# OWA operators for decision support *

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**Abstract**

R. Yager [6] introduced a new aggregation technique based on the ordered weighted averaging (OWA) operators. In this article we illustrate the applicability of OWA operators to a doctoral student selection problem at the Graduate School of Turku Centre for Computer Science.

## 1 OWA Operators

Ronald R. Yager [6] introduced a new aggregation technique based on the ordered weighted averaging operators.

**Definition 1.1** *An OWA operator of dimension $n$ is a mapping $F\colon R^n \to R$, that has an associated $n$ vector $w = (w_1, w_2, \dots, w_n)^T$ such as $w_i \in [0, 1]$, $1 \leq i \leq n$, and*

$$w_1 + \cdots + w_n = 1.$$

*Furthermore*

$$F(a_1, \dots, a_n) = w_1 b_1 + \cdots + w_n b_n$$

*where $b_j$ is the $j$-th largest element of the bag $< a_1, \dots, a_n >$.*

A fundamental aspect of this operator is the re-ordering step, in particular an aggregate $a_i$ is not associated with a particular weight $w_i$ but rather a weight is associated with a particular ordered position of aggregate. In order to classify OWA operators in regard to

*in: *Proceedings of EUFIT'97 Conference*, September 8-11, 1997 Aachen, Germany, Verlag Mainz, Aachen, Vol. II, 1997 1539-1544.

†Partially supported by OTKA T 14144 and OTKA T 019455.

their location between *and* and *or*, Yager [6] introduced a measure of *orness*, associated with any vector $w$ as follows

$$orness(w) = \frac{1}{n-1}\sum_{i=1}^{n}(n-i)w_i$$

It is easy to see that for any $w$ the $orness(w)$ is always in the unit interval. Furthermore, note that the nearer $w$ is to an *or*, the closer its measure is to one; while the nearer it is to an *and*, the closer is to zero. Generally, an OWA operator with much of nonzero weights near the top will be an *orlike* operator, ($orness(w) \geq 0.5$), and when much of the weights are nonzero near the bottom, the OWA operator will be *andlike*. The standard degree of orness associated with a **R**egular **I**ncreasing **M**onotone (RIM) linguistic quantifier $Q$

$$orness(Q) = \int_0^1 Q(r)\,dr$$

is equal to the area under the quantifier [9]. Consider the family of RIM quantifiers

$$Q_\alpha(r) = r^\alpha,\ \ \alpha \geq 0. \tag{1}$$

It is clear that

$$orness(Q_\alpha) = \int_0^1 r^\alpha\,dr = \frac{1}{\alpha+1}$$

and $orness(Q_\alpha) < 0.5$ for $\alpha > 1$, $orness(Q_\alpha) = 0.5$ for $\alpha = 1$ and $orness(Q_\alpha) > 0.5$ for $\alpha < 1$. In [6] Yager suggested an approach to the aggregation of criteria satisfactions guided by a regular non-decreasing quintifier $Q$. If $Q$ is RIM quantifier then we measure the overall success of the alternative $x = (a_1, \dots, a_n)$ by $F_Q(a_1, \dots, a_n)$, where $F_Q$ is an OWA operator derived from $Q$, i.e. the weights associated with this quantified guided aggregation are obtained as follows

$$w_i = Q(\frac{i}{n}) - Q(\frac{i-1}{n}) \tag{2}$$

for $i = 1, \dots, n$.

## 2 The case

The Graduate School of Turku Centre for Computer Science (TUCS) offers a programme for gaining the Doctoral (PhD) degree in Computer Science and Information Systems. It is open for students from everywhere. The teaching language of the school is English. Prerequisites are either a Master's or a Bachelor's degree in Computer Science or in a closely related field. Study time is expected to be 4 years when starting from Master's level and 6 years from Bachelor's level. Since the number of applicants (usually between 20 and 40) is much greater than the number of available scholarhips (around 6) we have to rank the candidates based on their performances. It can also happen that only a part of available scholarships will be awarded, because the number of *good* candidates is smaller than the number of available places.

The problem of selecting *young promising doctoral researchers* can be seen to consist of three components. The first component is a collection

$$X = \{x_1, \dots, x_p\}$$

of applicants for the Ph.D. program. The second component is a collection of 6 criteria (see Table 1) which are considered relevant in the ranking process.

For simplicity we suppose that all applicants are *young* and have Master's degree acquired more than one year before. In this case all the criteria are meaningful, and are of approximately the same importance.

| **Research interests** | (excellent) | (average) | (weak) |
|---|---|---|---|
| - Fit in research groups | ○ | ○ | ○ |
| - On the frontier of research | ○ | ○ | ○ |
| - Contributions | ○ | ○ | ○ |
| **Academic background** | | | |
| - University | ○ | ○ | ○ |
| - Grade average | ○ | ○ | ○ |
| - Time for acquiring degree | ○ | ○ | ○ |
| Letters of recommendation | Y | N | |
| Knowledge of English | Y | N | |

**Table 1** Evaluation sheet.

For applicants with Bachelor's degree the first three criteria *Fit in research groups*, *Contributions* and *On the frontier of research* are meaningless, because we have an undergraduate student without any research record. An applicant with Bachelor's degree or *just acquired* Master's degree should have excellent university record from a good university to be competitive. For *old* applicants we encounter the problem of trade-offs between the age and the research record, and in this case their ratings on the last three criteria *University*, *Grade average* and *Time for acquiring degree* do not really matter. An *old* applicant should have a very good research record and a history of scientific cooperation with a TUCS research group to be competitive.

The third component is a group of 11 experts whose opinions are solicited in ranking the alternatives. The experts are selected from 9 research groups.

So we have a **M**ulti **E**xpert-**M**ulti **C**riteria **D**ecision **M**aking (ME-MCDM) problem. The ranking system described in the following is a two stage process. In the first stage, individual experts are asked to provide an evaluation of the alternatives. This evaluation consists of a rating for each alternative on each of the criteria, where the ratings are chosen from the scale $\{1, 2, 3\}$, where 3 stands for *excellent*, 2 stands for *average* and 1 means *weak* performance. Each expert provides a 6-tuple $(a_1, \dots, a_6)$ for each applicant, where $a_i \in \{1, 2, 3\},\ i = 1, \dots, 6$. The next step in the process is to find the overall evaluation for an alternative by a given expert.

In the second stage we aggregate the individual experts evaluations to obtain an overall value for each applicant. Taking into consideration that we have 6 criteria (see Table 1) the weights derived from $Q_\alpha$ are determined by (2). Furthermore, whatever is the linguistic quantifier, $Q_\alpha$, representing the statement *most criteria are satisfied by* $x$, we see that

$$1 \leq F_\alpha(a_1, \ldots, a_6) \leq 3$$

holds for each alternative $x = (a_1, \ldots, a_6)$ since $a_i \in \{1, 2, 3\},\ i = 1, \ldots, 6$.

We search for an index $\alpha \geq 0$ such that the associated linguistic quantifier $Q_\alpha$ from the family (1) approximates the experts' preferences as much as possible. After interviewing the experts we found that all of them agreed on the following principles

(i) if an applicant has more than two weak performances then his overall performance should be less than two,

(ii) if an applicant has maximum two weak performances then his overall performance should be more than two,

(iii) if an applicant has all but one excellent performances then his overall performance should be about 2.75,

(iv) if an applicant has three weak performances and one of them is on the criterion *on the frontier of research* then his overall performance should not be above 1.5,

From $(i)$ and $(ii)$ we find

$$1 < \alpha \leq 1.293,$$

which means that $Q_\alpha$ should be *andlike* (or risk averse) quantifier with a degree of compensation just below the arithmetic average. It is easy to verify that (iii) and (iv) can not be satisfied by any quantifier $Q_\alpha,\ 1 < \alpha \leq 1.293$, from the family (1). In fact, (iii) requires that $\alpha \approx 0.732$ which is smaller than 1 and (iv) can be satisfied if $\alpha \geq 2$ which is bigger than 1.293. Rules (iii) and (iv) have priority whenever they are applicable.

In the second stage the technique for combining the expert's evaluation to obtain an overall evaluation for each alternative is based upon the OWA operators. Each applicant is represented by an 11-tuple

$$(b_1, \ldots, b_{11})$$

where $b_i \in [1, 3]$ is the unit score derived from the $i$-th expert's ratings. We suppose that the $b_i$'s are organized in descending order, i.e. $b_i$ can be seen as the worst of the $i$-th top scores.

Taking into consideration that the experts are selected from 9 different research groups there exists no applicant that scores overall well on the first criterion "Fit in research group". After a series of negotiations all experts agreed that the support of at least four experts is needed for qualification of the applicant.

Since we have 11 experts, applicants are evaluated based on their top four scores $(b_1, \ldots, b_4)$ and if at least three experts agree that the applicant is excellent then his final score should be 2.75 which is a cut-off value for the best student. That is

$$F_\alpha(3, 3, 3, 1) = 3 \times (w_1 + w_2 + w_3) + w_4 = 2.75 \iff \alpha \approx 0.464$$

So in the second stage we should choose an *orlike* OWA operator with $\alpha \approx 0.464$ for aggregating the top six scores of the applicant to find the final score.

If the final score is less than 2 then the applicant is disqualified and if the final score is at least 2.5 then the scholarship should be awarded to him. If the final score is between 2 and 2.5 then the scholarship can be awarded to the applicant pending on the total number of scholarships available.

**Example 1** *Let us choose $\alpha = 1.2$ for the aggregation of the ratings in the first stage. Consider some applicant with the following scores*

| *Criteria* | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ | $C_6$ |
|---|---|---|---|---|---|---|
| Expert 1 | 3 | 2 | 3 | 2 | 3 | 1 |
| Expert 2 | 2 | 3 | 3 | 2 | 3 | 2 |
| Expert 3 | 2 | 2 | 3 | 2 | 2 | 1 |
| Expert 4 | 3 | 2 | 3 | 3 | 3 | 2 |
| Expert 5 | 2 | 2 | 3 | 2 | 3 | 1 |
| Expert 6 | 3 | 2 | 3 | 2 | 3 | 1 |
| Expert 7 | 1 | 2 | 3 | 2 | 3 | 2 |
| Expert 8 | 1 | 2 | 3 | 2 | 3 | 1 |
| Expert 9 | 1 | 2 | 2 | 2 | 3 | 2 |
| Expert 10 | 1 | 2 | 2 | 3 | 3 | 1 |
| Expert 11 | 1 | 2 | 2 | 2 | 2 | 1 |

The weights associated with this linguistic quantifier are

$$(0.116, 0.151, 0.168, 0.180, 0.189, 0.196)$$

After re-ordering the scores in descending order we get the following table

| | | | | | | | *Unit score* |
|---|---|---|---|---|---|---|---|
| Expert 1 | 3 | 3 | 3 | 2 | 2 | 1 | 2.239 |
| Expert 2 | 3 | 3 | 3 | 2 | 2 | 2 | 2.435 |
| Expert 3 | 3 | 2 | 2 | 2 | 2 | 1 | 1.920 |
| Expert 4 | 3 | 3 | 3 | 3 | 2 | 2 | 2.615 |
| Expert 5 | 3 | 3 | 2 | 2 | 2 | 1 | 2.071 |
| Expert 6 | 3 | 3 | 3 | 2 | 2 | 1 | 2.239 |
| Expert 7 | 3 | 3 | 2 | 2 | 2 | 1 | 2.071 |
| Expert 8 | 3 | 3 | 2 | 2 | 1 | 1 | 1.882 |
| Expert 9 | 3 | 2 | 2 | 2 | 2 | 1 | 1.920 |
| Expert 10 | 3 | 3 | 2 | 2 | 1 | 1 | 1.882 |
| Expert 11 | 2 | 2 | 2 | 2 | 1 | 1 | 1.615 |

In the second stage we choose $\alpha = 0.464$ and obtain the following weights

$$(0.526, 0.199, 0.150, 0.125).$$

The best four scores of the applicant are

$$(2.615, 2.435, 2.239, 2.239).$$

The final score is computed as

$$F_\alpha(2.615, 2.435, 2.239, 2.239) = 2.475.$$

So the applicant has good chances to get the scholarship.

# 3 Summary and Conclusions

We have presented a two stage process for doctoral student selection problem. In the first stage we have used an *andlike* OWA operator to implement some basic rules derived from certain (extremal) situations. In the second stage we have applied an *orlike* OWA operator, because the final score of applicants should be high if at least three experts find his record attractive (we do not require support from *all experts*).

It can happen (and it really happened) that some experts (a minority) forms a coalition and deliberately *overrate* some candidates in order to qualify them even though the majority of experts finds these candidates overall weak. We can resolve this problem by adding an extra criterion to the set of criteria measuring the competency of individual experts, or we issue an alarm message about the attempted cheating. To determine the most appropriate linguistic quantifier in the first stage we can also try to *identify* interdependences between criteria [1, 2, 3].

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