

Rank and Conjugation for the Frobenius Representation of an Overpartition

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Abstract. We discuss conjugation and Dyson’s rank for overpartitions from the perspective of the Frobenius representation. More specifically, we translate the classical definition of Dyson’s rank to the Frobenius representation of an overpartition and define a new kind of conjugation in terms of this representation. We then use q -series identities to study overpartitions that are self-conjugate with respect to this conjugation.

Keywords: partitions, overpartitions, Frobenius partitions, Dyson’s rank, conjugation

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