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Lifting modules with indecomposable decompositions. (English summary)

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Rings are associative with identity and modules are unitary right modules. The Jacobson radical of a module M is denoted by $\mathrm{rad}(M)$. A module M is a lifting module if and only if any submodule A of M contains a direct summand B of M such that A/B is small in M/B. The authors are interested in the problem of when a lifting module M is a direct sum of indecomposable submodules. They give two sufficient conditions for this to happen: (i) when $\mathrm{rad}(M)^{(I)}$ is small in $M^{(I)}$ for any index set I; (ii) when R has the a.c.c. on subsets of the form $\mathrm{ann}_R(m)$, where mR is a local direct summand of M and $\mathrm{rad}(M)$ is small in M. In particular, this happens over right perfect rings and right Bass rings (namely rings over which $\mathrm{rad}(M)$ is small in M for every module M).

Several interesting examples are provided.

Reviewed by *Jonathan Golan*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.