TWO-PERIOD FINANCIAL CONTRACTS WITH PRIVATE INFORMATION AND COSTLY STATE VERIFICATION*

DAVID C. WEBB

INTRODUCTION

Townsend [1982] and others have argued that, in a multiperiod context, linked contracts may be Pareto efficient relative to the private information environment. The similarities with the supergame literature are well documented. Townsend [1982, p. 1169] remarks: ". . . the key idea of supergames is that future payoffs to the decision maker are tied to present actions of the decision maker." In this context, by linking contracts together, an informed agent may be induced to report more honestly than in a single-period agreement. In other words, "honesty may become the best policy." In recent work Allen [1985] and Fudenberg et al. [1990] show that in a private information environment with no precommitment the gains to long-term contracting (income smoothing) in models such as Townsend [1982] are in fact due to restrictions on borrowing and saving. In particular, if the entrepreneur can access capital markets freely and on the same terms as the bank, then long-term contracts will be no better than a sequence of short-term contracts in the repeated model. The role of tie-ins then reflects little more than the fact that opportunities depend upon wealth as in the full-information decentralized solution. In this paper the entrepreneur only cares about his final payoff so that we do not establish a role for long-term contracts as a means of smoothing income. Instead, as suggested by Hart and Holmstrom [1986], we argue that long-term contracts derive from an inability to costlessly verify contingencies. In particular, if output realizations of projects are not costlessly verifiable, a long-term contract may then be used to induce truthful revelations that cannot be supported by a sequence of short-term contracts. This leads to a saving of verification costs over allocations achieved with unlinked contracts. This argument contributes to understanding why firms with known future prospects do not finance projects separately but instead enter into long-term relationships with banks.

We examine a two-period version of the borrower-lender problem of Gale and Hellwig [1985] and Townsend [1979]. In this

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framework output realizations are private information to the entrepreneur and only verifiable by the bank at a cost. However, the properties of two successive projects are common knowledge. We assume risk-neutral agents. It is shown that the optimal contract is a two-period contract with, like equity, state-contingent payments at the first date. This contract structure is incentive compatible and is shown to dominate an unmodified sequence of standard-debt-contracts with fixed payments at both dates. This is because it saves verification costs. This is illustrated in a simple numerical example.

I. The Problem

Consider an entrepreneur with a two-period horizon. He has the following opportunities. At date $t = 0$ he has a project that requires an investment of K_0 which yields a random return stream at data $t = 1$ of $\tilde{X}_1$. This return is distributed continuously on the interval $[0,\overline{X}_1]$ with strictly increasing, differentiable distribution function $H_1(X_1)$. At date $t = 1$ he has a second project that requires an investment of K_1 and yields a random return stream at date $t = 2$ of $\tilde{X}_2$. This return is distributed continuously on the interval $[0,\overline{X}_2]$ with strictly increasing, differentiable distribution function $H_2(X_2)$. We assume that at each date the entrepreneur's project is of positive net value: $E_0\tilde{X}_1 > K_0$, and $E_1\tilde{X}_2 > K_1$. At each date, as an alternative to investing in his projects, he can invest in a perfectly divisible, liquid asset, which for simplicity yields no interest.

The entrepreneur is risk neutral. His objective at date $t = 0$ is to maximize the expected value of his wealth at date $t = 2$:

$$\max E_0\tilde{W}_2. \tag{1}$$

At date $t = 0$ the entrepreneur has initial wealth of W_0. However, $W_0 < K_0$, so that if he is to undertake his first project, he must obtain outside finance. This is raised from banks. Banks are risk neutral expected profit maximizers who raise their finance at an interest rate of zero. We assume two banks engaged in Bertrand competition. The entrepreneur's projects at each date, their inputs and return streams, are common knowledge. However, the realizations of project returns are private information to the entrepreneur. The banks can only observe returns if they pay a fixed verification cost C.[1] We assume that banks can precommit to audit

1. For a discussion of optimal auditing see Mookerjee and Png [1989].

in certain contingencies. The return streams at each date net of these costs are given, respectively, by

$$\tilde{Y}_1 = \tilde{X}_1 - \tilde{\lambda}_1 C \geq 0 \qquad (2)$$
$$\tilde{Y}_2 = \tilde{X}_2 - \tilde{\lambda}_2 C \geq 0,$$

where $\tilde{\lambda}_1$ and $\tilde{\lambda}_2$ are indicator variables for costly verification at dates $t = 1$ and $t = 2$, respectively, taking on the value of one if the bank inspects and zero otherwise.

Banks know how much entrepreneurs need to invest at each date. They know the entrepreneur's external financing requirements and how much equity the entrepreneur puts up. But since output realizations are private information, at date $t = 1$ the entrepreneur can invest unobserved in the safe asset. This fact is very important in determining what form contract offers will take.

II. A Sequence of Standard-Debt-Contracts

One possible solution to the financing problem is a sequence of standard-debt-contracts along the lines of the one-period problem of Gale and Hellwig [1985]. This will be the optimal solution if we restrict contracts to have fixed payments in solvency states at each date.

Let B_0 denote the amount of money the entrepreneur borrows and S_0 the entrepreneur's equity stake at date $t = 0$. Let $D_1(\omega_1)$ denote the contractual payment to be made by the entrepreneur to the bank at date $t = 1$ in state $\omega_1 \in \Omega_1$. $B_1(\omega_1)$ is the entrepreneur's borrowing at date $t = 1$ contingent upon state $\omega_1 \in \Omega_1$ and $D_2(\omega_2|\omega_1)$ the contractual payment at date $t = 2$ in state $\omega_2 \in \Omega_2$ on this borrowing. The entrepreneur's wealth and equity stake at date $t = 1$ in state $\omega_1 \in \Omega_1$ are, respectively, $W_1(\omega_1)$ and $S_1(\omega_1)$. The contract problem is as follows:

$$\max E_0 \tilde{W}_2 \qquad (3a)$$

subject to

$$W_2(\omega_2) = Y_2(\omega_2) - D_2(\omega_2|\omega_1) + [W_1(\omega_1) - S_1(\omega_1)] \qquad (3b)$$

$$W_1(\omega_1) = Y_1(\omega_1) - D_1(\omega_1) + (W_0 - S_0) \qquad (3c)$$

$$W_1(\omega'_1) \geq W_1(\omega_1;\omega'_1) \qquad \omega'_1 \neq \omega_1 \qquad (3d)$$

$$W_2(\omega'_2) \geq W_2(\omega_2;\omega'_2) \qquad w'_2 \neq \omega_2 \qquad (3e)$$

$$E_0\tilde{D}_1 \geq B_0 \qquad (3f)$$

$$E_1\tilde{D}_2 \geq B_1(\omega_1). \qquad (3g)$$

Here (3d) and (3e) are incentive-compatibility conditions. If these conditions are satisfied, then, for example, if the true state at date $t = 1$ is ω'_1 the entrepreneur is better off announcing this state than some other state $\omega_1 \neq \omega'_1$. Conditions (3f) and (3g) are nonnegative (expected) profit conditions for banks. Bertrand competition between banks at each date ensures that these conditions are satisfied as equalities.

The above problem can be solved recursively. Suppose that at date $t = 1$ the entrepreneur has wealth $W_1(\omega_1)$. Then we have to find a contract that maximizes $E_1\tilde{W}_2$ subject to (3b), (3e), and (3g). If $W_1(\omega_1) \geq K_1$, the entrepreneur self-finances; but if $W_1(\omega_1) < K_1$, he must raise outside finance. Under the assumption that the entrepreneur has limited liability in the event of default and that there is a positive probability of default on any fixed payment from Gale and Hellwig [1985], we know that the contract at date $t = 1$ has

(i) maximum equity participation, $S_1(\omega_1) = W_1(\omega_1) \geq 0$ and $B_1(\omega_1) = K_1 - S_1(\omega_1) > 0$ or $S_1(\omega_1) = K_1 < W_1(\omega_1) > 0$ and $B_1(\omega_1) = 0$;
(ii) a fixed payment to the bank in nondefault states of $\overline{D}_2(\omega_1) > 0$ if $B_1(\omega_1) > 0$;
(iii) default and inspection by the bank if $X_2(\omega_2) < \overline{D}_2(\omega_1)$;
(iv) the bank recovers the maximum amount in default states of $X_2(\omega_2) - C$.

Given this, the first-period contract must be chosen to maximize $E_0\tilde{W}_1$ subject to (3c), (3d), and (3f). Limited liability and a positive probability of default on any fixed payment at date $t = 1$ mean that the first-period contract must also be a standard-debt-contract with

(i) maximum equity participation, $S_0 = W_0 > 0$;
(ii) a fixed payment $\overline{D}_1 > 0$ in nondefault states;
(iii) default and inspection by the bank if $X_1(\omega_1) < \overline{D}_1$;
(iv) the bank recovers the maximum amount in default states of $X_1(\omega_1) - C$.

The basic features of this contract sequence are easy to understand. As proved by Gale and Hellwig [1985], fixed payments to the bank in nondefault states and inspection in default states are required for incentive-compatibility. The other properties are required so as to minimize verification costs incurred in default states.

III. A Two-Period Contract

With a sequence of standard-debt-contracts the existence of the second project is not used at date $t = 0$ to reduce the incidence of verification costs at date $t = 1$. Yet the entrepreneur only cares about his final payoff and should in principle be prepared to vary the promised payment to the bank across states at the first date so as to reduce the incidence of verification costs. If output is low, his promised payment is low with the bank being compensated by higher payments in good states. Since the return on his second project is independent of the first, the entrepreneur can precommit to honest payments if under the contract the benefits of making dishonestly low payments at date $t = 1$ are more than offset by a lost share of the value of the second project.

With competition between banks in the middle period, given the amount of borrowing the entrepreneur undertakes at this date, the second stage of the contract problem has to be a standard-debt-contract. However, because the amount the entrepreneur has to borrow at date $t = 1$ is a function of how much he has to repay at that date, there is scope for making these payments state contingent and thereby save verification costs.

Given that the second-stage contract is a standard-debt-contract and maximum equity participation at each date, we substitute (3b), (3c), (3f), and (3g) into (3a) to obtain the two-period objective function:

$$(4)\quad E_0\tilde{W}_2 = \int_0^{\overline{X}_2} X_2(\omega_2)dH_2 - \int_0^{\overline{X}_1}\int_0^{\overline{D}_2(\omega_1)} CdH_2dH_1 + \int_0^{\overline{X}_1} X_1(\omega_1)dH_1 - \int_0^{\overline{D}_1} CdH_1 - K_1 - K_0 + W_0.$$

Now choose the values of $D_1(\omega_1)$ with $\overline{D}_1 < D_1(\omega_1) \leq X_1(\omega_1)$ satisfying (3f) to maximize (4). We obtain the following marginal condition for choice of $D_1(\omega_1)$:

$$(5)\quad -C\frac{dH_2}{d\overline{D}_2(\omega_1^*)}\frac{d\overline{D}_2(\omega_1^*)}{d\overline{D}_1}\frac{dH_1}{d\overline{D}_1}\frac{d\overline{D}_1}{dD_1(\omega_1)} - C\frac{dH_2}{d\overline{D}_2(\omega_1)}\frac{d\overline{D}_2(\omega_1)}{dD_1(\omega_1)}\frac{dH_1}{dD_1(\omega_1)} - C\frac{dH_1}{d\overline{D}_1}\frac{d\overline{D}_1}{dD_1(\omega_1)} = 0.$$

The intuition behind this condition is that by raising $D_1(\omega_1)$ with $D_1(\omega_1) \leq X_1(\omega_1)$, $\overline{D}_1$ can be cut so as to still satisfy (3f). This saves verification costs at date $t = 1$, given by the third term. However, if ω_1 occurs and the higher $D_1(\omega_1)$ is paid, the entrepreneur must borrow more, $B_1(\omega_1)$, to finance K_1. This in turn implies that $\overline{D}_2(\omega_1)$ satisfying (3g) rises, increasing verification costs at date $t = 2$, given by the second term. Offsetting this is the effect of the reduction in $\overline{D}_1$ which reduces borrowing at the default margin ω_1^*, satisfying $X_1(\omega_1) = \overline{D}_1$, causing a reduction in $\overline{D}_2(\omega_1^*)$ satisfying (3g) and a saving of verification costs, at date $t = 2$ given by the first term.

Since varying $D_1(\omega_1)$ across solvency states at date $t = 1$ reduces verification costs at date $t = 1$, keeping the value of date $t = 0$ borrowing equal to B_0, the expected value of $W_1(\omega_1)$ must be increased. Substituting (3b) into (3a), it follows that the expected value of $W_2(\omega_2)$ is increased.

However, if we are to have any variation in the payments at date $t = 1$, the incentive-compatibility constraints in problem (3) must be altered. They must ensure that the entrepreneur is better off at the second stage if he tells the truth at date $t = 1$. That is, if at date $t = 1$ the true state is ω'_1,

$$(6) \qquad E_1\tilde{W}_2(\omega_2|\omega'_1) \geq E_1\tilde{W}_2(\omega_2|\omega_1;\omega'_1) \qquad \omega_1 \neq \omega'_1,$$

where, for example, $W_2(\omega_2|\omega_1;\omega'_1)$ is the entrepreneur's wealth at date $t = 2$ in state ω_2 if he announces state ω_1 when the true state is ω'_1. Suppose that the true state at date $t = 1$ is ω'_1 and that the payment to the bank in that state is $D_1(\omega'_1)$. Now imagine that under the contract, if the entrepreneur announces another state, $\omega_1 \neq \omega'_1$, he pays $D_1(\omega_1) < D_1(\omega'_1)$. Then, so as not to be found out, the entrepreneur must act as if his return is $X_1(\omega_1) < X_1(\omega'_1)$ and hence $W_1(\omega_1) < W_1(\omega'_1)$ so that his equity stake is $S_1(\omega_1) < S_1(\omega'_1)$. Then with a smaller equity stake, for positive borrowing at date $t = 1$, $B_1(\omega_1) > B_1(\omega'_1)$. This in turn implies that the fixed payment at the second stage satisfying (3g) will be higher, $\overline{D}_2(\omega_1) > \overline{D}_2(\omega'_1)$. The gain to lying is that the entrepreneur withholds $D_1(\omega'_1) - D_1(\omega_1)$. The cost arises from having to borrow more at date $t = 1$ and consequently having to make higher payments to the bank at the second stage. This cost will be larger, the greater $X_1(\omega'_1) - X_1(\omega_1) > 0$ and hence $B_1(\omega_1) - B(\omega'_1) > 0$. To ensure truth telling, the gain to lying must be less than the cost. Evaluating (6), we get

$$(7) \quad D_1(\omega'_1) - D_1(\omega_1) < \int_{\overline{D}_2(\omega'_1)}^{\overline{X}_2} (X_2(\omega_2) - \overline{D}_2(\omega'_1))dH_2 - \int_{\overline{D}_2(\omega_1)}^{\overline{X}_2} (X_2(\omega_2) - \overline{D}_2(\omega_1))dH_2.$$

Condition (7) imposes a limit on the variation in payments in nondefault states at date $t = 1$ without encouraging lying. That is $D_1(\omega_1) - \overline{D}_1$ in nondefault states cannot be too large.

Assuming that at each date the payments to the bank are made out of project income, $D_1(\omega_1) \leq Y_1(\omega_1)$, and $D_2(\omega_2) \leq Y_2(\omega_2)$. Then if there is a positive probability of default on any fixed payment to the bank at each date, we know that the two-period contract has the following general properties:

(i) a maximum equity stake for the entrepreneur, $S_0 = W_0$;
(ii) a stream of state contingent payments to the bank in nondefault states at date $t = 1$ with $D_1(\omega_1) \geq \overline{D}_1$ in these states;
(iii) if $X_1(\omega_1) < \overline{D}_1$, the entrepreneur defaults and the bank inspects;
(iv) maximum recovery of debt in default states at date $t = 1$ of $X_1(\omega_1) - C$;
(v) maximum equity participation at date $t = 1$, $S_1(\omega_1) = W_1(\omega_1) \geq 0$ and $B_1(\omega_1) \geq 0$ or $S_1(\omega_1) < W_1(\omega_1)$ and $B_1(\omega_1) = 0$, if the entrepreneur defaults at date $t = 1$ he undertakes his second project entirely with borrowed money;
(vi) a fixed payment in default states at date $t = 2$ of $\overline{D}_2(\omega_1) \geqslant 0$, where this payment is contingent upon how much is borrowed at date $t = 1$, $B_1(\omega_1) \geq 0$;
(vii) default and inspection by the bank at date $t = 2$ if $X_2(\omega_2) < \overline{D}_2(\omega_1)$;
(viii) maximum recovery of debt in default states at date $t = 2$ of $X_2(\omega_2) - C$.

The general principle that is at work here is interesting. Since the entrepreneur only cares about his terminal wealth, he is willing to delay his payoff from his projects until the final period. The average return to a sequence of independent projects will be fairly certain, however, thereby reducing the need for verification. Thus, taking into account incentive constraints, we are able to get closer to the first best. We conjecture, but do not prove, that the longer the sequence of projects the closer we can get to the first best.

IV. A Numerical Example

Here we shall illustrate both a sequence of standard-debt-contracts and the contract with state-contingent payments at date $t = 1$. The example we use has simple two-point distributions for project returns. Hence we impose maximum equity participation and maximum recovery of debt in default states and concentrate on the role of state-contingent versus fixed payments. For simplicity, and without loss, here assume an entrepreneur with no initial wealth who has two projects, one at date $t = 0$ and a second at date $t = 1$. For ease of exposition we also assume that both projects are identical. At date $t = 0$ the project has an input of 8 and yields an output at date $t = 1$ of 14 with probability ½ or 7½ with probability ½. At date $t = 1$ he has the same opportunity again. This information is known to the bank. Output realizations are private information of the entrepreneur; the bank can observe output at a fixed cost of 3½. The numbers have been picked for ease of computation, and no particular significance should be attached to them.

We outline two solutions to the financing problem. First, we illustrate the outcome with a sequence of standard-debt-contracts. With this solution, at date $t = 0$ the fixed payment $\overline{D}_1$ satisfies

$$\tfrac{1}{2}\overline{D}_1 + \tfrac{1}{2}\min\,[\overline{D}_1, 7\tfrac{1}{2}] - \tfrac{1}{2}\tilde{\lambda}_1 3\tfrac{1}{2} = 8, \tag{8}$$

where $\tilde{\lambda}_1 = 1$ if $\overline{D}_1 > 7\frac{1}{2}$. The solution to (8) is $\overline{D}_1 = 12$. It follows that $W_1 = (14 - 12) = 2$ with probability ½ or 0 with probability ½. Then at date $t = 1$, with maximum equity participation, the entrepreneur borrows $8 - W_1$. Then $\overline{D}_2$ satisfies the bank's zero profit condition,

$$\tfrac{1}{2}\overline{D}_2 + \tfrac{1}{2}\min\,[\overline{D}_2, 7\tfrac{1}{2}] - \tfrac{1}{2}\tilde{\lambda}_2 3\tfrac{1}{2} = 8 - W_1, \tag{9}$$

where $\tilde{\lambda}_2 = 1$ if $\overline{D}_2 > 7\frac{1}{2}$ and $\tilde{\lambda}_2 = 0$ if $\overline{D}_2 < 7\frac{1}{2}$. If $W_1 = 2$, the solution to (9) is $\overline{D}_2 = 6$. However, if $W_1 = 0$, then it follows that $\overline{D}_2 = 12$. Then at date $t = 1$ the entrepreneur's expected return at date $t = 2$ is either $E_1\tilde{W}_2 = \frac{1}{2}(14 - 6) + \frac{1}{2}(7\frac{1}{2} - 6) = 4\frac{3}{4}$ with probability ½, or $E_1\tilde{W}_2 = \frac{1}{2}(14 - 12) = 1$ with probability ½. Hence at date $t = 0$,

$$E_0\tilde{W}_2 = \tfrac{1}{2}(4\tfrac{3}{4}) + \tfrac{1}{2}(1) = 2\tfrac{7}{8}. \tag{10}$$

It is readily checked that this solution is incentive compatible.

Second, consider the possibility that the first project is financed with a contract that has state-contingent payments at date $t = 1$. Consider the contract at date $t = 0$ that has a payment of $D_1(\omega_1) = 8\frac{1}{2}$ in the high-income state and 7½ in the low-income state. These payments satisfy the bank's zero profit condition, which since verification costs will not be incurred is

$$\tag{11} \tfrac{1}{2}(8\tfrac{1}{2}) + \tfrac{1}{2}(7\tfrac{1}{2}) = 8.$$

It follows that $W_1 = (14 - 8\frac{1}{2}) = 5\frac{1}{2}$ with probability $\frac{1}{2}$ or 0 with probability $\frac{1}{2}$. Then at date $t = 1$, with maximum equity participation of $W_1 = 5\frac{1}{2}$, $\bar{D}_2$ satisfies

$$\tag{12} \tfrac{1}{2}\,\bar{D}_2 + \tfrac{1}{2}\min\,[\bar{D}_2, 7\tfrac{1}{2}] - \tfrac{1}{2}\,\tilde{\lambda}_2 C = 8 - 5\tfrac{1}{2},$$

where since $\bar{D}_2 < 7\frac{1}{2}$, $\tilde{\lambda}_2 = 0$ so that $\bar{D}_2 = 2\frac{1}{2}$. But if $W_1 = 0$, $\bar{D}_2$ satisfies

$$\tag{13} \tfrac{1}{2}\,\bar{D}_2 + \min\,(\bar{D}_2, 7\tfrac{1}{2}) - \tfrac{1}{2}\,\tilde{\lambda}_2 3\tfrac{1}{2} = 8,$$

where $\tilde{\lambda}_2 = 1$ if $\bar{D}_2 > 7\frac{1}{2}$ and the solution to (13) is $\bar{D}_2 = 12$. Then at date $t = 1$ the entrepreneur's expected return at date $t = 2$ is either $E_1\tilde{W}_2 = \frac{1}{2}(14 - 2\frac{1}{2}) + \frac{1}{2}(7\frac{1}{2} - 2\frac{1}{2}) = 8\frac{1}{4}$ with probability $\frac{1}{2}$ or $E_1\tilde{W}_2 = \frac{1}{2}(14 - 12) = 1$ with probability $\frac{1}{2}$. Hence, at date $t = 0$,

$$\tag{14} E_0\tilde{W}_2 = \tfrac{1}{2}(8\tfrac{1}{4}) + \tfrac{1}{2}(1) = 4\tfrac{5}{8}.$$

Finally, we must check that this solution is incentive compatible at date $t = 1$ so that (6) is satisfied. Clearly if output is $7\frac{1}{2}$, the entrepreneur cannot announce an output of 14. However, if output is 14, we have to ensure that the entrepreneur will not announce that output is $7\frac{1}{2}$. Suppose that he did: then he must hold back $14 - 7\frac{1}{2} = 6\frac{1}{2}$ and borrow as if $W_1 = 0$ so that $\bar{D}_2 = 12$; then $E_1\tilde{W}_2 = \frac{1}{2}(14 - 12) + 6\frac{1}{2} = 7\frac{1}{2}$. If he acts honestly, he gets $8\frac{1}{4}$ so the postulated solution is incentive compatible at date $t = 1$. Incentive compatibility at date $t = 2$ is also guaranteed.

Comparing (14) with (10), we see that the contract with state-contingent payments at date $t = 1$ dominates a sequence of standard-debt-contracts. Now compare these outcomes with the full information outcome. The total social value of the entrepreneur's projects is $2(\frac{1}{2}(14) + \frac{1}{2}(7\frac{1}{2}) - 8) = 5\frac{1}{2}$. With the sequence of standard-debt-contracts the entrepreneur realizes a total net expected return of $2\frac{7}{8}$. The shortfall from the social value is $2\frac{5}{8}$ which is accounted for by the expected value of verification costs. These are $\frac{1}{2}(3\frac{1}{2})$ at date $t = 1$ and $\frac{1}{4}(3\frac{1}{2})$ at date $t = 2$ which add to $2\frac{5}{8}$. With the second contract the entrepreneur realizes a total net expected return of $4\frac{5}{8}$. Here the verification costs at date $t = 1$ are avoided, although there is the same incidence at date $t = 2$ so that total expected verification costs are $\frac{1}{4}(3\frac{1}{2})$.

V. Comment on Rolling Over Debt Payments

Before concluding the paper, I shall comment on the restriction that $D_1(\omega_1) \leq Y_1(\omega_1)$ which was imposed on the contractual payments at date $t = 1$. It is interesting to consider the implications of the removal of this constraint. The restriction that all contractual obligations are made out of current income means that the contract cannot specify that in some state $\omega_1 \in \Omega_1$ the entrepreneur should have an obligation of $D_1(\omega_1)$, but if $D_1(\omega_1) > X_1(\omega_1)$, $X_1(\omega_1)$ is paid and the residual $R_1(\omega_1) = D_1(\omega_1) - X_1(\omega_1) > 0$ is rolled over, alongside new borrowing, as a claim against the second project without the bank inspecting. The problem is that the entrepreneur would always like to declare the state with the lowest obligation. However, along similar lines to the analysis in Section III, the payments $D_1(\omega_1)$, $\omega_1 \in \Omega_1$, could be made incentive compatible. The entrepreneur could be made to reveal high values of $X_1(\omega_1)$ when $D_1(\omega_1) > X_1(\omega_1)$ by having a smaller payment rolled over. But there will be a lower limit on the actual amount that can be paid at date $t = 1$, $\overline{D}_1$. If this amount is not paid, the bank will inspect. Then $\overline{D}_1 - Y_1(\omega_1) > 0$, where $Y_1(\omega_1) = X_1(\omega_1) - C$ will be rolled over.

The last point raises a problem. Consider the case when $\overline{D}_1 - Y_1(\omega_1) > 0$ is rolled over. This is consistent with minimizing verification costs. However, there is a limit to how much can be rolled over, which is when there is too much debt for it to be in the entrepreneur's interests to do his second project. Then at this limit nothing further is rolled over. In a more general model which allowed for endogenous effort, this may raise moral hazard problems. An entrepreneur whose first project is going badly may have an incentive to fail in a big way.

In the case of the numerical example in Section IV, the reader should be able to see that allowing for rollover of payments at date $t = 1$ yields exactly the same outcome as that we have given. It involves setting $\overline{D}_1 = 8$ in both states at date $t = 1$ and rolling over $8 - 7\frac{1}{2}$ in the bad state. The rest of the example follows along the same lines as before.

VI. Conclusion

In this paper we assumed private information and costly state verification and showed that multiperiod contracts may be preferable to a sequence of one-period contracts. If the return distributions of projects are common knowledge, with costly state verification, the optimal contract is a two-period contract. Like equity, it has state-contingent payments at the first stage and a second-stage contract linked to the first-stage output relevation. The payment at the second stage, however, is fixed in nondefault states. The paper explains why firms with known future prospects may deviate from simple project financing and enter into long-term relationships with banks.

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