DESIGNING DIDACTICAL TOOLS AND MICROWORLDS FOR MATHEMATICS EDUCATIONS

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Abstract

This paper reports on the extensive design activities of the Freudenthal Institute in the last few years on the area of small didactical tools and microworlds (java applets). New technologies led to new ideas for visualising and learning mathematics. The effort made in several development- and implementation projects resulted in a collection of robust and well-tested didactical tools. A lot of these applets has found it way to the practice of mathematics education and a lot of school work with these new tools. Still, a lot of research is still needed (and planned) to get a detailed image of the possibilities and constraints of these new tools and their role in longer learning trajectories.

Key words: microworlds, technology, educational tools.

Introduction

The objectives of the Freudenthal Institute are research and curriculum development for innovation and improvement of mathematics education. From the very start of the introduction of the computer in mathematics education the possibilities of this medium were seriously investigated. Software development was practised at the institute from the beginning. About 8 years ago new technologies and programming tools that facilitate faster computer graphics led to an overwhelming amount of new ideas for visualising and new ways of learning mathematics and creative programmers and designers produced quite exciting prototypes. Off course this was just a first stage of the development of robust, well-tested and researched didactical tools. In the next stage further development, field testing and refinement was performed in several research-, development- and implementation projects, in real educational settings and in close co-operation with teachers. This paper reports on the different experiences, the successes but also the encountered problems in this development and implementation process. Although this process is still ongoing, a considerable collection of applets has found its way to the practice of mathematics education and there is much appreciation for these new tools at schools. Still a lot of research is needed (and planned) to get a detailed

image of the possibilities and constraints of these new tools, especially of their role in longer learning trajectories.

To give insight in the design activities and the development process, some paradigmatic applet examples are described. First a distinction is drawn between three different kind of applets. The borders between these three categories are not very sharp, but this classification helps to structure the underlying design choices.

- Applets that offers a 'virtual reality'. These applets are used for representing and simulating real-world objects and processes that form the basis of mathematical reasoning.

- Applets that facilitate the use of 'models'. These applets offer interactive models that can be helpful in building and understanding the more abstract mathematical objects and concepts.

- Applets that offer a mathematical microworld. In these applets mathematical objects like formulas, equations and graphs can be constructed and transformed.

For each category one or more examples of these applets are described and used to illustrate en discuss the typical issues and problems in the design process.

Theoretical Framework

The theoretical foundation is mainly based upon the theory of Realistic Mathematics Education (RME). The main idea of RME is to create opportunities for students to come to regard the knowledge they acquire as their own knowledge. Therefore, contextual problems which students recognise as relevant and real, and which also evoke productive solution strategies, play an important role. In RME the intended goals are built progressively upon the students' informal ideas and strategies (Freudenthal, 1991).

For RME to work, it is necessary to know how students model new situations. Students are confronted with problem situations for which they do not have the appropriate models at their disposal; i.e. models which describe possible structures or patterns in the situations. At this point, theories on symbolising – individual as well as social aspects – are useful (Gravemeijer et al., 2002).

Applets that offer a 'virtual reality'.

In the RME theory real-world situations and experiences form the starting point of mathematical activities. Educational designs should for that reason contain carefully chosen real-world situations that are suitable for being mathematised. Software that simulates these situations can be used to wake up old experiences and extend these with new ones by purposefully provoked interactions.

3D-Geometry showed to be a very fruitful subject for this approach. We developed quite some applets on this subject for students of all levels.
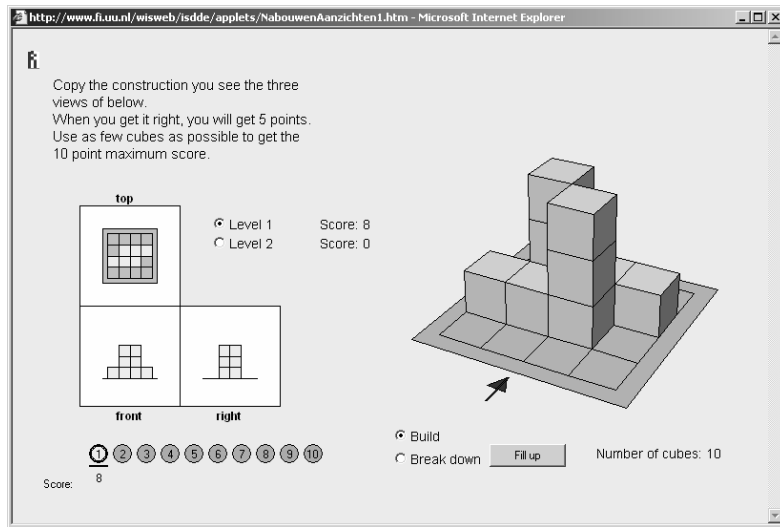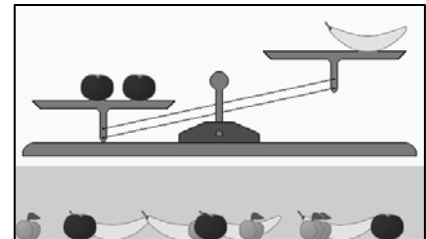

Figure 1

Figure 2

The term 'virtual reality' is possibly somewhat misleading. In common virtual reality software the aim is to create a 'world' that should be experienced as real as possible. In de applets the aim is to make firm connection with real-world experiences, but also to make the step to more model-like representations, in which certain mathematical concepts can emerge. The design challenge lays in finding a balance between giving the user freedom in his constructions and explorations in the virtual environment and imposing constraints to guide the user to intended experiences. For example the block building environment (figure 1) gives the user the freedom in making his own constructions, but the environment also enforces a cubic structure that draws the attention more easily to orthogonal co-ordinates as a mean to model space.

A quite different example of this category is the applet 'Fruit Balance` (figure 2). It offers a ´real´ balance that can be used to compare the weights of three different pieces of fruit. The task is to find the weight of all of them, when one is given. It's an nice example of the RME approach. A meaningful problem is introduced, without offering the ready-made mathematical tools like standard solving algorithms for a set of equations with multiple variables, but by offering a tool to explore the problem situation. The, often informal strategies that students put forward can become the starting of a learning trajectory on solving equations, based on 'guided reinvention', one of the important aspects of RME. Off course the most important thing is the design of such a trajectory, in which the student's explorations, reasoning and solutions are shared, discussed and generalised to other

problems. Else the applet activity will only be a nice puzzle without a substantial role in the learning of mathematics.

Applets that facilitate the use of 'models'

Geometrical shapes and objects are not used in the mathematical domain of geometry only. Often they appear as model or tool for representing more abstract mathematical concepts and processes. A nice example of such a model is the number line that can be used in arithmetic. Processes like addition and subtraction can be modelled by making step or jumps on the number line. For a developer of it-tools there lays the challenge in making those models dynamically and in developing interactions or games in which especially the dynamic processes can be made visible and controllable. The applet "Jump tool" offers such a dynamic and interactive version of the more static representations of the number line.

Another example of a geometrical way to model mathematical objects is the 'area-model' for algebraic quadratic expressions. Variables are used in the width and height of rectangles and quadratic expressions represent their areas. Several applets were designed, using this model. 'Geometric Algebra 2d' is one of them (figure 3).
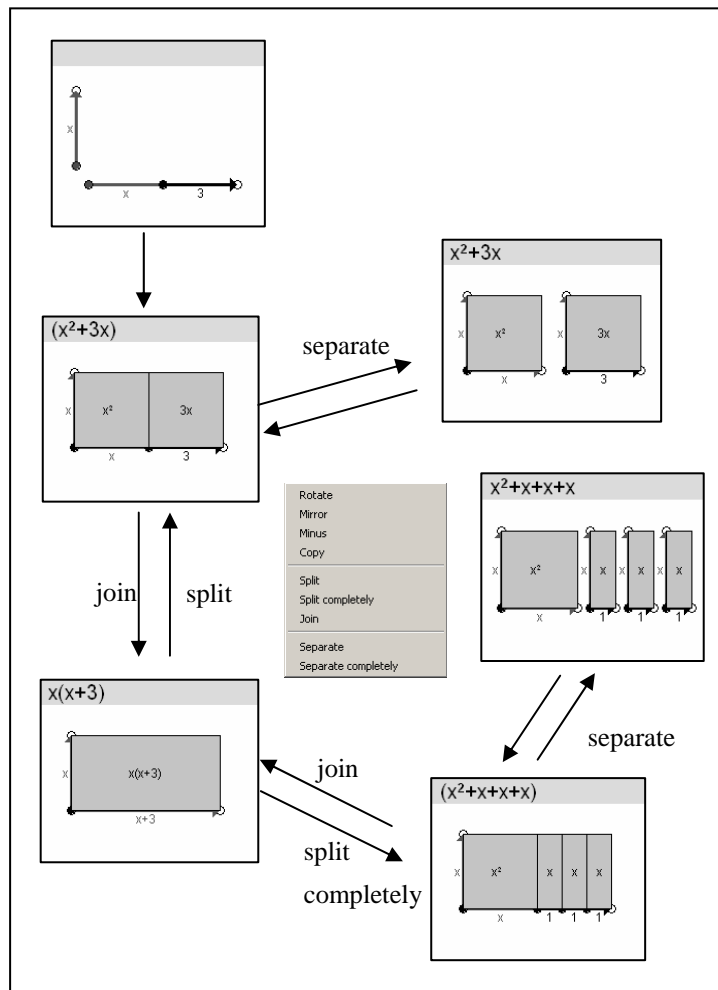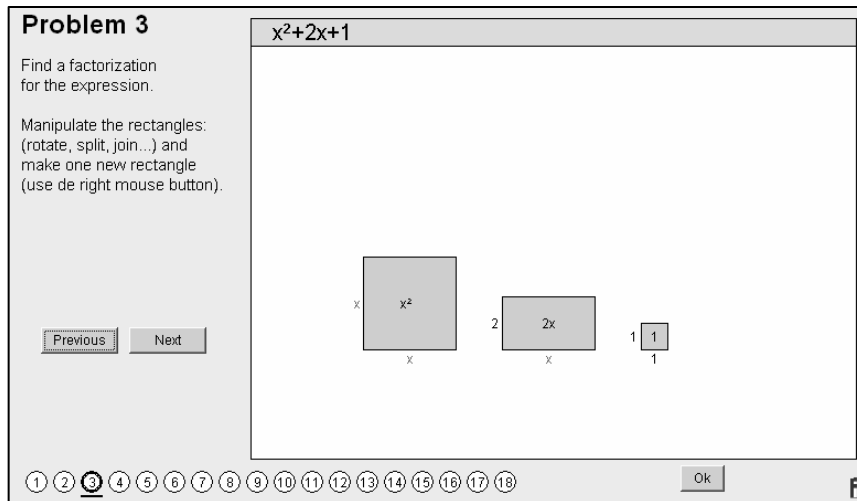

Figure 3

Figure 4

In this applet line segments of both constant and variable length can be combined to compound line segments and rectangles. By constructing these rectangles, the resulting algebraic expressions for the area are produced. Other, equivalent expressions can be made by splitting and joining the rectangles. Some interactive game activities were designed in which students have to make factorisations by combining a number of small rectangles to a single one. (figure 4)

Although students liked to work with these applets, it appeared that their skills in manipulating the rectangles didn't automatically enhance their abilities with and insight in algebraic manipulations. Explanations of these shortcomings could be found in the RME theory. Especially the part of emergent modelling was useful. In the RME view it is important that the formal mathematics can be developed as a tool for solving problems that are real for students. One should start with investigating phenomena's and problem situations that can be structured and solved by the mathematics that has to be learned. In working on these problems models can be used as mediating tools, that bridge the gap between informal solving strategies and the formal mathematics. So a model is more than a (visual) representation of a piece of abstract mathematics. First, it has to be developed as a tool for structuring a real situation or problem. The strong relation between the rectangles in the applet and the algebra is for experts obvious, but for learners for whom the algebra is still a unknown area, the world of rectangles can become isolated and not transferable. This is especially the case if there is no proper attention for the problems for which the model and in the end the algebra offers a generic tool. The sketched model development requires a carefully designed learning trajectory of which the applet activities were just a part.

Another applet 'Algebra Arrows' is based on the well-known 'machine' model for functions and formulas, in which functions are represented as input-output machines. In this applet the designer has tried to facilitate the model development, as described above, more extensive. It offers students a tool to build their own
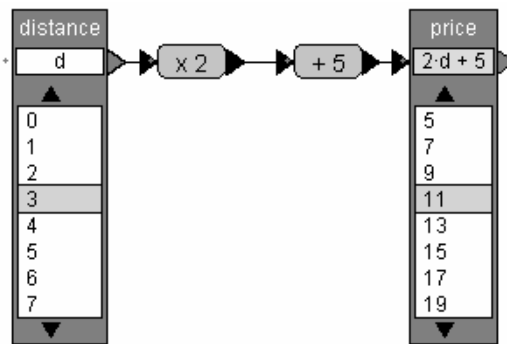

Figure 5

models in order to structure and solve problems. It has several levels of application and it is meant to facilitate a step by step development of the function- and formula concept and get acquainted with the commonly used mathematical objects and representations.

Applets that offer a mathematical microworld.

The third category contains applets that work on formal mathematical objects, like formulas, graphs equations etc. In this sense they are comparable with mathematical tool like the graphic- of symbolic calculator or CAS. The difference is that they are didactically designed and do not aim at merely carrying out the standard operations and algorithms. Most of them allow interactions and transformations on the objects that are not common in the standard mathematical tools. They often have a build-in task for students to perform. Some of the routine work is performed by (CAS) facilities of the applet, other parts, always carefully chosen, are left for the student to do, supported by proper feedback. In this way it is possible to focus on certain concepts and skills.

An example of an applet in this category is 'Solving equation with the cover-up strategy' (figure 6). Using this applet students can solve a certain equation type by subsequently selecting a part of the expression with the mouse and giving it a value. (Figure 6 shows a solving scenario.) The applet doesn't impose a blind algorithm. It lets the student interact with the expression and gives insight in its structure. With a growing insight it becomes possible to make larger and more efficient steps in the solving process.

In the design of these environments it's important to find a balance in directing the student to desirable skills and giving room to personal strategies and exploration. When the latter is missing the applet becomes just a training tool for ready-made

algorithms. Some of the applets from this category resembles environments like APLUSIX (Nicaud et al., 2004) and L'ALGEBRISTA (Cerulli and Mariotti). Nicaud speaks of "Mixing microworld and CAS features".

For understanding the features of this kinds of tools, it's important to reflect on the role of (solving)algorithms in mathematics education. In a mechanical approach of mathematics education the algorithms are seen as the starting-point of a learning trajectory. The learning objectives are a proper application of these given algorithms. In the RME approach algorithms themselves and the understanding of how they work are products of the learning trajectory too. So it's important to work on problems that involve mathematical objects in a way that permits explorations and student own alternative strategies. Off course there should be guidance and negotiation toward the known and most efficient strategies, but not by imposing them from the start. Another important requirement of the RME approach is that the elementary mathematical objects that are the 'building blocks' of the activities in the applets are 'real' for student, that is to say they have already a meaningful foundation, based on processes of lower order.
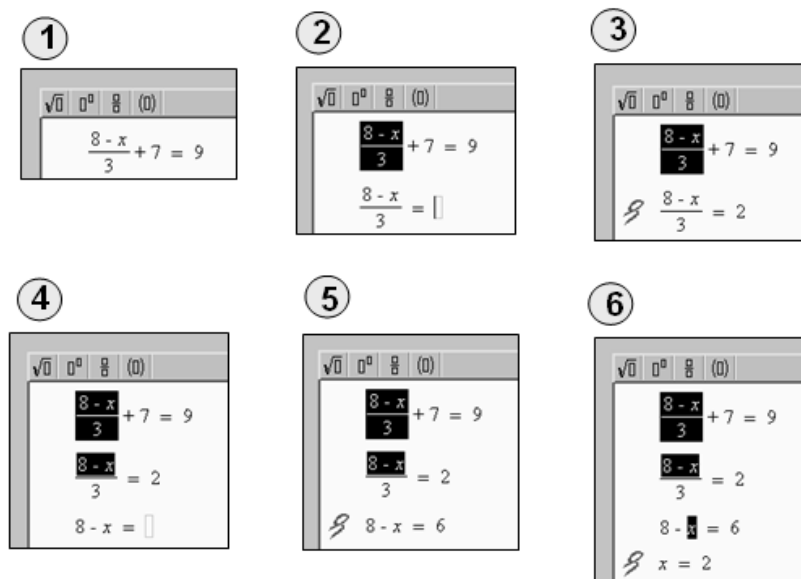

Figure 6

Challenges and future development

In the different development and research projects carried out at the Freudenthal Institute, the applets discussed in this paper showed to have add-on value in RME-designed learning trajectories. It's noticed too that a proper design of the whole

learning trajectory is vital for exploiting the benefits of the single applets. The division we made in three different categories also defines more or less their possible places in such trajectories.

Especially learning trajectories with applets that help students in developing the founding models for 'inventing' the formal mathematical tools are interesting, but difficult to design as well. One of the problems is that applets that support model development, often tend to impose a ready-made model that is not flexible enough to evolve. For applet designers it's a challenge to exhaustively explore the dynamics of this new medium and apply them to overcome this problem. For development researchers the challenge lays in the design of convincing learning trajectories that integrate these new tools.

Applets are distributed by the world wide web, which makes them accessible quite easy. But he web offers other features, especially easy to use and place independent communication and registration facilities, that can be used to enhance education as well. At the Freudenthal Institute an environment was developed that can be used by schools and individual students to store the results their applet activities. It was noticed that working with the applet could be fleeting because of the lack of persistence of the achieved results, the impossibility to reflect on them later and to share them with the teacher or other students. In a new planned research project on tool use, these new features will be integrated with the applets and other activities in a new innovative learning arrangement for mathematics.

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