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Topologies Induced by Relations with Applications

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Abstract: Some methods for inducing topological structures by relations were initiated and their importance in applications were indicated. Topologies generated by equivalence relations were all quasi-discrete spaces. We induced the topologies generated using similarity relations and pre-order relations. Also, the topologies generated using general binary relations on the universe of discourse were initiated. Finally, rheumatic fever data reduction using topologies induced by relations were studied.

Key words: Topological spaces, binary relations, information system, rough sets, data reduction

INTRODUCTION

Topology is an important and interesting area of mathematics, the study of which will not only introduce you to new concepts and theorems but also put into context old ones like continuous functions. It is so fundamental that its influence is evident in almost every other branch of mathematics. This makes the study of topology relevant to all who aspire to be mathematicians whether their first love is algebra, analysis, category theory, chaos, continuum mechanics, dynamics, geometry, industrial mathematics, mathematical biology, mathematical economics, mathematical finance, mathematical modeling, mathematical physics, mathematics of communication, number theory, numerical mathematics, operation research or statistics. Topological notions like compactness, connectedness and denseness are as basic to mathematicians of today as sets and functions were to those of last century[3,8,9].

For a long time, many individuals believed that abstract topological structures have limited application in the generalization of real line and complex plane or some connections to Algebra and other branches of mathematics. And it seems that there is a big gap between these structures and real life applications. We noticed that in some situations, the concept of relation is used to get topologies that are used in important applications such as computing topologies[15], recombination spaces[2,7,17] and information granulation[21] which are used in biological sciences and some other fields of applications.

The aim of rough set theory is to give a description of the set of objects by logical, set-theoretical, topological etc. tools in terms of similarity relations and derived notions related by these relations. The description of the set of objects entails as well relationships and functional or near to functional dependencies among various similarity relations generated by various sets of the set of objects.

Rough sets were first introduced by[10,11] and are based on approximation spaces. An approximation space is a pair A = (Ob, R). Here, R is an equivalence relation, also called indiscernibility relation, imposing a granularity on the universe Ob such that $R \subseteq Ob \times Ob$. Furthermore, we assume Ob to be finite. For $x \in Ob$, let $[x]_R$ be the equivalence class containing x, i.e., $[x]_R = \{y: y R x\}$.

Given an arbitrary set $X \subseteq Ob$, we wish to describe X in terms of elements or granules of Ob/R. Pawlak proposed the use of lower and upper approximations of a set X, denoted $\underline{R}(X)$ and $\overline{R}(X)$, respectively. Lower and upper approximations are defined as:

$$\underline{R}(X) = \{x \in Ob: [x]_R \subseteq X\}$$
$$\overline{R}(X) = \{x \in Ob: [x]_R \cap X \neq \phi\}$$

The semantics of the approximations of sets may be defined as follows:

- Elements of the universe that belong to $\underline{R}(X)$ are those elements that surely belong to the set X
- Elements that belong to $\overline{R}(X)$ possibly belong to the set X
- Elements that belong to $Ob/\overline{R}(X)$ are elements of the universe that surely do not belong to the set X. Hence, the uncertainty lies in $\overline{R}(X)/\underline{R}(X)$ which is also called area of uncertainty. Elements of the area of uncertainty may, or may not, belong to X

The approximation operators can also be considered using membership functions. It is possible to define a rough membership function as presented in[12].

MATERIALS AND METHODS

Topologies induced by relations: Let A = (Ob, R) be an approximation space. The equivalence classes Ob\R of the relation R will be called elementary sets (atoms) in A. Every finite union of elementary sets in A will be called a composed set in A. The family of all composed sets in A will be denoted by com (A).

The family com (A) in the approximation space A = (Ob, R) is a topology on the set Ob.

Since the approximation space A = (Ob, R), defines uniquely the topological space τ (A) = (Ob, com (A)) and com (A) is the family of all open sets in τ (A) and U/R is a basis for τ (A), then τ (A) is a quasi-discrete topology on Ob and com (A) is both the set of all open and closed sets in τ (A). Thus, the lower approximation and the upper approximation of any subset $X \subseteq Ob$ can be interpreted as the interior and the closure of the set X in the topological space τ (A), respectively.

Lemma 1: If β is a base for a topological space (Ob,τ), where β is a partition of Ob, then for every subset $X \subseteq Ob$:

- $int_\tau(X) = \bigcup\{B \in \beta : B \subseteq X\}$
- $cl_\tau(X) = \bigcup\{B \in \beta : B \cap X \neq \varphi\}$

Proof: Only we prove (ii) because (i) is trivial. Let $x \in cl_\tau$ (X) then for every open set G containing x, $X \cap G \neq \varphi$. But $G = \bigcup_{B \in \beta} B$, then there exists $B_o \subseteq G$ such that $x \in B_o \subseteq G$. But B_o is an open set containing x, hence $B_o \cap X \neq \varphi$ and $x \in \bigcup\{B \in \beta: B \cap X \neq \varphi\}$.

Conversely, if $x \in \bigcup\{B \in \beta : B \cap X \neq \varphi\}$ and G is an open set containing x and β is a partition of Ob, $x \in \bigcup$, then x belongs to only one element of β say $x \in B_o$. Then must $B_o \subseteq G$, i.e., $x \in B_o \subseteq G$ but $B_o \cap X \neq \varphi$, hence $G \cap X \neq \varphi$. Then $x \in cl_\tau(X)$.

Let A_1 = (Ob, R_1) and A_2 = (Ob, R_2) be two approximation spaces. Then we say that the partition Ob/R depends on the partition Ob/R_2 denoted Ob/R_1 if and only if $B = \bigcup_{S \in Ob/R_2} S, \forall B \in Ob/R_1$.

Proposition 2: Let τ_1 and τ_2 be the topologies induced by the partitions Ob/R_1 and Ob/R_2 respectively. Then $Ob/R_1 \leq Ob/R_2$ iff $\tau_2 \subseteq \tau_1$.

Example: Consider the partitions $\beta_1 = \{\{x_1, x_2\}, \{x_3\}, \{x_4\}\}$ and $\beta_2 = \{\{x_1, x_2\}, \{x_3, x_4\}\}$ of the set Ob = $\{x_1, x_2, x_3, x_4\}$. Then $\beta_1 \leq \beta_2$ and $\tau_2 \subseteq \tau_1$ where $\tau_1 = \{Ob, \phi, \{x_3\}, \{x_4\}, \{x_{1,2}\}, \{x_3, x_4\}, \{x_1, x_2 x_3\}, \{x_1, x_2 x_4\}\}$, $\tau_2 = \{Ob, \phi, \{x_1, x_2\}, \{x_3, x_4\}\}$ are the topologies generated by β_1 and β_2 respectively.

For any topological space (Ob,τ), we define the equivalence relation E(τ) on the set Ob by $(x, y) \in E(\tau)$ iff $cl_\tau(\{x\}) = cl_\tau(\{y\}), \forall x, y \in Ob$. The set of all equivalence classes of E(τ) is denoted by Ob/ E(τ).

Proposition 3: Let A = (Ob, R) be an approximation space and let τ_R be the topology generated by the base B_R = Ob/R. If (Ob, τ) is the quasi-discrete topological space has Ob/E(τ) as a base. Then $\tau_R = \tau$ iff for all $x \in B_R \in \beta_R$ there exists $B \in Ob/E(\tau)$ such that $x \in B$.

Lemma 4[15]: For any topology τ on a set Ob and for all x, $y \in Ob$, if $y \in cl_\tau(\{x\})$ and $x \in cl_\tau(\{y\})$ then $cl_\tau(\{x\}) = cl_\tau(\{y\})$.

Lemma 5[15]: If τ is a quasi-discrete topology on a set Ob, then $y \in cl_\tau(\{x\})$ implies $x \in cl_\tau(\{y\})$ for all x, $y \in Ob$.

Lemma 6[15]: If τ is a quasi-discrete topology on a set Ob, then the family $\{cl_\tau(\{x\}): x \in Ob\}$ is a partition of Ob.

Proposition 7: Let τ be the topology induced by the partition β_R = Ob/R . Then β_R = Ob/E(τ).

Proof: $x \in B$, $B \in \beta_R$:

- iff $x \in cl_\tau(B) = \bigcup_{y \in B} cl_\tau(\{y\})$
- iff $y_o \in B$ and $x \in cl_\tau(\{y_o\})$ iff $cl_\tau(\{x\}) = cl_\tau(\{y_o\})$ (Lemma 2.2)
- iff $(x, y_o) \in E(\tau)$
- iff $A \in Ob/E(\tau)$ such that $x \in A$
- iff β_R = Ob/E(τ)

For any n approximation spaces A_1 = (Ob, R_1), A_2 = (Ob, R_2),…, A_n = (Ob, R_n) we define the partition $Ob/E(\tau_{ind}) = \bigcap_{i=1,2,\ldots,n} Ob/E(\tau_i)$.

Theorem 8: $\tau_i \subseteq \tau_{ind}$, i = 1, 2,…, n where τ_i and τ_{ind} are the topologies generated by the partitions Ob/(τ_i) and Ob/E(τ_{ind}) respectively.

Proof: Since $Ob/E(\tau_{ind}) \leq Ob/E(\tau_i)$ for all i = 1, 2,…, n then $\tau_I \subseteq \tau_{ind}$.

Example: Consider the topological space (Ob, τ) where Ob = {x_1, x_2, x_3, x_4} and b = {{x_1}, {x_2, x_3}, {x_4}} is the base of τ, then τ is a quasi-discrete topology and:

$$cl_\tau(\{x_1\}) = \{x_1\},\ cl_\tau(\{x_2\}) = \{x_2, x_3\},\ cl_\tau(\{x_3\}) = \{x_2, x_3\}$$
$$cl_\tau(\{x_4\}) = \{x_4\}$$

Then Ob/E(τ) = {{x_1}, {x_2, x_3}, {x_4}} = β.

Example: Consider the approximation spaces $A_1 = (Ob, R_1)$, $A_2 = (Ob, R_2)$ and $A_3 = (Ob, R_3)$ where $Ob = \{x_1, x_2, x_3, x_4\}$ and Ob/E(τ_1) = {{x_1}, {x_2, x_3}, {x_4}}, Ob/E (τ_2) = {{x_1, x_2}, {x_3, x_4}} and Ob/E (τ_3) = {{x_1}, {x_2}, {x_3, x_4}}are the bases of τ_1, τ_2 and τ_3 respectively, then $Ob/E(\tau_{ind}) = (Ob/E(\tau_1)) \cap (Ob/E(\tau_2)) \cap (Ob/E(\tau_3)) = \{\{x_1\},\{x_2\},\{x_3\},\{x_4\}\}$ is the partition induced by E(τ_{ind}). Then $\tau_\iota \subset \tau_{ind}$, i = 1, 2, 3.

Topologies generated using similarity relations: A similar relation R on Ob is any relation satisfies:

- For any x∈Ob, xRx (reflexive)
- For any x, y∈Ob, if xRy then yRx (Symmetric)

For x∈Ob, we define the similar class containing x by R(x) = {y∈Ob: xRy}.

The relation R on Ob defined by xRy iff d(x,y)<n where (Ob, d) is a metric space with a metric function d defined as: d(x,y) = $|x-y|$ and n = card (Ob), is a similar relation.

Proposition 9: For any similar relation R defined on Ob we have:

- x∈R(x)
- y∈R(x) iff x∈R(y)
- xRy iff x∈R(y) and y∈R(x)

The class β = {B(x): x∈X} is called a symmetric covering of a set X if x∈B(y) iff y∈B(x). Then the class β = {R(x): x∈Ob} is a symmetric covering of the set of objects Ob.

Let β is the symmetric covering of Ob by the similar relation R. Then we define a relation R_β induced by β by x R_β y iff there exist B∈β and x,y∈B.

Proposition 10: The relation R_β is a similar relation on the set of objects Ob.

Since β is a covering of Ob, then for any x∈Ob there exists B∈β such that x∈B hence x, x∈B∈β then $xR_\beta x$. Let $xR_\beta y$ then there exists B∈β such that x, y ∈B then y, x∈B hence $yR_\beta x$.

Proposition 10 For every x∈Ob we have:

$$R_\beta(x) = \bigcup_{B\in\beta(x)} B \text{ , where } \beta(x) = \{B\in\beta: x\in B\}$$

Proof:

$$\begin{aligned} y\in R_\beta(x) &\Leftrightarrow \exists\, B\in \beta \text{ and } x,y\in B \\ &\Leftrightarrow \exists\, B\in \beta \text{ and } x\in B \text{ and } y\in B \\ &\Leftrightarrow \exists\, B\in \beta \text{ and } y\in B \\ &\Leftrightarrow \bigcup_{B\in\beta(x)} B \end{aligned}$$

Let β is the covering of Ob. Then we define the class β^* = {R_β(x): x∈Ob}.

Proposition 11: The class β^* is a symmetric covering of the set of objects Ob and $R_\beta \subseteq R_{\beta^*}$.

Proof:

- $x\in R\beta(y) \Leftrightarrow \exists\, B\in\beta(y)$ and $x\in B \Leftrightarrow \exists\, B\in\beta(x)$ and $y\in B \Leftrightarrow y\in R\beta(x)$
- Let $(x,y) \in R_\beta \Rightarrow \exists\, B\in\beta$ and x, y ∈B
  $\Rightarrow B\in \beta(x)$ and $B\in \beta(y)$
  $\Rightarrow B\in\beta(x)\cap \beta(y)$
  $\Rightarrow B\in \bigcup_{B\in\beta(x)} B = R_\beta(x)\in\beta^*$
  $\Rightarrow x,y\in B\in\beta^*$
  $\Rightarrow x,y\in R_{\beta^*}$

Let A⊆Ob be any non empty subset of the set of objects. Then A is called a similar pre-class of R if for any x, y∈A $\Rightarrow$ (x, y)∈R.

Proposition 12: Every similar class R(x) is a maximal similar pre-class.

For an element x∈Ob we define a class called the pre-similar class of x as follows:

$L_R(x)$ = {A⊆Ob: x∈A and A is similar pre-class of R}. Let L_R = {$L_R(x)$: x∈Ob} be the family of all pre-similar classes. Then we define a relation R* on L_R by for any $L_R(x)$, $L_R(y) \in L_R$, $L_R(x)R^*L_R(y)$ iff there exist $A\in L_R(x)$ and $B\in L_R(y)$ and $A \cap B \neq \varphi$.

Proposition 13:

- The relation R* on L_R is a similar relation
- xRy iff $L_R(x)R^*L_R(y)$ for any x,y∈Ob

Proof:

- Since for any $L_R(x) \in L_R$ and $A \in L_R(x)$, $A \cap A \neq \phi$ then $L_R(x)R^*L_R(x)$ hence R^* is reflexive. Also if $L_R(x)R^*L_R(y)$ then there exist $A \in L_R(x)$ and $B \in L_R(y)$ such that $A \cap B \neq \phi$, hence $B \cap A \neq \phi$, hence $L_R(y)R^*L_R(x)$. then R^* is symmetric.
- Firstly, we will prove that $xRy \Rightarrow L_R(x)R^*L_R(y)$

Let $(x, y) \in R \Rightarrow \{x, y\}$ is a similar pre-class of R. $\Rightarrow$ there exist a similar class R(x) such that $\{x,y\} \subseteq R(x)$ and $R(x) \in L_R(x)$ but R is symmetric then $R(x) \in L_R(y)$, then there exist $A = R(x) \in L_R(x)$ and $B = R(x) \in L_R(y)$ and $A \cap B \neq \varphi$, hence $L_R(x)R^*L_R(y)$.

Conversely, let for some $x,y \in Ob$, $L_R(x)R^*L_R(y)$ then there exist $R(z) \in L_R(x)$ and $R(z) \in L_R(y)$ a similar class of R. hence $x \in R(z)$ and $y \in R(z)$ then $x,y \in R(z)$ hence xRy.

Let $L_R(x)$ be the pre-similar class of $x \in Ob$. Then we define a set $L^*_R(x) = \bigcup_{A \in L_R(x)} A$ called the R-link of x, where $A \in L_R(x)$ and $A \neq R(x)$.

If $L^*_R(x) = Ob$ then it is called open R- link of x and if $L^*_R(x) \subset Ob$ then it is called closed R- link of x.

The class $M = \{L^*_R(x) : x \in Ob\}$ of all R- links of $x \in Ob$ is a subbase of a topology on Ob called the linked topology and denoted $\tau_{L^*_R}$.

Proposition 14:

- For any $x \in Ob$, $L^*_R(x) \subseteq R(x)$
- The class M is a symmetric covering of Ob

Proof:

- Let $y \in L^*R(x) \Rightarrow y \in \bigcup_{A \in L_R(x)} A \Rightarrow$ there exists $A = R(x) \in LR(x)$ and $y \in A$ then $y \in R(x) \Rightarrow L^*R(x) \subseteq R(x)$
- For any $x \in Ob$, $x \in L^*_R(y) \Rightarrow$ M is covering of Ob

Now let $x \in L^*_R(y) \Rightarrow x \in \bigcup_{A \in L_R(x)} A$:

- there exist $A = R(x) \in L_R(y)$ and $x \in R(x)$
- $x, y \in R(x)$
- $y \in L^*_R(x)$

then M is a symmetric covering of Ob.

Proposition 15:

- The linked topology $\tau_{L^*_R}$ is finer than the similar topology τ_R, where τ_R is the topology generated by the subbase $\{R(x) : x \in Ob\}$
- $xRy \Rightarrow \exists$ open set $u \in \tau_{L^*_R}$ and $x,y \in u$

Example: Let $Ob = \{c_1, c_2, \ldots, c_7\}$ be the set of objects which is seven computers in a local network in a certain company. Let τ be the irregular topology on the set of objects which induced by a general relation on Ob which makes the following graph. We define a similar relation R on the set of objects by: Two computers x and y are in relation by R iff the computer x has a copy of a certain program in the computer y.

Then we can define the similar classes of R as follows:

- $R(c_1) = \{c_1, c_2, c_4\}$, $R(c_2) = \{c_1, c_2, c_3, c_4, c_5\}$, $R(c_3) = \{c_2, c_3, c_5\}$, $R(c_4) = \{c_1, c_2, c_5, c_6, c_4\}$, $R(c_5) = \{c_2, c_3, c_4, c_6, c_5\}$, $R(c_6) = \{c_4, c_5, c_6, c_7\}$, $R(c_7) = \{c_6, c_7\}$. Then we have $L_R(c_1) = \{\{c_1\}, \{c_1, c_2\}, \{c_1, c_4\}, \{c_1, c_2, c_4\}\}$, $L_R(c_2) = \{\{c_2\}, \{c_2, c_1\}, \{c_2, c_4\}, \{c_2, c_5\}, \{c_2, c_3\}, \{c_2, c_3, c_5\}, \{c_2, c_1, c_4\}, \{c_2, c_4, c_5\}, \{c_2, c_1, c_3, c_4, c_5\}\}$, $L_R(c_3) = \{\{c_3\}, \{c_3, c_2\}, \{c_3, c_5\}, \{c_3, c_2, c_5\}\}$, $L_R(c_4) = \{\{c_4, \{c_4, c_1\}, \{c_4, c_2\}, \{c_4, c_5\}, \{c_4, c_6\}, \{c_4, c_1, c_2\}, \{c_4, c_2, c_5\}, \{c_4, c_5, c_6\}, \{c_4, c_1, c_2, c_5, c_6\}\}$, $L_R(c_5) = \{\{c_5\}, \{c_5, c_2\}, \{c_5, c_4\}, \{c_5, c_3\}, \{c_5, c_6\}, \{c_5, c_2, c_3\}, \{c_5, c_2, c_4\}, \{c_5, c_4, c_6\}, \{c_2, c_5, c_4, c_5, c_6\}\}$, $L_R(c_6) = \{\{c_6\}, \{c_6, c_4\}, \{c_6, c_5\}, \{c_6, c_7\}, \{c_6, c_4, c_7\}, \{c_6, c_5, c_7\}, \{c_6, c_4, c_5\}, \{c_4, c_5, c_6, c_7\}\}$
- $L_R(c_7) = \{\{c_7\}, \{c_6, c_7\}\}$. Also, we have: $L^*_R(c_1) = \{c_1, c_2, c_4\}$, $L^*_R(c_2) = \{c_1, c_2, c_3, c_4, c_5\}$, $L^*_R(c_3) = \{c_2, c_3, c_5\}$ $L^*_R(c_4) = \{c_1, c_2, c_5, c_6, c_4\}$, $L^*_R(c_5) = \{c_2, c_3, c_4, c_5, c_6\}$, $L^*_R(c_6) = \{c_4, c_5, c_6, c_7\}$, $L^*_R(c_7) = \{c_7\}$

Then the linked topology $\tau_{L^*_R}$ is finer than the similar topology, such that $L^*_R(c_i) \subseteq R(c_i)$ for all $i = 1,2,\ldots,7$.

For any subset A of the set of objects, we define two sets $\underline{R}(A)$ and $\overline{R}(A)$, they are called the lower and upper similar classes of A by:

$$\underline{R}(A) = \bigcup\{R(x) : R(x) \subseteq A\}$$

and

$$\overline{R}(A) = \bigcup\{R(x) : R(x) \cap A \neq \phi\}$$

Let $\underline{\tau}_R$ be the topology induced by the subbase $\{\underline{R}(A): A \subseteq Ob\}$ this topology called the lower similar topology. Also we define the topology $\bar{\tau}_R$ which is called the upper similar topology and generated by the subbase $\{\overline{R}(A): A \subseteq Ob\}$.

Proposition 16: Let $\underline{\tau}_R$ and $\bar{\tau}_R$ be the lower and upper similar topologies then:

- $\bar{\tau}_R \subseteq \underline{\tau}_R$ if R is an equivalence relation
- $\bar{\tau}_R \subseteq \underline{\tau}_R$ if R is a similar relation
- $\underline{\tau}_R$ and $\bar{\tau}_R$ are in general not comparable if R is a general relation

The following proposition present another way to generate topologies from similarity relations.

Proposition 17: $\tau^{**}{}_R = \{A \subseteq Ob : \forall x \in A, R(x) \subseteq A\}$ is a topology on Ob.

Proof:

- $Ob, \varphi \in \tau^{**}{}_R$ is clearly
- If $A_1, A_2, \ldots \in \tau^{**}{}_R$ and $x \in \bigcup_i A_i$ for some $i_\circ$ then $R(x) \subseteq A_{i_\circ}$ then $R(x) \subseteq \bigcup_i A_i$ hence $\bigcup_i A_i \in \tau^{**}{}_R$
- Let $A_1, A_2 \in \tau^{**}{}_R$, then $\forall x \in A_1 \cap A_2$ we have $R(x) \subseteq A_1$ and $R(x) \subseteq A_2$ hence $R(x) \subseteq A_1 \cap A_2$ then $A_1 \cap A_2 \in \tau^{**}{}_R$

Example: Consider Ob = {a, b, c, d} be the set of objects with a similar relation R its similar classes are:

- R(a) = {a, c}, R(b) = {b, d}, R(c) = {a, c, d} and R(d) = {b, c, d}. Then: $\underline{\tau}_R$ = {Ob, φ {c}, {d}, {c, d}, {a, c}, {b, d}, {a, c, d}, {b, c, d}}, $\bar{\tau}_R$ = {Ob, φ, {d, c}, {a, c, d}, {b, c, d}} and $\tau^{**}R$ = {Ob, φ} then $\tau^{**}R \subset \bar{\tau}_R \subset \underline{\tau}_R$
- The conjugate relation $\overline{R}$ of R is defined by (x, y) $\in \overline{R}$ iff (x, y) $\notin$ R or x = y

Proposition 18:

- $\underline{R} \cap \overline{R} = I$, I is the identity relation
- $\overline{R}$ is a similar relation
- $\overline{\overline{R}} = R$

Proof:

- $(x,y) \in R \cap \overline{R}$ iff $x = y \Rightarrow R \cap \overline{R} = I$
- $(x,x) \in \overline{R}$ such that x = x and if $(x,y) \in \overline{R}$ then $(x,y) \notin R$ or x = y then $(y,x) \notin R$ or y = x hence $(y,x) \in \overline{R}$
- $(x, y) \in \overline{\overline{R}} \Leftrightarrow (x,y) \notin \overline{R}$ or $x = y \Leftrightarrow (x,y) \in R$ or $x = y \Leftrightarrow (x,y) \in R$

Example: Let Ob = {a, b, c, d} be the set of objects with the similar relation R = {(a, a), (b, b), (c, c), (d, d), (d, c), (c, d), (d, b), (b, d), (c, b), (b, c), (b, a), (a, b)}. Then $\overline{R} = I \cup (Ob \times Ob - R)$ = {(a,a), (b,b), (c,), (d,d), (c,a), (a,c), (d,a), (a,d)}.

Topologies generated using dominance (pre-order) relations: For a long time, many mathematicians believed that there is a large deviation between abstract topological structures and computing[12-14].

A relation R on a set Ob is called a dominance relation (pre-order) whenever R is both reflexive and transitive . If x is related to y, we write xRy and say that x dominances y. The set $R(y) = \{x : yRx\}$ is called the before set.

Example: Let $Ob = \{1, 2, 3, 4, 5, 6\}$ and $(x, y) \in R$ if and only if $x \mid y$, $x, y, \in Ob$.

R = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (4, 4), (5, 5), (6, 6), (1,6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6)}. The relation R is a dominance relation.

In a finite space (Ob, τ), it is clear that $\tau^c = \{Ob - G : G \in \tau\}$ is also a topology.

A subset $F \subset X$ is a closed set iff $F = \bigcup_{y \in F} Fy$ such that $Fy = \{x : yRx\}$ (Fy is the smallest closed set about x). This is the dual of our representation of open sets.

If R is a dominance relation on a set Ob, then its dual R_D is defined by the requirement yR_Dx if and only if xRy.

A point x in a subset U of Ob is insulated from $Ob - U$ if and only if there is no point y in $Ob - U$ such that y dominates x.

Lemma 19: Let R be a dominance relation on a set Ob, $U \subset Ob$, $P \in U$ the following are equivalent:

- P is insulated from $Ob - U$
- $P \in U$, $(y, P) \in R$, then $y \in U$

Table 1: Application for Openness algorithm

P(Ob)	Insulated points	Open sets
{1}	√	√
{2}		
{3}		
{4}		
{1, 2}	√	√
{1, 3}		
{1, 4}		
{2, 3}		
{2, 4}		
{3, 4}		
{1, 2, 3}	√	√
{1, 2, 4}		
{2, 3, 4}		
{1, 3, 4}		
Ob	√	√
φ	√	√

Proof: First, consider $p\in U$ is insulated from Ob-U, (y, p) $\in R$. Let $y\notin U$, then $y\in$ Ob-U, So $(y, p)\notin R$, but $(y, p)\in R$, a contradiction, then $y\in U$.

Second, consider $p\in U$, $x\in$ Ob-U, suppose $(x, p) \in R$. Then $x\in U$ contradicts that $x\notin U$, then $\tau_R = \{U \subset Ob : x \text{ is insulated from } Ob - U, \forall\ x \in U\}$ $(x, p)\notin R$.

Proposition 20: If R is a dominance relation on a set Ob, then is topology on Ob.

Proof: Clearly Ob and ϕ are elements in τ_R let $U_i\in \tau_R$ for every $i\in I$. For any $x \in \bigcup_{i\in I} U_i$ and $(y, x)\in R$, there is $i_0\in I$ such that $x \in U_{i_0}$. By openness of U_{i_0}, we have $y \in U_{i_0} \subset \bigcup_{i=I} U_i$. Therefore $\bigcup_{i\in I} U_i \in \tau_R$. Also, if A and B are elements of τ_R, then $\bar{\underline{\tau}}^{2} = \{Ob, \varphi, \{d\}\}$.

According to the above proposition we give the following algorithm to check the openness of a subset $U \subset Ob$ with respect to a dominance relation R.

Openness algorithm:

i- Find Ob-U
ii- Investigates the existence of any pair (a, b) $\in R$, $a \in Ob - U$, $b\in U$, we have two cases:
  - If there exists such pair, then U is not open
  - If there is not such pair (a, b), then U is open

The following example (Table 1) is an application for the above algorithm.

Example 4: Consider Ob = {1, 2, 3, 4}, $(x, y)\in R$ whenever $x \le y$, $\forall\ x, y \in Ob$, then R = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4), (3, 3), (3, 4), (4, 4)}.

Table 2: Closure space generated by a general relation

A	Cl_R (A)	int_r (A)
{a}	{a}	ϕ
{b}	{b, c}	{b}
{c}	{a, c}	ϕ
{a, b}	Ob	{b}
{a, c}	{a, c}	ϕ
{b, c}	Ob	{b, c}
Ob	Ob	Ob
ϕ	ϕ	ϕ

Then the induced topology is τ_R = {Ob, φ, {1}, {1, 2}, {1, 2, 3}}.

Let Ob be the set of objects and let R be any binary relation on Ob. The relation R gives rise to a closure operator cl_R as follows:

$$cl_R(A) = A\cup\{y\in X\}|\exists x\in A: (y, x)\in R\} \text{ for every } A\subseteq Ob$$

Lemma 21: The interior operator corresponding to cl_R is given by:

$$int_R(A) = \{y\in A: \forall x\in A^c, \sim yRx\}$$

Proof: $int_R(A) = [cl_R(A^c)]^c$

$= \{Ac \cup \{y\in X: \exists x\in A^c, (y, x)\in R\}\}^c$
$= A \cap \{y\in X: \exists x\in A^c, (y, x)\in \}^c$
$= A \cap \{y\in X: \forall x\in A^c, (y, x)\notin R\}^c$
$= \{y\in A: \forall x\in A^c, \sim yRx\}$

Thus the interior operator of A consist of those elements of A which are not R-related to any elements outside A.

Lemma 22: For any relation R on Ob, (Ob, cl_R) is closure space.

In the following we will give an example (Table 2) for closure space generated by a general relation.

Example: Consider Ob{a, b, c} and R is a binary relation on Ob, R = {(a, b), (c, b), (a, c)}. Then we have the Table 2 for closures and interiors of the subsets of Ob: We note from Table 2 that:

- $cl_R(\phi) = \phi$
- $A \subseteq cl_R(A)$
- $cl_R(A\cup B) = cl_R(A) \cup cl_R(B)$ for all $A, B \subseteq Ob$

Lemma 23 If Ob be non-empty set and R is transitive relation, then (x, cl_R) is topological space.

Example 24 Consider the relation R = {(1, 1), (2, 3), (3, 2), (2, 2)} on Ob = {1, 2, 3}. Table 3 shows closures and interiors of the subsets of Ob.

Table 3: Closures and interiors of the subsets of Ob

A	$Cl_R(A)$	$int_R(A)$
{1}	{1}	{1}
{2}	{2, 3}	ϕ
{3}	{2, 3}	ϕ
{1, 2}	Ob	{1}
{1, 3}	Ob	{1}
{2, 3}	{2, 3}	{2, 3}
Ob	Ob	Ob
ϕ	ϕ	ϕ

From Table 3, we have:

- $clR(\phi) = \phi$
- $A \subseteq cl_R(A$
- $cl_R(A \cup B) = cl_R(A) \cup cl_R(B)$ for all $A, B \subseteq Ob$
- $cl_R(cl_R(A)) = cl_R(A)$ for all $A \subseteq Ob$

Topologies generated using general binary relations: The basic aim of this section is to generate topological structures using the lower and the upper approximations of any binary relation. Given general approximation space $A = (Ob, R)$ where R here is any general binary relation on Ob. For any subset X of Ob we define lower and upper approximations as follows[18,19,22]:

$$\underline{R}(X) = \{x \in Ob : \forall y((x, y) \in R \Rightarrow y \in X)\}$$

and

$$\overline{R}(X) = \{x \in Ob : \exists y((x, y) \in R \wedge y \in X)\}$$

Then the following structures are topologies on Ob:

$$\underline{\tau}_1 = \{G \subseteq Ob : \underline{R}(G) = G\}$$

$$\underline{\tau}_2 = \{G \subseteq Ob : \underline{R}^2(G) = \underline{R}(\underline{R}(G)) = G\}$$

$$\underline{\tau}_3 = \{G \subseteq Ob : \underline{R}^3(G) = \underline{R}(\underline{R}(\underline{R}(G))) = G\} \ldots..$$

$$\underline{\tau}_{n-1} = \{G \subseteq Ob : \underline{R}^{n-1}(G) = G\ ,\ n = |Ob|\}$$

These topologies have the property that: $\underline{\tau}_1 \subseteq \underline{\tau}_2 \subseteq \ldots. \subseteq \underline{\tau}_{n-1}$.

Also, if we deal with the upper approximation instead of the lower approximation we can construct the following topologies:

$$\overline{\tau}_1 = \{G \subseteq Ob : \overline{R}(G) = G \vee \overline{R}(G) = \varphi\}$$

$$\overline{\tau}_2 = \{G \subseteq Ob : \overline{R}^2(G) = \overline{R}(\overline{R}(G)) = G \vee \overline{R}(\overline{R}(G)) = \phi\}$$

$$\overline{\tau}_3 = \{G \subseteq Ob : \overline{R}^3(G) = \overline{R}(\overline{R}(\overline{R}(G))) = G \vee \overline{R}(\overline{R}(\overline{R}(G))) = \phi\} \ldots\ldots$$

$$\overline{\tau}_{n-1} = \{G \subseteq Ob : \overline{R}^{n-1}(G) = G \vee \overline{R}^{n-1}(G) = \phi\ , n = |Ob|\}$$

These topologies have the property that $\overline{\tau}_1 \subseteq \overline{\tau}_2 \subseteq \ldots. \subseteq \overline{\tau}_{n-1}$.

In the following we will give some illustrative examples and remarks.

Example: Let $Ob = \{a, b, c, d\}$ be the universe and let $R = \{(a, b), (c, d), (b, d), (d, a), (c, b)\}$ be a general binary relation on Ob. Then we have the following topologies on Ob using the lower approximation:

$$\underline{\tau}_1 = \{Ob, \varphi, \{a, b, d\}\}$$

$$\underline{\tau}_2 = \{Ob, \varphi, \{a, b, d\}\}$$

$$\underline{\tau}_3 = \{Ob, \phi, \{a\}, \{b\}, \{d\}, \{a, b\}, \{a, d\}, \{b, d\}, \{a, b, d\}\}$$

If we made more iteration to introduce more topologies using the lower approximation we will obtain that: $\underline{\tau}_4 = \underline{\tau}_1$, $\underline{\tau}_5 = \underline{\tau}_2$ and $\underline{\tau}_6 = \underline{\tau}_3$ and so on.

Also we have the following topologies on Ob using the upper approximation:

$$\overline{\tau}_1 = \{Ob, \varphi, \{c\}\}$$

$$\overline{\tau}_2 = \{Ob, \varphi, \{c\}\}$$

$$\overline{\tau}_3 = \{Ob, \phi, \{d\}, \{c\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}\}$$

If we made more iteration to introduce more topologies using the upper approximation we will obtain that : $\overline{\tau}_4 = \overline{\tau}_1$, $\overline{\tau}_5 = \overline{\tau}_2$ and $\overline{\tau}_6 = \overline{\tau}_3$ and so on.

Remark: If the relation R on the universe Ob is constant, then all topologies induced by the lower or the upper approximations are indiscrete.

If the relation R on the universe Ob is identity or contain the identity relation, then all topologies induced by the lower or the upper approximations are discrete.

If we made more iteration to introduce more topologies using the lower approximation or the upper approximation, then all new iterations will introduce the same topologies we before obtained.

Another method for constructing topologies using the lower and the upper approximations is presented bellow:

All the following are topologies on Ob:

$$\underline{\tau}^{1} = \{\underline{R}(G) : \underline{R}(G) = \overline{R}(G)\}$$

$$\underline{\tau}^{2} = \{\underline{R}^2(G) : \underline{R}^2(G) = \overline{R}^2(G)\}$$

$\underline{\bar{\tau}}^{3} = \{\underline{R}^{3}(G) : \underline{R}^{3}(G) = \bar{R}^{3}(G)\}$,

$\ldots\ldots, \underline{\bar{\tau}}^{n-1} = \{\underline{R}^{n-1}(G) : \underline{R}^{n-1}(G) = \bar{R}^{n-1}(G)\}$

Also, all the following structures are topologies on Ob. $\tau^1 = \{\underline{R}(\bar{R}(G)) : \underline{R}(\bar{R}(G)) = \bar{R}(\underline{R}(G))\}$, $\tau^2 = \{\underline{R}(\bar{R}(\underline{R}(G))) : \underline{R}(\bar{R}(\underline{R}(G))) = \bar{R}(\underline{R}(\bar{R}(G)))\}$ and so on.

Example: According to Example 4.1 we have:

$\underline{\bar{\tau}}^{1} = \{Ob, \varphi, \{d\}, \{a, b, c\}\}$

$\underline{\bar{\tau}}^{2} = \{Ob, \varphi, \{d\}\}$

$\underline{\bar{\tau}}^{3} = \{Ob, \varphi, \{d\}\}$

RESULTS AND DISCUSSION

Here we will give the main conventions that we will apply in this work. These conventions will be indicated by examples.

We briefly describe the rheumatic fever datasets used in our example. No doubt that, the rheumatic fever is a very common disease. It has many symptoms differs from patient to another but though the diagnosis it is the same. So, we obtained the following data on seven rheumatic fever patients from Banha fever hospital, Egypt. All patients are between 9-12 years old with history of Arthurian began from age 3-5 years. This disease has many symptoms and it is usually started in young age and still with the patient along his life. Table 4 introduced the seven patients characterized by 8 symptoms (Attributes) using them to decide the diagnosis for each patient (Decision Attribute). Table 5 shows the rheumatic fever information system.

Let us consider the topological space τ_a generated using binary relation defined on the attribute a. Also, using the same terminology the topological space τ_B is the topology generated using general relation defined on a subset of attributes B of all condition attributes At. The decision attribute generates the topology τ_D.

Now, we will use the following suggestion:

- The set of attributes $B \subseteq At$ is called a reduct if $\tau_B \le \tau_D$ and B is minimal, where:

  $(\tau_B \le \tau_D$ iff $\forall G \in \tau_B, \exists G' \in \tau_D$ s.t. $G \subset G'$, $G, G' \ne U)$

Table 4: Rheumatic fever data

Attribute name	Attribute values	Attribute refers to
Sex (S)	S_1	Male
	S_2	Female
Pharyngitis (P)	yes	Yes
	no	No
Arthritis (A)	a_0	No arthritis
	a_1	Began in the knee
	a_2	Began in the ankle
Carditis (C)	r_1	Affected
	r_2	Not affected
Chorea (Ch)	yes	Yes
	no	No
ESR	e_1	Normal
	e_2	High
Abdonominal pain (Ap)	p_1	Absent
	p_2	Present
Headache (H)	yes	Yes
	no	No
Diagnosis (D)	d_1	Rheumatic arthritis
	d_2	Rheumatic carditis
	d_3	Rheumatic arthritis and carditis

Table 5: Rheumatic fever information system

Attributes	S	P	A	C	Ch	ESR	Ap	H	D
Patients									
p1	S_2	yes	a_1	r_1	yes	e_1	p_1	no	d_3
p2	S_1	yes	a_1	r_1	yes	e_2	p_1	yes	d_3
p3	S_2	yes	a_2	r_1	no	e_1	p_1	no	d_3
p4	S_1	yes	a_1	r_2	no	e_1	p_1	no	d_1
p5	S_1	no	a_0	r_1	no	e_1	p_2	no	d_2
p6	S_1	yes	a_1	r_1	no	e_2	p_1	no	d_3
p7	S_1	yes	a_2	r_1	no	e_1	p_1	yes	d_3

- The attribute $a \in At$ is called the core if $|\tau_a| \succ |\tau_b|$, $\forall$ $a, b \in At$, $b \ne a$

When the classical technique of rough set theory (ROSETTA software) used to obtain reducts and core of our data we found that we have 8 reducts of Table 5 with out any intersections among them. So, we do not have any core of Table 5. The set of obtained reducts is as follows:

$$\begin{aligned}&Red(At) = \{\{S \vee C \vee Ch\}, \{S \vee A \vee C\},\\ &\{S \vee P \vee A \vee Ap\}, \{P \vee A \vee C \vee K \vee Ap\},\\ &\{C \vee ESR\}, \{P \vee A \vee ESR \vee Ap\},\\ &\{A \vee C \vee H\}, \{P \vee A \vee Ap \vee H\}\}\end{aligned}$$

Now, after getting the reducts of Table 5 using the ROSETTA software. We will convert Table 4-7 using Table 6.

Now we will apply the above contributions on Table 6 where $Ob = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$ is the set of objects, the set of condition attributes is $At = \{\alpha, \beta, \delta\}$ and the decision attribute is the diagnosis D.

Table 6: Convert table

Attribute symbol	Refers to?	Attribute values	Refers to?
α	{S, CH}	α_1	S takes s_1
		α_2	K takes k_1
		α_3	Each of {S,K}takes $\{s_2, k_2\}$
β	{P, A, ESR}	β_1	F takes f_1
		β_2	A takes a_1
		β_3	A takes a_2
		β_4	E takes e_4
		B_5	Each of {F, A, E}takes $\{f_2, a_0, e\}$
δ	{C, Ap, H}	δ_1	R takes r_1
		δ_2	P takes p_1
		δ_3	H takes h_3
		δ_4	Each of {R,P,H} takes $\{r_2, p_2, h_2\}$
D	Diagnosis	d_1	Rheumatic arthritis
		d_2	Rheumatic carditis
		d_3	Rheumatic arthritis and carditis

Table 7: Multi-valued information system

U	α	β	δ	D
x_1	$\{\alpha_2\}$	$\{\beta_1, \beta_2, \beta_3\}$	$\{\delta_1, \delta_2\}$	$\{d_3\}$
x_2	$\{\alpha_1, \alpha_2\}$	$\{\beta_1, \beta_2\}$	$\{\delta_1, \delta_2, \delta_3\}$	$\{d_3\}$
x_3	$\{\alpha_3\}$	$\{\beta_1, \beta_3, \beta_4\}$	$\{\delta_1, \delta_2\}$	$\{d_3\}$
x_4	$\{\alpha_1\}$	$\{\beta_1, \beta_2, \beta_4\}$	$\{\delta_2\}$	$\{d_1\}$
x_5	$\{\alpha_1\}$	$\{\beta_4\}$	$\{\delta_1\}$	$\{d_2\}$
x_6	$\{\alpha_1\}$	$\{\beta_1, \beta_2\}$	$\{\delta_1, \delta_2\}$	$\{d_3\}$
x_7	$\{\alpha_1\}$	$\{\beta_1, \beta_3, \beta_4\}$	$\{\delta_1, \delta_2, \delta_3\}$	$\{d_3\}$

According to the binary relation $R_B = \{(x,y), f_B(x)^c \subseteq f_B(y), B \subseteq At\}$ we can construct the following topologies:

$\tau_\alpha = \{Ob, \varphi, \{x_2\}, \{x_3\}, \{x_2, x_3\}\}$, $\tau_\beta = \{Ob, \varphi\}$, $\tau_\delta = \{Ob, \varphi\}$, $\tau_{\alpha\beta} = \{Ob, \varphi\}$, $\tau_{\alpha\delta} = \{Ob, \varphi\}$, $\tau_{\beta\delta} = \{Ob, \varphi\}$, $\tau_{\alpha\beta\delta} = \{Ob, \varphi\}$

Now we will apply the relation $R_D = \{(x,y), f\ (x) \subseteq f\ (y)\}$ to deal with the decision attribute D and we can construct the following topology:

$$\tau_D = \{Ob, \varphi, \{x_1, x_2, x_3, x_6, x_7\}, \{x_1, x_2, x_3, x_4, x_6, x_7\}, \{x_1, x_2, x_3, x_5, x_6, x_7\}\}$$

We observe that, $\tau_\alpha \le \tau_D$, this leads to from the above contributions that $\{\alpha\}$ is the reduct and it is the core.

Then we can get the degree of dependency for each attribute as follows:

For $a = \alpha$, we get $\gamma(\alpha, D) = \frac{2}{7}$, for $a = \beta$, we get $\gamma(\beta, D) = 0$ and for $a = \delta$, $\gamma(\delta, D) = 0$. But if we get the degree of dependencies for the other attributes we will find that:

$$\gamma(\{\alpha,\beta\}, D) = \gamma(\{\alpha,\delta\}, D) = \gamma(\{\beta,\delta\}, D) = \gamma(C, D) = 0$$

Thus, the set of attributes of equal highest degree of dependency is the reduct of our system. So we conclude that $\{\alpha\}$is the reduct of our data using the topological method also, $\{\alpha\}$ is the core of our system.

Now, we observe that the reduction that we got by using the GMIS is contained in the reduction that we got using the discernibility matrix and this clears for us that our method for getting the reduction is more precise than using the ROSETTA method. Because, the ROSETTA method can not apply on general binary relations.

Topological reduction of single valued datasets: By reduction we mean if we can remove some data from the data table given in our information system preserving its basic properties. To express this idea more precisely, let S = (Ob, At, $\{V_a: a \in At\}$, f_a) be an information system (numerical system). Let r be a positive real, for each object $x \in$ Ob and for $a \in$ At, $N_a(x, r)$ is the a-neighborhood of x and defined by:

$$N_a(x, r) = \{y \in Ob: |f_a(x) - f_a(y)| \le r\}$$

For any subset B of At, the B-neighborhood of x is defined by:

$$N_B(x, r) = \{y \in Ob: |f_a(x) - f_a(y)| \le r \quad \forall a \in B\}$$

For any subset X of Ob, we define two mappings $Int, Cl: P(Ob) \longrightarrow P(Ob)$ as follows:

$$Int_B(X) = \{x \in Ob: N_a(x, r) \subseteq X, \forall a \in B\},s$$
$$Cl_B(X) = \{x \in Ob: N_a(x, r) \cap X \ne \phi, \forall a \in B\}.$$

The classes $\{Int_B(X): X \subseteq Ob, B \subseteq At\}$, $\{Cl_B(X): X \subseteq Ob, B \subseteq At\}$ and $\{N_B(x,r): x \in Ob, B \subseteq At\}$ are subbases of a topological spaces denoted τ_I, τ_C and τ_N respectively.

Now let At = $\{a_1, a_2, ..., a_n\}$ and let $\tau_{I_{a_1}}$, $\tau_{I_{a_2}}$, …, $\tau_{I_{a_n}}$, $\tau_{C_{a_1}}$, $\tau_{C_{a_2}}$, …, $\tau_{C_{a_n}}$ and $\tau_{N_{a_1}}$, $\tau_{N_{a_2}}$, …, $\tau_{N_{a_n}}$ be the topologies induced by the subbases{Int_{a1} (X): $\subseteq$ Ob},{Int_{a2} (X): $\subseteq$ Ob},…, {Int_{an} (X): $\subseteq$ Ob}, {Cl_{a1} (X): $\subseteq$ Ob}, {Cl_{a2} (X): $\subseteq$ Ob},…,{Cl_{an} (X): $\subseteq$ Ob} and {N_{a1} (X): $\subseteq$ Ob},{N_{a2} (X): $\subseteq$ Ob},…, {N_{an} (X): $\subseteq$ Ob}, respectively. These topologies called interior, closure and neighborhood topologies respectively.

Table 8: The information system

Ob	a1	a2	a3	a4
x_1	1	2	9	6
x_2	3	2	6	2
x_3	3	6	3	3
x_4	4	2	2	3
x_5	6	6	5	4

One of the two attributes a_i, a_j , $i \neq j$ is called interior-dispensable in At if $\tau_{I_{a_i}} = \tau_{I_{a_j}}$,otherwise , a_i or a_j is indispensable in At. Let $\tau_{1,2}$, $\tau_{1,3}$,…, $\tau_{n-1,n}$ be the topologies induced by $\tau_{I_{a_1}} \cup \tau_{I_{a_2}}$, $\tau_{I_{a_1}} \cup \tau_{I_{a_3}}$,…, $\tau_{I_{a_{n-1}}} \cup \tau_{I_{a_n}}$ if interior topologies are used (the same terminology used if closure topologies or neighborhood topologies is replaced).

Now if $\tau_{I_{At}}$ is the topology induced by $\{Int_{At}(X): X \subseteq Ob\}$ ($\tau_{C_{At}}$ or $\tau_{N_{At}}$ can be used alternately), then when $\tau_{i,j} = \tau_{I_{At}}$ the set $\{a_i, a_j\}$ is a second order reduct of At in S. On the other hand, if $\tau_{i,j} \neq \tau_{I_{At}}$ for all i, j = 1, 2,…, n we must calculate the highest topologies $\tau_{1,2,3}$,…, $\tau_{n-2,n-1,n}$ and the subset $\{a_i, a_j, a_k\}$ is a third order reduct of At in S when $\tau_{i,j,k} = \tau_{I_{AT}}$. By the same manner, we can define a highly order reducts of At in S.

In each case, the topological core of At in S is the intersection of all reducts (intersection of all the same order reducts). This core called the interior core and denoted $Core_{Int}$ (At). By the same terminology, we can define the closure core ($Core_{Cl}$ (At))and the neighborhood core ($Core_N$ (At)).

Illustrated Example Consider the information system given by Table 8 and if we choose r = 2, then $N_{a_i}(x,r) = \{y \in Ob: |f_{a_i}(x) - f_{a_i}(y)| \leq 2\}$,hence we have the following subbases:

ς_1 = {{x_1, x_2, x_3}, {x_1, x_2, x_3, x_4}, {x_2, x_3, x_4,x_5}, {x_4, x_5}}
ς_2 = {{x_1,x_2,x_4}, {x_3,x_5}}
ς_3 = {{x_1}, {x_3, x_4, x_5}, {x_2, x_5}, {x_3, x_4}, {x_2, x_3, x_5}}
ς_4 = {{x_2, x_3, x_4, x_5}, {x_1, x_5}, Ob }

The corresponding bases are:

β_1 = {{x_1, x_2, x_3}, {x_1, x_2, x_3, x_4}, {x_2, x_3, x_4, x_5}, {x_4, x_5}}, {x_4}, {x_2, x_3}, {x_2, x_3, x_4}}
β_2 = {{x_1, x_2, x_4}, {x_3, x_5}}
β_3 = {{x_1}, {x_3, x_4, x_5}, {x_2, x_5}, {x_3, x_4}, {x_2, x_3, x_5}, {x_5}, {x_3}, {x_3, x_5}}
β_4 = {{x_2, x_3, x_4, x_5}, {x_1, x_5}, {x_5}, Ob }

The corresponding topologies are:

τ_1 = {Ob, φ, {x_1, x_2, x_3}, {x_1, x_2, x_3, x_4}, {x_2, x_3, x_4, x_5}, {x_4, x_5}, {x_4}, {x_2, x_3}, {x_2, x_3, x_4}}
τ_2 = {Ob, φ, {x_1, x_2, x_4}, {x_3, x_5}}
τ_3 = {Ob, φ, {x_1}, {x_3, x_4, x_5}, {x_2, x_5}, {x_3, x_4}, {x_2, x_3, x_5}, {x_5}, {x_3}, {x_3, x_5},{x_1, x_2, x_5},{x_1, x_3, x_4, x_5},{x_1, x_2, x_3, x_5},{x_1, x_3, x_4}, {x_1, x_5}, {x_1, x_3, x_5},{x_1, x_3},{x_2, x_3, x_4, x_5},{x_2, x_3, x_5}, {x_3, x_4, x_5}}
τ_4 = {Ob, φ, {x_2, x_3, x_4, x_5}, {x_1, x_5}, {x_5}}

If we considered the set of all attributes then $\tau_{N_{At}}$ is the discrete topology, but the second order topologies are given such that: $\tau_{1,2} \neq \tau_{N_{At}}$, $\tau_{1,3} = \tau_{N_{At}}$, $\tau_{1,4} \neq \tau_{N_{At}}$, $\tau_{2,3} = \tau_{N_{At}}$, $\tau_{2,4} \neq \tau_{N_{At}}$, $\tau_{3,4} \neq \tau_{N_{At}}$ Then $\{a_1, a_3\}$ and $\{a_2, a_3\}$ are second order reducts of At and the second order core is given by $Core_N(At) = \{a_3\}$.

CONCLUSION

There are many approaches for obtaining topologies by relations and we used some of them in data reduction. These approaches were generalizations to Pawlak approaches namely, we ignored the notion of equivalence relations. Also, these approaches open the way for other approximations if we use the general topological recent concepts such as pre-open sets or semi-open sets. Make use of this terminology to obtain the missing values in incomplete datasets will be a good future work[1,4-6,16,20]. Implementing software for large data sets reduction using advanced programming languages will be also a good future work.

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