Optimal Power Control in Rayleigh-Fading Heterogeneous Wireless Networks

Chee Wei Tan, *Senior Member, IEEE*

***Abstract*—Heterogeneous wireless networks provide varying degrees of network coverage in a multi-tier configuration in which low-powered small cells are used to enhance performance. Due to the ad-hoc deployment of small cells, optimal resource allocation is important to provision fairness and enhance energy efficiency. We first study the worst outage probability problem in Rayleigh-fading channels, and solve this nonconvex stochastic program using mathematical tools from nonlinear Perron-Frobenius theory. As a by-product, we solve an open problem of convergence for a previously proposed algorithm in the interference-limited case. We then address a total power minimization problem with outage specification constraints and its feasibility issue. We propose a dynamic algorithm that adapts the outage probability specification in a heterogeneous wireless network to minimize the total energy consumption and to simultaneously provide fairness guarantees in terms of the worst outage probability. Finally, we provide numerical evaluation on the performance of the algorithms and the effectiveness of deploying closed-access small cells in heterogeneous wireless networks to address the tradeoff between energy saving and feasibility of users satisfying their outage probability specifications.**

***Index Terms*—Energy efficiency, optimization, outage probability, Perron-Frobenius theorem, power control, small cell networks.**

I. Introduction

THE NEXT-GENERATION heterogeneous wireless networks have to support many users with diverse quality of service requirements and to improve the overall system performance by using a mix of higher-tier macrocells and lower-tier small cells to enhance performance and network coverage [2]–[7]. As small cells utilize spectrum currently employed by macro-cellular networks and due to the broadcast nature of the wireless medium, interference is a major source of performance impairment. Also, small cells are deployed in an ad hoc manner, and this can lead to undesirable interference between cells. For example, a macrocell with a small cell or a small cell with another small cell. Wireless resources thus need to be shared fairly in a collaborative and distributed manner.

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The author is with the Department of Computer Science, City University of Hong Kong, Hong Kong (e-mail: cheewtan@gmail.com).

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Wireless resource sharing under interference is however far from perfect. Maintaining a balanced operation in the macro-small cell heterogeneous wireless network is difficult, because interference rises rapidly with increasing small cell density [2]–[5]. Without appropriate resource control, the network can become unstable or operate in a highly inefficient and unfair manner [8], [9]. In addition, wireless transmission depends on other factors such as statistical channel fading that is typically modeled by a Rayleigh, a Ricean or a Nakagami distribution depending on the wireless environment [10], [11]. In this paper, we focus on optimal power control for wireless resource sharing under Rayleigh fading that is relevant to in-building coverage model and urban environments (where small cells are mostly deployed). Optimal wireless resource sharing requires that the performance gain is not outweighed by interference and unfairness [2], [3].

Power control is important to resource allocation as it determines the appropriate Signal-to-Interference-Noise Ratio (SINR) at the receiver to meet the performance requirements [10], [12]. The SINR, as a nonlinear function of powers (neither convex nor concave), couples the transmit powers of all the users together. Now, a transmission outage is declared when the received SINR falls below a given threshold. Due to statistical channel fading, the SINR fluctuates over time, thus leading to a positive outage probability that increases with multi-user interference. Consequently, an optimal power allocation that incorporates these statistics of the SINR is important to provide some form of fairness guarantees in heterogeneous wireless network performance. Our objectives in this paper are to guarantee egalitarian fairness (commonly known as max-min fairness) and to minimize the power consumption in the context of outage probabilities. In general, finding the power to optimize the outage probabilities is mathematically challenging (a nonconvex stochastic program). Furthermore, the tight coupling of powers in the SINR complicates the design of power control algorithms with good convergence performance and low complexity.

II. Related Work

Existing work on power control with Rayleigh fading mainly fall into two categories. The first category assumes that transmitters have perfect knowledge of the channel state information and can track the Rayleigh fading states over time in order to allocate power for each state realization. For example, in [13], the authors proposed tracking Rayleigh-fading fluctuation to reduce outage in a macrocell network. The second category assumes that transmitters have only channel distribution information since instantaneous state information may not be practically available. Once optimized using the distribution information, powers are fixed regardless of the fading states. In the seminal work in [14], the authors studied power control problems to minimize the worst-case outage probability and the total power consumption in an interference-limited system (i.e., without background noise) using geometric programming. There are extensions to other power control problems with outage constraints, see e.g., [15], [16], but the geometric programming approach may not be scalable in practice as it requires a centralized solver to solve the geometric programs [14], [17]. The authors in [18] studied a total power minimization problem under a more general setting (with background noise and individual power constraints), but the feasibility issue of this problem is left open.

Our work here belongs to the second category. In particular, we address the general solution and resolve several open issues in [14], [18] using mathematical tools from the *nonlinear Perron-Frobenius theory* in [19]–[21] and nonnegative matrix theory [22], [23]. This leads to simple algorithms that converge to the global optimal solution. More generally, we show that these mathematical tools are sufficiently sharp enough to analytically solve these seemingly nonconvex problems and to link seemingly unrelated problems, e.g., the worst-case outage probability problem and the max-min weighted SINR problem (also known as the certainty-equivalent margin (CEM) in [14]), and characterize them as *nonlinear* eigenvalue problems, whose optimal value and solution are associated with a *nonlinear* Perron-Frobenius eigenvalue and eigenvector, respectively. This sheds important insights to solving a useful class of nonconvex power control problems.

There are also several implications for heterogeneous wireless network design. First, the developed theories and algorithms can be used to guarantee fairness among users [6]. Second, since it is imperative that small cell users *adapt* the power allocation to be energy efficient even when the system is infeasible [2], [3], [7], we also address the feasibility issue in the total power minimization problem of [14], [18]. In particular, we propose a distributed power control algorithm that adapts the outage probability specification to minimize the total energy consumption and to simultaneously guarantee fairness in terms of the worst-case outage probability. This leads to a decentralized dynamic algorithm without the need of a centralized admission controller (a desirable fact due to the ad-hoc architecture of small cell networks).

Overall, the contributions of the paper are as follows:

1) We solve analytically for the general solution of the worst outage probability minimization problem with power constraints, and propose an iterative algorithm to compute this solution. As a by-product, we resolve an open issue of convergence for a previously proposed algorithm and its fixed-point existence in [14] for the interference-limited special case.
2) We establish a tight relationship between the worst outage probability problem and its certainty-equivalent margin counterpart, and utilize the connection to find useful bounds and insights. A by-product of this analysis resolves an open issue of convergence for a previously proposed algorithm in [6] for a max-min weighted SINR problem without fading.

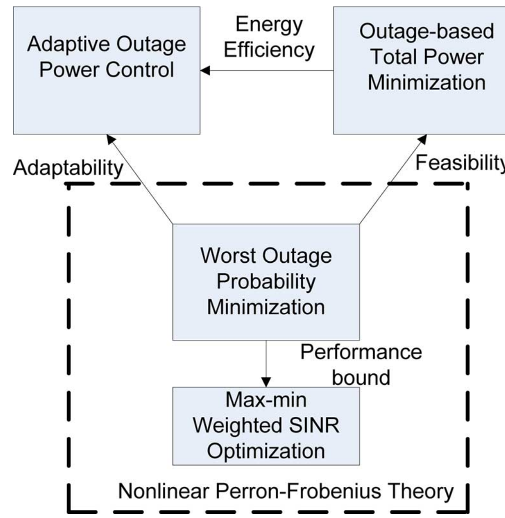

Fig. 1. An overview connecting the different optimization problems in this paper. The optimization problems contained in the dotted box are solved by the nonlinear Perron-Frobenius theory. Central to this paper is the worst outage probability problem. It can be connected to a non-fading max-min weighted SINR problem to deduce useful performance bounds. The solution to the worst outage probability problem is then leveraged to address the infeasibility issue of a total power minimization problem with outage specification constraints and to design an adaptable power control algorithm.

3) We characterize analytically the feasibility condition of the total power minimization problem with both outage specification and individual power constraints, and provide useful feasibility bounds.
4) Based on the established feasibility conditions, we propose a dynamic algorithm for the graceful handling of infeasibility in the network. In particular, the algorithm optimizes the overall energy consumption by adapting the outage probability specification based on our proposed worst outage probability algorithm. Numerical simulations demonstrate that the dynamic algorithm enables more users to meet their outage probability specification in comparison to a baseline non-adaptive algorithm when there is infeasibility.

Fig. 1 gives an overview of the connection between the various optimization problems and topics studied in this paper.

This paper is organized as follows. We introduce the system model in Section III. In Section IV, we study the problem of minimizing the worst outage probability analytically and propose a fast algorithm. In Section V, we study the feasibility condition of a total power minimization problem, and propose a dynamic power control algorithm that adapts the outage probability specification to minimize the total energy. In Section V-D, we discuss practical implementation issues related to our proposed algorithms and further extensions. In Section VI, we illustrate the numerical performance of our algorithms. We conclude the paper in Section VII.

The following notation is used. Boldface uppercase letters denote matrices, boldface lowercase letters denote column vectors, italics denote scalars, and $\mathbf{u} \geq \mathbf{v}$ denotes componentwise inequality between vectors $\mathbf{u}$ and $\mathbf{v}$. We also let $(\mathbf{Bu})_l$ denote the lth element of $\mathbf{Bu}$. Let $\mathbf{u}/\mathbf{v}$ denote the vector $[u_1/v_1, \ldots, u_L/v_L]^\top$. We write $\mathbf{B} \geq \mathbf{F}$ if $B_{ij} \geq F_{ij}$ for all i, j. The Perron-Frobenius eigenvalue of a nonnegative matrix

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$\mathbf{F}$ is denoted as $\rho(\mathbf{F})$, and the right and left eigenvector of $\mathbf{F}$ associated with $\rho(\mathbf{F})$ are denoted by $\mathbf{x}(\mathbf{F}) \geq \mathbf{0}$ and $\mathbf{y}(\mathbf{F}) \geq \mathbf{0}$ (or, simply $\mathbf{x}$ and $\mathbf{y}$, when the context is clear) respectively. Recall that the Perron-Frobenius eigenvalue of $\mathbf{F}$ is the eigenvalue with the largest absolute value. Assume that $\mathbf{F}$ is an irreducible nonnegative matrix.[1] Then $\rho(\mathbf{F})$ is simple and positive, and $\mathbf{x}(\mathbf{F}), \mathbf{y}(\mathbf{F}) > \mathbf{0}$ [24]. The super-script $(\cdot)^\top$ denotes transpose. We denote $\mathbf{e}_l$ as the lth unit coordinate vector and $\mathbf{I}$ as the identity matrix. For any vector $\tilde{\boldsymbol{\gamma}} = [\tilde{\gamma}_1, \ldots, \tilde{\gamma}_L]^\top \in \mathbb{R}^L$, let $e^{\tilde{\boldsymbol{\gamma}}} = [e^{\tilde{\gamma}_1}, \ldots, e^{\tilde{\gamma}_L}]^\top$.

III. System Model

Consider a multiuser communication system with L users (logical transmitter and receiver pairs) sharing a common frequency. This system can be modeled by a Gaussian interference channel with an additive white Gaussian noise power n_l at the lth receiver that employs singe-user decoding. The lth transmitter has a power p_l. Under Rayleigh fading, the power received from the jth transmitter at lth receiver is given by $G_{lj} R_{lj} p_j$ where G_{lj} represents the nonnegative path gain between the jth transmitter and the lth receiver (it may model the path-loss and longer timescale shadow fading). In particular, R_{lj} models Rayleigh fading and is independent and exponentially distributed with unit mean. The distribution of the received power from the jth transmitter at the l receiver is then exponential with mean value $\mathbb{E}[G_{lj} R_{lj} p_j] = G_{lj} p_j$.

Next, let us define a nonnegative matrix $\mathbf{F}$ with the entries:

$$F_{lj} = \begin{cases} 0, & \text{if } l = j \\ \frac{G_{lj}}{G_{ll}}, & \text{if } l \neq j \end{cases} \tag{1}$$

and

$$\mathbf{v} = \left(\frac{n_1}{G_{11}}, \frac{n_2}{G_{22}}, \ldots, \frac{n_L}{G_{LL}} \right)^\top. \tag{2}$$

Moreover, we assume that $\mathbf{F}$ is irreducible. This assumption can be easily satisfied by a sufficient condition that $F_{lj} > 0$ for all $l \neq j$, i.e., all the users interfere with one another.

Using the above notations, the Signal-to-Interference-Noise Ratio (SINR) at the lth receiver (e.g., a linear matched filter) can be expressed as [12], [14]:

$$\mathsf{SINR}_l(\mathbf{p}) = \frac{R_{ll} p_l}{\sum\limits_{j \neq l} F_{lj} R_{lj} p_j + v_l}. \tag{3}$$

Now, (3) is a random variable that depends on Rayleigh fading. The transmission from the lth transmitter to its receiver is successful if $\mathsf{SINR}_l(\mathbf{p}) \geq \beta_l$ (no outage), where β_l is a given threshold for reliable communication. An outage occurs at the lth receiver whenever $\mathsf{SINR}_l(\mathbf{p}) < \beta_l$. We express this outage probability of the lth user by

$$P(\mathsf{SINR}_l(\mathbf{p}) < \beta_l). \tag{4}$$

The transmit powers in wireless networks are typically constrained, e.g., by a power constraint set $\mathcal{P}$, due to resource budget consideration [10]. For example, in a cellular uplink system, we often have individual power constraints, i.e.,

$$\mathcal{P} = \{\mathbf{p} \mid \mathbf{p} \geq \mathbf{0},\, p_l \leq \bar{p} \;\; \forall l\}. \tag{5}$$

Power constraints can also be used to model interference management. For example, a basic premise in small cells is that the following two conditions are satisfied [6]:

1) A small cell user receives adequate levels of transmission quality within the small cell.
2) The small cell users do not cause unacceptable levels of interference to the macrocell users.

To satisfy the second condition above, a possibility is to explicitly impose power constraints on the small cell users. Let us illustrate using an example of a single macrocell user and multiple small cells in [6]. Assume that there is no fading between this single macrocell receiver and all the small cell users. This assumption holds only in this paragraph for illustration purpose. Let us denote this macrocell user by the index 0 and the small cell users by indices $1, \ldots, L$. The macrocell user transmits with a fixed power P_0, where $P_0 \geq \gamma_0 v_0$, i.e., the macrocell user can satisfy the SINR threshold γ_0 even when there is no interference from the small cells. In the presence of small cells' interference, the SINR of this macrocell user has to satisfy $\frac{P_0}{\sum_{j=1}^L F_{0j} p_j + v_0} \geq \gamma_0$, which can be rewritten as a single power constraint to yield

$$\mathcal{P} = \left\{ \mathbf{p} \mid \mathbf{p} \geq \mathbf{0},\, \sum_{j=1}^{L} F_{0j} p_j \leq (P_0/\gamma_0 - v_0) \right\} \tag{6}$$

that must be satisfied by the transmit powers of all the small cell users. Note that (6) is feasible when $P_0 \geq \gamma_0 v_0$. In general, a feasible power constraint of the form $\mathbf{a}^\top \mathbf{p} \leq 1$ for some positive constant vector $\mathbf{a}$ can be used to model interference management requirements.

To satisfy the first condition above on adequate levels of transmission quality, we focus on the *worst outage probability* that reflects the minimum adequate level of fairness experienced by all the users, i.e., minimizing the maximum outage probability given in (4) among L users. The transmit power vector $(p_1, \ldots, p_L)^\top$ is the optimization variable of interest.

IV. Worst Outage Probability Minimization

The problem of minimizing the worst outage probability can be formulated as

$$\begin{aligned} &\text{minimize} && \max\nolimits_{l=1,\ldots,L} \; P(\mathsf{SINR}_l(\mathbf{p}) < \beta_l) \\ &\text{subject to} && \mathbf{p} \in \mathcal{P}, \\ &\text{variables:} && \mathbf{p}. \end{aligned} \tag{7}$$

Let us denote the optimal worst outage probability, i.e., the optimal value of (7), by $O^\star$.

Assuming independent Rayleigh fading at all the signals, the outage probability of the lth user can be given analytically by [14]:[2]

$$P(\mathsf{SINR}_l(\mathbf{p}) < \beta_l) = 1 - e^{\frac{-v_l \beta_l}{p_l}} \prod_{j=1}^{L} \left(1 + \frac{\beta_l F_{lj} p_j}{p_l}\right)^{-1}. \tag{8}$$

Observe that the probability of successful transmission, i.e., the complement of (8), is simply the product of two factors, namely, $e^{-v_l \beta_l / p_l}$ and $\prod_{j=1}^{L} \left(1 + \frac{\beta_l F_{lj} p_j}{p_l}\right)^{-1}$, which are

[1]A nonnegative matrix $\mathbf{F}$ is said to be irreducible if there exists a positive integer m such that the matrix $\mathbf{F}^m$ has all entries positive.

[2]A closed form expression was first derived in [25], but we use another equivalent form derived in [14].

the probability of successful transmission in a noise-limited Rayleigh-fading channel (i.e., no interference) and an interference-limited Rayleigh-fading channel (i.e., no additive white Gaussian noise) respectively.

By using (8) and defining a deterministic function:

$$\phi_l(\mathbf{p}) = 1 - e^{\frac{-v_l\beta_l}{p_l}} \prod_{j=1}^{L} \left(1 + \frac{\beta_l F_{lj} p_j}{p_l}\right)^{-1} \quad \forall l, \tag{9}$$

then the stochastic program in (7) simplifies to a deterministic problem:

$$\begin{aligned} &\text{minimize} && \max_{l=1,\ldots,L} \ \phi_l(\mathbf{p}) \\ &\text{subject to} && \mathbf{p} \in \mathcal{P}. \end{aligned} \tag{10}$$

Note that (10) is always feasible as long as the given power budget constraints in (5) and (6) are feasible, and its optimal solution is strictly positive. Previous work in the literature, e.g., [14], only considered (10) for the interference-limited case, i.e., $\mathbf{v} = 0$ and *without any power constraint*. In this special case, [14] showed that (10) can be reformulated as a geometric program (a special class of convex optimization), and then solved numerically by the interior point method [17].

In the following, we give a reformulation of (10) as a convex optimization problem (not a geometric program but reduces to one in the interference-limited special case). By exploiting the nonlinear Perron-Frobenius theory, we propose a fast algorithm[3] (no parameter tuning whatsoever and orders of magnitude faster than standard convex optimization algorithms, e.g., interior point method) to solve (10) optimally. As a by-product, it resolves an open problem on the convergence of a previously proposed heuristic algorithm in [14]. Furthermore, we characterize analytically the optimal value and solution of (10) in terms of a Perron-Frobenius eigenvalue and eigenvector of a specially constructed nonnegative matrix respectively.

Let us introduce an auxiliary variable τ and write (10) in the epigraph form (augmenting the constraint set with additional L constraints):

$$\begin{aligned} &\text{minimize} && \tau \\ &\text{subject to} && 1 - e^{\frac{-v_l\beta_l}{p_l}} \prod_{j=1}^{L} \left(1 + \frac{\beta_l F_{lj} p_j}{p_l}\right)^{-1} \le \tau \ \ \forall l, \\ &&& \mathbf{p} \in \mathcal{P}, \\ &\text{variables:} && \mathbf{p}, \ \tau. \end{aligned} \tag{11}$$

Let us introduce a new variable

$$\alpha = -\log(1-\tau), \tag{12}$$

and then, by rewriting the L augmented constraints in (11), (11) is equivalent to the following problem:

$$\begin{aligned} &\text{minimize} && \alpha \\ &\text{subject to} && \frac{v_l\beta_l}{p_l} + \sum_{j=1}^{L} \log\left(1 + \frac{\beta_l F_{lj} p_j}{p_l}\right) \le \alpha \ \ \forall l, \\ &&& \mathbf{p} \in \mathcal{P}, \\ &\text{variables:} && \mathbf{p}, \ \alpha. \end{aligned} \tag{13}$$

[3]Some key computational considerations are the extremely fast signal processing requirement at the transceiver chip and the decentralized environment.

We call the first L constraints of (13) the *outage constraints*, and denote the optimal solution to (13) by $(\mathbf{p}^\star, \alpha^\star)$. Note that $\mathbf{p}^\star$ is also the optimal solution to (10).

Now, (13) is nonconvex in $(\mathbf{p}, \alpha)$. However, by making a logarithmic change of variable in $\mathbf{p}$, i.e., $\tilde{p}_l = \log p_l$ for all l, (13) can be converted into the following convex optimization problem in $(\tilde{\mathbf{p}}, \alpha)$:[4]

$$\begin{aligned} &\text{minimize} && \alpha \\ &\text{subject to} && v_l\beta_l e^{-\tilde{p}_l} + \sum_{j=1}^{L} \log\left(1 + \beta_l F_{lj} e^{\tilde{p}_j - \tilde{p}_l}\right) \le \alpha \ \ \forall l, \\ &&& e^{\tilde{\mathbf{p}}} \in \mathcal{P}, \\ &\text{variables:} && \tilde{\mathbf{p}}, \ \alpha. \end{aligned} \tag{14}$$

Though solving the nonconvex problem in (13) is equivalent to solving the convex problem[5] in (14), we shall use a nonlinear Perron-Frobenius theory-based approach to solve (13) optimally. Using nonnegative matrix theory, we then connect (13) to the Lagrange duality of (14) (cf. Lemma 2 later) to shed further insights to the solution.

Lemma 1: At optimality of (13), the outage constraints in (13) are tight:

$$\frac{v_l\beta_l}{p_l^\star} + \sum_{j=1}^{L} \log\left(1 + \frac{\beta_l F_{lj} p_j^\star}{p_l^\star}\right) = \alpha^\star \ \ \forall l. \tag{15}$$

Furthermore, if $\mathcal{P} = \{\mathbf{p} \mid \mathbf{a}^\top \mathbf{p} \le \bar{P}\}$, we have $\mathbf{a}^\top \mathbf{p}^\star = \bar{P}$, and if $\mathcal{P} = \{\mathbf{p} \mid p_l \le \bar{p} \ \forall l\}$, we have $p_i^\star = \bar{p}$ for some i.

Proof: First, we note that it has been pointed out in [14] that all the outage constraints are tight for the interference-limited case, i.e., $\mathbf{v} = \mathbf{0}$. We prove the first part of Lemma 1 for the general case here. Clearly, the function on the lefthand side of the lth outage constraint in (13) is monotone increasing in p_j, $j \neq l$, and monotone decreasing in p_l. Suppose the lth constraint is not tight at optimality, i.e., $v_l\beta_l/p_l^\star + \sum_{j=1}^{L} \log\left(1 + \frac{\beta_l F_{lj} p_j^\star}{p_l^\star}\right) < \alpha^\star$. Then, we choose a feasible power $p_l < p_l^\star$ such that the evaluated value of $v_l\beta_l/p_l + \sum_{j=1}^{L} \log\left(1 + \frac{\beta_l F_{lj} p_j^\star}{p_l}\right)$ is still less than $\alpha^\star$. Now, $v_j\beta_j/p_j^\star + \sum_{k\neq l} \log\left(1 + \frac{\beta_j F_{jk} p_k^\star}{p_j^\star}\right) + \log\left(1 + \frac{\beta_j F_{jl} p_l}{p_j^\star}\right)$ for all $j \neq l$. This implies that the value of α can be further decreased, i.e., $\alpha < \alpha^\star$, which contradicts the assumption. Hence, the lth constraint must be tight at optimality for all l.

We next prove the second part for $\mathcal{P} = \{\mathbf{p} \mid p_l \le \bar{p} \ \forall l\}$. Suppose $p_l^\star < \bar{p}$ at optimality for all l. Let a positive scalar $a = \min_{l=1,\ldots,L} \bar{p}/p_l^\star > 1$, and choose a feasible power $\mathbf{p} = a\mathbf{p}^\star$, which evaluates the outage constraints as

$$v_l\beta_l/p_l + \sum_{j} \log\left(1 + \frac{\beta_l F_{lj} p_j}{p_l}\right)$$

[4]Note that (13) cannot be rewritten as a geometric programming formulation, as has been done in [14]. Nevertheless, after a logarithmic change of variables, a convex form can still be obtained as shown here.

[5]The optimization problem in (14) is convex, because the objective function is linear and the constraint set is convex. In particular, the function $\sum_{j=1}^{L} \log\left(1 + \beta_l F_{lj} e^{\tilde{p}_j - \tilde{p}_l}\right)$ is convex because the log-sum-exp function is convex [17]. Thus, the constraint set in (14) that consists of exponentials and log-sum-exp functions is a convex one.

TABLE I
CHARACTERIZATION OF MAX-MIN OPTIMIZATION PROBLEMS IN THIS AND PREVIOUS WORK USING THE NONLINEAR PERRON-FROBENIUS THEORY

Concave Self-mapping $f_l(\mathbf{p})$	*Perron-Frobenius eigenvalue* $\alpha^\star$	*Right eigenvector* $\mathbf{p}^\star$	Remark
$(\mathrm{diag}(\boldsymbol{\beta})(\mathbf{Fp} + (1/\bar{p})\mathbf{v}))_l$, $i = \arg\max_{l=1,\ldots,L} \rho\left(\mathrm{diag}(\boldsymbol{\beta})(\mathbf{F} + (1/\bar{p})\mathbf{v}\mathbf{e}_l^\top)\right)$	$\rho\left(\mathrm{diag}(\boldsymbol{\beta})(\mathbf{F} + (1/\bar{p})\mathbf{v}\mathbf{e}_i^\top)\right)$	$\mathbf{x}\left(\mathrm{diag}(\boldsymbol{\beta})(\mathbf{F} + (1/\bar{p})\mathbf{v}\mathbf{e}_i^\top)\right)$	[21]
$(\mathrm{diag}(\boldsymbol{\beta})(\mathbf{Fp} + (1/\bar{P})\mathbf{v}))_l$	$\rho\left(\mathrm{diag}(\boldsymbol{\beta})(\mathbf{F} + (1/\bar{P})\mathbf{v}\mathbf{a}^\top)\right)$	$\mathbf{x}\left(\mathrm{diag}(\boldsymbol{\beta})(\mathbf{F} + (1/\bar{P})\mathbf{v}\mathbf{a}^\top)\right)$	[30]
$v_l\beta_l + \sum_{j=1}^{L} p_l \log\left(1 + \frac{\beta_l F_{lj} p_j}{p_l}\right)$, $i = \arg\max_{l=1,\ldots,L} \rho\left(\mathrm{diag}(\boldsymbol{\beta})(\mathbf{B}(\mathbf{p}^\star) + (1/\bar{p})\mathbf{v}\mathbf{e}_l^\top)\right)$	$\rho\left(\mathrm{diag}(\boldsymbol{\beta})(\mathbf{B}(\mathbf{p}^\star) + (1/\bar{p})\mathbf{v}\mathbf{e}_i^\top)\right)$	$\mathbf{x}\left(\mathrm{diag}(\boldsymbol{\beta})(\mathbf{B}(\mathbf{p}^\star) + (1/\bar{p})\mathbf{v}\mathbf{e}_i^\top)\right)$	Herein, Corollary 3
$v_l\beta_l + \sum_{j=1}^{L} p_l \log\left(1 + \frac{\beta_l F_{lj} p_j}{p_l}\right)$	$\rho\left(\mathrm{diag}(\boldsymbol{\beta})(\mathbf{B}(\mathbf{p}^\star) + (1/\bar{P})\mathbf{v}\mathbf{a}^\top)\right)$	$\mathbf{x}\left(\mathrm{diag}(\boldsymbol{\beta})(\mathbf{B}(\mathbf{p}^\star) + (1/\bar{P})\mathbf{v}\mathbf{a}^\top)\right)$	Herein
$(\mathbf{Fp} + \mathbf{v})_l$	$\rho\left(\mathbf{F} + \mathbf{v}\mathbf{a}^\top\right)$	$\mathbf{x}\left(\mathbf{F} + \mathbf{v}\mathbf{a}^\top\right)$	[6], Remark 4

$$
\begin{aligned}
&= v_l\beta_l/ap_l^\star + \sum_{j=1}^{L} \log\left(1 + \frac{\beta_l F_{lj} p_j^\star}{p_l^\star}\right) \\
&< v_l\beta_l/p_l^\star + \sum_{j=1}^{L} \log\left(1 + \frac{\beta_l F_{lj} p_j^\star}{p_l^\star}\right) = \alpha^\star
\end{aligned}
$$

for all l. This implies that α can be further decreased, i.e., $\alpha < \alpha^\star$, which contradicts the assumption. Hence, $p_i^\star = \bar{p}$ for some i. A similar proof can be given when $\mathcal{P} = \{\mathbf{p} \mid \mathbf{a}^\top\mathbf{p} \le \bar{P}\}$ and is omitted. ■

Hence, by using Lemma 1, we have the optimal worst outage probability

$$
O^\star = \phi_l(\mathbf{p}^\star) \quad \forall\, l, \tag{16}
$$

and also $O^\star = 1 - e^{-\alpha^\star}$.

Remark 1: Now, finding the fixed-point solution in (15) in Lemma 1 may seem nontrivial. However, by exploiting a connection between the nonlinear Perron-Frobenius theory in [19], [20] (that includes unveiling a hidden convexity in (15)), we can give an analytical solution to (15) and, equivalently, the optimal value and the optimal solution of (10) in Section IV-D (see Table I later). Interestingly, $\alpha^\star$ in (13) (equivalently the optimal value of (10)) and the optimal solution $\mathbf{p}^\star$ can be viewed as a *nonlinear Perron-Frobenius eigenvalue* and its *nonlinear eigenvector*.[6]

We propose the following algorithm (with geometric convergence rate and no parameter tuning whatsoever) that computes the optimal solution of (13). We let k index discrete time slots.

[6]In summary, the intuition of using the nonlinear Perron-Frobenius theory to solve nonconvex optimization problem such as that in (10) lies in examining the fixed-point equations corresponding to the set of primal constraints that are tight at optimality. In particular, the fixed-point equations exhibit special properties such as nonnegativity, monotonicity and convexity. We refer the readers to [26] for an overview of this Perron-Frobenius theory approach to solving other similar nonconvex optimization problems in wireless network.

[7]Let $\|\cdot\|$ be an arbitrary vector norm. A sequence $\{\mathbf{p}(k)\}$ is said to converge geometrically fast to a fixed point $\mathbf{p}'$ if and only if $\|\mathbf{p}(k) - \mathbf{p}'\|$ converges to zero geometrically fast, i.e., there exists constants $A \ge 0$ and $\eta \in [0, 1)$ such that $\|\mathbf{p}(k) - \mathbf{p}'\| \le A\eta^k$ for all k [27].

Algorithm 1 (Worst Outage Probability Minimization):

1) Update power $\mathbf{p}(k+1)$:

$$
p_l(k+1) = -\log\left(1 - \phi_l(\mathbf{p}(k))\right) p_l(k) \quad \forall\, l. \tag{17}
$$

2) Normalize $\mathbf{p}(k+1)$:

$$
\mathbf{p}(k+1) \leftarrow \frac{\mathbf{p}(k+1)\cdot\bar{P}}{\mathbf{a}^\top\mathbf{p}(k+1)} \ \text{ if } \ \mathcal{P} = \{\mathbf{p} \mid \mathbf{a}^\top\mathbf{p} \le \bar{P}\}. \tag{18}
$$

$$
\mathbf{p}(k+1) \leftarrow \frac{\mathbf{p}(k+1)\cdot\bar{p}}{\max_{j=1,\ldots,L} p_j(k+1)} \ \text{ if } \ \mathcal{P} = \{\mathbf{p} \mid p_l \le \bar{p}\,\forall\, l\}. \tag{19}
$$

Theorem 1: Starting from any initial point $\mathbf{p}(0)$, $\mathbf{p}(k)$ in Algorithm 1 converges geometrically fast[7] to the optimal solution of (10).

Proof: Let us write the left-hand side of the lth outage constraint in (13) as $\frac{f_l(\mathbf{p})}{p_l} \le \alpha$, where

$$
f_l(\mathbf{p}) = v_l\beta_l + \sum_{j=1}^{L} p_l \log\left(1 + \frac{\beta_l F_{lj} p_j}{p_l}\right). \tag{20}
$$

In the following, we show that

$$
\mathbf{f}(\mathbf{p}) = [f_1(\mathbf{p}),\ f_2(\mathbf{p}), \ldots,\ f_L(\mathbf{p})]^\top \tag{21}
$$

is a positive *concave self-mapping* on the standard cone $K = \mathbb{R}_+^L$. The definition of a concave self-mapping is given in [20] as follows.

Definition 1 (Concave Self-Mapping [20]): A mapping $\mathbf{f} : K \to K$ is concave if

$$
\mathbf{f}(a\mathbf{c}+(1-a)\mathbf{z}) \ge a\mathbf{f}(\mathbf{c})+(1-a)\mathbf{f}(\mathbf{z}) \ \ \forall\, \mathbf{c}, \mathbf{z} \in K, \ \ a \in [0, 1],
$$

and monotone if $\mathbf{0} \le \mathbf{c} \le \mathbf{z}$ implies $\mathbf{0} \le \mathbf{f}(\mathbf{c}) \le \mathbf{f}(\mathbf{z})$.

Let $\|\cdot\|$ be a norm on $\mathbb{R}^L$ that is monotone, i.e., $\|\mathbf{c}\| \leq \|\mathbf{z}\|$. A concave self-mapping of K is monotone on K and continuous on the interior of K with respect to $\|\cdot\|$ [20].

We first show that $\mathbf{f}(\mathbf{p}) = [f_1(\mathbf{p}),\ f_2(\mathbf{p}), \ldots,\ f_L(\mathbf{p})]^\top$ is a *cone mapping*[8] with respect to the interior of K. Taking the derivative of $f_l(\mathbf{p})$ with respect to p_l, the jth entry of the first derivative $\triangledown f_l(\mathbf{p})$ is given by:

$$(\triangledown f_l(\mathbf{p}))_j = \begin{cases} \sum_{k=1}^{L} \left(\log\left(1 + \frac{\beta_l F_{lk} p_k}{p_l}\right) - \frac{\beta_l F_{lk} p_k}{p_l + \beta_l F_{lk} p_k} \right), & \text{if } j = l \\ \frac{\beta_l F_{lj} p_l}{p_l + \beta_l F_{lj} p_j}, & \text{if } j \neq l. \end{cases} \tag{22}$$

Since $z/(1+z) \leq \log(1+z)$ for $z \geq 0$, $(\triangledown f_l(\mathbf{p}))_l \geq 0$. Thus, $(\triangledown f_l(\mathbf{p}))_j \geq 0$ for all j, i.e., $f_l(\mathbf{p})$ increases monotonically in $\mathbf{p}$. Now, we state the following result [28].

Theorem 2 (Proposition 3.2 in [28]): Let $\mathcal{K}$ be the set of cone mappings with respect to the interior of the positive standard cone. Suppose $\mathbf{f} : K \to K$ is differentiable and the following inequalities hold for the component mappings: $f_l : K \to \mathbb{R}_+$ for all l: $\sum_j p_j \mid (\triangledown f_l(\mathbf{p}))_j \mid \leq f_l(\mathbf{p})$ on K. Then $\mathbf{f} \in \mathcal{K}$.

Now, we have

$$\begin{aligned} &\sum_{j=1}^{L} p_j \mid (\triangledown f_l(\mathbf{p}))_j \mid \\ &= \sum_{j=1}^{L} p_j (\triangledown f_l(\mathbf{p}))_j \\ &= \sum_{k=1}^{L} \left(p_l \log\left(1 + \frac{\beta_l F_{lk} p_k}{p_l}\right) - \frac{\beta_l F_{lk} p_k p_l}{p_l + \beta_l F_{lk} p_k} \right) \\ &\quad + \sum_{j \neq l} \frac{\beta_l F_{lj} p_l p_j}{p_l + \beta_l F_{lj} p_j} \\ &= \sum_{k=1}^{L} p_l \log\left(1 + \frac{\beta_l F_{lk} p_k}{p_l}\right) \\ &\leq f_l(\mathbf{p}). \end{aligned} \tag{23}$$

Hence, by Theorem 2, $f_l(\mathbf{p})$ is a strictly positive and monotone cone mapping on K.

We next show that $\mathbf{f}(\mathbf{p}) = [f_1(\mathbf{p}),\ f_2(\mathbf{p}), \ldots,\ f_L(\mathbf{p})]^\top$ is a concave self-mapping. Taking the second derivative, we obtain the Hessian $\triangledown^2 f_l(\mathbf{p})$ with entries given by:

$$(\triangledown^2 f_l(\mathbf{p}))_{jk} = \begin{cases} -\frac{(\beta_l F_{lj})^2 p_j}{(p_l + \beta_l F_{lj} p_j)^2}, & \text{if } j = k, k \neq l \\ \frac{(\beta_l F_{lj})^2 p_j}{(p_l + \beta_l F_{lj} p_j)^2}, & \text{if } j \neq k, \text{ either } k \text{ or } j = l \\ -\sum_{m=1}^{L} \frac{(\beta_l F_{lm})^2 p_m^2 / p_l}{(p_l + \beta_l F_{lm} p_m)^2}, & \text{if } j = k = l \\ 0, & \text{otherwise.} \end{cases} \tag{24}$$

Now, the Hessian $\triangledown^2 f_l(\mathbf{p})$ is indeed negative definite: for all real vectors $\mathbf{z}$, we have

$$\begin{aligned} \mathbf{z}^\top \triangledown^2 f_l(\mathbf{p}) \mathbf{z} &= -\frac{1}{p_l} \sum_{k=1}^{L} \frac{(\beta_l F_{lk})^2 (p_l z_k - p_k z_l)^2}{(p_l + \beta_l F_{lk} p_k)^2} \\ &< 0. \end{aligned} \tag{25}$$

Another proof is to observe that $t \log(1 + x/t)$ is strictly concave in (x, t) for strictly positive t, as it is the perspective function of the strictly concave function $\log(1 + x)$ [17]. Hence, $f_l(\mathbf{p})$ is a sum of strictly concave perspective function, and therefore $\mathbf{f}(\mathbf{p})$ is strictly concave in $\mathbf{p}$.

We note that any concave self-mapping of K is in $\mathcal{K}$ and it is monotone and continuous [28]. Indeed, $f_l(\mathbf{p})$ is monotone increasing in $\mathbf{p}$ as has been shown earlier.

We first state the following key theorem in [20].

Theorem 3 (Krause's Theorem [20]): Let $\|\cdot\|$ be a monotone norm on $\mathbb{R}^L$. For a concave mapping $f : \mathbb{R}_+^L \to \mathbb{R}_+^L$ with $f(\mathbf{z}) > 0$ for $\mathbf{z} \geq \mathbf{0}$, the following statements hold. The conditional eigenvalue problem $f(\mathbf{z}) = \lambda \mathbf{z}$, $\lambda \in \mathbb{R}$, $\mathbf{z} \geq \mathbf{0}$, $\|\mathbf{z}\| = 1$ has a unique solution $(\lambda^*, \mathbf{z}^*)$, where $\lambda^* > 0$, $\mathbf{z}^* > \mathbf{0}$. Furthermore, $\lim_{k \to \infty} \tilde{f}(\mathbf{z}(k))$ converges geometrically fast to $\mathbf{z}^*$, where $\tilde{f}(\mathbf{z}) = f(\mathbf{z})/\|f(\mathbf{z})\|$.

The weighted sum and individual power constraints in (5) are the monotone weighted ℓ_1 norm constraint $\|\mathrm{diag}(\mathbf{a})\mathbf{p}\|_1 = \bar{P}$ and the ℓ_∞ norm constraint $\|\mathbf{p}\|_\infty = \bar{p}$ respectively. By Theorem 3, the convergence of the iteration

$$\mathbf{p}(k+1) = \frac{\mathbf{f}(\mathbf{p}(k))}{\|\mathbf{f}(\mathbf{p}(k))\|} \tag{26}$$

to the *unique* fixed point $\mathbf{p} = \mathbf{f}(\mathbf{p})/\|\mathbf{f}(\mathbf{p})\|$ is geometrically fast, regardless of the initial point. ■

Remark 2: We remark that Algorithm 1 is a purely deterministic algorithm, and, upon convergence, all the users will transmit at the optimal power solution *keeping the power fixed regardless of the Rayleigh-fading random realization over time*. This means that no channel realization, e.g., the random variable R_{lj} for all l, j, is required in the update at each iteration (what is needed is only the additive white Gaussian noise power and the channel gains G_{lj} for all l, j which is assumed to be fairly static and does not vary at the Rayleigh fading timescale). The update in (17) is obtained by applying the nonlinear Perron-Frobenius theory to (15) of Lemma 1, which is then rewritten using the notation given in (9). To compute $\phi_l(\mathbf{p}(k))$ in (17), the lth user measures separately the received interfering power $\{F_{lj} p_j(k)\}, j \neq l$. The normalization at Step IV-A.0.a can be made distributed using gossip algorithms to compute either $\max_{l=1,\ldots,L} p_l(k+1)$ or $\mathbf{a}^\top \mathbf{p}(k+1)$ [29].

In the following, we first derive useful bounds to $O^\star$ given in terms of the problem parameters of (10). Then, we solve (10) analytically in the interference-limited special case without any power constraint, and then extend the analysis to the general case with power constraints.

A. Worst Outage Probability Bounds

We now develop lower and upper bounds for the worst outage probability $O^\star$ using a certainty-equivalent margin

[8]A self-mapping $\mathbf{f}$ of a cone K is called a *cone mapping* if for every $\mathbf{c}, \mathbf{z} \in K$ and $1 \leq \lambda$ from $\lambda^{-1}\mathbf{c} \leq \mathbf{z} \leq \lambda \mathbf{c}$, it follows that $\lambda^{-1}\mathbf{f}(\mathbf{c}) \leq \mathbf{f}(\mathbf{z}) \leq \lambda \mathbf{f}(\mathbf{c})$, that is a cone mapping $\mathbf{f}$ maps any interval $[\lambda^{-1}\mathbf{z}, \lambda \mathbf{z}]$ into $[\lambda^{-1}\mathbf{f}(\mathbf{z}), \lambda \mathbf{f}(\mathbf{z})]$.

(CEM) problem[9] as proxy. In particular, the CEM problem replaces the statistical variation in the desired signal and the interference of (3) by their respective mean values, i.e., replace the random variables in (3) by the unit mean. This yields a purely deterministic function of powers that can suitably be interpreted as the SINR when there is no fading. As such, let us consider the following problem:[10]

$$\begin{aligned} \text{maximize} \quad & \min_{l=1,\ldots,L} \frac{1}{\beta_l} \frac{p_l}{\sum_{j\neq l} F_{lj} p_j + v_l} \\ \text{subject to} \quad & \mathbf{p} \in \mathcal{P},\ \mathbf{p} \geq \mathbf{0}. \end{aligned} \tag{27}$$

Now, the optimal value and solution of (27) can be obtained analytically [21], [30], and this allows us to deduce the following bounds using the CEM analytical optimal value (also in terms of the constant problem parameters of (13)).

Corollary 1: If $\mathcal{P} = \{\mathbf{p} \mid p_l \leq \bar{p}\ \forall l\}$, the worst outage probability $O^\star$ satisfies

$$\begin{aligned} \frac{\rho(\mathrm{diag}(\boldsymbol{\beta})(\mathbf{F} + (1/\bar{p})\mathbf{v}\mathbf{e}_i^\top))}{1 + \rho(\mathrm{diag}(\boldsymbol{\beta})(\mathbf{F} + (1/\bar{p})\mathbf{v}\mathbf{e}_i^\top))} &\leq O^\star = 1 - e^{-\alpha^\star} \\ &\leq 1 - e^{-\rho(\mathrm{diag}(\boldsymbol{\beta})(\mathbf{F}+(1/\bar{p})\mathbf{v}\mathbf{e}_i^\top))}, \end{aligned} \tag{28}$$

$$\text{where } i = \arg\max_{l=1,\ldots,L} \rho(\mathrm{diag}(\boldsymbol{\beta})(\mathbf{F} + (1/\bar{p})\mathbf{v}\mathbf{e}_l^\top)) \tag{29}$$

and $\alpha^\star$ is the optimal value to (13).

Proof: Using the inequalities $1 + \sum_{l=1}^L z_l \leq \prod_{l=1}^L (1 + z_l) \leq e^{\sum_{l=1}^L z_l}$ for nonnegative $\mathbf{z}$ (cf. [14]), a lower and upper bound on $\alpha^\star$ can be given by $1/(1+\mathsf{cem}) \leq \alpha^\star \leq 1 - e^{-1/\mathsf{cem}}$, where cem is the optimal value of (27) and is analytically given by $1/\rho(\mathrm{diag}(\boldsymbol{\beta})(\mathbf{F} + (1/\bar{p})\mathbf{v}\mathbf{e}_i^\top))$, where i is given by (29) [21]. The bounds on $O^\star = 1 - e^{-\alpha^\star}$ can thus be obtained, hence proving Corollary 1. ■

Remark 3: Note that the lower bound in Corollary 1 is not necessarily the tightest, but the bounds in Corollary 1 illustrate that the CEM problem, i.e., the spectral information of a concave self-mapping $\mathbf{f}(\mathbf{p}) = \mathrm{diag}(\boldsymbol{\beta})(\mathbf{F}\mathbf{p} + \mathbf{v})$ (cf. [21]) can provide useful quick bounds to the worst outage probability. Corollary 1 reduces to a result in [14] in the interference-limited case. Results similar to Corollary 1 can also be obtained for the case when $\mathcal{P} = \{\mathbf{p} \mid \mathbf{a}^\top \mathbf{p} \leq \bar{P}\}$, but is omitted here.

Remark 4: A special case of (27) is when $\boldsymbol{\beta} = \mathbf{1}$ and when $\mathcal{P}$ in (27) is given by (6), which corresponds to a non-fading max-min SINR problem for a macro-small cell network studied in [6]. In particular, (6), written in a general form as $\mathbf{a}^\top \mathbf{p} \leq 1$ for some positive $\mathbf{a}$, can be associated with a weighted ℓ-1 monotone norm ($\|\mathrm{diag}(\mathbf{a}) \cdot \|_1$), and this permits the use of Theorem 3 (cf. [21], [30] and also Table I). Its analytical closed-form solution is given by $\mathbf{p} = \mathbf{x}(\mathbf{F} + \mathbf{v}\mathbf{a}^\top)$ (up to a scaling constant). In addition, the fixed-point iteration given by

$$\mathbf{p}(k+1) = \frac{\mathbf{F}\mathbf{p}(k) + \mathbf{v}}{\mathbf{a}^\top(\mathbf{F}\mathbf{p}(k) + \mathbf{v})} \tag{30}$$

converges geometrically fast to the optimal solution of the macro-small cell problem in [6], thereby resolving an open problem in [6].

[9]The certainty-equivalent margin (CEM) problem, in its simplified case where $v_l = 0$ for all l and without power constraint, was first used in the seminal work [14] to relate to the worst outage probability problem in the interference-limited special case (i.e., assuming no additive white Gaussian noise and no power constraint). We retain the CEM terminology here to be consistent with [14].

[10]Interestingly, (27) can be viewed as a max-min SINR problem when there is no fading, and this nonconvex problem can also be solved optimally by the nonlinear Perron-Frobenius theory [21], [30]. In addition, see Remark 4 later.

[11]The geometric programming approach in [14] to this interference-limited special case is to first rewrite (11) as a standard geometric program and then solve it numerically using the interior point method [17].

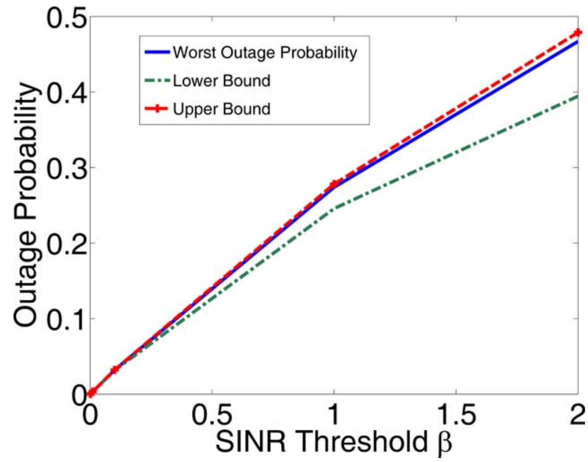

Fig. 2. The worst outage probability versus the SINR threshold β for a system with 20 small cell users, where each user has a common SINR threshold β. The corresponding lower and upper bounds in Corollary 1 are also illustrated.

1) Example 1: Fig. 2 plots the worst outage probability and the bounds for a system with 20 small cell users with individual power constraints using parameters in [2], where each user has a common SINR threshold β. We make several observations. First, the worst outage probability value and its bounds are concave in β. Second, the worst outage probability is very close to its upper bound. In fact, our numerical observations indicate that the optimal power $\mathbf{p}^\star$ is also close in value to the optimal solution of (27), i.e., $\mathbf{x}(\mathrm{diag}(\boldsymbol{\beta})(\mathbf{F} + (1/\bar{p})\mathbf{v}\mathbf{e}_i^\top))$, where i is given by (29) (cf. [21]). Further, for small β, the upper and lower bounds are very close to each other. This suggests that, in low-powered networks, the CEM solution can give sufficiently good approximation to the worst outage probability.

B. Interference-Limited Case

We now turn to solve (10) analytically for the interference-limited case, i.e., $\mathbf{v} = \mathbf{0}$.[11] In this case, (20) is in addition a *primitive positive homogeneous function of degree one*. Next, we define the nonnegative matrix $\mathbf{B}$ with the entries (that are functions of $\mathbf{p}$):

$$B_{lj} = \begin{cases} 0, & \text{if } l = j \\ \frac{p_l}{\beta_l p_j} \log\left(1 + \frac{\beta_l F_{lj} p_j}{p_l}\right), & \text{if } l \neq j. \end{cases} \tag{31}$$

Note that $\mathbf{B}(\mathbf{p})$ is irreducible whenever $\mathbf{F}$ is. Using (31), as shown in [14], we can write the optimal value $\alpha^\star$ and optimal solution $\mathbf{p}^\star$ of (13) in the interference-limited special case as

$$\alpha^\star = \rho(\mathrm{diag}(\boldsymbol{\beta})\mathbf{B}(\mathbf{p}^\star)) \tag{32}$$

$$\text{and } \mathbf{p}^\star = \mathbf{x}(\mathrm{diag}(\boldsymbol{\beta})\mathbf{B}(\mathbf{p}^\star)) \text{ (up to a scaling constant)} \tag{33}$$

respectively. Thus, $\mathbf{p}^\star$ is a fixed point of

$$\mathbf{p} = \frac{1}{\rho(\mathrm{diag}(\boldsymbol{\beta})\mathbf{B}(\mathbf{p}))} \mathrm{diag}(\boldsymbol{\beta})\mathbf{B}(\mathbf{p})\mathbf{p}. \tag{34}$$

Further, it is interesting to note the following result of $\mathbf{p}^\star$.

Corollary 2: In the interference-limited case, the optimal power of (13), $\mathbf{p}^\star$, satisfies

$$\mathbf{p}^\star = \arg\max_{\mathbf{p}>\mathbf{0}} \rho(\mathrm{diag}(\boldsymbol{\beta})\mathbf{B}(\mathbf{p})). \quad (35)$$

Proof: Using (31), we can rewrite the outage constraints in (13) in matrix form as

$$\mathrm{diag}(\boldsymbol{\beta})\mathbf{B}(\mathbf{p})\mathbf{p} \le \alpha\mathbf{p}. \quad (36)$$

Next, we need the following result from [24].

Theorem 4 (Theorem 1.6, [24] (Subinvariance Theorem)): Let $\mathbf{A}$ be an irreducible nonnegative matrix, s a positive number, and $\mathbf{z} \ge \mathbf{0}$, a vector satisfying $\mathbf{Az} \le s\mathbf{z}$. Then, (i) $\mathbf{z} > \mathbf{0}$; (ii) $s \ge \rho(\mathbf{A})$. Moreover, $s = \rho(\mathbf{A})$ if and only if $\mathbf{Az} = s\mathbf{z}$.

Applying Theorem 4 to (36) (let $\mathbf{A} = \mathrm{diag}(\boldsymbol{\beta})\mathbf{B}(\mathbf{p})$, $\mathbf{z}$ be a feasible $\mathbf{p}$ and $s = \alpha$), we have $\rho(\mathrm{diag}(\boldsymbol{\beta})\mathbf{B}(\mathbf{p})) \le \alpha$ for any feasible $\mathbf{p}$ and α. But $\alpha^\star = \rho(\mathrm{diag}(\boldsymbol{\beta})\mathbf{B}(\mathbf{p}^\star))$. Hence, $\rho(\mathrm{diag}(\boldsymbol{\beta})\mathbf{B}(\mathbf{p})) \le \rho(\mathrm{diag}(\boldsymbol{\beta})\mathbf{B}(\mathbf{p}^\star))$. This proves Corollary 2. ■

The following method was first proposed in [14] to compute the optimal solution for the interference-limited case without any power constraint:

$$\mathbf{p}(k+1) = \frac{1}{\rho(\mathrm{diag}(\boldsymbol{\beta})\mathbf{B}(\mathbf{p}(k)))}\mathrm{diag}(\boldsymbol{\beta})\mathbf{B}(\mathbf{p}(k))\mathbf{p}(k). \quad (37)$$

The authors in [14] observed that this iteration converges numerically to an acceptable accuracy with a fixed initial vector $\mathbf{p}(0) = \mathbf{x}(\mathrm{diag}(\boldsymbol{\beta})\mathbf{F})$ (optimal solution of (27) without power constraints and noise power). The issues of convergence and existence of a fixed point were however left open in [14].

Our Algorithm 1 in fact reduces to an update similar to (37) when $\mathbf{v} = \mathbf{0}$, and computes a solution that is equal to the fixed point of (34) up to a scaling constant. The scaling factor in (37) tends to $\rho(\mathrm{diag}(\boldsymbol{\beta})\mathbf{B}(\mathbf{p}^\star))$ with increasing k. From Theorem 1, this means that (37) converges from *any initial point* to the fixed point in (34) geometrically fast, thus resolving the open issues of convergence and fixed-point existence in [14].

C. Duality by Lagrange and Perron-Frobenius

We now show that the optimal value $\alpha^\star$ and optimal solution $\mathbf{p}^\star$ of (13) can be derived analytically from the spectral information of a specially constructed rank-one perturbation of $\mathrm{diag}(\boldsymbol{\beta})\mathbf{B}$, where $\mathbf{B}$ is given in (31). The following result is obtained based on the nonlinear Perron-Frobenius theory in [19] and the Friedland-Karlin inequality[12] in [22], [23].

Lemma 2: The optimal solution $(\mathbf{p}^\star, \alpha^\star)$ of (13) satisfies

$$\begin{aligned}\log\alpha^\star &= \log\rho(\mathrm{diag}(\boldsymbol{\beta})(\mathbf{B}(\mathbf{p}^\star) + \mathbf{v}\mathbf{c}_*^\top))\\ &= \max_{\|\mathbf{c}\|_D=1} \log\rho(\mathrm{diag}(\boldsymbol{\beta})(\mathbf{B}(\mathbf{p}^\star) + \mathbf{v}\mathbf{c}^\top))\\ &= \max_{\boldsymbol{\lambda}\ge\mathbf{0},\mathbf{1}^\top\boldsymbol{\lambda}=1}\ \min_{\mathbf{p}\in\mathcal{P}} \sum_{l=1}^L \lambda_l \log\frac{\beta_l(\mathbf{B}(\mathbf{p})\mathbf{p}+\mathbf{v})_l}{p_l} \quad (38)\\ &= \min_{\mathbf{p}\in\mathcal{P}}\ \max_{\boldsymbol{\lambda}\ge\mathbf{0},\mathbf{1}^\top\boldsymbol{\lambda}=1} \sum_{l=1}^L \lambda_l \log\frac{\beta_l(\mathbf{B}(\mathbf{p})\mathbf{p}+\mathbf{v})_l}{p_l}, \quad (39)\end{aligned}$$

where the optimal $\mathbf{p}$ in (38) and (39) are both given by $\mathbf{x}(\mathrm{diag}(\boldsymbol{\beta})(\mathbf{B}(\mathbf{p}^\star)+\mathbf{v}\mathbf{c}_*^\top))$ (which is equal to $\mathbf{p}^\star$ up to a scaling constant), and the optimal $\boldsymbol{\lambda}$ in (38) and (39) are both given by the Hadamard product of $\mathbf{x}(\mathrm{diag}(\boldsymbol{\beta})(\mathbf{B}(\mathbf{p}^\star) + \mathbf{b}\mathbf{c}_*^\top))$ and $\mathbf{y}(\mathrm{diag}(\boldsymbol{\beta})(\mathbf{B}(\mathbf{p}^\star) + \mathbf{b}\mathbf{c}_*^\top))$.

Furthermore, $\mathbf{p} = \mathbf{x}(\mathrm{diag}(\boldsymbol{\beta})(\mathbf{B}(\mathbf{p}^\star) + \mathbf{v}\mathbf{c}_*^\top))$ is the dual of $\mathbf{c}_*$ with respect to $\|\cdot\|_D$.[13]

Proof: The proof outline of Lemma 2 is to first consider the Lagrange duality of (14) and then apply the nonnegative matrix theory result in [19], [22]. We first state the following lemma that extends a result in [30]:

Lemma 3: Let $\mathbf{A}$ be an irreducible nonnegative matrix, $\mathbf{b}$ a nonnegative vector and $\|\cdot\|$ a norm on $\mathbb{R}^L$ with a corresponding dual norm $\|\cdot\|_D$. Then,

$$\begin{aligned}&\log\rho(\mathbf{A} + \mathbf{b}\mathbf{c}_*^\top)\\ &\quad= \max_{\|\mathbf{c}\|_D=1} \log\rho(\mathbf{A} + \mathbf{b}\mathbf{c}^\top)\\ &\quad= \max_{\boldsymbol{\lambda}\ge\mathbf{0},\mathbf{1}^\top\boldsymbol{\lambda}=1}\ \min_{\|\mathbf{p}\|=1} \sum_{l=1}^L \lambda_l \log\frac{(\mathbf{Ap}+\mathbf{b})_l}{p_l} \quad (40)\\ &\quad= \min_{\|\mathbf{p}\|=1}\ \max_{\boldsymbol{\lambda}\ge\mathbf{0},\mathbf{1}^\top\boldsymbol{\lambda}=1} \sum_{l=1}^L \lambda_l \log\frac{(\mathbf{Ap}+\mathbf{b})_l}{p_l}, \quad (41)\end{aligned}$$

where the optimal $\mathbf{p}$ in (40) and (41) are both given by $\mathbf{x}(\mathbf{A} + \mathbf{b}\mathbf{c}_*^\top)$, and the optimal $\boldsymbol{\lambda}$ in (40) and (41) are both given by $\mathbf{x}(\mathbf{A} + \mathbf{b}\mathbf{c}_*^\top) \circ \mathbf{y}(\mathbf{A} + \mathbf{b}\mathbf{c}_*^\top)$.

Furthermore, $\mathbf{p} = \mathbf{x}(\mathbf{A} + \mathbf{b}\mathbf{c}_*^\top)$ is the dual of $\mathbf{c}_*$ with respect to $\|\cdot\|_D$.

First, we express the outage constraints in (14) using the matrix $\mathbf{B}$ and rewrite (14) as the following equivalent problem (let $\tilde{\alpha} = \log\alpha$):

$$\begin{aligned}&\text{minimize} \quad \tilde{\alpha}\\ &\text{subject to} \quad \log\left(\frac{\beta_l(v_l + \mathbf{B}(e^{\tilde{\mathbf{p}}})e^{\tilde{\mathbf{p}}})_l}{e^{\tilde{p}_l}}\right) \le \tilde{\alpha}\ \ \forall\, l,\ e^{\tilde{\mathbf{p}}} \in \mathcal{P},\\ &\text{variables :} \quad \tilde{\mathbf{p}},\ \tilde{\alpha}. \quad (42)\end{aligned}$$

Next, by augmenting only the outage constraints, the partial Lagrangian function of (42) is given by

$$\begin{aligned}L(\tilde{\alpha}, \tilde{\mathbf{p}}, \boldsymbol{\lambda}) &= \left(1 - \sum_{l=1}^L \lambda_l\right)\tilde{\alpha}\\ &\quad + \sum_{l=1}^L \lambda_l \log\left(\frac{\beta_l(v_l + \mathbf{B}(e^{\tilde{\mathbf{p}}})e^{\tilde{\mathbf{p}}})_l}{e^{\tilde{p}_l}}\right). \quad (43)\end{aligned}$$

[12]The Friedland-Karlin inequalities are important fundamental results in nonnegative matrix theory related to the linear Perron-Frobenius theorem [22], [31]. A Friedland-Karlin inequality result (cf. Theorem 3.1 in [22]) states that for any irreducible nonnegative matrix $\mathbf{A}$, $\prod_l ((\mathbf{Az})_l/z_l)^{x_l y_l} \ge \rho(\mathbf{A})$ for all strictly positive $\mathbf{z}$, where $\mathbf{x}$ and $\mathbf{y}$ are the Perron-Frobenius right and left eigenvectors of $\mathbf{A}$ respectively. Equality holds if and only if $\mathbf{z} = a\mathbf{x}$ for some positive a. Also, see [23] for its generalization and applications to inverse problems in nonnegative matrix theory.

[13]A pair $(\mathbf{u}, \mathbf{v})$ of vectors of $\mathbb{R}^L$ is said to be a dual pair with respect to $\|\cdot\|$ if $\|\mathbf{v}\|_D\|\mathbf{u}\| = \mathbf{v}^\top\mathbf{u} = 1$.

Now, the Lagrange dual function of (42) is finite only if $\sum_{l=1}^{L}\lambda_l = 1$ for all feasible $\boldsymbol{\lambda}$. Hence, the Lagrange dual function is given by

$$\min_{\tilde{\alpha}, e^{\tilde{\mathbf{p}}}\in\mathcal{P}} L(\tilde{\alpha},\tilde{\mathbf{p}},\boldsymbol{\lambda}) = \sum_{l=1}^{L}\lambda_l \log\left(\frac{\beta_l(v_l+\mathbf{B}(e^{\tilde{\mathbf{p}}^\star})e^{\tilde{\mathbf{p}}^\star})_l}{e^{\tilde{p}_l^\star}}\right), \tag{44}$$

where $\tilde{\mathbf{p}}^\star$ is the optimal solution to (42).

Now, we are ready to apply Lemma 3 to (44). In particular, we can compute $\mathbf{c}_*$ explicitly depending on the choice of $\mathcal{P}$. For the case when $\mathcal{P} = \{\mathbf{p}|\mathbf{a}^\top\mathbf{p} \le \bar{P}\}$, let $\mathbf{A} = \mathrm{diag}(\boldsymbol{\beta})\mathbf{B}(\mathbf{p}^\star)$, $\mathbf{b} = \mathrm{diag}(\boldsymbol{\beta})\mathbf{v}$ and $\mathbf{c}_* = (1/\bar{P})\mathbf{a}$ in Lemma 3. For the case when $\mathcal{P} = \{\mathbf{p}|p_l \le \bar{p}\,\forall l\}$, let $\mathbf{A} = \mathrm{diag}(\boldsymbol{\beta})\mathbf{B}(\mathbf{p}^\star)$, $\mathbf{b} = \mathrm{diag}(\boldsymbol{\beta})\mathbf{v}$ and $\mathbf{c}_* = (1/\bar{p})\mathbf{e}_i$, where $i = \arg\max_{l=1,\ldots,L}\rho(\mathrm{diag}(\boldsymbol{\beta})(\mathbf{B}(\mathbf{p}^\star) + \mathbf{v}\mathbf{e}_l^\top))$ in Lemma 3. ■

Note that the optimal dual variable in (14) is equal to the optimal $\boldsymbol{\lambda}$ in (38) and (39). Using Lemma 2, we can now give analytically the optimal value and solution of (13) when $\mathcal{P} = \{\mathbf{p} \mid p_l \le \bar{p}\ \forall l\}$.

Corollary 3: The optimal value and solution of (13) are given respectively by

$$\alpha^\star = \rho\left(\mathrm{diag}(\boldsymbol{\beta})(\mathbf{B}(\mathbf{p}^\star) + (1/\bar{p})\mathbf{v}\mathbf{e}_i^\top)\right) \tag{45}$$

and

$$\mathbf{p}^\star = \mathbf{x}\left(\mathrm{diag}(\boldsymbol{\beta})(\mathbf{B}(\mathbf{p}^\star) + (1/\bar{p})\mathbf{v}\mathbf{e}_i^\top)\right), \tag{46}$$

$$\text{where } i = \arg\max_{l=1,\ldots,L}\rho\left(\mathrm{diag}(\boldsymbol{\beta})(\mathbf{B}(\mathbf{p}^\star) + (1/\bar{p})\mathbf{v}\mathbf{e}_l^\top)\right). \tag{47}$$

Furthermore, $p_i^\star = \bar{p}$ for the i (not necessarily unique) in (47).

Remark 5: In general, the optimal index i in (47) differs from (29). Our simulations show that both are empirically the same when the CEM solution is close to $\mathbf{p}^\star$, especially so in the low-powered regime (cf. Fig. 2). Unlike (47), (29) can be computed *a priori* from the problem parameters.

Table I summarizes the connection between the Perron-Frobenius spectrum (of the respectively different concave self-mappings) and the optimal value and solution of the optimization problems under the CEM model and the Rayleigh-fading model subject to the different power constraints (individual and total power constraints). The second and third row of Table I tabulate the CEM case for individual and total power constraints respectively. The fourth and fifth row of Table I tabulate the worst outage probability case for individual and total power constraints respectively. The sixth row of Table I tabulates the results for the max-min SINR problem with a weighted power constraint for small cell users in a single macrocell.

V. Total Power Minimization and Adaptive Outage Power Control

In this section, we first study the total power minimization problem subject to both outage specification and individual power constraints, and address its feasibility conditions using the results in the previous sections. An adaptive algorithm is then proposed to minimize the total power consumption and simultaneously guarantee a min-max fairness in terms of worst outage probability.

The problem of minimizing the total power subject to given outage specification under Rayleigh fading and individual power constraints can be formulated as

$$\begin{aligned} &\text{minimize} \quad \mathbf{1}^\top\mathbf{p} \\ &\text{subject to} \quad 1 - e^{\frac{-v_l\beta_l}{p_l}}\prod_{j=1}^{L}\left(1+\frac{\beta_l F_{lj}p_j}{p_l}\right)^{-1} \le \bar{O}_l \quad \forall\ l, \\ &\qquad \mathbf{p}\in\{\mathbf{p}\mid p_l\le\bar{p}\ \ \forall\, l\}, \\ &\text{variables:} \quad \mathbf{p}, \end{aligned} \tag{48}$$

where $0 < \bar{O}_l < 1$ is a given outage probability bound for the lth user. Depending on the given parameters $\bar{O}_l$ for all l, (48) may or may not be feasible. This is unlike the worst outage probability problem in (7), which is always feasible.

Next, using (20), (48) can be rewritten as

$$\begin{aligned} &\text{minimize} \quad \mathbf{1}^\top\mathbf{p} \\ &\text{subject to} \quad \frac{f_l(\mathbf{p})}{p_l} \le \alpha_l \quad \forall\ l, \\ &\qquad \mathbf{p}\in\{\mathbf{p}\mid p_l\le\bar{p}\ \ \forall\, l\}, \end{aligned} \tag{49}$$

where we have

$$\alpha_l = -\log(1-\bar{O}_l) \quad \forall\ l. \tag{50}$$

If (48) is feasible, it can be shown that all the L outage constraints in (48) are tight at optimality [18]. We deduce the feasibility condition of (48) from (49) in the following result.

Lemma 4: There is a unique optimal $\mathbf{p}$ in (48) if and only if

$$\max_{\mathbf{p}\in\{\mathbf{p}|p_l\le\bar{p}\ \forall\, l\}} \rho(\mathrm{diag}(\boldsymbol{\beta}/\boldsymbol{\alpha})\mathbf{B}(\mathbf{p})) < 1. \tag{51}$$

Furthermore, $\rho(\mathrm{diag}(\boldsymbol{\beta}/\boldsymbol{\alpha})\mathbf{B}(\mathbf{p})) < 1$ if $\alpha^\star < \min_{l=1,\ldots,L}\alpha_l$, i.e., $1-e^{-\alpha^\star} < \bar{O}_l$ for all l.

Proof: To show the feasibility condition, we examine the condition under which there is a fixed point to

$$f_l(\mathbf{p}) = \beta_l((\mathbf{B}(\mathbf{p})\mathbf{p})_l + v_l) = \alpha_l p_l \ \text{ for all } \ l, \tag{52}$$

which can be rewritten in matrix form as

$$(\mathbf{I} - \mathrm{diag}(\boldsymbol{\beta}/\boldsymbol{\alpha})\mathbf{B}(\mathbf{p}))\mathbf{p} = \mathrm{diag}(\boldsymbol{\beta}/\boldsymbol{\alpha})\mathbf{v}. \tag{53}$$

We first state the following result from [24].

Theorem 5: A necessary and sufficient condition for a solution $\mathbf{z}\ge\mathbf{0}, \mathbf{z}\ne\mathbf{0}$ to the equations $(\mathbf{I}-\mathbf{A})\mathbf{z} = \mathbf{c}$ to exist for any $\mathbf{c}\ge\mathbf{0}, \mathbf{c}\ne\mathbf{0}$ is that $\rho(\mathbf{A}) < 1$. In this case there is only one solution $\mathbf{z}$, which is strictly positive and given by $\mathbf{z} = (\mathbf{I}-\mathbf{A})^{-1}\mathbf{c}$.

Since $\mathrm{diag}(\boldsymbol{\beta}/\boldsymbol{\alpha})\mathbf{B}$ is an irreducible nonnegative matrix, it follows from Theorem 5 (letting $\mathbf{A} = \mathrm{diag}(\boldsymbol{\beta}/\boldsymbol{\alpha})\mathbf{B}$, $\mathbf{c} = \mathrm{diag}(\boldsymbol{\beta}/\boldsymbol{\alpha})\mathbf{v}$) that $\mathbf{p}$ in (53) is unique and strictly positive if and only if $\rho(\mathrm{diag}(\boldsymbol{\beta}/\boldsymbol{\alpha})\mathbf{B}(\mathbf{p})) < 1$ for all $\mathbf{p}\in\{\mathbf{p}\mid p_l\le\bar{p}\,\forall l\}$. This is equivalent to stating

$$\max_{\mathbf{p}\in\{\mathbf{p}|p_l\le\bar{p}\ \forall\, l\}} \rho(\mathrm{diag}(\boldsymbol{\beta}/\boldsymbol{\alpha})\mathbf{B}(\mathbf{p})) < 1, \tag{54}$$

thus proving the first part.

To show the second part, we note that

$$\begin{aligned} \rho(\mathrm{diag}(\boldsymbol{\beta}/\boldsymbol{\alpha})\mathbf{B}(\mathbf{p})) &\le (1/\min_{l=1,\ldots,L}\alpha_l)\rho(\mathrm{diag}(\boldsymbol{\beta})\mathbf{B}(\mathbf{p})) \\ &\le (1/\min_{l=1,\ldots,L}\alpha_l)\alpha^\star. \end{aligned}$$

Thus, a sufficient condition that $(1/\min_{l=1,\dots,L}\alpha_l)\alpha^\star < 1$ implies that $\rho(\mathrm{diag}(\boldsymbol{\beta}/\boldsymbol{\alpha})\mathbf{B}(\mathbf{p})) < 1$. ■

A. Feasibility Bounds

From (51) in Lemma 4, we see that verifying the feasibility of (48) requires solving a Perron-Frobenius eigenvalue maximization problem (a nonconvex problem). We next provide useful (tight) bounds on this nonconvex optimal value $\rho(\mathrm{diag}(\boldsymbol{\beta}/\boldsymbol{\alpha})\mathbf{B}(\mathbf{p}))$ in (51) that exploit the optimal value and solution of the worst outage probability problem in Section IV.

Theorem 6: Let $\alpha^\star$ and $\mathbf{p}^\star$ be given in (32) and (33), respectively. Then, we have

$$\begin{aligned}&\prod_{l=1}^{L}(\alpha_l)^{-x_l(\mathbf{B}(\mathbf{p}^\star))y_l(\mathbf{B}(\mathbf{p}^\star))}\alpha^\star\\ &\quad\le \max_{\mathbf{p}\in\{\mathbf{p}|p_l\le\bar{p}\ \forall l\}}\rho(\mathrm{diag}(\boldsymbol{\beta}/\boldsymbol{\alpha})\mathbf{B}(\mathbf{p}))\\ &\quad\le \max_{l=1,\dots,L}(\alpha^\star/\alpha_l). \end{aligned} \tag{55}$$

Further, equality is achieved in the lower and upper bounds when α_l's are equal for all l.

Proof: By applying the Friedland-Karlin inequalities in [22], [23], the function $\rho(\mathrm{diag}(\boldsymbol{\beta}/\boldsymbol{\alpha})\mathbf{B}(\mathbf{p}))$ can be bounded by

$$\begin{aligned}&\prod_{l=1}^{L}(\alpha_l)^{-x_l(\mathbf{B}(\mathbf{p}))y_l(\mathbf{B}(\mathbf{p}))}\rho(\mathrm{diag}(\boldsymbol{\beta})\mathbf{B}(\mathbf{p}))\\ &\quad\le\rho(\mathrm{diag}(\boldsymbol{\beta}/\boldsymbol{\alpha})\mathbf{B}(\mathbf{p}))\\ &\quad\le \max_{l=1,\dots,L}(1/\alpha_l)\rho(\mathrm{diag}(\boldsymbol{\beta})\mathbf{B}(\mathbf{p})) \end{aligned} \tag{56}$$

for any feasible $\mathbf{p}\in\{\mathbf{p}\mid p_l\le\bar{p}\ \forall l\}$. Now, from the lower bound in (56), we have

$$\begin{aligned}&\prod_{l=1}^{L}(\alpha_l)^{-x_l(\mathbf{B}(\mathbf{p}^\star))y_l(\mathbf{B}(\mathbf{p}^\star))}\rho(\mathrm{diag}(\boldsymbol{\beta})\mathbf{B}(\mathbf{p}^\star))\\ &\le \max_{\mathbf{p}\in\{\mathbf{p}|p_l\le\bar{p}\ \forall l\}}\left\{\prod_{l=1}^{L}(\alpha_l)^{-x_l(\mathbf{B}(\mathbf{p}))y_l(\mathbf{B}(\mathbf{p}))}\rho(\mathrm{diag}(\boldsymbol{\beta})\mathbf{B}(\mathbf{p}))\right\}\\ &\le \max_{\mathbf{p}\in\{\mathbf{p}|p_l\le\bar{p}\ \forall l\}}\rho(\mathrm{diag}(\boldsymbol{\beta}/\boldsymbol{\alpha})\mathbf{B}(\mathbf{p})), \end{aligned} \tag{57}$$

where $\mathbf{p}^\star\in\{\mathbf{p}\mid p_l\le\bar{p}\ \forall l\}$ is given by (33). Using $\alpha^\star$ given in (32), we thus establish (55). The condition under which equalities in (56) are achieved follows from the application of the Friedland-Karlin inequalities in [22], [23]. ■

These bounds can be easily computed in the two-user case.[14]

1) Example 2: In the two-user case, we have

$$\begin{aligned}\frac{\alpha^\star}{\sqrt{\alpha_1\alpha_2}} &\le \max_{\mathbf{p}\in\{\mathbf{p}|p_l\le\bar{p}\ \forall l\}}\rho(\mathrm{diag}(\boldsymbol{\beta}/\boldsymbol{\alpha})\mathbf{B}(\mathbf{p}))\\ &\le \max\left\{\frac{\alpha^\star}{\alpha_1},\ \frac{\alpha^\star}{\alpha_2}\right\}. \end{aligned} \tag{58}$$

Combining Theorem 6 with Corollary 1, simplified bounds in terms of the CEM solution (and more directly in terms of the problem parameters) can be obtained:

$$\begin{aligned}&\prod_{l=1}^{L}(\alpha_l)^{-x_l(\mathbf{B}(\mathbf{p}^\star))y_l(\mathbf{B}(\mathbf{p}^\star))}\\ &\quad\times\log(1+\rho(\mathrm{diag}(\boldsymbol{\beta})(\mathbf{F}+(1/\bar{p})\mathbf{v}\mathbf{e}_i^\top)))\\ &\le \max_{\mathbf{p}\in\{\mathbf{p}|p_l\le\bar{p}\ \forall l\}}\rho(\mathrm{diag}(\boldsymbol{\beta}/\boldsymbol{\alpha})\mathbf{B}(\mathbf{p}))\\ &\le \max_{l=1,\dots,L}(1/\alpha_l)\rho(\mathrm{diag}(\boldsymbol{\beta})(\mathbf{F}+(1/\bar{p})\mathbf{v}\mathbf{e}_i^\top)), \end{aligned} \tag{59}$$

where i is given by (29) and we have made use of the fact that

$$\begin{aligned}&\max_{l=1,\dots,L}\rho(\mathrm{diag}(\boldsymbol{\beta})(\mathbf{B}(\mathbf{p}^\star)+(1/\bar{p})\mathbf{v}\mathbf{e}_l^\top))\\ &\qquad\le \max_{l=1,\dots,L}\rho(\mathrm{diag}(\boldsymbol{\beta})(\mathbf{F}+(1/\bar{p})\mathbf{v}\mathbf{e}_l^\top))\end{aligned}$$

in the last inequality.

We now state the following algorithm proposed in [18].

Algorithm 2 (Total Power Minimization):

$$p_l(k+1)=\min\{-\log(1-\phi_l(\mathbf{p}(k)))\,p_l(k)/\alpha_l,\ \bar{p}\}\ \forall\, l. \tag{60}$$

We now establish the necessary and sufficient condition under which Algorithm 2 converges. This condition is also necessary and sufficient for (48) to be feasible.

Corollary 4: Starting from any initial point $\mathbf{p}(0)$, $\mathbf{p}(k)$ in Algorithm 2 converges geometrically fast to the optimal solution of (48) if and only if $\rho(\mathrm{diag}(\boldsymbol{\beta}/\boldsymbol{\alpha})\mathbf{B}(\mathbf{p}))<1$ for all $\mathbf{p}\in\{\mathbf{p}\mid p_l\le\bar{p}\ \forall l\}$.

Proof: The necessary and sufficient condition under which (48) is feasible is given in Lemma 4. If (48) is feasible, the convergence proof for Algorithm 2 can be found in [18]. Hence, Corollary 4 is proved. ■

B. Adaptive Outage-Based Power Control

Heterogeneous wireless networks have to be adaptive in order to be spectral and energy efficient. When the system is infeasible, resource allocation has to be adapted to resolve the infeasibility issue. By leveraging the results in the previous sections, we propose the following Adaptive Outage-based Power Control (AOPC) algorithm for total energy minimization.

Algorithm 3 (Adaptive Outage-based Power Control):

---

1) Update the auxiliary variable $\mathbf{z}(k+1)$:

$$z_l(k+1)=-\log(1-\phi_l(\mathbf{z}(k)))\,z_l(k)\quad\forall\, l. \tag{61}$$

2) Normalize $\mathbf{z}(k+1)$:

$$\mathbf{z}(k+1)\leftarrow\frac{\mathbf{z}(k+1)\cdot\bar{p}}{\max_{j=1,\dots,L}z_j(k+1)}. \tag{62}$$

3) Update the transmit power $\mathbf{p}(k+1)$:

$$p_l(k+1)=\min\left\{\frac{-\log(1-\phi_l(\mathbf{p}(k)))\,p_l(k)}{\max\{\alpha_l,-\log(1-\phi_l(\mathbf{z}(k)))\}},\ \bar{p}\right\}\forall\, l. \tag{63}$$

Corollary 5: Starting from any initial point $\mathbf{z}(0)$ and $\mathbf{p}(0)$, $\mathbf{p}(k)$ in Algorithm 3 converges geometrically fast to the op-

[14]The Schur product of the Perron-Frobenius right and left eigenvectors of a zero-diagonal 2×2 positive matrix equals $[1/2, 1/2]$, simplifying the computation in (55).

timal solution of (49), where the righthand side of the outage constraints in (49) are replaced by

$$\max\left\{\alpha_l,\ \rho(\mathrm{diag}(\boldsymbol{\beta})(\mathbf{B}(\mathbf{p}^\star)+(1/\bar{p})\mathbf{v}\mathbf{e}_i^\top))\right\},$$

where $\mathbf{p}^\star$ and i are given by (46) and (47) respectively, for all l.

Proof: Theorem 1 proves the convergence of $\mathbf{z}(k)$ in Step 1 and 2 of Algorithm 3, and also $\lim_{k\to\infty} -\log(1-\phi_l(\mathbf{z}(k)))\to \rho(\mathrm{diag}(\boldsymbol{\beta})(\mathbf{B}(\mathbf{p}^\star)+(1/\bar{p})\mathbf{v}\mathbf{e}_i^\top))$. From Corollary 4, $\mathbf{p}(k)$ converges to a point $\mathbf{p}'$ that satisfies

$$\begin{aligned}-\log(1-\phi_l(\mathbf{p}')) &= \boldsymbol{\alpha}'\\ &= \max\{\boldsymbol{\alpha},\ \rho(\mathrm{diag}(\boldsymbol{\beta})(\mathbf{B}(\mathbf{p}^\star)+(1/\bar{p})\mathbf{v}\mathbf{e}_i^\top))\},\end{aligned}$$

which always satisfies $\rho(\mathrm{diag}(\boldsymbol{\beta}/\boldsymbol{\alpha}')\mathbf{B}(\mathbf{p})) < 1$. This proves Corollary 5. ■

Remark 6: If $\alpha_l < \rho(\mathrm{diag}(\boldsymbol{\beta})(\mathbf{B}(\mathbf{p}^\star)+(1/\bar{p})\mathbf{v}\mathbf{e}_i^\top))\}$ for all l, then $\lim_{k\to\infty}\mathbf{z}(k) = \lim_{k\to\infty}\mathbf{p}(k) = \mathbf{p}^\star$ in Algorithm 3.

Remark 7: We remark that, whenever the total power minimization problem in (48) is infeasible, the output from Algorithm 2 or AOPC only satisfies the power budget constraints, e.g., in (5) and (6), but they do not satisfy the outage probability specification constraint set in (48). We will numerically evaluate the performance of Algorithm 2 and AOPC in terms of the total power consumption obtained by the algorithm output and the number of users that do not satisfy the outage probability specification constraint in Section VI.

C. Practical Implementation Issues and Extensions

In this section, we discuss practical implementation issues related to Algorithm 1 and Algorithm AOPC. Algorithm 1 solves the worst outage probability problem in the general setting. From an algorithm complexity viewpoint, this has improved the state-of-art in [14], in which the algorithm design methodology is based on the interior point method to solve convex optimization problems (particularly geometric programs in convex form). From a practical perspective, Algorithm 1 is more attractive than interior point methods in terms of computational complexity and ease of decentralized implementation. This means that Algorithm 1 can handle large-scale problem setting, which is a key feature of next-generation heterogeneous wireless networks. It is interesting to study how to obtain a fully-distributed Algorithm 1 and Algorithm AOPC, i.e., without requiring message passing and normalization of the iterates (in Step 2 of Algorithm 1 and Algorithm AOPC), as future work.

VI. Numerical Evaluation

In this section, we evaluate the performance of Algorithms 1, 2 and AOPC. Fig. 3 illustrates the basic macro-small cell interference network model used in [2] for our simulations. A small cell user is a link between the home access terminal and access point (box of Fig. 3), and interference comes from the macrocell base station/access terminals and other small cell users. We consider a single macrocell base station with 50 access terminals that consist of both macrocell and small cell users (closed-access mode with one user in each small cell). Each user communicates with its respective base station over independent Rayleigh

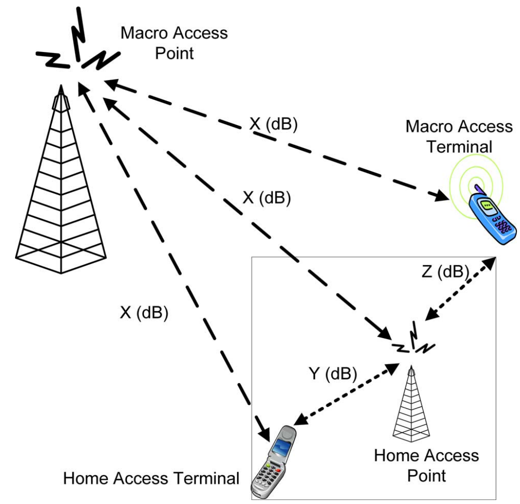

Fig. 3. A macro-small cell model. The parameter X dB, Y dB and Z dB denote path gain between MAP and HAP, between HAP and HAT and between HAP and MAT respectively (cf. [2, Table 2] for values of X, Y, Z).

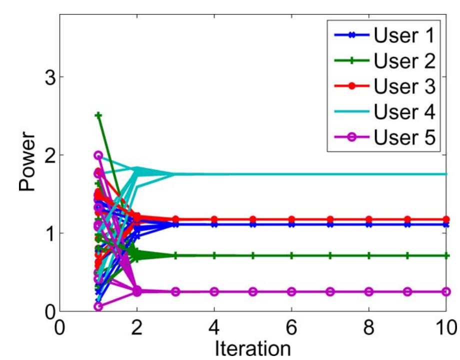

Fig. 4. Experiment 1. Convergence of power (in Watts and illustrated above in logarithmic scale) from different initial points for five small cell users using Algorithm 1.

fading channels, and experience interference between macro and small cells. We use the dense-urban propagation parameters in [2, Table 2]. Typical values of the channel coupling (X) between the macro access point and the other network entities, e.g., home access terminal and macro access terminal, ranges from 100 dB to 140 dB, the channel coupling between the home access point and the home access terminal (Y) or the coupling between the home access point and the macro access terminal (Z) are typically 80 dB.

A. Experiment 1 (Convergence of Algorithm 1)

Fig. 4 shows the convergence of Algorithm 1 for five small cell users from ten different initial points. To verify the optimality correctness of the converged solution of Algorithm 1, we have also solved (14) using an interior point method algorithm for comparison purpose. Simulations show that convergence happens in less than ten iterations even for thousands of users and a large power range, e.g., 125 mW to 2 W (maximum output of UMTS/3G Power Class 4 to Class 1 mobile phone, respectively).

From Theorem 3, Algorithm 1 can be viewed as a *nonlinear* power method in linear algebra. It is well known that the convergence rate of the power method is determined by the ratio of

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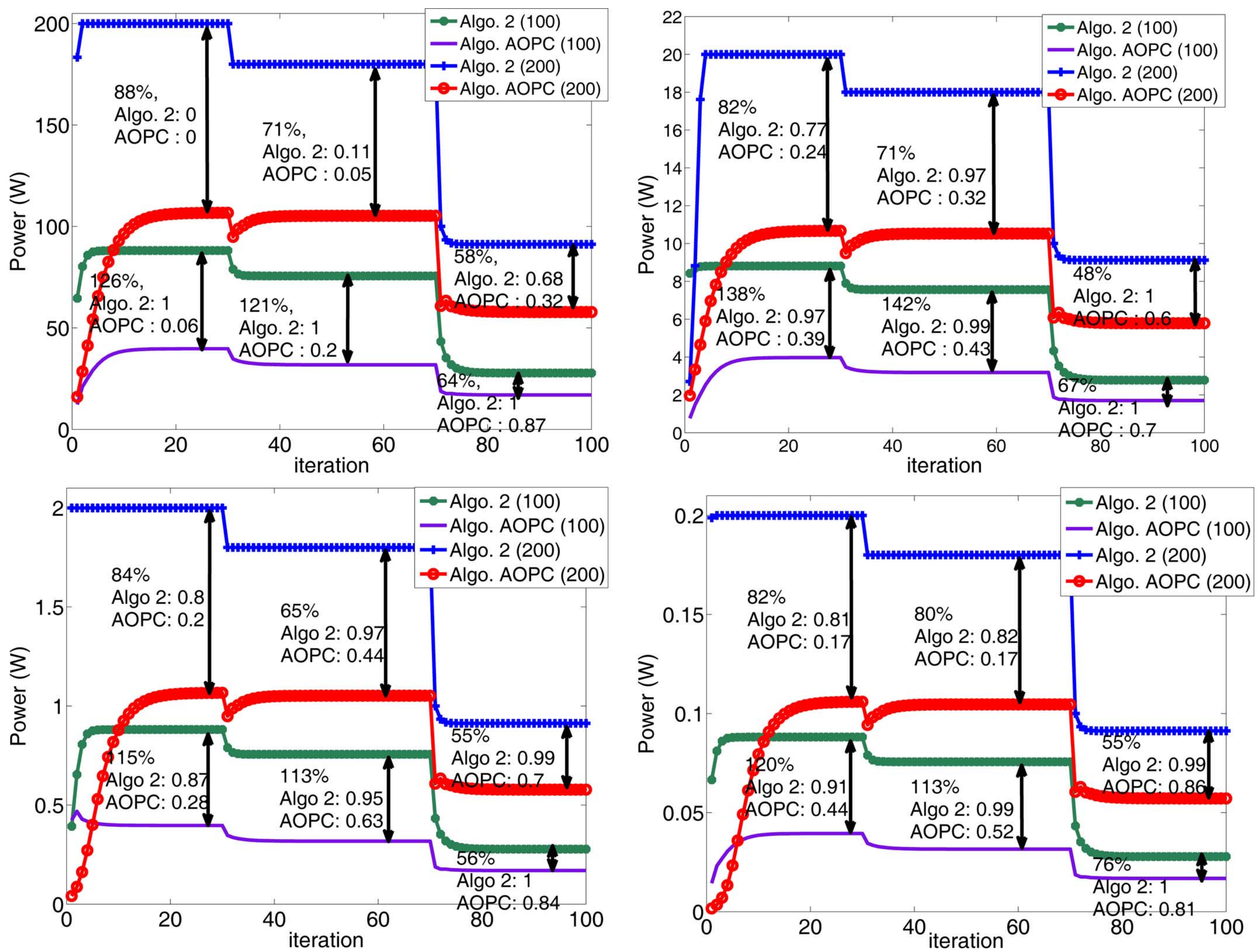

Fig. 5. Experiment 2. We plot the evolution of the total power consumption for two networks with 100 and 200 small cell users as we vary $\bar{p}$. A time slot on the x-axis of each graph refers to a power update iteration. Ten and fifty percent of the users are removed at time slot 30 and 70 respectively. The top left graph, top right graph, bottom left graph and bottom right graph show the case for $\bar{p} = 1$ W, $\bar{p} = 0.1$ W, $\bar{p} = 10$ mW and $\bar{p} = 1$ mW respectively. The power consumption difference between Algorithms 2 and AOPC, and the fraction of the number of users meeting the given threshold $\boldsymbol{\alpha}$ are in text form.

the second dominant eigenvalue to the Perron-Frobenius eigenvalue [24]. The method converges slowly if this ratio is close to one. Now, this ratio for $\mathbf{B}(\mathbf{p}^\star)$ (cf. fourth and fifth row of Table I) determines only the *local convergence rate* of Algorithm 1 near the fixed point $\mathbf{p}^\star$. Since the CEM solution is numerically observed to give good approximation to $\mathbf{p}^\star$ (especially in the regime of low power and small $\boldsymbol{\beta}$, cf. Fig. 2), this ratio is conceivably close to those computed using the CEM solution (cf. second and third row of Table I), which on the other hand determines a *global convergence rate* for the CEM case. We empirically determine this ratio to be in the range of 0.2 – 0.4 using the parameters in [2]. The worst outage probability bounds in Corollary 1 are thus useful for numerically analyzing the overall convergence rate of Algorithm 1.

B. Experiment 2 (Comparison Between Algorithms 2 and AOPC)

We next provide numerical examples to compare the total power consumption using Algorithms 2 and AOPC to find a power solution when (48) is infeasible. Fig. 5 shows the total power evolution in a network with initially 100 and 200 small cell users. Then, ten and fifty percent of the users are removed at time slot 30 and 70 respectively. On each graph, the difference in total power and the fraction of users that satisfy their outage probability threshold $\boldsymbol{\alpha}$ are recorded. As illustrated, in comparison to AOPC, Algorithm 2 can lead to an increased total power consumption of 50% or more in all cases at the expense of a smaller number of users meeting $\boldsymbol{\alpha}$. On the other hand, the infeasibility of (48) leads to the phenomenon that some users who run Algorithm 2 are not able to achieve $\boldsymbol{\alpha}$ and thus have to transmit at $\bar{p}$. By enforcing a worst outage probability fairness across all users, Algorithm AOPC computes power that are typically smaller than $\bar{p}$, thus leading to a smaller total power consumption.

C. Experiment 3 (Performance Comparison Between Macrocell and Small Cell Users)

In this experiment, we evaluate the performance of Algorithms 2 and AOPC in a macrocell network with a cell radius of 1.4 km and with fifteen randomly located small cell users.

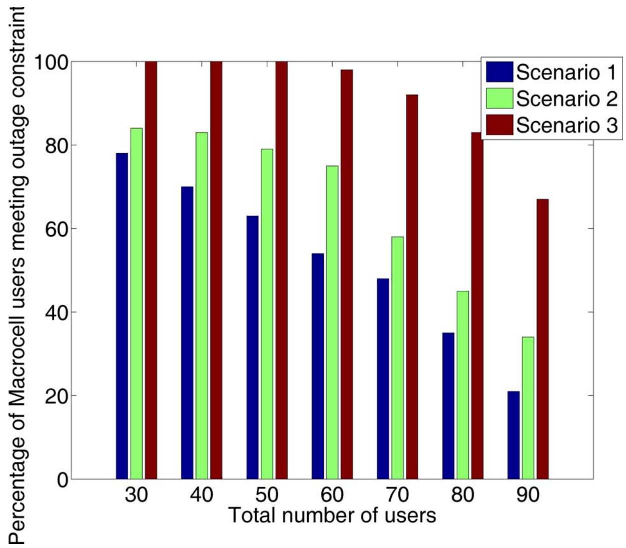

Fig. 6. Performance comparison of the three scenarios in terms of the average percentage of macrocell users meeting the outage constraints over 5000 simulations.

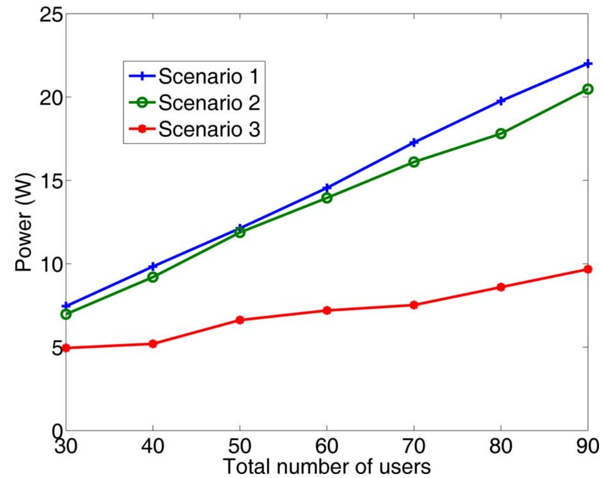

Fig. 7. Performance comparison of the three scenarios in terms of the average total power consumption of all the users over 5000 simulations.

Each user in the small cell and macrocell network communicates with its base station over channels with pass loss of $L = 128.1 + 37.6\log_{10}(d)$ dB and $L = 98.5 + 20\log_{10}(d)$ dB, where d is the distance in kilometers, respectively. Each user has a maximum power constraint of 500 mW (27 dBm), and each user is served one independent data stream from its base station. The noise power spectral density is set to -162 dBm/Hz. We assign an outage probability specification of 2% and SINR thresholds of 5 dB to all the macrocell and small cell users.

We analyze the performance of three scenarios, namely Scenario 1: when all the small cell basestations are switched off, i.e., all the access terminals are macrocell users and use Algorithm 2, Scenario 2: when all the small cell basestations are switched on and all the users use Algorithm 2 and Scenario 3: all macrocell users use Algorithm 2 and all small cell users use Algorithm AOPC[15]. The normalization of $\mathbf{z}(k)$ at Step 2 of Algorithm AOPC can be performed by the small cell users in a distributed manner assuming that there is perfect exchange of coordination between the small cell base stations and the macrocellular network. The three scenarios are run over 5000 numerical simulations with the fifteen small cell users randomly placed in the cell at each simulation run.

We compare the average percentage of macrocell users that meet the given outage constraints among the three scenarios in Fig. 6. Fig. 7 compares the total power consumption for the three scenarios. By comparing Scenarios 1 and 2 as shown in Figs. 6 and 7, the deployment of small cell improves the percentage of users meeting the given outage constraints, but the total power consumption may not be appreciable. By comparing the performance of Scenarios 2 and 3, we observe that, by allowing small cell users to have a min-max outage probability fairness, the power consumption difference between Algorithm 2 and AOPC can be as much as 50%. This saving of energy consumption comes at the expense of a larger number of small cell users not meeting their outage constraints as compared to that of Algorithm 2.

[15]By letting macrocell users and small cell users to run Algorithm 2 and Algorithm AOPC respectively essentially places a higher priority on macrocell users as compared to small cell users since the outage probability of macrocell users should not deviate from given specification whereas that of the small cell users can be higher than the given specification.

VII. Conclusion

We studied the worst outage probability problem that have power constraints in a multiuser Rayleigh-faded network using tools from the nonlinear Perron-Frobenius theory and nonnegative matrix theory. The optimal value and solution can be characterized by the spectral property of matrices induced by a particular positive mapping. We then proposed a geometrically fast convergent algorithm, free of parameter tuning, to solve it optimally in a distributed manner. As a by-product, we solved an open problem of convergence for a previously proposed algorithm in the interference-limited case. We also established a tight relationship between the worst outage probability problem and its certainty-equivalent margin counterpart, and utilized the connection to find useful bounds and to evaluate the fairness of resource allocation. We then addressed a total power minimization problem with outage specification constraints and its feasibility condition. We proposed a dynamic algorithm that adapted its outage probability specification to minimize the total power in a heterogeneous wireless network. Numerical results showed that the dynamic algorithm can be effective for deploying closed-access small cells in a macrocell in terms of total power consumption and the percentage of users satisfying their outage probability specification.

As future work, it is interesting to generalize our analysis and algorithm design methodologies based on the nonlinear Perron-Frobenius theory to solve the worst outage probability problem for other practical fading channel models such as the Ricean, Weibull and Nakagami distributions. It is also interesting to solve this problem for more general nonlinear power constraints or to extend it for other wireless utility objectives (see, e.g., recent efforts in [32] on extending the nonlinear Perron-Frobenius theory for this purpose). For the total power minimization problem, the infeasibility issue can possibly be tackled using a joint admission control and power control scheme, i.e., removing some constraints in (48) adaptively (see, e.g., [33]). In other words, the AOPC algorithm can also be combined with admission control protocols to overcome the barrier of infeasibility in a system.

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Chee Wei Tan (M'08–SM'12) received the M.A. and Ph.D. degrees in electrical engineering from Princeton University, Princeton, NJ, USA, in 2006 and 2008, respectively.

He is an Associate Professor with the City University of Hong Kong, Hong Kong. He was a Visiting Faculty with Qualcomm R&D, San Diego, CA, USA, in 2011. His research interests are in networks, inference in online large data analytics, and optimization theory and its applications.

Dr. Tan was the recipient of the 2008 Princeton University Wu Prize for Excellence, the 2011 IEEE Communications Society AP Outstanding Young Researcher Award, and the 2015 National Research Foundation Fellowship. He was the Chair of the IEEE Information Theory Society Hong Kong Chapter in 2014–2015. He was twice selected to participate at the U.S. National Academy of Engineering China–America Frontiers of Engineering Symposium in 2013 and 2015. He currently serves as an Editor for the IEEE TRANSACTIONS ON COMMUNICATIONS.