Dynamic Sampling Rate Adjustment for Compressive Spectrum Sensing over Cognitive Radio Network

Ching-Chun Huang and Li-Chun Wang

***Abstract*—In this paper, a dynamic sampling rate adjustment scheme is proposed for compressive spectrum sensing in cognitive radio network. Nowadays, compressive sensing (CS) has been proposed with a revolutionary idea to sense the sparse spectrum by using a lower sampling rate. However, many methods for compressive spectrum sensing assume that the sparse level is static and a fixed compressive sampling rate is applied over time. To adapt to time-varying sparse levels and adjust the sampling rate, we proposed to model sparse levels as a dynamic system and treat the dynamic rate selection as a tracking problem. By introducing the Sequential Monte Carlo (SMC) algorithm into a distributed compressive spectrum sensing framework, we could not only track the optimal sampling rate but determine the unoccupied channels accurately in a unified method.**

***Index Terms*—Cognitive radio, dynamic system, sequential Monte Carlo, compressive spectrum sensing.**

I. INTRODUCTION

An emerging idea to solve the problem of spectrum drought relies on cognitive radio (CR) networks where the unlicensed users are allowed to cleverly utilize the spectrum holes when the induced interference causes harmless effects. To dynamically switch among available spectrum holes, each CR user has to sense the wideband spectrum in a fast and efficient manner. To fulfill the goal, compressive spectrum sensing has been proposed to sense the spectrum at a sub-Nyquist sampling rate.

Compressive spectrum sensing methods could be roughly divided as single-CR and multiple-CR approaches. For single-CR approaches, Tian and Giannakis [1] proposed to sense the wideband spectrum based on the concept of compressive sampling (CS) [2] for a single CR user. Because the spectrum is found to be sparse, the original wireless signals could be reconstructed even when the problem formulation is under-constrained. Meanwhile, Polo et al. [3] proposed an analog-to-information converter (AIC) to sense the target signal at the information rate rather than Nyquist rate. Recently, unlike [1][3], which reconstruct the complete spectrum and then apply a wavelet approach [4] to estimate the energy of each channel, the authors in [5] introduced a novel compressive detection method which bypasses the step of spectrum reconstruction and estimates the channel energies directly.

However, a single-CR approach may not accurately sense the activities of primary users due to channel fading effects. To better the results, a novel way is to rely on a multiple-CR approach to well utilize the spatial diversity gain. Among those approaches, Tian [6] proposed a distributed compressed spectrum sensing approach, where CS is performed at each local CR and one-hop local communications are developed for collaborative sampling and data fusion. In [7], instead of performing CS in each CR, the authors proposed to collect the autocorrelations of the compressed signals from all CRs. The CS reconstruction algorithm is finally operated at the fusion center in order to well exploit joint sparsity among CRs.

Many CS methods assume that the sparse level of spectrum is static. However, the sparse level is varying; the optimal sampling rate for CS should be adaptive from time to time. Recently, some researchers start to discuss dynamic CS. Zhang et al. [8] proposed a correlation-based method to classify the status of spectrum as sparse or non-sparse statuses. Still, the method mainly switches between a fixed compressive rate and Nyquist rate. Yin et al. [9] proposed a dynamic CS, which merely needs to recover the recent change of channels. Comparing with static CS, the method is fast and requires fewer measurements. However, correct results of previous estimation are the precondition. While false detection and rejection are inevitable, error propagation may occur.

In this paper, we focus on the adaptation of sampling rate adjustment. Our goal is similar to Wang et al.'s work in [10], which proposed an adaptive CS method for a single CR to dynamically select the optimal sampling rate. Their system, before locating the convergence point of the optimal rate, has to experience a transient time period. However, during the period, the detection of channel status presents unreliable performance owing to the incorrect setting of the sampling rate. If the sparse level keeps varying, the transient time may be prolonged, which will lead to the degradation of the detection performance. To secure stable detection performance and adjust the sampling rate simultaneously, we suggest exploiting the spatial diversity gain over a multiple-CR network. Here, we model the dynamic rate adaptation as a tracking problem and propose a Sequential Monte Carlo (SMC) approach to integrate distributed compressive spectrum sensing, data fusion, channel status estimation, and optimal rate selection into a unified framework. By using the proposed framework, our system could reduce the requirement of overall samples and ensure the accuracy of spectrum sensing. In contrast to previous works, our scheme presents the capability of adapting to dynamic sparse level without error propagation and unreliable transient periods.

II. SYSTEM MODEL

The system model we consider is shown in Fig. 1. Here, there are M neighboring CRs randomly located in a region and

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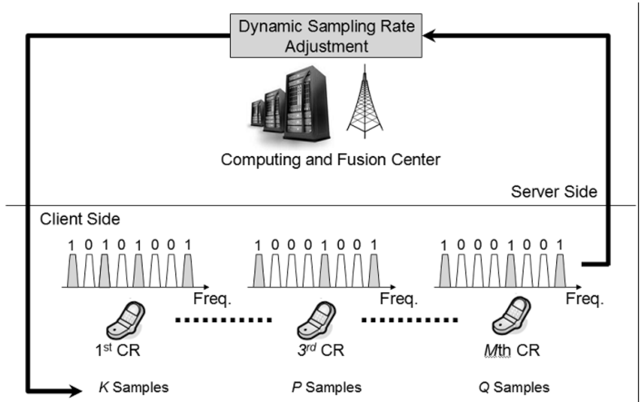

Fig. 1. Our system model for cooperative compressive spectrum sensing with dynamic sampling rate adjustment.

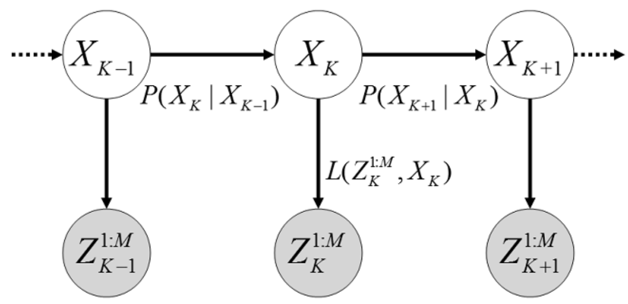

Fig. 2. The dynamic system for sampling rate tracking.

there is a computing-and-fusion center working as a server. The server collects M statue reports of N channels from M local CRs to infer the final channel statuses. At the same time, the server also computes the optimal sampling rate and determines the assignments of sampling rates to local CRs for the next sensing process. On the other hand, based on the assigned sample rate, each CR senses the spectrum and performs the CS algorithm to detect the statuses of N local channel. Specifically, for a local CR, say the ith CR, it outputs a status vector $Z^i_{N\times 1}$, where the nth element $z_n \in \{0,1\}$ indicates the status of the nth channel.

III. The Proposed Method

To model the dynamic sampling rate adjustment, we treat sampling rate X as a random variable and attempt to approximate its posterior probability distribution $P(X_K|CZ_{1:K})$ at sensing index K given all collected channel statuses $CZ_{1:K}$ from index 1 to K. Here, $CZ_{1:K} = \{Z^i_{N\times 1,t}\}^{i=1:M}_{t=1:M}$ is the channel status set reported by M CRs up to sensing index K. With the estimated $P(X_K|CZ_{1:K})$, the optimal sampling rate is determined by securing the maximum of the distribution. In our system, to efficiently estimate $P(X_K|CZ_{1:K})$, we make the Markov assumption and apply a dynamic system as shown in Fig. 2. With the model, the estimation problem could be re-formulated by the Bayesian rule as

$$P(X_k|CZ_{1:K}) \sim L(Z^{1:M}_K, X_K) \times P(X_K|CZ_{1:K-1}) \times P(X_K), \tag{1}$$

where $L(Z^{1:M}_K, X_K)$ is the likelihood term of current status observation $Z^{1:M}_K$, $P(X_K|CZ_{1:K-1})$ illustrates the temporal prior of X_K propagated from previous processes, and $P(X_K)$ represents the preference prior.

Below, we illustrate how to derive the channel statuses $Z^i_{N\times 1}$ based on CS for each local CR. Next, the formulations of the likelihood term and prior terms are to be detailed. Finally, we explain the proposed SMC method to numerically estimate $P(X_K|CZ_{1:K})$ and track the dynamic sampling rate.

A. Compressive Spectrum Sensing (CS) for a Local CR

Before the inference process in (1), the detection of channel statuses in local CRs should be determined. Many previous single-CR approaches could be applied to achieve the goal. In our system, due to the robustness, we adopt the compressive sensing method proposed by Polo et al. [3] to reconstruct the spectrum; exploit the wavelet-based edge detector [4] to identify the number of channels and their frequency ranges; finally apply an energy-based detection method [11] to determine the channel status. From the process, we could directly identify the channel number N and the frequency range of each channel. Also, we could systematically determine the channel statuses. Since those methods are well-known, we only provide the gist of this process.

For each local CR, at sensing index K, based on previous interference of sparse level, the fusion-and-computing center assigns a sampling rate x^i to the ith CR. Adopting the AIC method in [3] and according to x^i, the ith CR senses $S_i = R(x^i)$ samples instead of S_{Nyq} samples and generates a compressed autocorrelation vector r_C with size $2S_i \times 1$; function $R(.)$ here maps an assigned sampling rate to its sample number. In [1], it showed that the edge spectrum E_n, which is the derivative of the power spectrum density (PSD) F_n, presents the sparse property. Based on the CS theory [3][4], we can formulate an l_1-norm optimization problem and recover the edge spectrum E_n. Hence, we have

$$\hat{E}_n = \operatorname*{argmin}_{E_n} \|E_n\|_1 \quad s.t.\ r_C = PW^{-1}F^{-1}D^{-1}E_n, \tag{2}$$

where P is the $2S_i \times 2S_{Nyq}$ projection matrix relating the compressed autocorrelation vector r_C and the uncompressed autocorrelation vector r_N. The derivation of P could be found in [3]. Also, W is a wavelet-based smoothing used for noise suppression; F represents the Fourier transform in a matrix form; D could be any derivative operation such as first-order difference matrix; finally, $(.)^{-1}$ denotes matrix inverse. Note that W, F, and D have the size $2S_{Nyq} \times 2S_{Nyq}$. To solve the optimization problem in (2), many linear programming methods, such as basis pursuit [12], could be applied. Next, PSD F_n is reconstructed for the detection of channel occupancy by $F_n = D^{-1}E_n$. By applying an adaptive threshold [11], the N channel statuses $Z^i_{N\times 1}(x^i) = \{z^i_n\}_{n=1:N}$ reported by the ith CR with sampling rate x^i is determined.

B. Formulations of Probability Models in the Server End

On the server side, the fusion center collects the channel statuses $CZ_{1:K}$ from M CRs and next computes the three system models to estimate the posterior $P(X_K|CZ_{1:K})$ in (1). Below we illustrate the three system models.

1) Evaluation of Likelihood Term $L(Z_K^{1:M}, X_K)$: To compute the likelihood, we well utilize the joint consistency property of channel statuses among multiple CRs. Ideally, the channel statuses reported by neighboring CRs should be similar. This spatial correlation property enables our system to adjust the sampling rate. Here, for $X_K = x^i$ is a sampling rate with high likelihood, the corresponding channel statuses $Z_{N\times 1}^i | x^i$ reported by the ith CR under sampling rate x^i should be consistent with the status reports from other CRs. Hence, we could measure the likelihood of $X_K = x^i$ by evaluating the consistency level. Assume we receive M statues reports, where each report indicates the status measurements of N channels. Based on the M statues reports, we evaluate the occupied probability $P_n(1)$ of the nth channel by calculating the ratio of occupied observations to the total observations M. That is

$$P_n(1) = \frac{\sum_i^M z_n^i}{M}. \quad (3)$$

The available probability $P_n(0)$ is then estimated by $1 - P_n(1)$. It should be noted that $z_n^i \in \{0, 1\}$ indicates the status of the nth channel reported by the ith CR. The likelihood value of the sampling rate $X_K = x^i$ is then evaluated by

$$L(Z_K^{1:M}, X_K = x^i) = \prod_{n=1}^{N} P_n(1)^{z_n^i | x^i} P_n(0)^{1 - z_n^i | x^i}. \quad (4)$$

Moreover, based on the fused statues reports, we decide the status of nth channel to be free (0) if $P_n(1) < 0.5$; otherwise the channel is occupied.

2) Formulation of the Temporal Prior: Based on the proposed dynamic system as shown in Fig. 2, the temporal prior is decoupled as

$$P(X_K | CZ_{1:K-1}) = \int P(X_K | X_{K-1}) P(X_{K-1} | CZ_{1:K-1}) dX_{K-1}, \quad (5)$$

where $P(X_K | X_{K-1})$ introduces a temporal prediction process of the sampling rate and $P(X_{K-1} | CZ_{1:K-1})$ represents the previous posterior distribution estimated at sensing index $K - 1$. To determine the temporal prior, we need to calculate the probability $P(X_K | X_{K-1})$. In this paper, we define the prediction process of a sampling rate as

$$\begin{cases} X_K = U(X_{K-1}, X_{K-1}^*) + n_{k,CR} & \text{if } X_{K-1} \neq X_{K-1}^*, \\ X_K = n_{k,ER} & \text{if } X_{K-1} = X_{K-1}^*, \end{cases} \quad (6)$$

where $U(X_{K-1}, X_{K-1}^*)$ indicates the uniform sampling process modeling a random walk behavior between the rate X_{K-1} and the optimal rate X_{K-1}^* at sensing index $K - 1$; $n_{K,CR}$ is a uniform random variable between $[0, CR]$, representing the uncertainty of prediction inside a predefined confidence range CR; $n_{K,ER}$ is also a uniform random sample between $[0, -ER]$, designed to explore some possible smaller sampling rate inside the predefined explore range ER. In this process, the prediction of X_{K-1} is set to move toward but still larger than X_{K-1}^* to ensure the correct reconstruction. On the other hand, we allow an exploration for some possible smaller sampling rates while $X_{K-1} = X_{K-1}^*$. With the temporal prediction process, the probability distribution $P(X_K | X_{K-1})$ could then be determined and so does $P(X_K | CZ_{1:K-1})$.

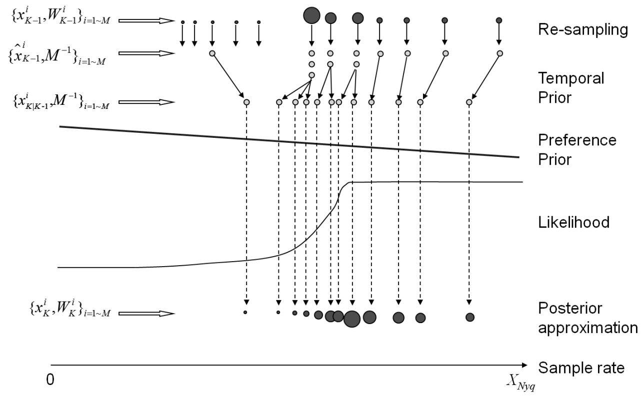

Fig. 3. Estimation flow of posterior distribution based on SMC.

3) Formulation of the Preference Prior: Besides the temporal prior, we also applied the preference prior $P(X_K)$ of the sampling rate for the optimal solution finding. Intuitively, our system prefers a smaller sampling rate for efficiency. Hence, the preference prior, $P(X_K)$, is designed to be a linear function as

$$P(X_K) = sX_K + Offset, \quad (7)$$

where $s = -0.01$ is a predefined slope of the linear function; $Offset$ is determined by the normalization process to make the probability summation of $P(X_K)$ equal to 1. Note that the interesting range of sampling rates is $[0, X_{Nyq}]$. With the prior $P(X_K)$, the proposed system tends to search the minimum sampling rate and meantime preserve the accuracy.

C. Estimation of Posterior Distribution Based on SMC

Since the prior and likelihood distributions are non-linear, direct computing of the posterior $P(X_K | CZ_{1:K})$ in equation (1) is non-trivial. Without a closed-form solution, we propose a SMC-based method to estimate $P(X_K | CZ_{1:K})$ in a numerical manner. To illustrate our method, without loss of generality, we detail how we operate the drawn particles to represent the evaluation of posterior distributions from the $K - 1$th to Kth sensing index. At the initial index where $K = 1$, we set all sample rates equal to Nyquist rate.

The operation flow is illustrated in Fig. 3. Assume that, at the $K - 1$th index, we have M equal-weighted particles to numerically approximate the posterior $P(X_{K-1} | CZ_{1:K-1})$. To simulate the temporal prior $P(X_K | CZ_{1:K-1})$ at the Kth sensing index, our system moves each of the M particles to a predicted location based on the prediction process in (6). Moreover, we evaluate the values of the likelihood term L^i in (4) and the preference prior term P^i in (7) with the corresponding sampling rate of the ith particle. The product of L^i and P^i forms the new weight W^i of the ith particle, and the new particle set $\{x_K^i, W_K^i\}_{i=1\sim M}$ approximates the posterior distribution $P(X_K | CZ_{1:K})$ at the Kth sensing index. Finally, a resample process is introduced. In the resample process, samples with larger weights are converted to more equal-weighted samples, while samples with smaller weights are converted to fewer equal-weighted samples or null samples. With this process, particles could be redistributed to the high-likelihood regions for the search of the optimal rate. The

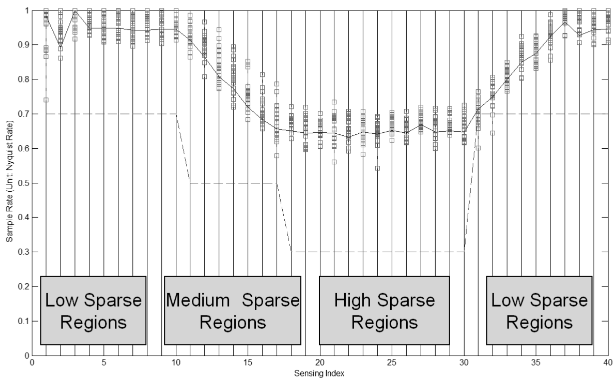

Fig. 4. The proposed dynamic rate adjustment scheme over a cognitive radio network with time-varying sparse levels. Sensing index $(K) \in [1, 40]$.

TABLE I
STATUS DETECTION ACCURACY (ACC) AND THE AVERAGE SENSING RATE (ASR) USED FOR SPECTRUM RECONSTRUCTION. NYQ: NYQUIST RATE.

Sensing Method	High sparse level		Medium sparse level		Low sparse level	
	ACC	ASR	ACC	ASR	ACC	ASR
Nyq Rate	100%	1.00 Nyq	100%	1.00 Nyq	100%	1.00 Nyq
3/4 Nyq Rate	100%	0.75 Nyq	97%	0.75 Nyq	82%	0.75 Nyq
Dynamic Rate	100%	0.66 Nyq	100%	0.77 Nyq	98%	0.91 Nyq

resample process is defined in (8).

$$\{\hat{x}^j\}_{j=1:M} = resample(\{x^i, CW^i\}_{i=1:M}), \quad (8)$$

where $\hat{x}^j = x^i$ if $CW^{i-1} < \frac{j}{M} \leq CW^i$. CW^i is a normalized and cumulated weight, calculated by

$$CW^i = \frac{\sum_{c=1}^{i} W^c}{\sum_{c=1}^{M} W^c}. \quad (9)$$

After the re-sampling process, we have a new equal-weighted set with again M particles for the next sensing index.

IV. SIMULATION RESULTS AND CONCLUSION

We simulate a cognitive radio network where 20 channel bands ($N = 20$) uniformly share the interesting frequency range $[50, 405]$ MHz for licensed users. White Gaussian process is used to model the environment noise with PSD at level 100. The PSD of transmitted signals is set at level 40. Moreover, we set the CR number $M = 20$. To emulate the time-varying sparse levels of a communication environment, we change the number of occupied channels over time as the red-and-dotted line shown in Fig. 4. Here, high, medium, and low sparse levels are defined as 30%, 50%, and 70% of the channel bands currently being used. Also, there are totally 40 sensing indices $(K) \in [1, 40]$ in our simulation.

As could be found in Fig. 4, the sparse level dynamically changes and our proposed sampling rate adjustment scheme could adaptively determines the optimal rate according to the sparse level as the blue line. In Fig. 4, the small blocks at each sensing index represent the drawn particles for different sampling rates. If comparing with some typical sensing methods which use a fixed sampling rate such as Nyquist rate or sub-Nyquist rate, our system could look for the optimal balance between system accuracy and efficiency. A quantitative evaluation of system accuracy and efficiency is listed in Table I. Here, we measure the status detection accuracy (ACC) to assess the system accuracy, which is defined as

$$ACC = \frac{True\ occupied\ channel\ detection + True\ free\ channel\ detection}{Total\ channel\ number \times Sensing\ index\ number}. \quad (10)$$

Besides, we calculate the average sensing rate (ASR) during different periods with different sparse levels to assess the system efficiency. Here, we use Nyquist rate of our transmitted signal as the reference unit.

In summary, we find that the sparse level of the environment spectrum is not static but time-varying. To adapt to the dynamic changing, we proposed an adjustment scheme for dynamic sampling rate selection. Here, the optimal dynamic rate selection is treated as a tracking problem. By combining SMC-based tracking concept with a distributed compressive spectrum sensing framework, the optimal sampling rate is well-tracked and the free channels are well-determined.

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