WORKING PAPERS
RESEARCH DEPARTMENT

WORKING PAPER NO. 13-32/R
ON THE WELFARE PROPERTIES OF FRACTIONAL RESERVE BANKING

Daniel R. Sanches
Federal Reserve Bank of Philadelphia

December 2013

RESEARCH DEPARTMENT, FEDERAL RESERVE BANK OF PHILADELPHIA
Ten Independence Mall, Philadelphia, PA 19106-1574 • www.philadelphiafed.org/research-and-data/

On the Welfare Properties of Fractional Reserve Banking

Daniel R. Sanches
Federal Reserve Bank of Philadelphia

December 2013

The author thanks Mitchell Berlin, Douglas Gale, Chao Gu, Todd Keister, Guido Menzio, Will Roberds, and David Skeie for excellent comments.

Correspondence to Sanches at Research Department, Federal Reserve Bank of Philadelphia, Ten Independence Mall, Philadelphia, PA 19106-1574; phone: (215) 574-4358; Fax: (215) 574-4303; e-mail: Daniel.Sanches@phil.frb.org.

Abstract

Monetary economists have long recognized a tension between the benefits of fractional reserve banking, such as the ability to undertake more profitable (long-term) investment opportunities, and the difficulties associated with fractional reserve banking, such as the risk of insolvency for each bank. The goal of this paper is to show that a specific form of private bank coalition (a joint-liability arrangement) allows the members of the banking system to engage in fractional reserve banking in such a way that the solvency of each member bank is completely guaranteed. Under this arrangement, I show that a lower reserve ratio usually translates into a higher exchange value of bank liabilities, benefiting the consumers who use them as a means of payment.

Keywords: bank coalition, fractional reserve banking, interbank credit, reserve management

JEL classifications: E42, G21

1. INTRODUCTION

One of the main characteristics of the modern banking system is the small amount of reserves in lawful money that banks hold relative to the amount of short-term liabilities (such as demand deposits) they issue. Economists usually refer to this practice as fractional reserve banking. The proponents of fractional reserve banking have argued that a fractional system allows banks to economize on non-interest-bearing reserves, permitting them to increase the return on their assets and, in the case of a competitive market for bank liabilities, pay a higher return to their liability holders. Implicit in this argument is the conjecture that a lower level of reserves necessarily results in a higher return (or smaller discount) paid on a particular class of bank liabilities: those that facilitate payments and settlement (such as bank notes and demand deposits). This is usually viewed as a socially desirable outcome because one of the main functions of banks is to provide transaction services.

Fractional reserve banking is indeed a superior form of banking provided that each bank relies on an interbank market to borrow reserves in case it suffers an unusual number of withdrawals. The fact that each bank holds only a fraction of its demandable liabilities in the form of highly liquid assets makes it prone to failure. A typical concern is whether fractional reserve banking renders the banking system insolvent in the event that interbank markets, for some reason, fail to perform the function of transferring reserves from more liquid banks to illiquid banks.[1] Thus, there is a clear tension between the benefits of fractional reserve banking, such as the ability to undertake more profitable (long-term) investment opportunities, and the difficulties associated with the implementation of fractional reserve banking, such as the risk of insolvency for each bank.

The goal of this paper is to investigate whether it is possible to implement a fractional reserve system that allows member banks to take advantage of profitable investment oppor-

[1] For instance, Friedman (1959) has argued in favor of a banking system in which each member bank holds in reserve the full value of its demandable liabilities. His main concern is the stability of the banking system.

tunities and that is safe and sound. My main result is to show that a specific form of private bank coalition (a joint-liability arrangement) allows the members of the banking system to engage in fractional reserve banking in such a way that the solvency of each individual member is completely guaranteed. As opposed to markets, this bank coalition involves the monitoring and supervision of the activities of member banks. Finally, I argue that this kind of bank coalition resembles the clearinghouse associations that developed in the U.S. in the 19th century, as described in Friedman and Schwartz (1963), Gorton (1984, 1985), Gorton and Mullineaux (1987), and Moen and Tallman (1992, 2000).

I construct a random-matching model in which privately issued liabilities circulate as a medium of exchange. People meet in pairs and rely on bank liabilities to trade. The redemption of bank liabilities happens periodically in a centralized location where sellers who have sold goods to buyers take their bank liabilities to claim their face value. The key incentive problem within the banking system arises due to hidden action: It is necessary to provide banks with incentives to induce them to voluntarily report the creation of bank liabilities and hold the appropriate level of reserves. To deal with this incentive problem, there exists a clearinghouse association (a recordkeeping and safekeeping device) that requires member banks to report their transactions, imposes reserve requirements on each one of them, and supervises the settlement and clearing of bank liabilities at each date. Thus, the kind of monitoring provided by the clearinghouse allows each member bank to issue liabilities that effectively circulate as a medium of exchange.

I initially characterize an equilibrium allocation in the absence of any interbank credit or insurance scheme. In this case, a safe and sound banking system (i.e., one in which note holders do not suffer losses due to bank failures) necessarily involves an institutional arrangement in which each banker is required to hold in reserve the full value of his demandable liabilities. As should be expected, each member bank is fully solvent at any moment so that this form of banking ensures the stability of the payment system. However, I show that such a system costs something for the members of society. First, the banking system as a whole holds excess reserves at the end of each date, which is clearly inefficient because these resources could have been invested in higher-return assets. Second, the rate of return paid on bank liabilities is relatively low because it is necessary to induce bankers to truthfully report the creation of bank liabilities and voluntarily hold the appropriate level of reserves. Because each consumer holds her wealth in the form of bank liabilities, the equilibrium value of these liabilities has a first-order effect on the welfare of consumers. Thus, it is desirable to investigate the existence of an incentive-feasible scheme within the banking system that results in a higher equilibrium value of bank liabilities and that preserves the safety of bank liabilities as a means of payment (so that traders do not discount privately issued notes because of the possibility of losses due to bank failures).

In particular, I characterize the properties of a banking system in which each banker voluntarily chooses to become a member of a coalition that will issue bank liabilities that are effectively joint obligations of its members. Each banker continues to issue liabilities that identify him as a debtor, but the coalition publicly announces that, in the event an individual banker is unable to keep his promises, other members will honor any obligation of that member, according to their joint capacity. This joint-liability arrangement will allow member banks to reduce the share of funds invested in non-interest-bearing assets and, consequently, to increase the share of funds invested in interest-bearing assets. As a result, it is possible to eliminate excess reserves in the banking system and induce each banker to pay a higher return on bank liabilities, benefiting the consumers who use these liabilities as a means of payment.

Most important, I show that this outcome can be achieved in such a way that the stability of the banking system is preserved (i.e., neither bank failures nor losses to note holders occur on the equilibrium path) despite the implementation of fractional reserve banking. Specifically, I show that a joint-liability arrangement of the kind described above is an effective mechanism that permits the *ex post* transfer of reserves from liquid banks to illiquid banks in order to ensure the solvency of each individual member bank.

Finally, I argue that the analysis of the properties of a joint-liability arrangement is extremely relevant to the study of the evolution of the U.S. banking industry because the possibility of engaging in a risk-sharing arrangement of the kind described above is extremely valuable to banks that have historically had limited opportunities to expand and diversify their balance sheets given restrictions on the establishment of nationwide branches. My analysis incorporates this restriction on the size of each bank and concludes that the formation of a bank coalition that issues joint liabilities is indeed socially beneficial.

The rest of the paper is structured as follows. Section 2 discusses the related literature. Section 3 presents the basic framework. Section 4 carefully describes the exchange mechanism. In Section 5, I characterize equilibrium allocations in the case of a fully backed system. In Section 6, I discuss the welfare implications of a joint-liability arrangement. Section 7 concludes.

2. RELATED LITERATURE

My analysis is clearly related to the vast literature on inside money. Some prominent papers studying the properties of inside money include those by Kahn and Roberds (1998, 1999), Cavalcanti and Wallace (1999a, 1999b), Williamson (1999), Azariadis, Bullard, and Smith (2001), Li (2001, 2006), Martin and Schreft (2006), Berentsen (2006), Mills (2007), He, Huang, and Wright (2008), Andolfatto and Nosal (2009), Huangfu and Sun (2011), Araujo and Minetti (2011), and Gu, Mattesini, Monnet, and Wright (forthcoming), among many others. In these papers, reserve management is not the focus of the analysis, so the welfare properties of alternative reserve policies are not studied.

One prominent paper that explicitly accounts for reserve management is that of Cavalcanti, Erosa, and Temzelides (1999).[2] In this paper, the authors characterize an equilibrium allocation corresponding to a banking system for which regulation is weaker than 100% reserve requirements. In contrast to their work, my analysis focuses on the welfare properties of interbank arrangements as a means of better managing banking reserves. Also, my framework allows me to fully characterize the effects on prices and quantities (in their model, prices are exogenous).

My results can also be viewed as a response to the narrow banking proposal, as described in Wallace (1996). That author uses the Diamond-Dybvig model (see Diamond and

[2] See also Cavancanti, Erosa, and Temzelides (2005).

Dybvig, 1983) to show that a banking system that issues liabilities fully backed by safe short-term assets is socially undesirable. In his concluding remarks, Wallace points out that, in reality, bank liabilities serve as a means of payment (something not captured in the Diamond-Dybvig framework) and raises some concerns about how this property would influence the conclusions. My analysis emphasizes precisely the role that bank liabilities play in facilitating transactions.

A recent paper that also studies the benefits of fractional reserve banking is that of Chari and Phelan (2012). These authors find that, under some circumstances, a fully backed system is socially desirable because of the existence of a social cost (in terms of resources devoted to the banking system) associated with the private creation of government currency substitutes. In my analysis, the desirability of a fractional reserve system relies on the existence of an incentive-feasible interbank arrangement that permits member banks to transfer reserves among themselves based on their individual trading histories. In this respect, my analysis is in line with that of Kocherlakota (1998) and Kocherlakota and Wallace (1998), who characterize the effects of different forms of recordkeeping on trading arrangements. In the case of a fully backed system, there is no need for any kind of credit arrangement among banks, so society does not take advantage of its recordkeeping possibilities. Under a fractional reserve system, society benefits from a sophisticated credit arrangement that permits member banks to economize on non-interest-bearing reserves due to the creation of long-term credit relationships. The possibility of keeping track of trading histories within the banking sector is essential for the implementation of such an arrangement.

In my framework, a key assumption for tractability is that the clearing and settlement of bank liabilities occur in a centralized location, as in Koeppl, Monnet, Temzelides (2008) and Deviatov and Wallace (2009).[3] The assumption that the clearing and settlement of privately issued liabilities take place periodically in a centralized location allows me to fully characterize the effects of alternative reserve policies on equilibrium prices and quantities.

[3] In this respect, my model bears a resemblance to those of Lagos and Wright (2005) and Rocheteau and Wright (2005).

Consequently, I can focus on the key incentive problem influencing the decision to hold reserves: namely, hidden action.

Finally, it is important to mention that my results have a similar flavor to those obtained by Berentsen, Camera, and Waller (2007) in that the introduction of an interbank arrangement of the kind described above allows society to better allocate resources. In my analysis, the welfare gains come entirely from better management of banking reserves, which can only be achieved through the implementation of an incentive-feasible scheme among the members of the banking sector.

3. MODEL

Time $t = 0, 1, 2, ...$ is discrete, and the horizon is infinite. Each period is divided into three subperiods or stages. There are two physical commodities, referred to as good x and good y, that are perfectly divisible. There are three types of agents, indexed by $i = 1, 2, 3$, who are infinitely lived. There is a $[0, 1]$ continuum of each type.

Types 2 and 3 want to consume good x, whereas type 1 wants to consume good y. If good x is not properly stored in the subperiod it is produced, it will depreciate completely. Good y is perishable and cannot be stored, so it must be consumed in the subperiod it is produced. Type 1 is able to produce good x only in the first subperiod. Type 2 is able to produce good y only in the second subperiod. Type 3 is unable to produce either good but has access to the technology to perfectly store good x at any moment. In the first subperiod, each type 3 also has access to a (divisible) investment technology that requires good x as an input and yields a fixed return $\rho > 1$ (in terms of good x) only at the beginning of the following date. Finally, each type 3 has access to a technology that allows him to create, at zero cost, an indivisible and durable object, referred to as a note, that perfectly identifies him. This means that notes issued by each type 3 are perfectly distinguishable from those issued by other people so that counterfeiting will not be a problem.

I now explicitly describe preferences. Let $x_t \in \{0, 1\}$ denote type 1's production of good x at date t, and let $y_t \in \mathbb{R}_+$ denote his consumption of good y at date t. Type 1's preferences are represented by

$$u(y_t) - \gamma x_t,$$

where $\gamma \in \mathbb{R}_+$, and $u : \mathbb{R}_+ \to \mathbb{R}$ is continuously differentiable, increasing, and strictly concave, with $u(0) = 0$ and $u'(0) = \infty$. We assume the production technology of good x allows type 1 to produce either zero or one unit of good x at each date, even though good x is perfectly divisible.

Let $y_t \in \mathbb{R}_+$ denote type 2's production of good y at date t, and let $x_t \in \mathbb{R}_+$ denote his consumption of good x at date t. Type 2's preferences are represented by

$$v(x_t) - \omega y_t,$$

where $\omega \in \mathbb{R}_+$, and $v : \mathbb{R}_+ \to \mathbb{R}$ is continuously differentiable, increasing, and concave, with $v(0) = 0$. Type 3 derives utility x_t if his consumption of good x at date t is $x_t \in \mathbb{R}_+$. Finally, let $\beta \in (0,1)$ denote the common discount factor over periods. I assume $\rho < \beta^{-1}$.

In each subperiod, there is a distinct round of interactions. In the first subperiod, each type 1 is randomly matched with a type 3. In the second subperiod, each type 1 is randomly matched with a type 2 with probability $\lambda \in (0,1)$. In the third subperiod, all type 2 and all type 3 meet in a centralized location. I assume that, after meeting with a type 1 bilaterally in the first subperiod, all type 3 immediately move to the centralized location. All type 2 arrive at the centralized location only in the third subperiod.

4. EXCHANGE MECHANISM

To describe the exchange process, it is convenient to refer to type 1 as a buyer, to type 2 as a seller, and to type 3 as a banker. To better understand these labels, it is easier to start with the second stage. In this stage, each buyer is randomly matched with a seller with probability λ. Because the buyer wants good y but is unable to produce good x for the seller at that time, the pair will be able to trade only if a medium of exchange is made available. As will become clear, each banker will be able to provide such a medium of exchange in the form of personal liabilities redeemable on demand. I refer to these tradable liabilities as bank notes (or simply notes). Thus, the objects that a buyer and a seller trade are good y and notes.

Each buyer will be able to acquire a note in the first stage when each one of them is randomly matched with a banker. In this stage, each buyer has access to the technology to produce good x, so the objects a buyer and a banker trade are good x and notes. Finally, in the third stage, all sellers and all bankers interact in a centralized location. In this stage, each merchant has an opportunity to redeem any note (i.e., convert a privately issued liability into good x) he has received from a consumer (if any) in the previous stage, so we can think of this stage as the settlement stage. Thus, two objects can be traded: good x and notes. Note that no production takes place during the settlement stage.

This payment arrangement works perfectly well provided that each banker is willing to set aside (i.e., invest in the storage technology) the appropriate amount of good x to have enough resources, referred to as reserves, to retire a note in case it is presented for redemption in the settlement stage (an event that happens with a positive probability). What makes the implementation of such an arrangement difficult is that not all trades in the economy are perfectly observable.

Let me now describe the information structure of this economy. Each banker is able to observe the actions of other bankers in the centralized location. In the first and second stages, the bilateral trades are privately observable, i.e., only the pair of agents participating in the meeting knows the amounts traded. This means that the creation of bank notes is privately observable. As a result, each banker may have an individual incentive to issue private liabilities without fully securing them with the storage technology (the only available safe, short-term asset).

In addition, it is important to note that each banker may want to opportunistically access previously accumulated reserves. This means that the possibility of a banker having many notes outstanding following a history of successful trading meetings and few redemptions creates a problem. In particular, a banker who has issued notes that remain in circulation (those issued to buyers who have not had an opportunity to trade with a seller) and who has held reserves to secure these notes may want to opportunistically consume these reserves in case they become very large. The short-term payoff of defection for the banker will be enormous in some cases, making him more likely to renege on his promises.

In view of these difficulties, let me explain how each banker will be able to issue private liabilities that effectively circulate as a medium of exchange. At the beginning of date zero, all bankers agree to establish a clearinghouse association that will work merely as a recordkeeping and safekeeping institution. It will accept deposits from member banks and will coordinate the clearing and settlement of privately issued liabilities. Each banker can be a member of the clearinghouse at no cost but has to follow its rules. The clearinghouse requires each banker to report any meeting in the first stage in which a note has been issued. For each note issued, the banker is required to store a fraction of the face value of the note (in terms of good x), to be interpreted as reserves backing the issuance of his note. In particular, each banker is required to "deposit" reserves with the clearinghouse every time he announces the creation of a bank liability so that he cannot opportunistically access his reserves in future periods. Thus, the clearinghouse provides safekeeping services to all member banks.

Recall that, shortly after meeting with buyers bilaterally in the first subperiod, all bankers meet in the centralized location so that they have an opportunity to report the creation of bank notes and deposit the appropriate amount of reserves with the clearinghouse. Each one of them can also invest in the productive technology. Note that sellers arrive at the centralized location only in the third subperiod, so they do not observe the amounts deposited by each banker. (See Figure 1 for a sequence of events within each period and Figure 2 for a representation of the payment mechanism.)

Any banker who fails to report the issuance of a note will have his membership permanently revoked. Note that his deviation will be publicly observable to members of the clearinghouse only when an unreported note is presented for redemption in the settlement stage, which may take several periods to happen.

It is important to mention that the acceptability of privately issued notes is endogenously determined. A seller's decision to accept a note issued by a banker in exchange for his output (good y) is based on the available information he has about the issuer. In my framework, the available information for each seller is provided by the clearinghouse. In particular, the clearinghouse provides a record of compliance with the clearinghouse rules for each member bank, i.e., people observe the membership status of each banker. Each seller knows that the clearinghouse requires member banks to deposit reserves to secure bank notes and that it expels members issuing notes without depositing the appropriate amount of reserves (when the deviation is detected). Thus, the decision to become a member of the clearinghouse is viewed as a signal of "financial rectitude", which will certainly influence a seller's decision to accept notes issued by a member of the clearinghouse.

Finally, I assume that each agent can carry, at most, one indivisible unit of money at any moment. This means that individual note holdings are restricted to the set $\{0, 1\}$. In this respect, the model developed in this paper relates to the second generation of search-theoretic models of monetary exchange, following the ideas in Shi (1995) and Trejos and Wright (1995). On the other hand, there is *no* restriction on the number of notes that each banker is allowed to issue at any moment except for that imposed by the matching technology and people's willingness to trade. This means that the number of notes issued by any banker belongs to the set $\{0, 1, 2, ...\}$.

5. FULLY BACKED SYSTEM

In this section, I characterize an equilibrium allocation in case the members of the clearinghouse decide not to engage in any sort of credit or insurance scheme. Thus, to guarantee the solvency of each member bank, the clearinghouse will require each banker to keep in reserve the full face value of any note he has issued. In other words, each banker will have to adopt a 100% reserve policy in order to remain a member.

5.1. Equilibrium

Throughout the paper, I restrict attention to equilibria for which there exist an invariant distribution of note holdings across buyers, an invariant distribution of note holdings across sellers, and an invariant volume of note creation and note redemption by the members of the banking sector. These invariant distributions can be summarized as follows. Let m^1 denote the invariant measure of buyers holding a note at the end of the first stage, let m^2 denote the invariant measure of sellers holding a note at the end of the second stage, and let m^3 denote the invariant volume of notes that are retired in the third stage. I only consider equilibria for which $m^1 = 1$ and $m^2 = m^3 = \lambda$. Thus, if each banker truthfully reports the creation of bank notes, there will be no uncertainty with respect to the total volume of redemptions in the settlement stage.

One key feature of the model rendering it highly tractable is that one does not need to track each banker's individual history of note creation and note redemption. A banker who has issued a note is required to deposit the full face value of the note so that it is fully secured by a safe asset (storage). All that matters for the characterization of an equilibrium allocation is the aggregate amount of privately issued notes in circulation outside the banking sector.

Let me start by describing the Bellman equations for each buyer. Let V^0 denote the beginning-of-period expected discounted utility of a buyer not holding a note, and let V^1 denote the beginning-of-period expected discounted utility of a buyer holding a note. The Bellman equations for a buyer are given by

$$V^0 = -\gamma + \alpha\lambda \left[u\left(y\right) + \beta V^0\right] + \left(1 - \alpha\lambda\right)\beta V^1, \tag{1}$$

$$V^1 = \alpha\lambda \left[u\left(y\right) + \beta V^0\right] + \left(1 - \alpha\lambda\right)\beta V^1. \tag{2}$$

Here $y \in \mathbb{R}_+$ denotes the quantity of good y that he will be able to purchase from the seller with whom he is matched in exchange for a note, and $\alpha \in [0, 1]$ denotes the probability that the seller will accept a privately issued note in exchange for his output.

If the buyer starts the period without a note, then he will be able to obtain one from the banker with whom he is currently matched, in which case he will produce one unit of good x. A newly issued note costs one unit of good x and is a promise to pay $\phi \in [0, 1]$ unit of good x on demand to the note holder. Then, with probability λ, the buyer will be matched with a seller in the second stage, in which case the buyer will be able to consume $y \in \mathbb{R}_+$ units of good y with probability α (and will enter the following period without a note). With probability $1-\alpha\lambda$, the buyer will not trade in the second stage and will hold on to his note.

Each buyer is able to save in the form of liabilities issued by bankers (a store of value) until he has an opportunity to consume. Recall that, by assumption, each buyer will not be able to produce again until he has an opportunity to spend his current note holdings. This means that the buyer's wealth is completely determined by the equilibrium value of bank liabilities.

Let W^0 denote the expected discounted utility of a seller who does not find a trading partner in the second stage, and let W^1 denote the expected discounted utility of a seller who finds a trading partner. In a stationary equilibrium, the Bellman equations for a seller are given by

$$W^0 = \beta \left[\lambda W^1 + (1-\lambda) W^0\right], \tag{3}$$

$$W^1 = \max_{\alpha \in [0,1]} \alpha \left[-\omega y + v(\phi)\right] + \beta \left[\lambda W^1 + (1-\lambda) W^0\right]. \tag{4}$$

I have implicitly assumed that the banker will voluntarily deposit the face value of each note issued so that by accepting a banker's note in trade a seller receives ϕ. Below I carefully discuss the incentive problem.

Now consider the Bellman equations for each banker. Let J^0 denote the expected discounted utility of a banker who is currently matched with a buyer not holding a note in the first stage, and let J^1 denote the expected discounted utility of a banker who is currently matched with a buyer holding a note. For each note issued in the first stage, the banker will be required to set aside the amount $\phi \in [0,1]$ in order to meet his future obligations. In a stationary equilibrium, the Bellman equations for a banker are given by

$$J^0 = 1 - \phi + \beta \left[\alpha\lambda J^0 + (1-\alpha\lambda) J^1\right], \tag{5}$$

$$J^1 = \beta \left[\alpha\lambda J^0 + (1-\alpha\lambda) J^1\right]. \tag{6}$$

Note that a banker who has issued a note (and who has truthfully reported it to the clearinghouse) is able to immediately consume the amount $1-\phi$ of good x. The consumption decision is trivial: The banker will save exactly the required amount because $\rho < \beta^{-1}$.

As previously mentioned, the expected discounted utility of each banker does not depend on the number of notes he has issued. On the equilibrium path, each banker is willing to deposit with the clearinghouse the amount ϕ for each note issued so that he can immediately consume $1-\phi$ every time he issues a note. Because the clearinghouse will ensure the solvency of each individual member in this case or in any other case considered in this paper, the number of notes outstanding for each banker will not influence his probability of failure. In particular, the probability of failure will be zero because the clearinghouse either requires each member to deposit the full face value of each note or ensures the *ex post* transfer of reserves from liquid banks to illiquid banks if fractional reserve banking is allowed (see next section).

A banker who meets a buyer holding a note can offer his own note in exchange for the buyer's note. In this case, the banker can claim the face value of someone else's note only if he reports the acquisition of such a note to the clearinghouse, in which case the clearinghouse will require him to hold reserves due to the issuance of his own note. Thus, such a trade will bring no extra benefit to the banker unless the buyer gives him some extra amount of good x together with his note. But, in this case, the buyer will clearly be better off holding on to his (previously acquired) note. Thus, a swap of notes happens if and only if both agents are indifferent. For simplicity, I assume that both choose not to swap notes in this case.[4]

The terms of trade in the first and second stages are determined as follows. Start with the second stage. I assume that the buyer makes a take-it-or-leave-it offer to the seller, in which case he will be able to capture all surplus from trade. Suppose that the seller accepts privately issued notes with probability one, i.e., $\alpha = 1$. (Below I discuss the decision to accept notes.) In a bilateral meeting, the buyer's surplus from trade is given by

$$u(y) + \beta V^0 - \beta V^1 = u(y) - \beta\gamma,$$

[4] Even if both agents swapped notes, the total volume of reserves would remain unchanged because the redemption of a note by another banker simply means a transfer of reserves within the banking system, which does not affect the total stock of notes available to the nonbank public.

and the seller's surplus from trade is simply given by

$$-\omega y+v\left(\phi\right).$$

The buyer is willing to make any offer such that

$$u\left(y\right)-\beta\gamma\geq 0,$$

and the seller accepts the buyer's offer if and only if

$$-\omega y+v\left(\phi\right)\geq 0.$$

This participation constraint will always bind in the case in which the buyer has all the bargaining power, so the quantity of good y produced in each bilateral meeting will be given by

$$y=\omega^{-1}v\left(\phi\right). \tag{7}$$

Consider now the terms of trade in the first stage. In a trade meeting, the buyer's participation constraint is given by

$$-\gamma+\lambda u\left(\omega^{-1}v\left(\phi\right)\right)+\beta\left(1-\lambda\right)\left(V^{1}-V^{0}\right)\geq 0.$$

Using (1) and (2), we can rewrite this participation constraint as follows:

$$u\left(\omega^{-1}v\left(\phi\right)\right)\geq\frac{\gamma\left[1-\beta\left(1-\lambda\right)\right]}{\lambda}. \tag{8}$$

The banker's participation constraint is given by

$$J^{0}\geq\beta\left[\lambda J^{0}+\left(1-\lambda\right)J^{1}\right],$$

which simply requires

$$\phi\leq 1. \tag{9}$$

This means that an equilibrium value ϕ must satisfy both (8) and (9).

To construct an equilibrium, we have to keep in mind that each banker has the option of not reporting his newly issued note to the clearinghouse. The punishment for failing to report any newly issued note (and setting aside the required amount of reserves) is the immediate termination of membership when any deviation is detected. Thus, each banker truthfully reports the creation of a new note in the first stage if and only if

$$1 - \phi + \beta \left[\lambda J^0 + (1 - \lambda) J^1 \right] \geq J^d, \tag{10}$$

where J^d denotes the value associated with his best deviation. The left-hand side gives the banker's expected discounted utility in the event he chooses to truthfully report the creation of a note. The right-hand side gives his expected discounted utility if he adopts his best deviation. This means that each banker is willing to deposit with the clearinghouse the full face value of each note he has issued provided that the equilibrium value of bank notes is such that his expected discounted utility is at least the same as what he would obtain by adopting his best deviation strategy. His best deviation strategy may involve issuing some notes without holding the appropriate amount of reserves, i.e., engaging in fractional reserve banking.

My first step is to show that the value of deviation is bounded below by 1 and is bounded above by $(1 - \beta + \lambda\beta)^{-1} \left[1 - (1 - \lambda)^2 \beta \right]$, regardless of the required deposit amount ϕ. To verify that $J^d \geq 1$, note that a banker who decides to deviate at any given date is able to immediately consume one unit of good x. His decision to not deposit reserves with the clearinghouse will certainly affect his continuation value. But, in any case, his continuation value is at least zero. Thus, I have shown that $J^d \geq 1$. To show that J^d has an upper bound, consider the hypothetical case in which a banker who has deviated at some date t is able to deviate at each subsequent date without increasing his probability of failure (for instance, because each note holder will freely dispose of his notes). In this case, the maximum expected discounted utility he can obtain is given by

$$\bar{J} = 1 + \beta (1 - \lambda) J',$$

where the value J' satisfies

$$J' = \lambda + (1 - \lambda) \beta J'.$$

When he initially deviates at some date t, he is able to immediately consume one unit of good x. He will be able to continue trading only with probability $1-\lambda$, which is precisely the probability that the buyer who has acquired his note does not find a trading partner in the second stage. If his deviation is not detected at date t, he will be able to issue a new note at date $t+1$ with probability λ. After date t, his probability of failure will not increase (even though more than one note has been issued without the corresponding amount of reserves) because I have assumed that whoever acquires his notes after date t will freely dispose of them so that his probability of survival continues to be given by $1-\lambda$ at the end of each date. It is straightforward to show that

$$\bar{J} = \frac{1-(1-\lambda)^2\beta}{1-(1-\lambda)\beta}.$$

Thus, the value associated with his best deviation J^d is indeed bounded:

$$1 \leq J^d \leq \frac{1-(1-\lambda)^2\beta}{1-(1-\lambda)\beta}. \tag{11}$$

As I have previously mentioned, a banker's best deviation strategy may involve issuing some notes without holding the appropriate amount of reserves, which will increase his individual probability of failure. To illustrate this point, consider a deviation strategy (not necessarily his best deviation) in which a banker decides to issue notes without holding reserves. If he chooses not to report a newly issued note, he can immediately consume the amount of good x he has received in exchange for his note. With probability $1-\lambda$, his deviation will remain undetected, in which case he will be able to issue a new note in the following period with probability λ. With probability $(1-\lambda)^2$, his deviation will remain undetected in the following period (now two of his notes are in circulation), in which case he will be able to issue a third note in the subsequent period with probability λ. Note that his probability of failure increases over time. As long as his deviation remains undetected, he will be able to continue to issue notes without reporting their existence to the clearinghouse, immediately consuming the proceeds from the sale of these notes. The expected discounted utility associated with this deviation strategy is given by

$$\hat{J} = 1 + (1-\lambda)\beta\hat{J}_1,$$

where, for each $i = 1, 2, 3, ...$, we have

$$\hat{J}_i = \lambda \left[1 + (1-\lambda)^{i+1} \beta \hat{J}_{i+1} \right] + (1-\lambda)(1-\lambda)^i \beta \hat{J}_i.$$

This strategy is likely to be his best deviation strategy when λ is relatively low, which means that his defection will be detected only with a small probability, and the discount factor β is low. Even though a banker's best deviation strategy may not be unique, there exists a unique value J^d in the interval given by (11) that corresponds to the maximum value associated with a best deviation strategy.

To guarantee that each banker voluntarily participates in the clearinghouse, it is necessary to make membership sufficiently profitable such that his expected discounted utility is larger than that associated with his best deviation. The truth-telling constraint (10) guarantees that each individual banker holds enough reserves to pay the full face value of each outstanding note. Thus, it is individually rational for each seller to choose $\alpha = 1$ (i.e., to accept privately issued notes with probability one) provided that (10) is satisfied. Recall that the seller observes the face value ϕ associated with a note and a banker's membership status. When the value ϕ is such that (10) is satisfied, each seller knows that the members of the clearinghouse are willing to deposit the required amount of reserves for each note issued. Thus, he is willing to accept a note issued by a member bank with probability one.

In equilibrium, I require that (10) holds with equality, which will allow me to obtain the highest equilibrium value of bank liabilities consistent with truthful reporting. In section 6, I provide an interpretation for this particular choice and discuss the implications of alternative regimes.

Finally, note that the participation constraints (8) and (9) impose both a minimum and a maximum value of notes consistent with equilibrium:

$$v^{-1}\left(\omega u^{-1}\left(\frac{\gamma\left[1-\beta(1-\lambda)\right]}{\lambda}\right)\right) \leq \phi \leq 1. \tag{12}$$

The minimum value arises owing to the buyer's participation constraint, whereas the maximum value arises because of the banker's participation constraint. Finally, suppose that each buyer starts date zero without a note so that each banker has an opportunity to issue a note to the buyer with whom he is initially matched. Given these requirements, it is now straightforward to formally define a stationary equilibrium.

Definition 1 *A stationary monetary equilibrium for the economy described above is an array* $\{J^0, J^1, J^d, V^0, V^1, W^0, W^1, \phi, y, \alpha, m^1, m^2, m^3\}$ *satisfying* $m^1 = 1$*,* $m^2 = m^3 = \lambda$*,* $\alpha = 1$*, (1), (2), (3), (4), (5), (6), (7), (10) with equality, and (12). In addition,* J^d *is the value associated with a banker's best deviation strategy when each banker is required to deposit with the clearinghouse the amount* ϕ*, contingent on the creation of a note.*

In a stationary equilibrium, the measure of bankers who issue a note in the first stage is given by λ, so the total volume of reserves increases by the amount $\lambda\phi$ (in terms of good x). In the third stage, a fraction λ of all outstanding notes is retired, so the total volume of reserves decreases by $\lambda\phi$. This means that the total volume of reserves at the end of the period is exactly the same as the volume at the beginning of the period in the case of a stationary equilibrium.

Assumption 1 Assume $\beta > v^{-1}\left(\omega u^{-1}\left(\frac{\gamma[1-\beta(1-\lambda)]}{\lambda}\right)\right)$.

Under this assumption, I can establish the existence and uniqueness of a stationary equilibrium.

Proposition 2 *There exists a unique stationary monetary equilibrium in which the value of notes satisfies*

$$\bar{\phi} = \frac{1-\beta(1-\lambda)-(1-\beta)J^d}{1-\beta(1-\lambda)}. \tag{13}$$

In this equilibrium, the end-of-period excess reserves are given by $(1-\lambda)\bar{\phi}$ *at each date.*

Proof. To show the existence of the values $\bar{\phi}$ and J^d, I construct candidate sequences $\{\bar{\phi}_s\}_{s=0}^{\infty}$ and $\{J_s^d\}_{s=0}^{\infty}$ as follows. Choose

$$J_0^d = \frac{1-(1-\lambda)^2\beta}{1-(1-\lambda)\beta}.$$

If (10) holds with equality, then the value of notes is given by

$$\bar{\phi}_0 = \frac{1-\beta(1-\lambda)-(1-\beta)J_0^d}{1-\beta(1-\lambda)}.$$

Given this choice for the value of notes, there exists a value associated with a best deviation strategy, J_1^d. It follows that $J_1^d \leq J_0^d$. Given J_1^d, I can define $\bar{\phi}_1$ as follows:

$$\bar{\phi}_1 = \frac{1 - \beta (1 - \lambda) - (1 - \beta) J_1^d}{1 - \beta (1 - \lambda)}.$$

Note that $\bar{\phi}_1 \geq \bar{\phi}_0$. Following the same steps as those described above, I can define an increasing sequence $\left\{\bar{\phi}_s\right\}_{s=0}^{\infty}$ and a decreasing sequence $\left\{J_s^d\right\}_{s=0}^{\infty}$. Because $\left\{J_s^d\right\}_{s=0}^{\infty}$ is bounded, it converges to a unique limit $J^d > 1$. Because $\left\{\bar{\phi}_s\right\}_{s=0}^{\infty}$ is bounded, it converges to a unique limit $\bar{\phi} < 1$.

The amount of good y produced and traded in each bilateral meeting in the second stage is given by

$$y = \bar{y} \equiv \omega^{-1} v \left(\bar{\phi}\right).$$

Using (1) and (2), we obtain the values V^0 and V^1:

$$V^0 = \frac{\lambda \left[u \left(\bar{y}\right) - \gamma \beta\right]}{1 - \beta} - \gamma,$$

$$V^1 = \frac{\lambda \left[u \left(\bar{y}\right) - \gamma \beta\right]}{1 - \beta}.$$

Because of the assumption that the buyer has all bargaining power when trading with a seller, it follows that $W^0 = W^1 = 0$.

Finally, I need to show that the end-of-period excess reserves are $(1 - \lambda) \bar{\phi}$. First, note that, at the end of each date, all sellers who have acquired a note are able to convert it into $\bar{\phi}$ unit of good x. Note also that there is no reason for them to delay the redemption of a note. Because $m^1 = 1$ in a stationary equilibrium and note holdings are constrained to the set $\{0, 1\}$, the total volume of reserves at the end of the first stage must be $\bar{\phi}$. Because $m^2 = \lambda$, the total volume of reserves decreases by the amount $\lambda \bar{\phi}$ at the end of the third stage. This means that the end-of-period volume of excess reserves is $(1 - \lambda) \bar{\phi}$. ■

In a stationary equilibrium, each banker consumes $1 - \bar{\phi}$ unit of good x when he has an opportunity to issue a note, each buyer consumes $\omega^{-1} v \left(\bar{\phi}\right)$ units of good y when he has an opportunity to trade with a seller and produces one unit of good x when he acquires a note, and each seller produces $\omega^{-1}v\left(\bar{\phi}\right)$ units of good y and consumes $\bar{\phi}$ unit of good x when he has an opportunity to trade with a buyer.

Note that a sound banking system of the kind described in this section costs something for nonbanks. As I have shown, the equilibrium value $\bar{\phi}$ is determined in such a way that each banker obtains a flow of income derived from the note-issuing privileges that is sufficient to induce him to deposit reserves to fully back his demandable liabilities. Because bankers have to be induced to hold the appropriate level of reserves, it means that there exists an endogenous minimum value associated with the banking privileges that is consistent with an equilibrium without bank failures. This endogenous franchise value is necessary for the implementation of a banking system in which bankers fully secure their demandable liabilities with safe, short-term assets so that bank failures do not occur in equilibrium.

Note that no banker invests in the productive technology in the case of a fully backed system. The requirement of depositing the full face value of each note with the clearinghouse in the form of non-interest-bearing assets (storage) is imposed to guarantee the solvency of each individual banker. When each individual banker follows this policy, the banking system as a whole holds excess reserves at the end of each period. In the next section, I consider an institutional arrangement that allows the members of the banking system to invest at least some of their funds in the productive technology and that simultaneously guarantees the solvency of each individual banker. Thus, such an arrangement will promote efficiency without compromising stability.

6. FRACTIONAL RESERVE SYSTEM

The goal of this section is to characterize an incentive-feasible arrangement within the banking sector that preserves the stability of the banking system but permits the members of society to achieve a better allocation of resources by taking advantage of more profitable investment opportunities. Suppose now that, at the beginning of date zero, the members of the clearinghouse association agree to issue notes that are effectively joint obligations of its members. Each banker continues to issue notes that identify him as a debtor, but the clearinghouse publicly announces that, in the event that an individual banker is unable to redeem his own notes, other members will honor any obligation of such a member, according to their joint capacity. Under this arrangement, each banker is entitled to use other bankers' reserves to meet his own obligations in case he is called for redemption provided that he is willing to pledge his own reserves to redeem the notes issued by other bankers in case they need them.

The clearinghouse is responsible for supervising the required deposit amounts by the members of the coalition. When dealing with each individual member, the clearinghouse needs to induce him to truthfully report the issuance of notes and voluntarily deposit the appropriate amount of resources, which is the same as saying that it needs to ensure that the truth-telling constraint (10) is satisfied. The clearinghouse will also determine the amounts to be invested in storage and in the productive technology, which will form the portfolio of the coalition. Initially, each individual banker has an equal claim on the current and future assets of the bank coalition. However, depending on his individual history of note creation and note redemption, the banker may be a creditor or a debtor within the coalition, in which case his claim on the current and future assets of the coalition may increase or decrease over time.

The main difference from the previous case is that now the relevant measure to determine the solvency of each banker who is called for redemption at each date is the ratio of the value of *all* reserves of the coalition to the value of *all* notes that are presented for redemption. Because the members of the coalition know that not all outstanding notes will be presented for redemption at each date, it is possible to invest at least some fraction of the funds in the productive technology to obtain a higher rate of return provided that each member is willing to deposit the appropriate amount of resources with the clearinghouse, which means that he is willing to engage in a joint-liability arrangement.

6.1. Equilibrium

Suppose that each buyer starts date zero without any note. Thus, at date zero, each banker has an opportunity to issue a note to the buyer with whom he is initially matched. Suppose initially the clearinghouse determines that each banker is required to save the same amount (to be deposited with the clearinghouse) as that observed in the previous case (i.e., the amount $\bar{\phi}$) so that (10) continues to hold with equality. This means that the total amount of good x that all bankers set aside at date zero is given by $\bar{\phi}$. Suppose that the aggregate amount invested in the productive technology at date zero and at any other subsequent date is given by

$$\frac{(1-\lambda)\,\bar{\phi}}{\rho}. \tag{14}$$

Under this investment policy, the amount of good x that will be available for the members of the clearinghouse at the beginning of each date $t \geq 1$ is at least $(1-\lambda)\,\bar{\phi}$. Given that, after date zero, the aggregate amount of good x that all bankers set aside is given by $\lambda\bar{\phi}$, the total amount of good x available for investing in either the storage technology or the productive technology is given by $\bar{\phi}$ at each date. Because a fraction λ of all notes in circulation will be retired in the settlement stage, this means that the equilibrium value of notes is now given by

$$\phi' \equiv \frac{\bar{\phi}}{\lambda}\left(1-\frac{1-\lambda}{\rho}\right) \tag{15}$$

in case the clearinghouse follows the investment policy described above. The equilibrium value (15) is determined by aggregate feasibility in case the members of the clearinghouse agree to engage in the joint-liability arrangement described above, and the clearinghouse follows the policy of investing the amount (14) in the productive technology at each date. This means that the consumption of each buyer who finds a trading partner will be given by

$$y = \omega^{-1}v\left(\phi'\right).$$

Because $\phi' > \bar{\phi}$, each buyer is strictly better off when the members of the clearinghouse decide to implement a banking arrangement of the kind described above. Because each banker remains indifferent, this arrangement is incentive compatible. Because each seller also remains indifferent, I can conclude that the stationary allocation obtained in this case Pareto dominates the stationary allocation obtained in the case of a fully backed system. I summarize these findings in the following proposition.

Proposition 3 *Provided that ω is sufficiently small, there exists a stationary monetary equilibrium in which the value of notes is given by $\phi' > \bar{\phi}$. In this equilibrium, each banker sets aside the amount $\bar{\phi}$ when he has an opportunity to issue a note. This equilibrium Pareto dominates the stationary monetary equilibrium obtained under a fully backed system.*

Note that, before settlement, the ratio of the aggregate value of reserves (storage) to the aggregate value of bank liabilities is given by

$$\frac{\lambda\phi'}{\phi'} = \lambda < 1.$$

This means that the banking system as a whole is holding in reserve only a fraction of the aggregate value of bank liabilities. The members of the clearinghouse know that not all notes in circulation will be presented for redemption at the end of each date, so by creating a mechanism for pooling reserves to avoid individual insolvency, it is possible to reduce the amount of non-interest-bearing assets in the system and, consequently, increase the amount of interest-bearing assets held by the coalition. This enhanced management of reserves allows the banking system to pay a higher return on bank liabilities. Note that the banking system as a whole always has enough reserves to retire all notes that are presented for redemption at the end of each date, so it is completely solvent by construction. Each individual banker may not have enough reserves to retire his own notes at a particular date but is able to use other bankers' reserves to fulfill his promises.

In the case described above, each banker remained indifferent between adopting a joint-liability scheme and operating under a fully backed system, which implied that all welfare gains went entirely to buyers. This relies on the choice of a particular trading mechanism that requires the truth-telling constraint to hold with equality at each date. Alternatively, I can construct an equilibrium in which all welfare gains go entirely to bankers. In this case, the equilibrium value of notes continues to be given by $\bar{\phi}$ so that each buyer remains indifferent. This means that, given the policy of investing the amount (14) in the productive technology at each date, each banker now needs to set aside the amount

$$\phi'' \equiv \bar{\phi}\lambda \left(1 - \frac{1-\lambda}{\rho}\right)^{-1}$$

when he has an opportunity to issue a note. This means that, when he is able to issue a note, he immediately consumes the amount

$$1 - \bar{\phi}\lambda \left(1 - \frac{1-\lambda}{\rho}\right)^{-1}.$$

Because $\phi'' < \bar{\phi}$, the left-hand side of the truth-telling constraint (10) is higher. Because the value associated with a banker's best deviation depends on the amount each banker is required to deposit with the clearinghouse when he issues a note, J^d may vary with changes in the terms of trade. In this case, it is clear that J^d has to either decrease or remain the same because each banker is required to set aside a smaller amount. Thus, (10) continues to be satisfied. Each banker is strictly better off when the terms of trade are such that he is able to capture all welfare gains from the adoption of a fractional reserve system of the kind described above.

Proposition 4 *Provided that ω is sufficiently small, there exists a stationary monetary equilibrium in which the value of notes is given by $\bar{\phi}$. In this equilibrium, each banker sets aside the amount $\phi'' < \bar{\phi}$ when he has an opportunity to issue a note. This equilibrium Pareto dominates the stationary monetary equilibrium obtained under a fully backed system.*

Note that, in this case, each banker strictly prefers to be a member of a clearinghouse association that initially agrees to issue liabilities that are joint obligations of its members and follows the investment policy described above, which renders the scheme incentive compatible. Not only is the joint-liability arrangement described above an effective mechanism to ensure the solvency of each individual banker, but it also makes the construction of a higher-yielding portfolio for the bank coalition feasible. By providing a mechanism that permits member banks that are called for redemption to use the reserves of banks that have idle balances, a joint-liability arrangement makes it possible to invest some of the aggregate resources of the coalition in the productive technology, increasing the return obtained on banking assets. The monitoring and supervision services provided by the clearinghouse association are essential for the effective implementation of such a risk-sharing arrangement.

It is important to mention once again that the stability of the system (i.e., the fact that there is no bank failure in equilibrium) can be achieved only by promising a sufficiently high expected discounted utility to each banker, which is necessary to induce him to truthfully report the creation of bank liabilities and voluntarily deposit reserves with the clearinghouse. This means that there exists an endogenously determined value associated with the banking privileges (a franchise value) that is consistent with the stability of the banking system. Because this also means that there is an upper bound on the return each banker is willing to pay on his liabilities, I conclude that the stability of the banking system costs something for nonbanks. And a joint-liability arrangement of the kind described above makes it possible to raise this upper bound due to a more efficient use of interbank deposits.

6.2. Discussion

It is important to provide some interpretations of the kind of exchange mechanism I have described in this paper. It is possible to interpret the clearinghouse in my model as the kind of private bank coalition that was formed in the United States prior to the establishment of the Federal Reserve System. For instance, Gorton (1984, 1985) and Gorton and Mullineaux (1987) have provided evidence that the clearinghouse associations that were formed in some cities in the United States (e.g., New York, Boston, and Chicago) in the second half of the 19th century evolved into a coalition of banks that required each member bank to report its transactions, imposed reserve requirements on each member bank, and supervised the settlement process on a daily basis. This means that clearinghouse associations provided supervision and regulation services to member banks. In this case, the adoption of a fractional reserve system can be interpreted as a mechanism to induce cooperation within the members of the clearinghouse so that the system as whole can reduce the share of funds invested in non-interest-bearing reserves and, consequently, increase the share of funds invested in interest-bearing assets.

An important element of my analysis is the assumption that bankers can issue, at most, one note in each period. This characteristic of the model is of particular interest to the U.S. experience. The geographic restriction imposed on the establishment of branches was a distinguishing characteristic of the U.S. banking system until very recently. Some researchers have argued that this restriction on branching tended to generate a large number of poorly diversified small banks. I have shown that a joint-liability arrangement provides a risk-sharing scheme that allows banks to engage in fractional reserve banking in a safe way despite the restriction on the number of notes a bank is able to issue in each period.

Before I conclude, it is important to discuss the welfare properties of at least another obvious monetary arrangement: an equilibrium in which all trades are carried out with outside fiat money. In the appendix, I show that the stationary allocation under a fully backed system can also be implemented with fiat money (in the absence of a clearinghouse), but the stationary allocation under a fractional reserve system cannot be reproduced in an economy where fiat money is the only available medium of exchange. This result arises because the adoption of a fractional reserve system involves a relatively sophisticated interbank arrangement that cannot be implemented with fiat money. This is in line with the results in Kocherlakota (1998), who argues that fiat money works as an *imperfect* form of memory in the sense that the set of allocations that can be implemented with memory (i.e., perfect knowledge of trading histories) is larger than the set of allocations that can be implemented with fiat money.

7. CONCLUSION

This paper has emphasized the welfare properties of a specific type of bank coalition: a joint-liability arrangement. As opposed to markets, this form of bank coalition involves monitoring the activities of member banks. I have shown that this form of banking organization offers an effective response to the well-known tension between the benefits of fractional reserve banking and the risks associated with it. In particular, I have demonstrated that it is possible to allow member banks to take advantage of profitable (long-term) investment opportunities without compromising the stability of the banking system. The bank coalition described above ensures the transfer of reserves from more liquid banks to illiquid banks despite the existence of incentive problems. As a result, it is possible to increase the share of funds invested in interest-bearing assets, raising the rate of return obtained on banking assets. The way to achieve this goal without compromising the stability of the banking system is to ensure that the value attached to participation in the bank coalition is sufficiently high for each member bank to voluntarily hold the required amount of reserves. Thus, a joint-liability arrangement of the kind described above is a welfare-improving mechanism that allows the members of society to achieve a better allocation of resources (a truly superior form of banking).

Finally, it is important to mention that the model developed in this paper can be easily applied to the study of other issues in the field of money and banking. For instance, Sanches (2013) introduces an aggregate shock affecting the return to storage to study how an external shock to the value of banking assets can affect trading activity under alternative banking arrangements.

REFERENCES

[1] D. Andolfatto, E. Nosal. "Money, intermediation, and banking," *Journal of Monetary Economics* 56 (2009), pp. 289-294.

[2] L. Araujo, R. Minetti. "On the essentiality of banks," *International Economic Review* 52 (2011), pp. 679-691.

[3] C. Azariadis, J. Bullard, B. Smith. "Private and public circulating liabilities," *Journal of Economic Theory* 99 (2001), pp. 59-116.

[4] A. Berentsen. "On the private provision of fiat currency," *European Economic Review* 50 (2006), pp. 1683-1698.

[5] A. Berentsen, G. Camera, C. Waller. "Money, credit and banking," *Journal of Economic Theory* 135 (2007), pp. 171-195.

[6] R. Cavalcanti, A. Erosa, T. Temzelides."Private money and reserve management in a random-matching model," *Journal of Political Economy* 107 (1999), pp. 929-945.

[7] R. Cavalcanti, A. Erosa, T. Temzelides. "Liquidity, money creation and destruction, and the returns to banking," *International Economic Review* 46 (2005), pp. 675-706.

[8] R. Cavalcanti, N. Wallace."A model of private bank-note issue," *Review of Economic Dynamics* 2 (1999a), pp. 104-136.

[9] R. Cavalcanti, N. Wallace."Inside and outside money as alternative media of exchange," *Journal of Money, Credit and Banking* 31 (1999b), pp. 443-457.

[10] V. V. Chari, C. Phelan. "On the social usefulness of fractional reserve banking," mimeo, Federal Reserve Bank of Minneapolis (2012).

[11] A. Deviatov, N. Wallace. "A model in which monetary policy is about money," *Journal of Monetary Economics* 56 (2009), pp. 283-288.

[12] D. Diamond, P. Dybvig. "Bank runs, deposit insurance, and liquidity," *Journal of Political Economy* 91 (1983), pp. 401-419.

[13] M. Friedman. *A Program for Monetary Stability*, Fordham University Press, New York (1959).

[14] M. Friedman, A. Schwartz. *A Monetary History of the United States, 1867-1960*, Princeton University Press, 1963.

[15] G. Gorton. "Private clearinghouses and the origins of central banking," *Business Review*, Federal Reserve Bank of Philadelphia, January-February (1984), pp. 1-11.

[16] G. Gorton. "Clearinghouses and the origins of central banking in the United States," *Journal of Economic History* 45 (1985), pp. 277-283.

[17] G. Gorton, D. Mullineaux. "The joint production of confidence: endogenous regulation and the 19th century commercial bank clearinghouse," *Journal of Money, Credit and Banking* 19 (1987), pp. 457-468.

[18] C. Gu, F. Mattesini, C. Monnet, R. Wright. "Banking: a new monetarist approach," *Review of Economic Studies*, forthcoming.

[19] P. He, L. Huang, R. Wright. "Money, banking, and monetary policy," *Journal of Monetary Economics* 55 (2008), pp.1013-1024.

[20] S. Huangfu, H. Sun. "Private money and bank runs," *Canadian Journal of Economics* 44 (2011), pp. 859-879.

[21] C. Kahn, W. Roberds. "Demandable debts as a means of payment: banknotes versus checks," *Journal of Money, Credit and Banking* 31 (1999), pp. 500-525.

[22] C. Kahn, W. Roberds. "Payment system settlement and bank incentives," *Review of Financial Studies* 11 (1998), pp. 845-870.

[23] N. Kocherlakota. "Money is memory," *Journal of Economic Theory* 81 (1998), pp. 232-251.

[24] N. Kocherlakota, N. Wallace. "Incomplete record-keeping and incomplete payment arrangements," *Journal of Economic Theory* 81 (1998), pp. 272-289.

[25] T. Koeppl, C. Monnet, T. Temzelides. "A dynamic model of settlement," *Journal of Economic Theory* 142 (2008), pp. 233-246.

[26] R. Lagos, R. Wright."A unified framework for monetary theory and policy analysis," *Journal of Political Economy* 113 (2005), pp. 463-84.

[27] Y. Li. "A search model of money and circulating private debt with applications to monetary policy," *International Economic Review* 42 (2001), pp. 925-946.

[28] Y. Li. "Banks, private money, and government regulation," *Journal of Monetary Economics* 53 (2006), pp. 2067-2083.

[29] A. Martin, S. Schreft. "Currency competition: a partial vindication of Hayek," *Journal of Monetary Economics* 53 (2006), pp. 2085-2111.

[30] D. Mills. "A model in which outside and inside money are essential," *Macroeconomic Dynamics* 11 (2007), pp. 219-236.

[31] J. Moen, E. Tallman. "The bank panic of 1907: the role of trust companies," *Journal of Economic History* 52 (1992), pp. 611-630.

[32] J. Moen, E. Tallman. "Clearinghouse membership and deposit contraction during the panic of 1907," *Journal of Economic History* 60 (2000), pp. 145-163.

[33] G. Rocheteau, R. Wright. "Money in search equilibrium, in competitive equilibrium, and in competitive search equilibrium,"*Econometrica* 73 (2005), pp. 175-202.

[34] D. Sanches. "Banking crises and the role of bank coalitions," Federal Reserve Bank of Philadelphia working paper No. 13-28 (2013).

[35] S. Shi. "Money and prices: a model of search and bargaining," *Journal of Economic Theory* 67 (1995), pp. 467-496.

[36] A. Trejos, R. Wright. "Search, bargaining, money, and prices," *Journal of Political Economy* 103 (1995), pp. 118-141.

[37] N. Wallace. "Narrow banking meets the Diamond-Dybvig model," *Federal Reserve Bank of Minneapolis Quarterly Review* 20 (1996), pp. 3-13.

[38] S. Williamson."Private money,"*Journal of Money, Credit and Banking* 31 (1999), pp. 469-491.

APPENDIX

Here I show that the stationary allocation obtained under a fully backed system can be reproduced in case outside fiat money is the only available medium of exchange, whereas the stationary allocation obtained under a fractional reserve system cannot.

Suppose I initially endow each banker and a (random) measure $1-\lambda$ of buyers with one unit of outside fiat money. Thus, the total stock of money is given by $2-\lambda$. I want to show that there exists a stationary monetary equilibrium that implements the same allocation as that obtained under a fully backed system. In this equilibrium, the value of money is one in the first market, $\omega^{-1}v\left(\bar{\phi}\right)$ in the second, and $\bar{\phi}$ in the third.

In a stationary monetary equilibrium, there exists an invariant distribution of money holdings across different types of agents. Let $\hat{m}^1$ denote the invariant measure of buyers holding one unit of money at the end of the first market, let $\hat{m}^2$ denote the invariant measure of sellers holding one unit of money at the end of the second market, and let $\hat{m}^3$ denote the invariant measure of bankers holding one unit of money at the end of the third market. In particular, I consider the following distribution: $\hat{m}^1 = \hat{m}^3 = 1$ and $\hat{m}^2 = \lambda$. This implies that, in the first stage, a trade meeting occurs when a banker holding money finds a buyer without money, in which case the former gives the latter one unit of money in exchange for one unit of good x, and that, in the second stage, a trade meeting occurs when a buyer finds a seller, in which case the latter produces $\omega^{-1}v\left(\bar{\phi}\right)$ units of good y in exchange for one unit of money. In the third stage, there is a Walrasian market in which all sellers who traded in the previous stage supply money and all bankers who traded in the first stage demand money. The equilibrium value of money in this market will be $\bar{\phi}$.

It is straightforward to show that such an equilibrium exists if and only if

$$1-\bar{\phi}+\frac{\lambda\beta\left(1-\bar{\phi}\right)}{1-\beta}\geq 1. \tag{16}$$

This condition guarantees that, after trading in the first market, each banker finds it optimal to store the amount $\bar{\phi}$ of good x in order to sell it on the Walrasian market in exchange for one unit of money. Note also that a banker holding money in the Walrasian market could exchange his note for $\bar{\phi}$ unit of good x. In this case, he is willing to immediately consume it since he knows that he will not have anything of interest to trade with a consumer in subsequent periods. However, condition (16) also guarantees that he will be better off by holding on to his note.

Note that each banker plays an important intermediation role. The stock of money flows from the group of buyers to the group of sellers by means of direct (bilateral) trades in the second stage. The role of bankers is to ensure that the stock of money returns to the hands of buyers so that they can trade with sellers again in the following period. Of course, each banker obtains a flow of income by undertaking this intermediation activity.

Finally, to show that the stationary equilibrium allocation in the case of a fractional reserve system cannot be implemented with fiat money, it is sufficient to examine the budget constraint of each individual banker. In the case of a fractional reserve system, I have shown that it is possible to keep each banker's consumption unchanged and, simultaneously, increase the value of privately issued notes. This is possible because the transfer scheme adopted in the centralized location allows the clearinghouse to "aggregate" the individual budget constraints, reducing the amount of excess reserves in the system. In the case of fiat money, each banker has to save the full amount in order to buy back one unit of fiat money in the third stage (Walrasian market), so that he can continue trading in future periods. Thus, there is no aggregation of individual budget constraints. Therefore, the allocation obtained under a fractional reserve system cannot be reproduced in case fiat money is the only available medium of exchange.

Figure 1: Sequence of Events within a Period

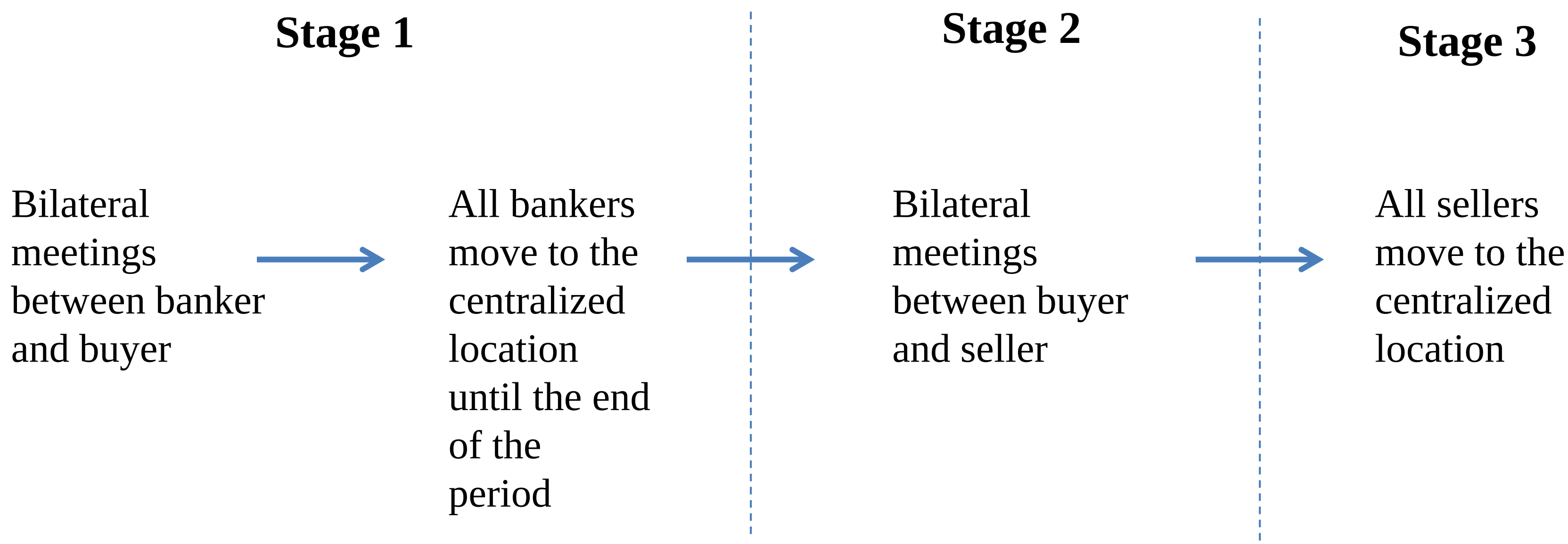

Figure 2: Creation and Redemption of Bank Notes