8]–[10], the OSS protocol indeed achieves diversity order two as expected by Corollary 1. In Figs. 2 and 3, we also see that the optimum power allocation scheme substantially reduces the outage probability of the OSS protocol even when the relay is at the center of the two sources.

In Fig. 4, we set $E_s = E$ and $E_r = 2E$ for the OSS protocol. Under this setting, we compare the exact outage probability and the lower bound $P_{\text{outage}}^{L,2}(R)$ when $R = 1.5$ bps/Hz. Once again, our lower bound is very tight for every considered average SNR. Only when the relay is at the center of the two sources, the lower bound slightly loses its accuracy. For all the other cases, the lower bound is extremely tight

⁶Unlike the OSS, ANC, and TDBC protocols, the PNC protocol is based on decode-and-forward. Thus, we do not compare the performance of the PNC protocol with other protocols in this paper.

⁷In the TDBC protocol, S_1 transmits with power E_1 at the first time slot, S_2 transmits with power E_2 at the second time slot, and R transmits with power E_r at the third time slot in order to achieve one information exchange. Therefore, we set $E_1 = E_2 = E_r = E$, and hence, the total power used by the cooperative network is $3E$ which is the same as for the OSS protocol. Similarly, we set $E_1 = E_2 = E_r = E$ for the ANC protocol.

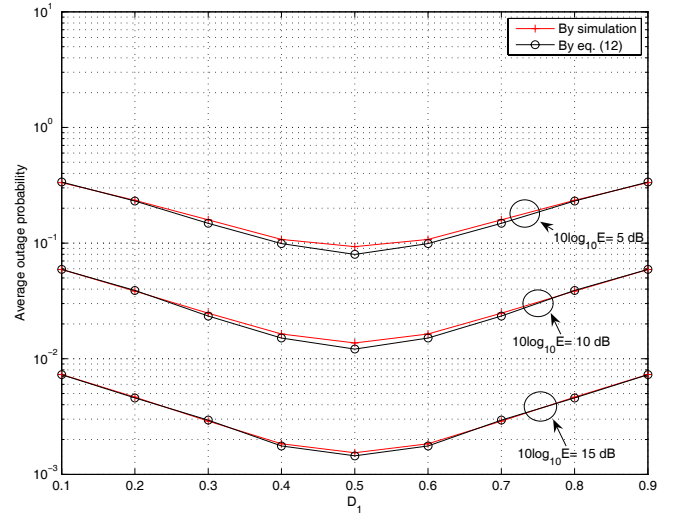


Fig. 4. Outage probability of the OSS protocol, $R = 1.5$ bps/Hz, $E_s = E$, and $E_r = 2E$.

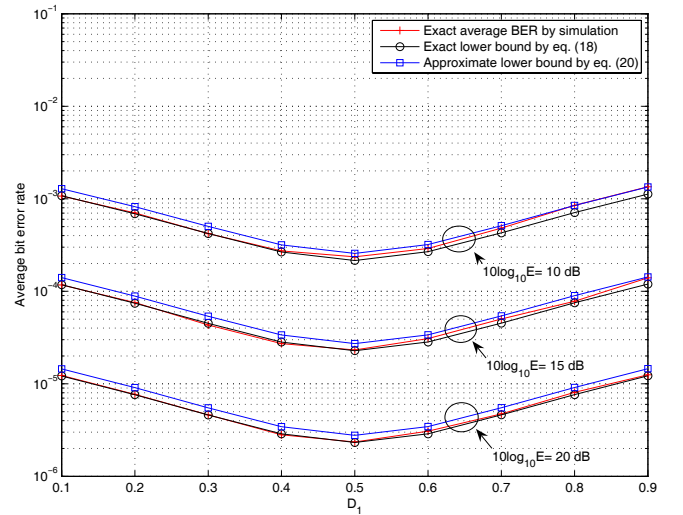


Fig. 5. Average BER at S_1 of the OSS protocol, 4-QAM, $E_s = 2E$, and $E_r = E$.

to the exact outage probability. This is because the bound $\min(x, y)$ of $xy/(x + y + 1)$ in (8) is tight when the values of x and y are quite different. When the relay is at the center of the two sources, the variances Ω_h and Ω_f are almost the same. The values of $|h|^2$ and $|f|^2$ are close to each other with a very high probability. The bound $\min(|h|^2, |f|^2)$ can not approximate $|h|^2|f|^2/(|h|^2 + |f|^2 + 1)$ very well, and hence, our lower bound slightly loses some accuracy for this case.

In Fig. 5, we demonstrate the exact average BER $P_{b,1}$ at S_1 , the exact lower bound $P_{b,1}^{L,1}$, and the approximate lower bound $\hat{P}_{b,1}^{L,1}$ when $E_s = 2E$ and $E_r = E$. Irrespective of the values of channel variances, the exact lower bound $P_{b,1}^{L,1}$ is very tight to the exact average BER for every considered average SNR. The approximate lower bound $\hat{P}_{b,1}^{L,1}$ is also very close to the exact average BER, although it is not as tight as $P_{b,1}^{L,1}$. However, the approximate lower bound is much easier to calculate because it is given in closed form.

In Fig. 6, we compare the exact average BERs of the OSS,

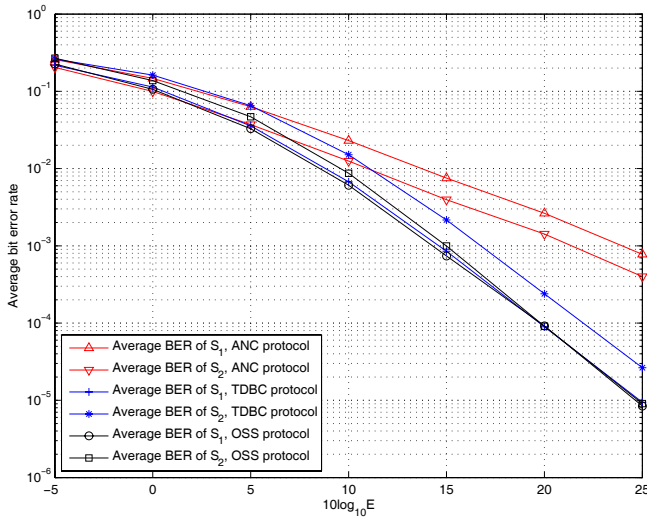


Fig. 6. Average BERs of the ANC, TDBC, and OSS protocols, when $D_1 = 0.3$ and bandwidth efficiency is 2 bps/Hz. For the ANC and TDBC protocols, $E_s = E_r = E$. For the OSS protocol, $E_s = 2E$, and $E_r = E$.

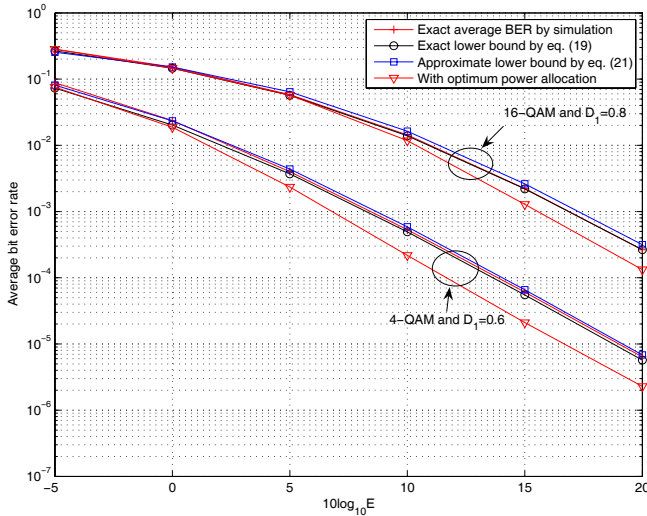


Fig. 7. Average BER at S_1 of the OSS protocol, $E_s = E$ and $E_r = 2E$.

ANC, and TDBC protocols. In order to make a fair comparison, we set the transmission powers as $E_1 = E_2 = E_r = E$ for the ANC and TDBC protocols. Moreover, we set that the OSS protocol uses 16-QAM, the TDBC protocol uses 8-QAM, and the ANC protocol uses 4-QAM. As a result, all protocols have the same bandwidth efficiency 2 bps/Hz. In the OSS protocol, the average BER at S_2 is just slightly larger than that at S_1 even when R is actually very close to S_1 . At high SNR range, the two sources have almost the same average BERs. However, for the ANC and TDBC protocols, one of the sources has a much worse average BER than the other, which may limit the practical applications of those two protocols.

In Fig. 7, we compare the exact average BER $P_{b,1}$ at S_1 , the exact lower bound $P_{b,1}^{L,2}$, and the approximate lower bound $\hat{P}_{b,1}^{L,2}$ when $E_s = E$ and $E_r = 2E$. The lower bounds $P_{b,1}^{L,2}$ and $\hat{P}_{b,1}^{L,2}$ both are very tight in the whole SNR range. Furthermore, the optimum power allocation scheme greatly

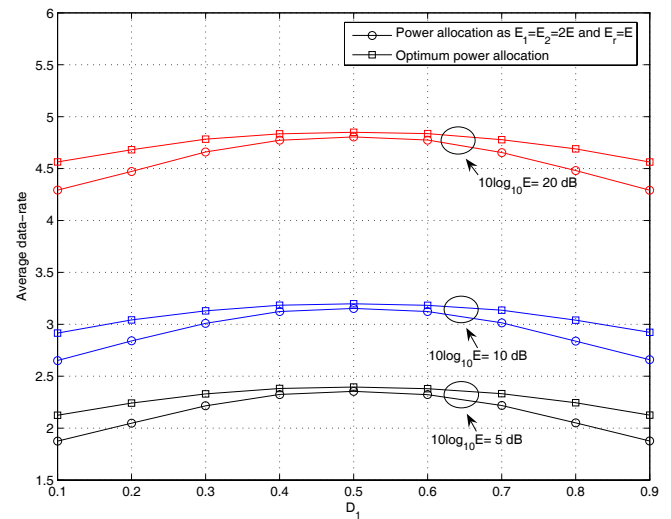


Fig. 8. Data-rate of the OSS protocol.

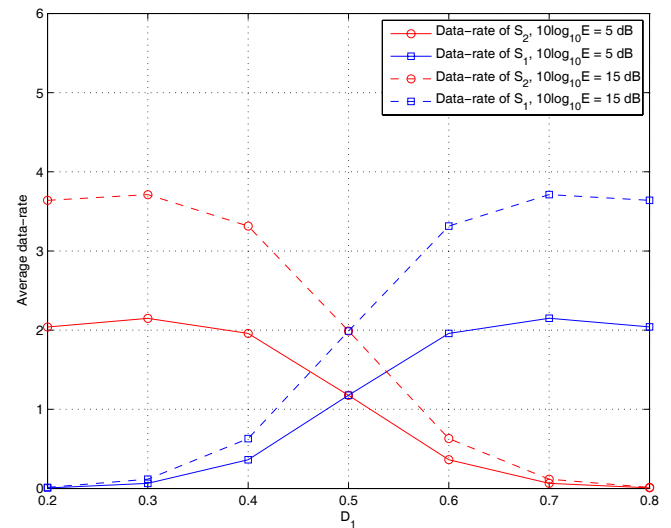


Fig. 9. Data-rates of S_1 and S_2 in the OSS protocol, $E_s = 2E$ and $E_r = E$.

reduces the average BERs. The effect of the optimum power allocation scheme can be seen in Fig. 8 as well. In this figure, the data-rate of the OSS protocol is considerably increased by the optimum power allocation scheme. Lastly, we show the data-rates of both S_1 and S_2 in Fig. 9 when $E_s = 2E$ and $E_r = E$. We notice that the data-rates of the two sources become asymmetric when the relay is close to one of the two sources. As we have discussed at the end of Section III, this issue is not a limitation of the OSS protocol, but it should be carefully addressed when we implement the OSS protocol in practical systems.

VII. CONCLUSION

In this paper, we study a new OSS protocol for the bidirectional cooperative networks. This protocol is designed in order to improve the reliability of such networks and it successfully achieves this goal by opportunistically supporting two traffic flows based on instantaneous channel conditions. In order to

evaluate the performance of the OSS protocol, we first analyze its outage probability. Specifically, a lower bound of the outage probability is derived and numerical results show that it is extremely tight, irrespective of the values of channel variances and average SNR. Based on this lower bound, we show that the OSS protocol achieves the full diversity order two in a bidirectional cooperative network with two sources and one relay. Then we derive the exact lower bounds of the average BERs at both sources in the OSS protocol. As demonstrated by numerical results, those exact lower bounds are very tight and they are not hard to calculate, since each of them contains only one integration over a finite range. Moreover, approximations of the exact lower bounds are derived as well and those approximate lower bounds are given in closed form. Lastly, an optimum power allocation scheme is developed for the OSS protocol. This scheme can simultaneously minimize the outage probability and average BER, and maximize the data-rate of the OSS protocol.

APPENDIX A

Proof of Theorem 1

It follows from (5)–(8) that a lower bound $P_{\text{outage}}^L(R)$ of $P_{\text{outage}}(R)$ can be defined as follows:

$$\begin{aligned} P_{\text{outage}}^L(R) &= \Pr\left(\frac{1}{2}\log_2\left(1 + \max\left(\min(E_s|f|^2, E_r|h|^2), \min(E_s|h|^2, E_r|f|^2)\right) + E_s|g|^2\right) < R\right). \end{aligned} \quad (\text{A.1})$$

Let $X = |h|^2$, $Y = |f|^2$, and $Z = |g|^2$. Thus, X , Y , and Z are exponential random variables with means Ω_h , Ω_f , and Ω_g , respectively. Moreover, let $h(x)$, $f(y)$, and $g(z)$ denote the probability density functions (PDFs) of X , Y , and Z , respectively. The lower bound $P_{\text{outage}}^L(R)$ can be expanded by using the law of total probability as follows:

$$\begin{aligned} P_{\text{outage}}^L(R) &= \Pr\left(\max(E_s Y, E_s X) + E_s Z < 2^{2R} - 1, E_s Y < E_r X, E_s X < E_r Y\right) \\ &+ \Pr\left(\max(E_s Y, E_r Y) + E_s Z < 2^{2R} - 1, E_s Y < E_r X, E_r Y < E_s X\right) \\ &+ \Pr\left(\max(E_r X, E_s X) + E_s Z < 2^{2R} - 1, E_r X < E_s Y, E_s X < E_r Y\right) \\ &+ \Pr\left(\max(E_r X, E_r Y) + E_s Z < 2^{2R} - 1, E_r X < E_s Y, E_r Y < E_s X\right). \end{aligned}$$

When $E_s \geq E_r$, we let $P_{\text{outage}}^{L,1}(R)$ denote the value of $P_{\text{outage}}^L(R)$ for this case and $P_{\text{outage}}^{L,1}(R)$ can be simplified in the following way

$$\begin{aligned} P_{\text{outage}}^{L,1}(R) &= \Pr\left(E_s X + E_s Z < 2^{2R} - 1, E_s X < E_r Y\right) \\ &+ \Pr\left(E_s Y + E_s Z < 2^{2R} - 1, E_s Y < E_r X\right) \\ &+ \Pr\left(E_r X + E_s Z < 2^{2R} - 1, E_s Y > E_r X > E_r Y\right) \\ &+ \Pr\left(E_r Y + E_s Z < 2^{2R} - 1, E_s X > E_r Y > E_r X\right). \end{aligned} \quad (\text{A.2})$$

The first probability in (A.2) can be solved by [18] as follows:

$$\begin{aligned} &\Pr\left(E_s X + E_s Z < 2^{2R} - 1, E_s X < E_r Y\right) \\ &= \int_{x=0}^{2^{2R}-1} \int_{z=0}^{2^{2R}-1-x} \int_{y=\frac{E_s x}{E_r}}^{\infty} f(x)h(y)g(z)dx dy dz \\ &= A_1\left(\frac{E_s}{2E_r}\Omega_h, \Omega_f, \frac{E_s}{2E_r}\Omega_g, \frac{2^{2R}-1}{E_r}\right). \end{aligned} \quad (\text{A.3})$$

Similarly, the second probability in (A.2) can be solved. The third probability in (A.2) is solved by [18] in the following way

$$\begin{aligned} &\Pr\left(E_r X + E_s Z < 2^{2R} - 1, E_s Y > E_r X > E_r Y\right) \\ &= \int_{x=0}^{2^{2R}-1} \int_{z=0}^{2^{2R}-1-\frac{E_r x}{E_s}} \int_{y=\frac{E_r x}{E_s}}^x f(x)h(y)g(z)dx dy dz \\ &= A_2\left(\Omega_h, \Omega_f, \frac{E_s}{2E_r}\Omega_f, \frac{E_s}{2E_r}\Omega_g, \frac{2^{2R}-1}{E_r}\right). \end{aligned} \quad (\text{A.4})$$

Similarly, the last probability in (A.2) can be solved. Therefore, when $E_s \geq E_r$, the exact outage probability is lower-bounded by $P_{\text{outage}}^{L,1}(R)$ as shown in (9).

When $E_r \geq E_s$, we let $P_{\text{outage}}^{L,2}(R)$ denote the value of $P_{\text{outage}}^L(R)$ for this case and $P_{\text{outage}}^{L,2}(R)$ can be simplified in the following way

$$\begin{aligned} P_{\text{outage}}^{L,2}(R) &= \Pr\left(E_r X + E_s Z < 2^{2R} - 1, E_r X < E_s Y\right) \\ &+ \Pr\left(E_r Y + E_s Z < 2^{2R} - 1, E_r Y < E_s X\right) \\ &+ \Pr\left(E_s X + E_s Z < 2^{2R} - 1, E_r Y > E_s X > E_s Y\right) \\ &+ \Pr\left(E_s Y + E_s Z < 2^{2R} - 1, E_r X > E_s Y > E_s X\right). \end{aligned} \quad (\text{A.5})$$

All the probabilities in (A.5) can be solved by using the techniques in (A.3) and (A.4). As a result, when $E_r \geq E_s$, the exact outage probability is lower-bounded by $P_{\text{outage}}^{L,2}(R)$ as shown in (12).

APPENDIX B

Proof of Lemma 1

Since X , Y , and Z are mutually independent, the conditional MGF $\text{MGF}_{W_1|W_1>W_2}(s)$ is given by

$$\begin{aligned} &\text{MGF}_{W_1|W_1>W_2}(s) \\ &= \text{MGF}_{U_1|W_1>W_2}(s)\text{MGF}_{C_s Z|W_1>W_2}(s). \end{aligned} \quad (\text{B.1})$$

The conditional MGF $\text{MGF}_{C_s Z|W_1>W_2}(s)$ is easy to obtain because Z is independent of X and Y ,

$$\text{MGF}_{C_s Z|W_1>W_2}(s) = \frac{1}{\frac{C_s \Omega_Z}{1 - C_s \Omega_Z} - s}. \quad (\text{B.2})$$

In order to obtain $\text{MGF}_{U_1|W_1>W_2}(s)$, we need the conditional cumulative distribution function (CDF) of U_1 , conditioned on $W_1 > W_2$. The conditional CDF $F_{U_1|W_1>W_2}(u_1)$ is defined as follows:

$$F_{U_1|W_1>W_2}(u_1) = \frac{\Pr(U_1 < u_1, W_1 > W_2)}{\Pr(W_1 > W_2)}. \quad (\text{B.3})$$

When $C_s \geq C_r$, by using the law of total probability, the denominator in (B.3) is given by

$$\begin{aligned} & \Pr(W_1 > W_2) \\ &= \Pr(C_s Y < C_r X) + \Pr(X > Y, C_r Y < C_s X < C_s Y) \\ & \quad + \Pr(X > Y, C_r Y < C_r X < C_s Y) \end{aligned} \quad (\text{B.4})$$

$$= \Pr(X > Y) \quad (\text{B.5})$$

$$= \frac{\Omega_X}{\Omega_X + \Omega_Y}. \quad (\text{B.6})$$

The numerator in (B.3) is given by

$$\begin{aligned} & \Pr(U_1 < u_1, W_1 > W_2) \\ &= \Pr(C_s Y < u_1, C_s Y < C_r X, X > Y) \\ & \quad + \Pr(C_r X < u_1, C_r X < C_s Y, X > Y) \end{aligned} \quad (\text{B.7})$$

$$\begin{aligned} &= \int_{y=0}^{\frac{u_1}{C_s}} \int_{x=\frac{C_s y}{C_r}}^{\infty} h(x) f(y) dx dy \\ & \quad + \int_{x=0}^{\frac{u_1}{C_r}} \int_{y=\frac{C_r x}{C_s}}^{\infty} h(x) f(y) dx dy \end{aligned} \quad (\text{B.8})$$

$$\begin{aligned} &= \frac{\Omega_X}{\Omega_X + \Omega_Y} - \exp\left(-u_1 \left(\frac{1}{C_r \Omega_X} + \frac{1}{C_s \Omega_Y}\right)\right) \\ & \quad + \frac{\Omega_Y}{\Omega_X + \Omega_Y} \exp\left(-u_1 \left(\frac{1}{C_r \Omega_X} + \frac{1}{C_r \Omega_Y}\right)\right) \end{aligned} \quad (\text{B.9})$$

Based on (B.3)–(B.9), we can derive the conditional CDF $F_{U_1|W_1>W_2}(u_1)$ for the case that $C_s \geq C_r$. By taking the derivative, the conditional PDF $f_{U_1|W_1>W_2}(u_1)$ is given by

$$\begin{aligned} & f_{U_1|W_1>W_2}(u_1) \\ &= \frac{\Omega_X + \Omega_Y}{\Omega_X} \left(\frac{1}{C_r \Omega_X} + \frac{1}{C_s \Omega_Y}\right) \\ & \quad \times \exp\left(-u_1 \left(\frac{1}{C_r \Omega_X} + \frac{1}{C_s \Omega_Y}\right)\right) \\ & \quad - \frac{\Omega_Y}{\Omega_X} \left(\frac{1}{C_r \Omega_X} + \frac{1}{C_r \Omega_Y}\right) \\ & \quad \times \exp\left(-u_1 \left(\frac{1}{C_r \Omega_X} + \frac{1}{C_r \Omega_Y}\right)\right). \end{aligned} \quad (\text{B.10})$$

Then, by the definition of MGF, it is not hard to show that

$$\begin{aligned} & \text{MGF}_{U_1|W_1>W_2}(s) \\ &= \int_{u_1=0}^{\infty} e^{su_1} f_{U_1|W_1>W_2}(u_1) du_1 \end{aligned} \quad (\text{B.11})$$

$$\begin{aligned} &= \frac{\Omega_X + \Omega_Y}{\Omega_X} \frac{\frac{1}{C_r \Omega_X} + \frac{1}{C_s \Omega_Y}}{\frac{1}{C_r \Omega_X} + \frac{1}{C_s \Omega_Y} - s} \\ & \quad - \frac{\Omega_Y}{\Omega_X} \frac{\frac{1}{C_r \Omega_X} + \frac{1}{C_r \Omega_Y}}{\frac{1}{C_r \Omega_X} + \frac{1}{C_r \Omega_Y} - s}. \end{aligned} \quad (\text{B.12})$$

Based on (B.2) and (B.11), we derive the conditional MGF $\text{MGF}_{W_1|W_1>W_2}(s)$ as given in (16) for the case that $C_s \geq C_r$.

When $C_r \geq C_s$, the denominator in (B.3) is given by

$$\begin{aligned} & \Pr(W_1 > W_2) \\ &= \Pr(C_r X < C_s Y) + \Pr(Y > X, C_s X < C_r Y < C_r X) \\ & \quad + \Pr(Y > X, C_s X < C_s Y < C_r X) \end{aligned} \quad (\text{B.13})$$

$$= \Pr(X < Y) \quad (\text{B.14})$$

$$= \frac{\Omega_Y}{\Omega_X + \Omega_Y}. \quad (\text{B.15})$$

The numerator in (B.3) is given by

$$\begin{aligned} & \Pr(U_1 < u_1, W_1 > W_2) \\ &= \Pr(C_s Y < u_1, C_s Y < C_r X, X < Y) \\ & \quad + \Pr(C_r X < u_1, C_r X < C_s Y, X < Y) \end{aligned} \quad (\text{B.16})$$

$$\begin{aligned} &= \int_{y=0}^{\frac{u_1}{C_s}} \int_{x=\frac{C_s y}{C_r}}^y h(x) f(y) dx dy \\ & \quad + \int_{x=0}^{\frac{u_1}{C_r}} \int_{y=\frac{C_r x}{C_s}}^{\infty} h(x) f(y) dx dy \end{aligned} \quad (\text{B.17})$$

$$\begin{aligned} &= \frac{\Omega_Y}{\Omega_X + \Omega_Y} - \exp\left(-u_1 \left(\frac{1}{C_r \Omega_X} + \frac{1}{C_s \Omega_Y}\right)\right) \\ & \quad + \frac{\Omega_X}{\Omega_X + \Omega_Y} \exp\left(-u_1 \left(\frac{1}{C_s \Omega_X} + \frac{1}{C_s \Omega_Y}\right)\right). \end{aligned} \quad (\text{B.18})$$

By following the same approach for the case of $C_s \geq C_r$ and using (B.15) and (B.18), it is not hard to obtain the conditional MGF

$$\begin{aligned} & \text{MGF}_{U_1|W_1>W_2}(s) \\ &= \frac{\Omega_X + \Omega_Y}{\Omega_Y} \frac{\frac{1}{C_r \Omega_X} + \frac{1}{C_s \Omega_Y}}{\frac{1}{C_r \Omega_X} + \frac{1}{C_s \Omega_Y} - s} \\ & \quad - \frac{\Omega_X}{\Omega_Y} \frac{\frac{1}{C_s \Omega_X} + \frac{1}{C_s \Omega_Y}}{\frac{1}{C_s \Omega_X} + \frac{1}{C_s \Omega_Y} - s}. \end{aligned} \quad (\text{B.19})$$

Lastly, by (B.2) and (B.19), we derive the conditional MGF $\text{MGF}_{W_1|W_1>W_2}(s)$ as given in (17) for the case that $C_r \geq C_s$.

APPENDIX C

Proof of Theorem 2

We first focus on the lower bounds of $P_{b,1}$. We define two new random variables $\bar{\gamma}_1$ and $\bar{\gamma}_2$ as $\bar{\gamma}_1 = \min(E_s |f|^2, E_r |h|^2) + E_s |g|^2$ and $\bar{\gamma}_2 = \min(E_s |h|^2, E_r |f|^2) + E_s |g|^2$, respectively. Then it follows from (8) that $\bar{\gamma}_1$ and $\bar{\gamma}_2$ are upper bounds of γ_1 and γ_2 , respectively. Furthermore, we define $\bar{P}_{b,1} = \mathbb{E}[P_b(\bar{\gamma}_1)|\bar{\gamma}_1 > \bar{\gamma}_2]$, where the function $P_b(x)$ is given by (14). As a result, $\bar{P}_{b,1}$ is a lower bound of $P_{b,1}$.

Let $f_{\bar{\gamma}_1|\bar{\gamma}_1>\bar{\gamma}_2}(x)$ denote the conditional PDF of $\bar{\gamma}_1$, conditioned on $\bar{\gamma}_1 > \bar{\gamma}_2$. Then the expectation in $\bar{P}_{b,1} = \mathbb{E}[P_b(\bar{\gamma}_1)|\bar{\gamma}_1 > \bar{\gamma}_2]$ can be rewritten in the following way

$$\begin{aligned} & \bar{P}_{b,1} \\ &= \int_{x=0}^{\infty} P_b(x) f_{\bar{\gamma}_1|\bar{\gamma}_1>\bar{\gamma}_2}(x) dx \end{aligned} \quad (\text{C.1})$$

$$\begin{aligned} &= \frac{2}{\sqrt{M} \log_2 M} \sum_{j=1}^{\log_2 \sqrt{M}} \sum_{i=0}^{(1-2^{-j})\sqrt{M}-1} A_{j,i}(M) \\ & \quad \times \int_{x=0}^{\infty} \frac{1}{\pi} \int_{\theta=0}^{\frac{\pi}{2}} \exp\left(-\frac{3(2i+1)^2 x}{2(M-1) \sin^2 \theta}\right) d\theta dx \end{aligned}$$

$$\begin{aligned} &= \frac{2}{\sqrt{M} \log_2 M} \sum_{j=1}^{\log_2 \sqrt{M}} \sum_{i=0}^{(1-2^{-j})\sqrt{M}-1} \frac{A_{j,i}(M)}{\pi} \\ & \quad \times \int_{\theta=0}^{\frac{\pi}{2}} \text{MGF}_{\bar{\gamma}_1|\bar{\gamma}_1>\bar{\gamma}_2}\left(-\frac{3(2i+1)^2}{2(M-1) \sin^2 \theta}\right) d\theta, \end{aligned} \quad (\text{C.2})$$

where the second equality is by using Craig's formula [21] and MGF $\bar{\gamma}_1|\bar{\gamma}_1>\bar{\gamma}_2(s)$ is the conditional MGF of $\bar{\gamma}_1$, conditioned on $\bar{\gamma}_1 > \bar{\gamma}_2$.

Let $P_{b,1}^{L,1}$ denote the value of $\bar{P}_{b,1}$ when $E_s \geq E_r$. For this case, the conditional MGF $\text{MGF}_{\bar{\gamma}_1|\bar{\gamma}_1>\bar{\gamma}_2}(s)$ can be easily obtained by using Lemma 1 and it is given by

$$\text{MGF}_{\bar{\gamma}_1|\bar{\gamma}_1>\bar{\gamma}_2}(s) = T_1(s; \Omega_h, \Omega_f, \Omega_g, E_s, E_r). \quad (\text{C.3})$$

Based on (C.2) and (C.3), it is not hard to show that $P_{b,1}^{L,1} = W_1(\Omega_h, \Omega_f, \Omega_g)$, where the function $W_1(x, y, z)$ is given in (18). On the other hand, let $P_{b,1}^{L,2}$ denote the value of $\bar{P}_{b,1}$ when $E_r \geq E_s$. For this case, the conditional MGF is given by

$$\text{MGF}_{\bar{\gamma}_1|\bar{\gamma}_1>\bar{\gamma}_2}(s) = T_2(s; \Omega_h, \Omega_f, \Omega_g, E_s, E_r). \quad (\text{C.4})$$

Based on (C.2) and (C.4), it is not hard to show that $P_{b,1}^{L,2} = W_2(\Omega_h, \Omega_f, \Omega_g)$, where the function $W_2(x, y, z)$ is given in (19).

Similarly, we derive the lower bounds $P_{b,2}^{L,1}$ and $P_{b,2}^{L,2}$ of $P_{b,2}$ based on (C.2) and Lemma 1. They are given by $P_{b,2}^{L,1} = W_1(\Omega_f, \Omega_h, \Omega_g)$ and $P_{b,2}^{L,2} = W_2(\Omega_f, \Omega_h, \Omega_g)$.

APPENDIX D

Proof of Corollary 2

In order to obtain approximations of the exact lower bounds in closed form, we approximate the Q -function in the following way

$$Q(x) \approx \frac{1}{12}e^{-\frac{x^2}{2}} + \frac{1}{4}e^{-\frac{2x^2}{3}}. \quad (\text{D.1})$$

This is a very tight approximation of Q -function as shown in [22].

We first show the approximations of $P_{b,1}^{L,1}$ and $P_{b,1}^{L,2}$. By using (D.1), $\bar{P}_{b,1}$ in (C.1) is approximated by

$$\begin{aligned} & \bar{P}_{b,1} \\ & \approx \frac{2}{\sqrt{M} \log_2 M} \sum_{j=1}^{\log_2 \sqrt{M}} \left\{ \sum_{i=0}^{(1-2^{-j})\sqrt{M}-1} A_{j,i}(M) \right. \\ & \quad \times \mathbb{E} \left[\frac{1}{12} \exp \left(-\frac{3(2i+1)^2 \bar{\gamma}_1}{2(M-1)} \right) \right. \\ & \quad \left. \left. + \frac{1}{4} \exp \left(-\frac{2(2i+1)^2 \bar{\gamma}_1}{M-1} \right) \right] \Big| \bar{\gamma}_1 > \bar{\gamma}_2 \right\} \quad (\text{D.2}) \end{aligned}$$

$$\begin{aligned} & = \frac{2}{\sqrt{M} \log_2 M} \sum_{j=1}^{\log_2 \sqrt{M}} \left\{ \sum_{i=0}^{(1-2^{-j})\sqrt{M}-1} A_{j,i}(M) \right. \\ & \quad \times \left[\frac{1}{12} \text{MGF}_{\bar{\gamma}_1|\bar{\gamma}_1>\bar{\gamma}_2} \left(-\frac{3(2i+1)^2 \bar{\gamma}_1}{2(M-1)} \right) \right. \\ & \quad \left. \left. + \frac{1}{4} \text{MGF}_{\bar{\gamma}_1|\bar{\gamma}_1>\bar{\gamma}_2} \left(-\frac{2(2i+1)^2 \bar{\gamma}_1}{M-1} \right) \right] \right\}. \quad (\text{D.3}) \end{aligned}$$

Let $\hat{P}_{b,1}^{L,1}$ denote the value of (D.3) when $E_s \geq E_r$, so $\hat{P}_{b,1}^{L,1}$ is an approximation of $P_{b,1}^{L,1}$ by definition. For this case, we have shown that the conditional MGF $\text{MGF}_{\bar{\gamma}_1|\bar{\gamma}_1>\bar{\gamma}_2}(s)$ is given by (C.3). Thus, $\hat{P}_{b,1}^{L,1} = V_1(\Omega_h, \Omega_f, \Omega_g)$, where $V_1(x, y, z)$ is defined in (20). On the other hand, let $\hat{P}_{b,1}^{L,2}$ denote the

value of (D.3) when $E_r \geq E_s$, so $\hat{P}_{b,1}^{L,2}$ is an approximation of $P_{b,1}^{L,2}$ by definition. For this case, we have shown that the conditional MGF $\text{MGF}_{\bar{\gamma}_1|\bar{\gamma}_1>\bar{\gamma}_2}(s)$ is given by (C.4). Thus, $\hat{P}_{b,1}^{L,1} = V_2(\Omega_h, \Omega_f, \Omega_g)$, where $V_2(x, y, z)$ is defined in (21).

Similarly, it can be shown that $P_{b,2}^{L,1}$ and $P_{b,2}^{L,2}$ are approximated by $\hat{P}_{b,2}^{L,1}$ and $\hat{P}_{b,2}^{L,2}$, respectively, where $\hat{P}_{b,2}^{L,1} = V_1(\Omega_f, \Omega_h, \Omega_g)$ and $\hat{P}_{b,2}^{L,2} = V_2(\Omega_f, \Omega_h, \Omega_g)$.

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