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Capacity of Wireless Communication Systems Employing Antenna Arrays, a Tutorial Study

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Abstract. A tutorial study is performed on the capacity of multiple antenna wireless communication systems. Multiple antenna structures can be classified into single-input multiple-outputs (SIMO), multiple-inputs single-output (MISO), and multiple-inputs multiple-outputs (MIMO) systems. Assuming that the channel is known at receiver, capacity expressions are provided for each structure, under the conditions of quasi-static flat fading. Also, information rate limits are provided in each case for some suboptimal structures or detection techniques that may be used in practice. Using simulations for the case of flat Rayleigh fading, capacities of optimal/suboptimal implementations are contrasted for each multi-antenna structure. Discussions are made on system design, regarding implementation complexity and practical limitations on achieving these capacities. In particular, the problem of fading correlation and required antenna spacing, effect of fast channel fading, and lack of channel knowledge at receiver are discussed. Providing the results of the most recent researches considering the capacity of multi-antenna systems, as well as some new results, this paper can give a good perspective for designing appropriate architectures in different wireless communication applications.

Keywords: antenna arrays, channel capacity, information rates, wireless channels, multipath propagation, channel fading, spatial diversity, MIMO systems.

1. Introduction

Application of antenna arrays in wireless communication has been of special interest, particularly in the last two decades. It has been shown by many studies that when an array is appropriately employed in a communication system, it helps in improving the system performance by increasing channel capacity and spectrum efficiency, extending range coverage, tailoring beam shape, steering multiple beams to track many subscribers, and compensating antenna aperture distortion electronically. It also reduces multipath fading, cochannel interferences (CCI), and bit error rate (BER) [1–3].

The antenna array may be used together with other methods such as channel coding, adaptive equalization, and interference cancelling to enhance the system performance.

A particular and important attraction of the use of antenna arrays is in high data rate wireless communication, such as transmission of high quality video information. A primary solution to this high data rate requirement may be the increase in bandwidth (BW) or the transmit power. However, these solutions are neither cost efficient nor satisfactory in practice.

An interesting alternative for increasing the channel capacity is to take use of multipath wireless channels, which has been of special interest in the past few years.

Since multipath propagation causes time-frequency signal fading, it is conventionally regarded as an impediment to reliable communication [4]. However, it has been recently known that multipath propagation can multiply the attaining information rate in a wireless communication system, provided that multiple antennas are used at both transmitter and receiver [5]. If multipath scattering is sufficiently rich and properly exploited, use of multiple antennas at both sides of the radio link can result in enormous channel capacities. Pioneer works on this type of systems have been performed in Bell Labs under a project named BLAST (Bell Laboratories Layered Space Time architecture) [6].

Multiple antenna structures can be divided into three groups: use of antenna array only at receiver, known as single-input multiple-outputs (SIMO) systems; use of antenna array only at transmitter, known as multiple-inputs single-output (MISO) systems; and use of antenna arrays at both transmitter and receiver, known as multiple-inputs multiple-outputs (MIMO) systems.

In this paper, our aim is to investigate how these different multi-antenna structures influence the channel capacity. We will concentrate on point-to-point (single-user) wireless communication systems all over the paper, except a brief discussion on multiuser systems made at the end of the paper. We try to present the most recent results of researches concerning the capacity of multi-antenna wireless systems. Meanwhile, some new concepts are discussed and some interesting techniques are studied more precisely. In particular, information rate limits of different multiple antenna structures are contrasted for different number of antennas; and complexity of various structures are discussed too. This can give to system designer a perspective to choose its preferred multi-antenna structure.

The reference list provided here is by no means complete, especially for the case of multiuser systems. However, it is tried to give a start point to the readers who want to follow some special subjects of interest. We have tried to present the most important and recent works related to the topics treated in this paper.

After defining a channel model and explaining some assumptions regarding the channel in Section 2, capacity expressions will be presented in Section 3 for the general case of MIMO systems. Next in Sections 4, 5, and 6, we will study particularly each one of SIMO, MISO, and MIMO systems, respectively. In each case, in addition to optimal structures and capacity bounds, some suboptimal implementations are also introduced, and information rate bounds are given. Also, the capacity of different implementations are contrasted with the single-input single-output (SISO) channel case. In Section 7, a comparison is made between capacities of SIMO, MISO, and MIMO systems. In Section 8, some practical concepts concerning the implementation of multi-antenna structures, as well as the effect of violation of each assumption made on the channel, will be discussed. At last, for the sake of completeness, a brief discussion is made in Section 9 on the implication of antenna arrays in multiuser systems.

2. Channel Model and Assumptions

The global scheme of a communication link is shown in Figure 1. The communication channel includes the effect of transmit/receive antennas and the propagation medium. *Far field* conditions are assumed, that is, dominant reflectors are assumed to be sufficiently far from the transmitter and the receiver. Under these conditions, angles of departure, angles of arrival and time delays can be assumed almost the same over the extent of arrays' apertures.

We will neglect the effect of antenna patterns and will assume that for both transmit and receive arrays, antenna gains are the same for all elements over all propagation paths (antenna sidelobes are neglected). This is well justified together with the far field assumption. In

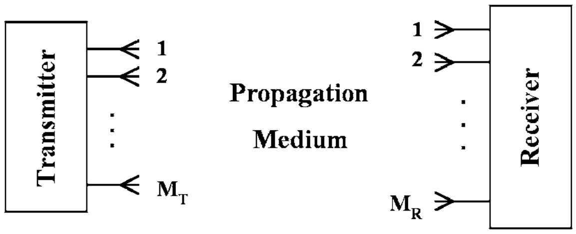

Figure 1. Global scheme of MIMO communication structure.

this way, the overall antenna power gain will be considered the same for all transmit/receive antenna pairs and will be denoted as g_{TR}.

2.1. Channel Model

A discrete time baseband channel model is considered. Considering a sampled model with one sample per symbol period, we will have a sufficient statistic for each transmitted/received symbol, provided that the pulse shaping satisfies the Nyquist criterion [7]. Also, flat channel model is considered. So, the channel can be represented by a channel matrix $\boldsymbol{H}$ of dimension $M_R \times M_T$, with M_R and M_T the number of antenna elements at receiver and transmitter, respectively. Entries of $\boldsymbol{H}$, h_{ij}, which are normalized[1] proper[2] complex random processes, represent the equivalent channel impulse response between jth transmit and ith receive antennas. Antenna gains and propagation loss are excluded from h_{ij}.

Let $\boldsymbol{x}$ be the vector of transmitted symbols on M_T antennas at one sample time. The vector of corresponding received symbols on the receiver array, $\boldsymbol{z}$, will be

$$\boldsymbol{z} = a\boldsymbol{H}\boldsymbol{x} + \boldsymbol{n} = \boldsymbol{y} + \boldsymbol{n}\,, \tag{1}$$

where $\boldsymbol{n}$ is the vector of receiver noise whose elements are considered as (proper complex) zero-mean additive white Gaussian noise (AWGN) samples, with power of σ^2. a is a constant that incorporates the effect of antenna gains and propagation loss,

$$a^2 = \alpha\, g_{TR} \tag{2}$$

with α the propagation power loss. Both α and g_{TR} are assumed to be the same for all transmit/receive antenna pairs, and to be constant over the signal BW.

2.2. Channel Fading

The statistics of h_{ij} depends on the fading conditions. Unless otherwise mentioned, we will always consider Rayleigh fading, which is valid for a rich scattering propagation environment, where no line-of-sight (LOS) exists between transmitter and receiver (denoted hereafter by Tx and Rx, respectively) [4]. Under such conditions, h_{ij} will be proper complex Gaussian random processes.

We suppose that the channel fading is not too rapid, so that $\boldsymbol{H}$ can be considered as constant during one or more bursts.[3] So, considering quasi-static conditions, the continuous channel

[1] In the sense that $E\{h_{ij}\, h_{ij}^*\} = 1$, with $(.)^*$ the complex conjugate operator

[2] For proper (circularly-symmetric) complex random processes, the real and imaginary parts of the system can be treated as two mutually orthogonal systems with the same dimensionality [8].

[3] Bursts are assumed to be long enough, so that the definition of capacity for a given $\boldsymbol{H}$ matrix is meaningful.

fading process is approximated as piece-wise constant. We define the coherence interval of the channel Δ (without dimension), as the number of symbol periods during which the propagation coefficients are almost constant; and they change to new independent values from an interval to another [9]. With quasi-static model, in fact we assume that $\Delta \to \infty$. In this way, we may speak of *random variables* instead of *random processes* for h_{ij}.

We also assume that the antenna elements at both Tx and Rx are spaced sufficiently apart, so that independent fading can be considered for each Tx/Rx antenna pair. In other words, h_{ij} are assumed to be independent.

2.3. Definition of Capacity

For a deterministic channel (constant h_{ij}), the channel capacity is also a deterministic value which gives an upper bound on the information rate for reliable communication, as states the Shannon theorem [10]. In other words, capacity is the maximum attainable mutual information between the channel input-output. It is also the case when channel is not deterministic, but each use of channel employs an independent realization of the $\boldsymbol{H}$ matrix [11, 12].

For the case of a randomly time-varying channel, the definition of the capacity depends on the bursts duration (codeword length) T. Here, the capacity is a random variable whose instant value depends on the corresponding $\boldsymbol{H}$ matrix. If $T \gg \Delta$, channel is said to behave ergodically,[4] and an ergodic (statistical average) capacity C_{av} is defined [13–15], which again means the maximum attainable mutual information. If $T \not\gg \Delta$, as is our case (see the previous paragraph), the maximum mutual information is not equal to the channel capacity, and the Shannon capacity of the channel may be even zero [12]. If we choose a transmission rate for communication, there is a non-zero probability that the realization $\boldsymbol{H}$ is incapable of supporting it. When the instant capacity is less than the preassumed value, a channel *outage* is said to be occurred. The mutual information can be regarded as a random quantity, giving rise to capacity-versus-outage considerations. The outage probability P_{out} is a useful parameter in studying channel capacity. Here, a tradeoff should be made between the expected throughput and outage [13, 14]. Note that in the literature, the capacity for a given outage probability is sometimes called *outage capacity*, however, it is not a correct terminology!

In contrast to capacity-versus-outage, a *delay-limited* capacity C_{DL} may be defined in the case of $T \not\gg \Delta$, which corresponds to zero-outage capacity. C_{DL} is zero for a SISO channel without power control; it can be a positive value using *optimal* power control and/or by exploiting time/space diversity [13–15]. In the limit of infinite time/space diversity, C_{DL} equals the ergodic capacity C_{av} [14].

In this paper, we will always consider capacity-versus-outage with $P_{out} = 0.01$, unless otherwise mentioned. The corresponding capacity values correspond to 99% percentage point of CCDF (Complementary Cumulative Distribution Function) of capacity.

It should be insisted that capacity is a limit to error-free bit rate that is provided by information theory. Any working system can only achieve a bit rate (at some desired small BER) that is only a fraction of capacity. With a given day's technology, the challenge is to design efficient coding/decoding algorithms which can approach the information theory bound on bit rate. In what will be seen throughout this paper, the term "capacity" will also be used to indicate the maximum deliverable bit rate when using some special (suboptimal) structures or detection techniques.

[4] This is the case, for example, for some submarine acoustics and avionic channels.

3. Capacity Expressions

It is assumed that the channel is estimated and tracked at Rx.[5] The expressions that are presented here, are valid under the assumption of perfect channel knowledge at Rx. The discussion on this assumption is made in Subsection 8.2.

We will consider the general MIMO system and will present the capacity expressions for this general case. The capacity of SIMO and MISO systems and other degenerate cases can be easily obtained using these general expressions, as it will be seen in next sections. We consider the constraint that the total transmit power at each sample time is equal to P_T.

3.1. Unknown-CSI Capacity

If the channel state information (CSI) is not known at Tx, we distribute the available power uniformly over the transmit antennas. In this case, the average received signal-to-noise ratio corresponding to each transmit antenna at the receiver array, ρ, is

$$\rho = \frac{P_T\, a^2}{M_T\, \sigma^2} = \frac{\rho_T}{M_T} \tag{3}$$

with ρ_T the total average SNR at the receiver array. In this paper, with SNR we mean ρ_T. Regarding the far field conditions, ρ is considered the same for all Tx antennas. As it is shown in the appendix, the capacity is given by

$$C = \sum_{i=1}^{M} \log_2(1 + \rho\, \lambda_{H,i}^2) \quad \text{bps/Hz}, \tag{4}$$

where $\lambda_{H,i}$ are the singular values of $\boldsymbol{H}$ and $M = \min(M_T, M_R)$. Similar expressions are obtained in [12, 24]. Notice that for this expression to hold, the inputs on M_T antennas must be statistically independent as it is shown in the appendix. Equation (4) can be interpreted as follows: in the case of flat fading, the MIMO channel can be reduced to a set of parallel independent SISO subchannels, or to a set of independent orthogonal modes of excitation [18], for which the capacity can be easily calculated. The number of these parallel channels equals the rank of $\boldsymbol{H}$, and the gain of the ith equivalent parallel channel is equal to the $\lambda_{H,i}^2$. Similar statements are given in [12, 19–21].

Equation (4) can be simplified as follows.

$$\begin{aligned} C &= \log_2 \prod_{i}^{M} (1 + \rho\, \lambda_{H,i}^2) = \log_2\, \det[\boldsymbol{I}_{M_R} + \rho\, \boldsymbol{\Lambda}_H \boldsymbol{\Lambda}_H^{\dagger}] \\ &= \log_2\, \det\left[\boldsymbol{I}_{M_R} + \frac{\rho_T}{M_T}\, \boldsymbol{H}\boldsymbol{H}^{\dagger}\right] \quad \text{bps/Hz} \end{aligned} \tag{5}$$

which is the expression presented in [5] for a flat fading MIMO channel. As defined before, $\rho_T = \rho\, M_T$. $\boldsymbol{I}_{M_R}$ is the identity matrix with the dimension of $(M_R \times M_R)$.

[5] See [16] for discussions on a completely unknown channel.

3.2. KNOWN-CSI CAPACITY

If the CSI is provided at the transmitter, we can optimally distribute the total available power P_T on M_T transmit antennas, a solution that is usually referred to as *water filling* (WF).[6] We assume that the CSI is provided for Tx in a causal manner, and no prediction can be made on it (see [15] for the opposite case). We also assume that the CSI is delivered to Tx with no delay.[7]

As it is shown in the appendix, the capacity in this case is given by

$$C = \sum_{i=1}^{M} \log_2 \left(1 + \frac{a^2}{\sigma^2} \lambda_{X,i} \, \lambda_{H,i}^2 \right) \quad \text{bps/Hz} \tag{6}$$

$$\lambda_{X,i} = \left(\psi - \frac{\sigma^2}{a^2 \, \lambda_{H,i}^2} \right)^+ , \tag{7}$$

where,

$$(s)^+ = \begin{cases} s, & \text{if } s > 0 \\ 0, & \text{if } s \leq 0 \end{cases} . \tag{8}$$

Similar expressions are obtained in [12, 24]. $\lambda_{X,i}$ are the eigenvalues of the transmit-symbols autocorrelation matrix, $\boldsymbol{R}_X$. ψ is determined so as to satisfy the constraint on the total transmit power,

$$\sum_{i=1}^{M_T} \lambda_{X,i} = P_T . \tag{9}$$

Consider the singular value decomposition (SVD) of $\boldsymbol{H}$ as in (10),

$$\boldsymbol{H} = \boldsymbol{U}_H \, \boldsymbol{\Lambda}_H \, \boldsymbol{V}_H^{\dagger} , \tag{10}$$

where $\boldsymbol{U}_H$ and $\boldsymbol{V}_H$ of dimensions $(M_R \times M_R)$ and $(M_T \times M_T)$, respectively, are unitary matrices, and $\boldsymbol{\Lambda}_H$ of dimension $(M_R \times M_T)$ contains $\lambda_{H,i}$. $(.)^{\dagger}$ denotes the transpose conjugate operator. As it is shown in the appendix, the optimum $\boldsymbol{R}_X$ to achieve the capacity of (6) is

$$\boldsymbol{R}_X = \boldsymbol{V}_H \, \boldsymbol{P}_X \, \boldsymbol{V}_H^{\dagger} . \tag{11}$$

$\boldsymbol{P}_X$ is a diagonal matrix with the diagonal entries of $\lambda_{X,i}$ in descending order. In order to satisfy (11), we should perform a *primary* power allotment[8] over the transmit antennas according to $\boldsymbol{P}_X$, followed by a weighting on symbols before transmission. These weight factors to be applied to the transmit array are given by the columns of $\boldsymbol{V}_H$. At the receiver, a weighting

[6] WF solution may be performed on time (over different channel realizations), as considered in [13, 14, 22, 23] for the case of SISO channels. It is shown that the availability of CSI at Tx in addition to Rx and performing WF in time, gives only little advantage in terms of C_{av}, and this advantage is more considerable in low SNR values.

[7] However, a more practical model would be to consider a delay in delivering the CSI to Tx, as is the case when CSI is fed back through an auxiliary communication link (see [13, 15, 22] and their references for the case of SISO channels and WF in time).

[8] For example, considering normalized-power symbols to be transmitted on antenna i, their amplitudes are multiplied by $\sqrt{\lambda_{X,i}}$.

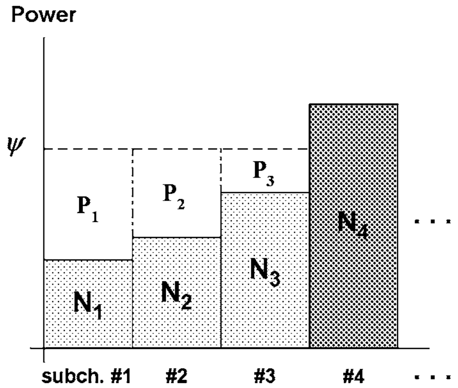

Figure 2. Understanding Water Filling principle over equivalent independent subchannels of a MIMO structure.

should be performed according to $\boldsymbol{U}_H^\dagger$. In this way, the channel is decomposed to M parallel independent subchannels over which signal transmission is performed in water filling method.

Equation (6) can be interpreted as follows: WF solution assigns more power to "better" subchannels, i.e. those with greater gain, and assigns less power (or probably no power) to "worse" subchannels, i.e. those with more attenuation.

Let $\frac{\sigma^2}{a^2\lambda_{H,i}^2} = N_i$ and $\lambda_{X,i} = P_i$. Equations (7) and (9) can be written in the following form,

$$\begin{cases} P_i = (\psi - N_i)^+ \\ \sum_i P_i = P_T \end{cases} . \tag{12}$$

The power allotment according to (12) is illustrated on Figure 2.

In the following, we will not consider the WF solution except a brief discussion in Subsection 8.3. So, we will assume that the channel is not known at Tx.

In the three following sections we will study each one of SIMO, MISO, and MIMO systems separately. In each case, we will first consider the *ideal* structure giving the real Shannon capacity bound. Then, we consider some suboptimal configurations/detection techniques, and will present the information rate bounds (which are also called capacity) for each case.

For SIMO and MISO cases, capacity expressions will be provided for the case of flat fading using the general expression of (5). Simulation results, however, are for the case of Rayleigh flat fading all over this paper, except a brief discussion on other propagation conditions in Subsection 8.5.

4. Reception Diversity: SIMO Systems

4.1. Ideal SIMO

For SIMO systems, $M = M_T = 1$, $\boldsymbol{H}$ is of dimension $(M_R \times 1)$, and hence, $rank(\boldsymbol{H}) = 1$ too. Under the conditions of flat fading, we have[9] [5]

$$C = \log_2 \left(1 + \rho_T \sum_{i=1}^{M_R} |H_i|^2 \right) \quad \text{bps/Hz} . \tag{13}$$

[9] For Rayleigh fading, $|H_i|^2$ are (normalized) centralized Chi-squared random variables with two degrees of freedom.

Remember that $\rho_T = \frac{P_T}{\sigma^2} a^2$. The capacity expressed in (13) can be achieved employing maximal ratio combining (MRC) [7, 25, 26]. In fact, this linear optimum[10] combining (OC) receiver maximizes the information that the output possesses about the input signals. It serves the dual role of capturing more the transmitter power and stabilizing channel (spatial) fluctuations. In the case of flat fading, MRC is equivalent to the minimum mean-square-error (MMSE) receiver [20, 27].

For a SIMO channel, as SNR increases, the capacity improvement compared to that of any underlying SISO channel approaches a constant [17, 24]. Figure 3 shows curves of capacity versus M_R for $P_{out} = 0.01$ and three different SNR values. $M_R = 1$ corresponds to the SISO channel case. To obtain Figure 3 as well as other simulation results to be presented, 10^5 channel realizations are used. As it can be seen from Figure 3, for values of $M_R > 4$ the increase in capacity may not be considerable taking into account the complexity added to the Rx with increase in the number of antennas. Yet, (as it will be explained in Section 9) in some applications such as cellular mobile radio, where the channel is shared between several users, increase in M_R provides substantial improvement against CCI [28].

4.2. Suboptimal Detection Techniques

4.2.1. *Selection Diversity*

Obviously, MRC requires the estimation of the channel at receiver. A suboptimal use of SIMO system is to select the best input signal of the M_R antennas and to discard the other signals [25, 26]. The capacity of this structure, named *maximum selection* diversity is given by

$$C = \max_i \ \log_2(1 + \rho_T \ |H_i|^2) = \log_2(1 + \rho_T \ \max_i |H_i|^2) \quad 1 \le i \le M_R \,. \tag{14}$$

For selection diversity, the criterion of the "best" input signal may be the signal with highest instantaneous (signal plus noise) power.[11] In fact, the selection of the strongest signal does not deteriorate the performance considerably, in comparison to the exact selection of maximum SNR signal, as shown in [29].

4.2.2. *Equal Gain Combining*

Another suboptimal implementation of the SIMO structure is to use equal gain combining (EGC) at the receiver for signal detection. In this approach, signals received on M_R antennas are simply added together (without any weighting) [25, 26]. Evidently, co-phasing of received signals should be performed before the combination [1]. However, EGC can be used with PAM (Pulse Amplitude Modulation) signaling, for example, where there is no information in the signal phase. In this case, EGC can be performed by adding the envelopes of the received signals.

Performance curves of EGC fall in between those of MRC and maximum selection detection techniques [26]. The bound on information rate for an EGC receiver is given by[12]

$$C = \log_2 \left(1 + \frac{\rho_T}{M_R} \left(\sum_{i=1}^{M_R} |H_i| \right)^2 \right) \,. \tag{15}$$

[10] To be more precise, it should be stated that MRC is the optimum detection for the case of no inter-symbol interference (ISI), i.e., the flat fading case; or with ideal equalization.

[11] Here we have the assumption that the average power of noise (plus probable CCI) on different antennas is the same.

[12] The division of ρ by M_R is due to the fact that with EGC, the power of noise is multiplied by M_R.

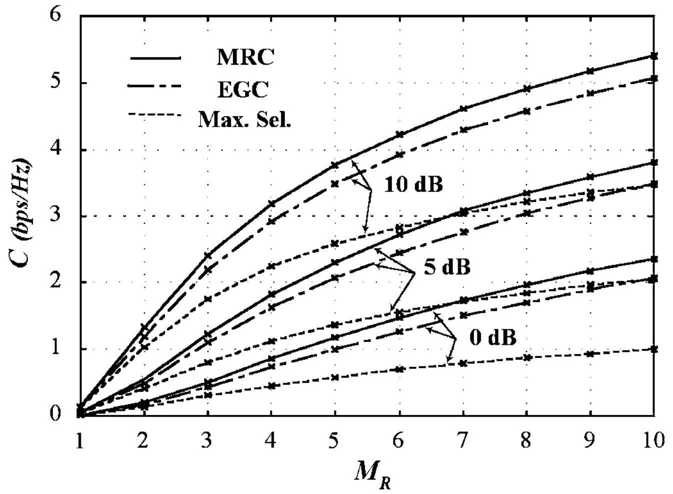

Figure 3. Capacity curves versus M_R for MRC, EGC (Equal Gain Combining), and maximum selection (Max. Sel.) SIMO detection methods, $P_{out} = 0.01$.

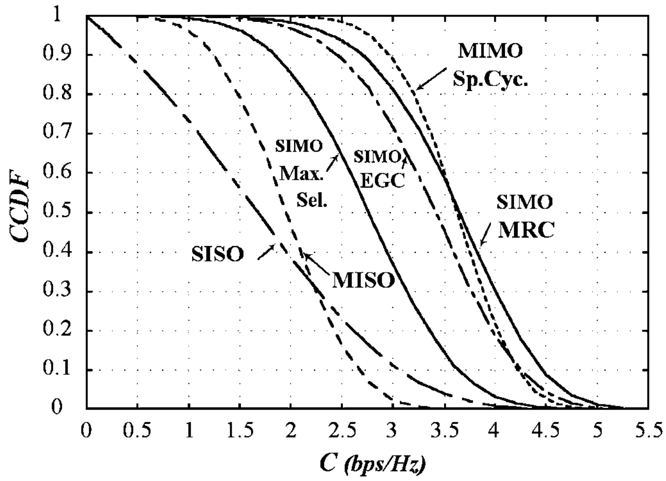

Figure 4. CCDF capacity curves for different structures: MRC SIMO, maximum selection (Max. Sel.) SIMO, EGC SIMO with $M_R = 4$; spatial cycling MIMO (Sp. Cyc.) with $M_R = 4$ and $M_T = 2$; MISO with $M_T = 4$; and SISO. SNR =5 dB.

As a comparison to the ideal SIMO structure, curves of capacity versus M_R are also shown in Figure 3. Capacities of EGC are very close to those of MRC, which corresponds well to the results presented in [26]. It should be noted again that EGC is of limited application. About selection diversity, it is seen that for small M_R values, the decrease in capacity compared to MRC can be well traded off with the considerable resulted simplicity in Rx.

4.3. Comparison with SISO

In Figure 4 we have contrasted CCDF curves of capacity for ideal SIMO, maximum selection SIMO, EGC SIMO, and SISO structures for $M_R = 4$ and SNR =5 dB. In addition, CCDF curves of two other structures are also shown, to be discussed later. We note that the smaller the variance of the capacity, the larger is the slope of the CCDF curve.

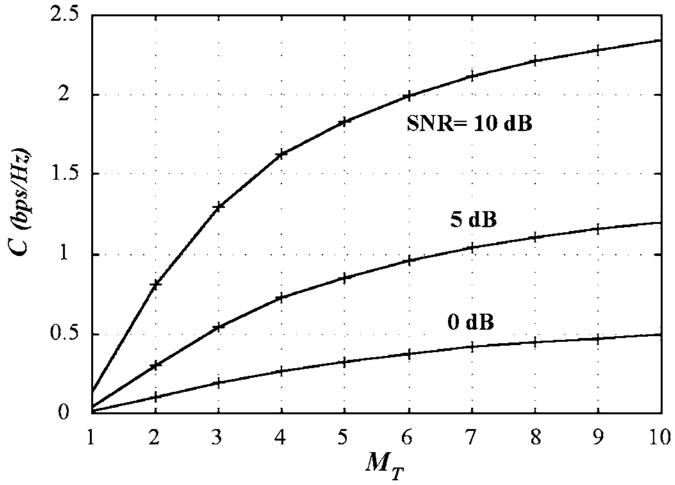

Figure 5. Capacity curves versus M_T for ideal MISO structure, $P_{out} = 0.01$.

5. Transmission Diversity: MISO Systems

5.1. Ideal MISO

For MISO structures, $M_R = M = 1$, $\boldsymbol{H}$ is of dimension $(1 \times M_T)$, and hence, $rank(\boldsymbol{H}) = 1$ too. Under the conditions of flat fading, we have [5]

$$C = \log_2\left(1 + \frac{\rho_T}{M_T}\sum_{i=1}^{M_T}|H_i|^2\right) \quad \text{bps/Hz}\,. \tag{16}$$

It can be shown that for MISO structure, asymptotically there is no additional capacity to be gained. For the case of Rayleigh fading, $\sum_{i=1}^{M_T}|H_i|^2$ has the $\chi^2_{2M_T}$ distribution (Chi-squared with $2M_T$ degrees of freedom). In (16) by strong law of large numbers, $\chi^2_{2M_T} \rightarrow 1$ in distribution, and the capacity approaches that of a Gaussian deterministic channel.

Figure 5 shows curves of capacity versus M_T for three different values of SNR. It is seen that much less capacity gain is achieved by increase in the number of antennas, compared to the case of SIMO structure. This is because the total transmitted power is constrained to P_T in both cases, but SIMO profits from a gain in SNR due to the use of the antenna array at Rx. This can also be seen comparing equations (13) and (16). From the point of view of fading reduction, SIMO and MISO structures have the same function.

It is important to know that here we have the assumption that the channel is unknown at transmitter. If the channel is known at Tx, the limit of capacity of a MISO structure is equal to that of a SIMO one with the same number of antennas, and is given by (13).

5.2. Comparison with SISO

As a comparison, CCDF curve of capacity for the ideal MISO structure is shown in Figure 4 together with CCDF curves for SISO and SIMO systems. It is seen that the variance of the capacity is less, compared to that for SISO and different SIMO structures.

An implementation method of MISO structures is proposed in [30–32], where M_T antennas transmit delayed versions of the signal. This creates *artificial* frequency selective fading at the

Rx, which uses equalization of type MLSE (Maximum Likelihood Sequence Estimation) or MMSE (Minimum Mean-Square Error) to profit from the spatial diversity gain. The reason of inserting this delay between different copies of symbols is to make it possible for the receiver to perform maximal ratio combining for signal detection. Otherwise, if for example the same symbols are transmitted from the transmit antennas, this optimal detection can not be performed since the transmitted symbols arrive simultaneously at Rx.[13]

Also, space-time codes (to be presented in Subsection 6.2 for the general case of MIMO) can be used for a MISO structure. This method is very effective, since it profits from error correction coding and transmission diversity at the same time. However, its processing complexity increases exponentially with BW efficiency and the required diversity order, and so, may not be practical or cost-efficient [33].

An interesting approach is that proposed by Alamouti where orthogonal codes are used (with rate 1) in a $M_T = 2$ structure [33]. Using an orthogonal space-time matrix of transmitted signals over two antennas and two time intervals at Tx, a relatively simple combination of received signals is proposed at Rx which gives the MRC detection for each transmitted signal.[14]

6. Transmit-Receive Diversity: MIMO Systems

6.1. Ideal MIMO

6.1.1. *Capacity versus Number of Antennas*

As previously shown in Section 3, Equations (4) and (5) represent the capacity of a flat fading MIMO channel. It can be shown that under high SNR conditions [24],

$$C \xrightarrow[\rho_T \to \infty]{} M \ \log_2 \rho_T \ + \sum_{i=1}^{M} \log_2 \left(|\lambda_{H,i}|^2 \right) - M \ \log_2 M \, . \tag{17}$$

That is, for high SNR values, the capacity can be increased almost linearly with M. Figure 6 shows capacity curves versus $M = M_R = M_T$ for several SNR values in the case of Rayleigh fading. It is seen that even for low SNR values, the increase in capacity is almost linear with increase in M.

The gain in capacity compared to the SISO channel case can be considered to be composed of two components [18]; the *array gain* at Rx which corresponds to the gain in the average power of the signal combination on M_R antennas, and the *diversity gain* which corresponds to the gain from increasing the system dimensionality (rank of $\boldsymbol{H}$) and depends highly on spatial correlation between antenna signals or the correlation between h_{ij} as it will be explained in Subsection 8.1. The diversity gain is given by $\min(M_R, M_T)$ under rich-scattering medium conditions.[15]

With constant M, to see concretely how the capacity changes with an increase only in M_R (or only in M_T), curves of capacity versus M_R (or M_T) for $M_T = 4$ (or $M_R = 4$) are shown in Figure 7 for three SNR values. Cases of $M_R = 1$ and $M_T = 1$ represent MISO

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[13] Notice that here we have the assumption of flat channel. If the channel is frequency selective, the inserted delay should be greater than the symbol duration plus the channel dispersion length [32].

[14] This idea is generalized to the case of MIMO structure in [33] with ($M_T = 2$, M_R).

[15] It is interesting to know that if we want to profit from the maximum array gain, the transmission should be performed only on the subchannel corresponding to the maximum $\lambda_{H,i}$ singular value of $\boldsymbol{H}$ [18].

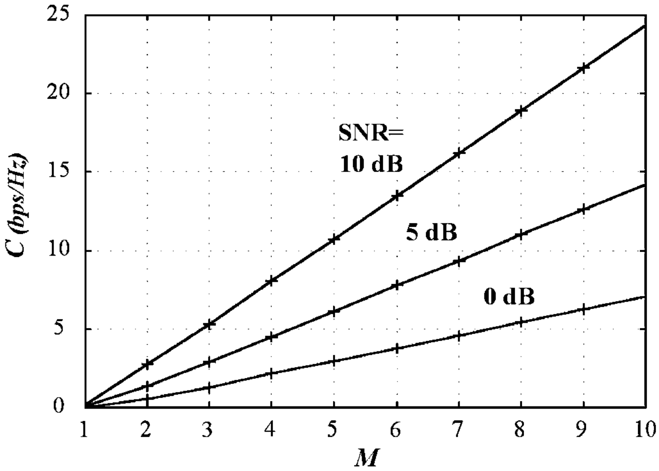

Figure 6. Ideal MIMO structure; capacity curves versus number of antennas $M = M_R = M_T$, $P_{out} = 0.01$.

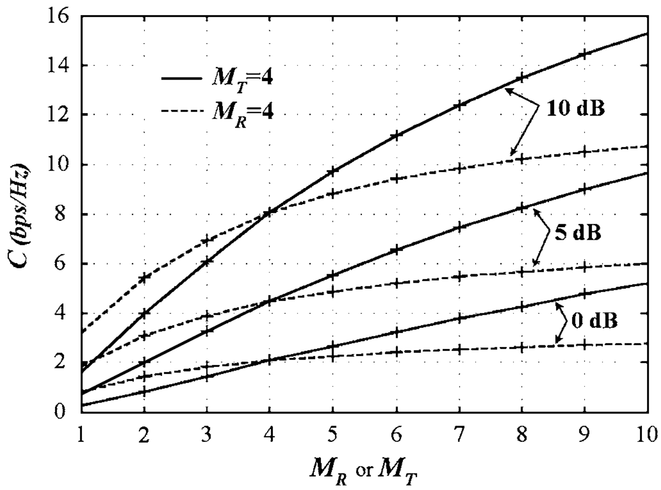

Figure 7. Ideal MIMO structure; capacity curves versus M_R or M_T; M_T or $M_R = 4$, $P_{out} = 0.01$.

and SIMO channels, respectively. As expected, with constant M, the increase in M_R is more efficient regarding the resulting increase in capacity, than the increase in M_T. In fact, with equal diversity order M, the case with $M_R > M_T$ profits from more array gain at Rx. Note that the increase in M_R may imply more system implementation complexity too.

It should be insisted again that the results presented here are for the case of unknown channel at transmitter. If channel is known at Tx, the limit of the MIMO capacity (obtained by water filling over the transmit antennas) is the same for (M_R, M_T) and (M_T, M_R) configurations.

6.1.2. *Capacity versus SNR*

If we define the capacity slope as the increase in capacity that results from the multiplication of SNR by a factor η,

$$Slope(\eta) = C(\eta\, SNR) - C(SNR) \tag{18}$$

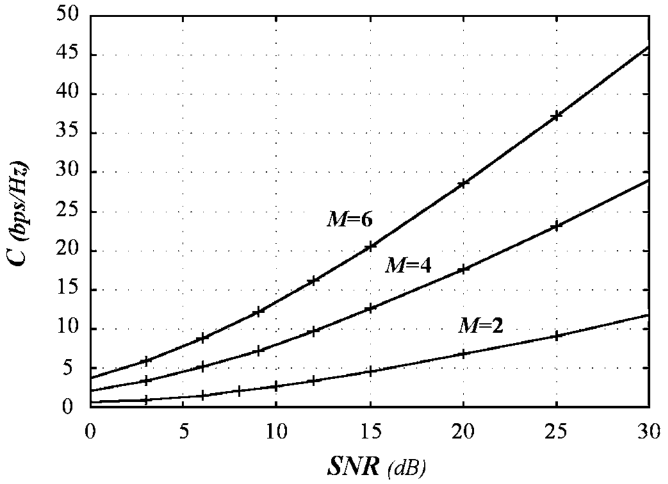

Figure 8. Ideal MIMO structure; capacity curves versus average SNR at receiver; $M = M_R = M_T$, $P_{out} = 0.01$.

it can be shown that [17, 24]

$$\lim_{\rho \to \infty} Slope(\eta) = M \log_2 \eta \,. \tag{19}$$

That is, under high SNR conditions, the capacity slope increases with increase in M. This can be seen from Figure 8 that shows curves of capacity versus SNR for $M = 2, 4, 6$.

The RHS of (19) equals $\log_2 \eta$ for a SIMO, MISO, or SISO channel.

6.1.3. *Rationality of the Obtained Capacity*

The obtained capacity improvement of MIMO systems may seem too large to be reasonable, regarding the number of constellation size that should be used to give the corresponding bit rates. However, it should be noticed that about $\frac{1}{M_T}$ of total bit rate should be considered in the design of signal constellations. In other words, the *per dimension* constellation size should be considered [5].

In contrast, SIMO systems may require a large and impractical constellation size for large channel capacities (apart from the complexity of Rx due to the increased M_R).

6.1.4. *Optimum Selection of Transmit Antennas*

The complexity of a MIMO system can be reduced by judicious selection of *fewer* transmit antennas without a considerable loss in the resulting channel capacity. Even, the capacity can be increased! Consider M_T transmit antennas in a MIMO structure. We select M_S antennas among M_T for signal transmission, in such a way that the selected antennas result in maximum capacity. This can be performed by an exhaustive search over all possible combinations of transmit antennas. For example, simulation results are given in Figure 9 for a (4,4) Rayleigh fading MIMO channel, where $M_S = 2$ antennas are selected for signal transmission. It is seen that for SNR $<$5 dB, signal transmission over M_S "best" antennas results even in an increase in channel capacity, and for SNR $<$ 10 dB, the difference in channel capacity is negligible, regarding the simplicity obtained.

A detailed analysis is performed in [34] on the effect of optimal selection of transmit antennas on channel capacity. Also, it is shown in [34, 35] that for a rank-deficient $\boldsymbol{H}$ (case of

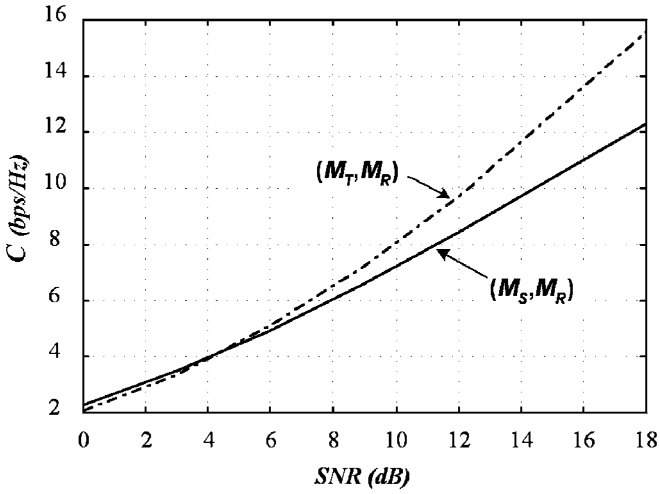

Figure 9. Optimal selection of M_S among M_T antennas for signal transmission, effect on MIMO channel capacity; $M_T = M_R = 4$, $M_S = 2$, $P_{out} = 0.01$.

insufficient scatterers in the propagation medium), it is optimal or close to optimal to use as many antennas for signal transmission as the rank of $\boldsymbol{H}$.

Notice that this optimal selection is of particular interest. We know that the cost of Tx is dominated by that of power amplifiers. Antennas are cheaper by typically two orders of magnitude [34]. So, it is economically advantageous to use a small number of amplifiers and a larger number of antennas, and to connect the amplifiers to a selected set of antennas[16] to achieve the maximum possible system capacity.

The exhaustive search method stated above may be computationally intensive, especially when M_T and M_S are relatively large. A computationally efficient *near-optimal* method of selection of transmit antennas is proposed in [36], which is based on the knowledge of CSI at Tx.

6.2. Attempts to Attain MIMO Capacity Limits

Equation (4) expresses the limit on information rate that can be ideally achieved in MIMO systems. However, to achieve this capacity complicated coding/decoding techniques should be employed. In particular, developing efficient spatio-temporal coding techniques has recently been of special interest in attaining the MIMO channel capacity. We try to give basic results of the most recent works on this subject. Interested reader may refer to these papers and the references cited therein for detailed discussions.

6.2.1. *BLAST Architecture*

One efficient processing architecture proposed is the diagonal-BLAST or D-BLAST, developed in Bell Labs and firstly presented by Foschini [37]. In this technique code blocks are dispersed across diagonals in space-time. $M_R = M_T = M$ is taken.

It is known that M diversity antennas can null out up to $M - 1$ interferers [28]. In D-BLAST architecture, separately encoded M data blocks are transmitted on each antenna. That is, one-dimensional (1-D) encoders are used to encode the data transmitted from each transmit

[16] Notice that it is well feasible under quasi-stationary conditions.

antenna, and the M encoders are assumed to function without sharing any information with each other. In each sequence duration, the receiver array detects the M received transmitted sequences : it nulls out the interference from yet undetected signals, and at the same time cancels out the interference from already-detected signals [37]. The order of detection is such that the sequence with higher SNR is detected first. In this way, the M-dimensional detection task is performed by M similar 1-D processing steps, and the Rx complexity grows only linearly with M.

Training sequences may be used to estimate the MIMO channel at Rx as considered in [38]. Notice that if joint detection of received signals is performed, as in multi-user detection methods, the complexity grows with m^M, with m the signal constellation size [39].

The information rate bound for D-BLAST architecture is given by[17] [37]

$$C = \sum_{k=1}^{M} \log_2 \left(1 + \frac{\rho}{M} \sum_{i=1}^{k} |H_{ik}|^2 \right) . \tag{20}$$

This capacity expression assumes *perfect* signal detection in already-processed layers, and hence, perfect cancellation of the contribution of the already-detected signals in the signal of the current layer being processed.

As it will be seen in the next section, the information rate bound approaches about 80% of the Shannon capacity given in (4). However, the implementation complexities of this approach has made the Bell Labs researchers to develop a simplified version of D-BLAST, named vertical BLAST or V-BLAST [40, 41]. It seems to be the first realized MIMO system reported. In their prototype, an (M_T, M_R) structure with $1 \leq M_T \leq 8$ and $M_T \leq M_R \leq 12$ is employed at $f_c = 1.9$ GHz (carrier frequency) between fixed Tx-Rx in an indoor propagation environment, under the conditions of quasi-static flat fading. No coding is considered for transmitter signals. Similar to D-BLAST, interference nulling from undetected signals, interference cancellation from already detected signals, and detection ordering (detecting maximum SNR signal in each step) is performed.

The capacity bounds for V-BLAST architecture are studied in [42]. It is shown that the asymptotic capacity grows linearly with the number of antennas, and that large fractions of D-BLAST rates (about 0.72 or even more, depending on the SNR) can be obtained with this simple and flexible approach. Up to 16 transmit/receive antennas are employed in the work of [42], under the same conditions of [40, 41] stated above.

A problem with V/D-BLAST architectures is that the decision errors produced in each layer affect signal detection in subsequent layers, which is critical in low SNR. A more recent work realized by Ariyavisitakul uses BLAST architecture with space-time codes, while applying *turbo* (iterative) detection technique to avoid this *error propagation* [39]. His results show that using space-time codes (almost similar to [43]) and turbo processing, the Shannon capacity can be achieved within about 3 dB in average SNR. It is shown that for a large number of Tx and Rx antennas, coding across the layers provides a better performance than independent coding within each layer.

[17] In fact, for $k \neq M$, any k entries of the kth column of $\boldsymbol{H}$ can be put in $\sum_i$, depending on the order of detection.

6.3. Space-Time Codes

Space time codes are apparently developed independently from the works relating to BLAST project. These codes firstly introduced by Tarokh et al. [43], combine spatial and temporal diversity techniques. The input data sequence is encoded by the channel encoder, and then, the encoded data pass through a serial-to-parallel converter, which splits it into M_T data streams. Each data stream is then transmitted simultaneously from different transmit antennas.[18] At the receiver, on each antenna, a superposition of M_T transmitted signals corrupted by noise and fading is received. Trellis codes can be used with Viterbi decoding [7] at Rx, where evidently the knowledge of $\boldsymbol{H}$ is necessary for computing branch metrics [43]. Under perfect channel knowledge and quasi-static conditions, it is shown in [43] that performances about within 2.5 dB of the capacity can be obtained (see also [45] where non-ideal conditions are considered). It is discussed in [43] how tradeoff should be considered between transmission rate, diversity advantage, signal constellation size, and trellis decoding complexity.

The case of large number of transmit antennas is considered in [46], where partitioning of transmit antennas and using space-time codes on each group of antennas is proposed, which helps to reduce the Rx complexity.

The case of MIMO structures under frequency-selective fading conditions is considered in [17, 24] (general comments on frequency-selective channels are given in Subsection 8.4). In [24] space-time vector coding (STVC) is proposed as a mean to approach to the channel capacity.[19] Notice that use of STVC necessitates the CSI knowledge at Tx. Also, a more practical space-frequency coding structure named multivariate discrete multitone (MDMT) is proposed in [24] which has a considerable complexity reduction compared to STVC method.

In [17], MDMT is proposed together with multivariate trellis-codes modulation for the case where Tx does not dispose the CSI. Using this coding technique, a MIMO structure is implemented with $1 \leq M_T \leq 3$ and $1 \leq M_R \leq 6$ at $f_c = 5.2$ GHz between mobile Tx-Rx with a maximum Doppler of ± 540 Hz [24].

The problem with trellis-based techniques is their decoding complexity. For a given number of transmit antennas, the decoding complexity of space-time trellis codes increases exponentially with the diversity level and the transmission rate. In contrast to these codes, space-time *block* codes (STBC) proposed by [48] have the property of having a very simple maximum likelihood decoding algorithm based on linear processing at Rx. These orthogonal codes can be considered as a generalization of Alamouti transmission scheme [33] for an arbitrary number of transmit antennas. The performance of STBCs as well as coding/decoding aspects are discussed in [49].

6.4. Suboptimal Structure: Spatial Cycling Technique

One simple implementation of MIMO system is to use only one Tx at a time, and to cycle through all M_T transmitters periodically with period of M_T [5]. In other words, at each sample time a SIMO structure is employed, and hence, it can be regarded as a generalized SIMO case. Note that the detection complexity is as in the case of ideal (MRC) SIMO explained in Section 4, but the transmitter is more complex here. With this technique, we profit from

[18] Tx can insert periodic orthogonal pilot sequences in each simultaneously transmitted burst, to permit to Rx to estimate the channel [44].

[19] This is a generalization of use of vector coding presented in [47] for the case of SISO frequency-selective channels.

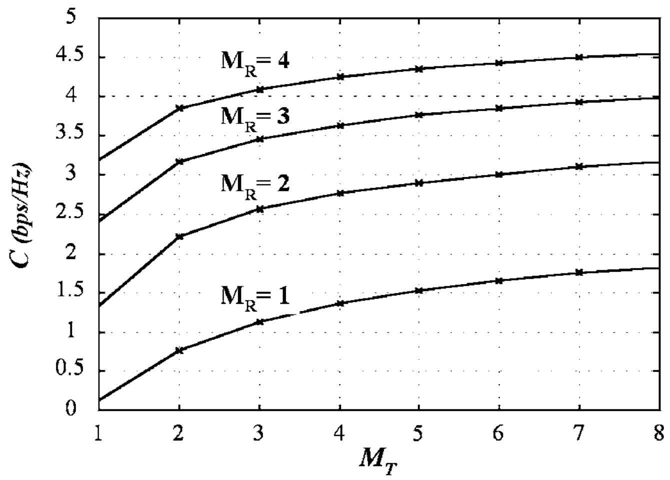

Figure 10. Spatial cycling MIMO structure; capacity curves versus M_T; SNR =10 dB, $P_{out} = 0.01$.

a simple implementation, there is no interference, and besides, the cycling ensures nontrivial dwelling on the better of M_T transmitters. The capacity will be the average of capacities in each Tx configuration [5].[20]

$$C = \frac{1}{M_T} \sum_{j=1}^{M_T} \log_2 \left(1 + \rho_T \sum_{i=1}^{M_R} |H_{ij}|^2 \right) \quad \text{bps/Hz}\,. \tag{21}$$

For four values of $M_R = 1, 2, 3, 4$ curves of capacity versus M_T are given in Figure 10 for a spatial cycling MIMO structure. $P_{\text{out}} = 0.01$ and SNR =10 dB is considered. It is seen that using just two antennas at Tx ($M_T = 2$), an important increase can be obtained in capacity, as compared to ideal SIMO case (points of $M_T = 1$ on the figure). Figure 11 contrasts values of capacity versus M_R, for two cases of ideal SIMO system, and MIMO system with spatial cycling technique assuming $M_T = 2$. $P_{out} = 0.01$ and SNR = 0, 5, 10 dB are considered. Also, CCDF curves of spatial cycling MIMO with $M_T = 2$ are contrasted to other SIMO structures in Figure 4.

6.5. Suboptimal Detection

To see the importance of use of an appropriate detection method, consider a very simple realization of MIMO system. The same number of antennas is used at Tx and Rx, i.e., $M_T = M_R = M$ is considered. The transmitted signal components are independently encoded; each receiver decodes the signal of one special antenna, and nulls out all signals that receives from the other antennas. We will call this detection method as *independent detection*. In this way, the capacity will be the sum of capacities of underlying SISO subchannels.[21]

$$C = \sum_{i=1}^{M} \log_2 \left(1 + \frac{\rho_T}{M_T} |H_{ii}|^2 \right) \quad \text{bps/Hz}\,. \tag{22}$$

[20] Notice that the improvement achieved is in capacity-versus-outage, and the average capacity is obviously the same as in SIMO case.

[21] Here, we have supposed that the ith Rx decodes the signal of the ith Tx.

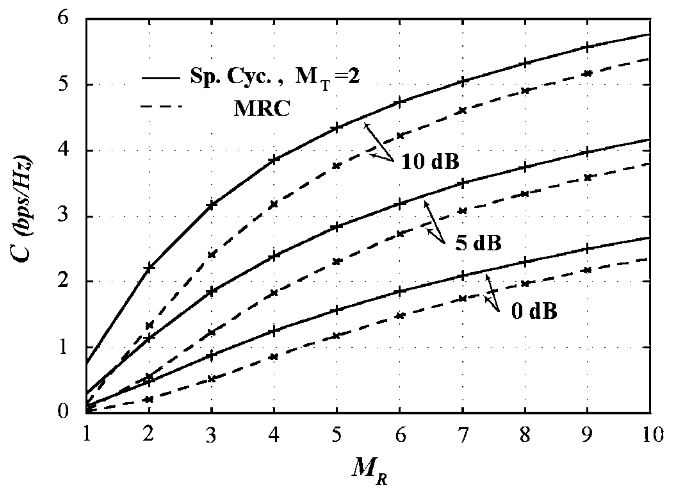

Figure 11. Capacity at $P_{out} = 0.01$ for two cases of SIMO-MRC and MIMO-Spatial cycling with $M_T = 2$.

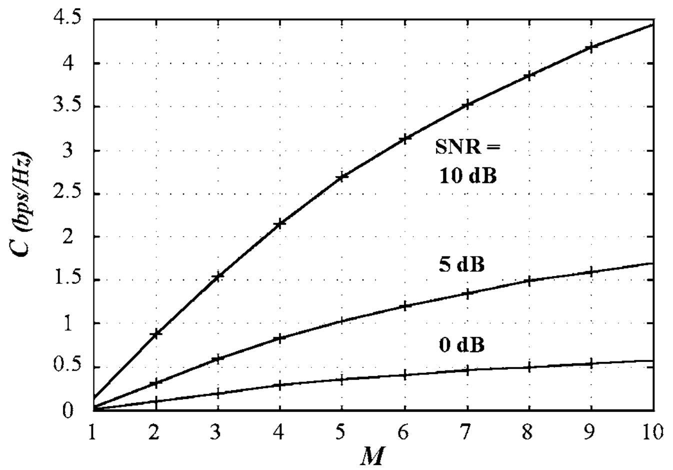

Figure 12. MIMO structure with independent detection; capacity curves versus number of antennas $M = M_R = M_T$, $P_{out} = 0.01$.

Curves of capacity versus M are shown in Figure 12 for SNR = 0, 5, 10 dB and $P_{out} = 0.01$. It is seen that although a MIMO structure is employed, the achieved capacity is not considerable.

Note that to achieve the capacity expressed in (22), different data rates should be used for each one of the underlying SISO subchannels. In practice, however, it may not be realizable. If equal bit rates are to be used for all transmit antennas, the capacity will be M times the minimum of capacities of SISO subchannels [5]. This results in an even smaller capacity-versus-outage.

In fact, as it was seen, there is no interest to employ this detection technique in practice, and our aim was just to show the necessity of use of a suitable detection technique.

6.6. Comparison with SISO

Figure 13 contrasts curves of capacity for ideal/suboptimal MIMO with $M_R = M_T = M = 2, 4$ and SISO structures. It is seen that the ideal MIMO capacity stands well superior to those

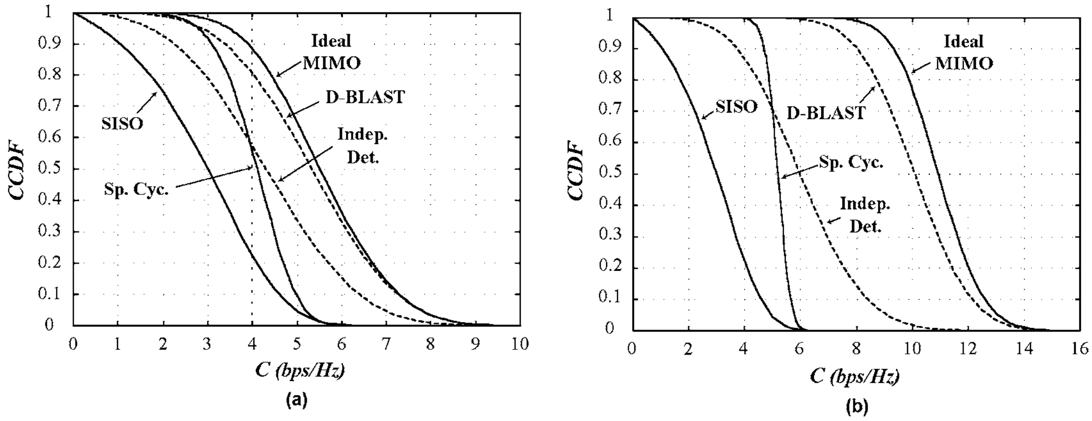

Figure 13. CCDF capacity curves for SISO and different MIMO structures: ideal, D-BLAST architecture, independent detection (Indep. Det.), spatial cycling (Sp. Cyc.) with $M_T = 2$; SNR =10 dB; (a) $M = 2$, (b) $M = 4$.

of the suboptimal structures. On the other hand, the small variance of capacity in the case of spatial cycling is noticeable ($M_T = 2$ is taken for this case).

7. Comparison between Ideal SIMO, MISO, and MIMO Capacities

Figure 14 contrasts capacities of different antenna array structures versus number of antenna elements employed. M on the figure represents M_T for MISO system, M_R for SIMO system, and $M_T = M_R$ for MIMO system (except for spatial cycling structure, where $M = M_R$ and $M_T = 2$). Ideal MISO structure is considered. For SIMO case, MRC and maximum selection detection techniques are taken. Also, for MIMO case, ideal detection, D-BLAST implementation, independent detection, and spatial cycling with $M_T = 2$ are considered. It is seen that the capacity of ideal MIMO stands well above those of MISO and SIMO structures.

As stated before, the D-BLAST architecture attains about 80% of the ideal MIMO capacity. The architecture of independent detection is practically of no interest; its capacity stands even below the MRC SIMO case.

It is seen that *for unknown channel at Tx*, the capacity of ideal MISO system stands below that of SIMO system. As it was explained in Section 5, the advantage of SIMO system comes from the constraint on transmitted power. Note that even a SIMO system with maximum selection has a greater capacity than an ideal MISO system.

8. Some Practical Aspects

A series of assumptions was made in Section 2 for obtaining the results of previous sections. Here, we will review each assumption and will discuss its rationality, as well as the effect on capacity when it is violated.

8.1. Fading Correlation and Antenna Elements Spacing

In our analyses we assumed independent fading for each pair of Tx-Rx antennas, i.e., independent entries for the channel matrix. However, in a real propagation environment, this

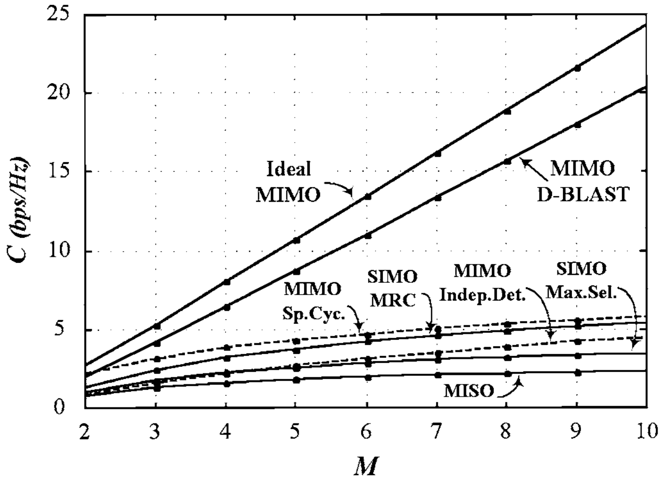

Figure 14. Contrasting capacities of multiple-antenna structures: ideal MIMO, MIMO with D-BLAST, MIMO with independent detection (Indep. Det.), MIMO with spatial cycling (Sp. Cyc.) with $M_T = 2$, ideal (MRC) SIMO, maximum selection SIMO (Max. Sel.), and ideal MISO; $P_{out} = 0.01$, SNR = 10 dB.

assumption is not completely satisfied and some correlation exists between fadings of different Tx-Rx antennas. In such a case, the capacity can be significantly smaller than in the independent fading case. Note that (4) and (6) hold also in the case of correlated fading. The effect of fading correlation on the capacity of SIMO and MIMO systems is discussed in [50] and [21], respectively. We will treat these two cases briefly in the following.

8.1.1. *SIMO Systems*

The independent fading assumption holds when multipath reflections are uniformly distributed around the receiver antennas that are spaced at least $\lambda/2$ apart (λ is the wavelength) [26, 50]. However, in some situations, signals arrive at the receiver antennas mainly from a given direction. There exist situations where the angle of arrival (AOA) approaches endfire (parallel to the array, considering a linear array) and the beamwidth of incident waves decreases. When the correlation is high (>0.8), the signals received on different antennas tend to fade at the same time, and the diversity benefit of the antenna array against fading is significantly reduced [50]. In such cases, antenna spacing must be increased in order to reduce correlation.

Consider the linear array at the receiver shown in Figure 15(a), where a local scattering is considered around the Tx [50]. ϕ is the AOA, δ the angle spread, and D_R the antenna spacing (ϕ and δ are considered almost the same for all array elements).

When signal arrives from other than the broadside, i.e. $\phi \neq 0$, the antenna spacing for low correlation increases, and the envelope correlation is never zero for almost all values of $\phi \neq 0$ and $\delta < 180°$. The required spacing is only a few λ even for small δ, unless ϕ is close to 90° [50].

8.1.2. *MIMO Systems*

Shiu et al. have assumed in [21] that one of the communicating parties (for example the Tx) is not obstructed by local scatterers, but the other one (the Rx) is surrounded by local scatterers. This case is shown in Figure 15(b). Here again, δ and ϕ are defined for whole transmit and receive arrays. As the angle spread δ approaches zero, the capacity of the (M, M) MIMO system approaches to that of $(1, M)$ SIMO system. The correlation between the columns of

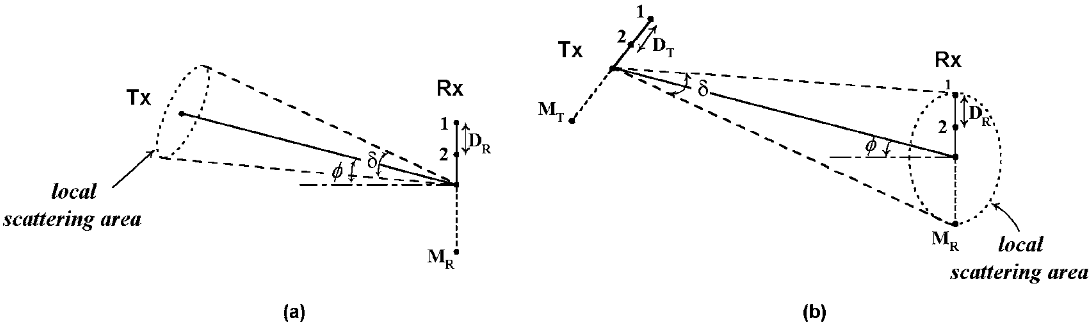

Figure 15. Scattering conditions considered for SIMO (a), and MIMO (b) systems, in [50] and [21], respectively.

$\boldsymbol{H}$ increases as δ decreases, and the disparity among $\lambda_{H,i}$ increases. Let D_T and D_R be the antenna spacing at the Tx and the Rx, respectively. It is shown that for a given $\delta \neq 180°$ at the receiver array, the capacity increases greatly as D_T increases. Equally, the capacity is increased by increasing D_R, but the increase is not as significant as when D_T is increased [21].

Reducing fading correlation may be a criterion on the choice of Rx antenna geometry. Under the conditions of correlated fading, For a fixed D_T, the capacity of a (M, M) broadside linear array is always higher than that of a (M, M) polygon array [21].

8.2. Channel Knowledge to Receiver and Quasi-Static Assumption

The analyses made in this paper assumed perfect channel knowledge at the Rx. This knowledge can be accomplished using channel estimation at the Rx, and to this purpose, some training sequences may be sent from Tx to Rx. On the other hand, we have assumed the quasi-static conditions for the channel, that is, the channel matrix $\boldsymbol{H}$ was assumed almost constant during one or more bursts.

If channel is not known to Rx, the presented expressions will be regarded as upper bounds for capacity. As $\Delta \to \infty$[22] the channel capacity approaches this upper bound, because with greater Δ, tracking the channel variations becomes more possible for the Rx [9]. Larger SNR values result in less difference between the capacity and the upper bound. For a fast varying channel, the capacity is far less than the perfect knowledge upper bound, because practically there is no possibility to estimate the channel at the Rx. Due to the same reason, the difference between the capacity and its upper bound increases for larger M_T and/or M_R [9].

It is also shown in [9] that for values of $M > \Delta$, no increase is achieved in capacity of MIMO channel by increase in M. Interested reader is referred to [15] for discussions on the case of unavailable CSI at Tx and Rx.

8.3. Effect of Water Filling on Capacity

As it was explained in Section 3, when the channel is known to the Tx, it can optimally allot the available power over the transmit antennas, so as to attain the known-CSI channel capacity. As shown in [51], for $M_R \geq M_T$, the increase in capacity achieved via WF may be of interest when small number of antennas are employed, and when low SNR is available at the Rx. The great interest of the WF solution is when $M_T > M_R$ [52].

[22] As defined in Section 2, Δ is the coherence interval of the channel in units of symbol period.

Notice that in order to provide the CSI for the Tx, an additional radio link needs to be established from the Rx to the Tx, so as to provide the channel estimation information for the Tx. From a practical point of view, this can be realized via a dedicated feedback channel or when the communication takes place in a duplex mode [14]. Note that such a feedback channel already exists in power control schemes currently implemented in some cellular standards [53]. In practice, vehicle movements or interference causes a mismatch between the state of the channel received by Tx and that estimated in Rx [43].

If the channel is frequency selective, WF should be performed in space and time [24, 54]. It is shown in [54] that the WF solution is much less interesting in this case, while the complexity of the realization is considerably greater.

Although WF at Tx may not be important regarding the increase in the channel capacity, in practice, knowledge of the channel can help us to perform beam forming on the transmitter array, and so, to send the available power in appropriate directions and to increase the channel capacity. This is not treated here, since we had assumed that no beam forming is performed at Tx.

8.4. Channel Dispersion

In fact, channel dispersion can be regarded as another source of diversity, which can be exploited to combat fading, and hence to increase capacity [15, 26]. Evidently, this statement assumes perfect equalization at the Rx. Using a spatio-temporal channel model, it is shown in [54] how channel dispersion can increase the channel capacity. Also, an explicit expression for capacity of MIMO frequency selective channels is given in [39] in terms of the frequency-domain correlation matrix of signals received on Rx antennas.

Concerning MIMO structures, they have firstly been envisaged to be used in non-dispersive media, i.e., media satisfying flat fading conditions, or to be used under OFDM (orthogonal frequency division multiplexing) signaling [17, 24]. That is because the equalization of MIMO channels is a very complicated task, and adds a non-negligible complexity to the system. Recent works have proposed the use of channel shortening filters in order to facilitate the task of channel equalization [55]. Also MMSE-DFE (decision feedback equalizer) equalization structures are studied in [56, 57].

With OFDM signaling, in the limiting case where the number of tones goes to infinity, the channel capacity approaches that of the underlying time-dispersive channel [58].

Notice that even under narrow-band signal transmission (large symbol duration relative to the channel delay spread), in practice some level of ISI is unavoidable due to the departure of the transmit and receive filters from their ideal Nyquist-based transfer functions [7, 42]. In fact, these filters have nonzero excess BW, and the resulting ISI should be cancelled.

8.5. Non-Rayleigh Fading Conditions

Analyses of previous sections assumed Rayleigh fading conditions, which is valid for a rich scattering wireless environment without any LOS between Tx and Rx. This is usually the case for media such as troposcatter, cellular, and indoor radio [4, 59]. Sometimes, there are few scatterers in the medium, and few multipaths contribute in signal propagation. For the number of multipaths L, at high SNR, the capacity increases almost linearly with $\min(M_T, M_R, L)$, which is the rank of $\boldsymbol{H}$ [24]. In the extreme case when there is no multipath ($L = 1$), $rank(\boldsymbol{H}) = 1$ and no transmit diversity can be exploited [61, 62].

In real propagation environments, there may exist a LOS between Tx and Rx, or fixed scatteres/signal reflectors may exist in addition to random main scatterers. In such cases, Ricean fading conditions hold [7].[23] It is shown in [61] that if the LOS contribution in signal propagation is not very significant, the increase in capacity by an increase in the number of antennas is still considerable.[24]

9. Multiuser Systems

Use of antenna arrays has recently been of special interest in multiuser systems. Although the current debate in these systems is on the use of TDMA (time division multiple access) or CDMA (code division multiple access) to achieve high capacity, a substantial additional gain in system capacity can be obtained by taking use of spatial diversity.[25] As the multiuser applications grow, the efficient use of spectral resources becomes more and more important.

The MIMO channel model is commonly used for many multiple access communication scenarios such as, DS-CDMA (direct-sequence), cellular mobile system with antenna array at the base station (BS), and multi-cellular system with joint multiuser detection [64].

Studying the capacity of these multi-user systems is beyond the scope of this paper, we try just to discuss these systems globally and to give some most recent important papers as reference for interested readers.

In multiuser systems, the capacity is usually given for two cases of *over-saturated* and *under-saturated* systems, where respectively, the number of users is greater than the system dimension and otherwise.

Most of models presented for multiuser systems are *basically* single-user models, in the sense that the interfering users in non-orthogonal accessing protocols are considered as additive noise (and usually Gaussian). To see the works performed in this approach, see the references in [65]. For a multiple-access model, the capacity of simple cellular systems is given in [66] for the case of a discrete-time Gaussian channel with cell-by-cell separate detection. The case of fading multiple-access channels is extensively studied in [65], where previous works can also be found in the references cited therein.

One special case is the case of RS-CDMA (random sequence) systems whose capacity is considered in [64, 67, 68]. In [68] the capacity is studied for different (suboptimal) detection techniques, and the gain resulting from optimal selection of random sequences is discussed. RS-CDMA with multiple antennas can be considered as to be equivalent to a multiuser system with antenna diversity and subject to channel fading [69].

Two important factors are the impact on capacity of the CSI information at Rx (which is affected by the imperfect channel estimation) [70], and the knowledge of Rx on the SNR of other interfering users [65].

Let us consider the case of cellular mobile radio and the use of multiple antennas in these systems in more detail. The general concept of application of antenna arrays in mobile com-

[23] Also, Nakagami fading model parameterized by the fading severity parameter m fits well to some urban multipath propagation data, and in particular to *microcellular* radio environments [60].

[24] Notice that LOS visibility is a desired parameter in radio mobile communication at high frequencies where the opacity of obstacles is high at high frequency bands and results in *shadowing*. If no LOS is available in general, fast power control algorithms and large dynamic ranges are necessary [63].

[25] In general, spread spectrum systems can be considered as systems which profit from frequency diversity. However, when the coherence BW of the channel is larger than the spreading BW (case of small channel delay spread), these techniques become ineffective in combating fading effect [33].

munication systems is extensively discussed in [1, 2]. Here, we will briefly discuss the case of cellular systems from the point of view of system capacity.

In cellular mobile radio, use of multiple antenna is suggested at the BS to achieve more system capacity, as well as to permit more interference suppression capability [26, 28, 71–73].

Of interest is the technique called *time division retransmission* proposed in [28, 71]. Using multiple antennas at the BS and a single antenna at mobile, the adaptive signal processing is performed at the base station where its cost can be amortized among many mobiles. During mobile-to-base transmission, the antenna elements weights are adjusted to maximize the SINR at the receiver output. During base-to-mobile transmission, the complex conjugate of the receiving weights are used, so that the signals from the base station antennas combine to enhance the reception of signal at the *desired mobile* and to reduce the power of this signal at other mobiles. In this way, as stated above, both the mobile and the base station receivers benefit from OC (and ideally, MRC) with the complexity and multiple antennas at the base station only [74].

Apart from the concept of capacity, an important interest of using multiple antennas is in interference cancelling. Assuming that the BS uses MRC for signal detection, it is known that using M_R antennas at the BS, we can profit from the nullification of N_i interferers, as well as $(M_R - N_i)$ diversity improvement against multipath fading [28]. This statement is also valid in the case of frequency selective fading, provided that ideal equalization is performed at the BS [28].

Study of MIMO systems in a cellular mobile concept is performed in [75].

At the BS, antennas are usually mounted above the *clutter*, and in order to have uncorrelated fading on the antennas, antenna spacings of the order of $10\lambda - 20\lambda$ (and even more, depending on the propagation conditions) should be used [26, 76]. A combination of space and polarization diversity may also be used, as considered in [77]. Notice that the interference cancelling capability of a multi-antenna BS holds even under the condition of completely correlated fading on the antenna elements [50]. So, antenna elements can be placed with small spacings, so as to permit an improvement in interference suppression, although they do not serve to reduce fading.

An interesting technique for limiting the adjacent cells' interference is to use sectorized antennas. For example using 120° beamwidth antennas, placed at three alternate corners of hexagonal cells, the number of possible interferers is reduced by a factor of three [71].

Antenna arrays can also be used in the concept of space-division multiple access (SDMA), where cells are divided into sectors (by means of highly directive antennas) in order to permit the reuse of the BW in the mutual interference-free sectors. In other words, each sector can be treated as a separate cell, and the frequency assignment may be performed in the usual manner. Mobiles are handled to the next sector as they leave the area covered by the current sector, as is done in a normal handoff process when mobiles cross the cell boundary [1].[26] Using sectorization, the problem of delay spread can be reduced noticeably too [74]. Capacity of mobile cellular radio SDMA systems is studied in [78].

[26] In total, increasing the system capacity by means of directive antennas results in a reduced required handoff rate in comparison to the conventional cell splitting technique.

10. Conclusion

The increasing demand for bandwidth in wireless networks may be satisfied by increasing the signal power and use of high gain antennas. However, in many applications, this is not the best solution, especially when we are faced to a randomly time-varying channel. Under such conditions, the most important techniques in providing reliable communication over wireless channels are diversity techniques, and of particular interest are the spatial diversity techniques. We have attempted in this paper to provide a clear image of the effect of using multiple antennas on the capacity of wireless communication systems. Three general structures of multi-antenna systems, i.e., SIMO, MISO, and MIMO structures were studied extensively, and several useful performance curves were provided. Moreover, the capacity of several suboptimal structures were studied and the increases in capacity of different structures with increase in the number of antennas were contrasted together.

The presented results were conditioned to some assumptions, particularly regarding channel fading. We have reviewed each assumption and explained its rationality in usual practical situations, as well as deteriorations in the presented results in the case of violation of the assumption.

If antenna elements are spaced sufficiently apart, use of multiple antenna elements at Rx is very effective in combating signal fading, as well as in interference cancellation when the channel BW is shared among several users. On the other hand, use of multiple antenna elements at both Tx and Rx makes it possible to attain high information rates in a rich scattering environment. In this case, the channel capacity can be considerably increased by adding antenna elements at both sides of the radio link. In fact, one can speak of multipath exploitation instead of multipath mitigation. Some key applications are fixed wireless and wireless LANs (Local Area Network).

The possibility of increasing the number of antennas in order to increase the channel capacity depends on the desired system complexity and cost, as well as the permitted size of Tx/Rx modules. Notice that use of multiple antennas requires circuitry in each diversity branch, resulting in an increased cost and power consumption, as well as the unit size. Use of higher frequencies may be a solution since it permits using smaller antennas and antenna spacings.

However, the serious problem is the system complexity, from the points of view of channel coding/decoding, detection, and synchronization. Especially, for MIMO systems, the problems such as efficient spatio-temporal coding and channel equalization/estimation, make its implementation very complex. Other complexities arise from timing errors, phase noise, and carrier frequency offset common in most wireless communication systems, which degrade the performance in practice. Fading correlation can also cause significant degradation in performance, as explained previously.

There stays still a large area of research on the implication of antenna arrays in future communication systems. Of particular is to improve the quality and the spectral efficiency of wireless systems by developing efficient modulation, coding, and signal processing techniques. Also, in multiuser systems, developing efficient techniques for sharing the available spectrum among different users, and particularly robust receiver design to confront the structured interference from other users of the multi-access channel, are important research subjects.

More specifically, about signal processing techniques in MIMO systems, current researches consider the implementation of MIMO wireless systems under OFDM signaling, the equalization of MIMO channels under delay-dispersive channel conditions, and more important, the elaboration of new efficient, yet computationally reasonable detection techniques and space-time codes in order to increase the spectral efficiency. Although the use of MIMO structures for communication between two points seems too complicated now, they are very interesting candidates for short future high bit rate communication systems.

Use of antenna arrays is also of special interest in the context of new generations of mobile systems, such as EDGE (Enhanced Data GSM Environment), IS-136, and UMTS (Universal Mobile Telecommunication System). The rising demand for personal communication services needs high data rate and high quality information exchange between portable terminals. New standards for the third generation (3G) of mobile systems which are very different from those of the current systems [63, 79] will necessitate the development of more efficient signal processing techniques.

It seems that more plausible trade space to satisfy the requirements of the 3G wireless systems are the base stations, rather than portable telephones. For the latter, there exist more limitations, particularly due to the limited size, and the requirement of powerful hardware for signal processing requirement. Also, efficient signal processing techniques which require significant processing power can not be used for low power devices, although advances in VLSI (Very Large Scale Integration) and integrated circuit technology for low power applications will provide a *partial* solution to this problem.

Use of multiple antenna systems is also an interesting subject in applications such as HF communication and submarine acoustics, where a serious limitation exists on the available channel bandwidth. Also, as pointed out previously, multiple antennas can be used in multi-access channels (TDMA or CDMA) as well as in multi-carrier systems to attain additional channel capacity and improvement in the system performance. Antenna arrays may also be used in geographically different locations, usually referred to as *macro-diversity* or distributed antenna systems, in order to combat larger scale fading effects.

On the whole, future wireless communication systems will perform a breakthrough in system performance, by taking use of antenna arrays at both sides of the communication link. Current researches focus on the design of new architectures that can take use of this potential capacity as much as possible. We conclude this paper with the statement of Marconi in 1932 that, “It is dangerous to put limits on wireless”.

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Appendix: Proof of Capacity Expressions

In this appendix we give the proofs of the capacity expressions given in (4) and (6) for the case of non-dispersive channel.

Let $\boldsymbol{R}_Z$, $\boldsymbol{R}_Y$, and $\boldsymbol{R}_n$ be the autocorrelation matrices of the vectors $\boldsymbol{z}$, $\boldsymbol{y}$, and $\boldsymbol{n}$, respectively, defined by (1). The input power constraint can be considered as $tr(\boldsymbol{R}_X) \leq P_T$ with $tr(.)$ the trace of matrix. The capacity of the Gaussian channel can be written as follows.

$$C = \max_{\text{pdf of } \boldsymbol{x}} [h(\boldsymbol{y})] - h(\boldsymbol{n}) \tag{23}$$

$h(.)$ is the differential entropy. The maximum of $h(y)$ is for Gaussian $\boldsymbol{y}$ which is achieved when $\boldsymbol{x}$ is Gaussian. Consequently [80],

$$C = \max_{\boldsymbol{R}_X} \log_2 \frac{\det(\boldsymbol{R}_Z)}{\det(\boldsymbol{R}_n)} = \max_{\boldsymbol{R}_X} \log_2 \frac{\det(\boldsymbol{R}_Y + \boldsymbol{R}_n)}{\det(\boldsymbol{R}_n)}\,. \tag{24}$$

Here det(.) denotes matrix determinant. As denoted, the maximization should be performed over $\boldsymbol{R}_X$. Consider the SVD of $\boldsymbol{H}$ as $\boldsymbol{H} = \boldsymbol{U}_H \boldsymbol{\Lambda}_H \boldsymbol{V}_H^{\dagger}$. We have

$$\boldsymbol{R}_Y = a^2 \boldsymbol{U}_H \boldsymbol{\Lambda}_H \boldsymbol{V}_H^{\dagger} \boldsymbol{R}_X \boldsymbol{V}_H \boldsymbol{\Lambda}_H^{\dagger} \boldsymbol{U}_H^{\dagger}\,. \tag{25}$$

On the other hand, $\boldsymbol{R}_n = \sigma^2 I$. So, the problem is to maximize $\det(\boldsymbol{R}_Y + \boldsymbol{R}_n)$, or equivalently to maximize

$$\det[a^2 \boldsymbol{\Lambda}_H \boldsymbol{V}_H^{\dagger} \boldsymbol{R}_X \boldsymbol{V}_H \boldsymbol{\Lambda}_H^{\dagger} + \sigma^2 \boldsymbol{I}] = \det(\boldsymbol{B})\,, \tag{26}$$

where we have named the matrix in the brackets as $\boldsymbol{B}$. We know that for a positive defined square matrix $\boldsymbol{A}$ we can write $\det(\boldsymbol{A}) \leq \prod_i A_{ii}$. In other words, for a family of matrices $\boldsymbol{A}$ with the same diagonal entries of A_{ii}, the maximum determinant is obtained for the diagonal matrix. So, to maximize $\det(\boldsymbol{B})$, we choose $\boldsymbol{R}_X = \boldsymbol{V}_H \boldsymbol{P}_X \boldsymbol{V}_H^{\dagger}$, with $\boldsymbol{P}_X$ a diagonal matrix with elements of $\lambda_{X,i}$ (positioned on the matrix diagonal in descending order). In this way, $\lambda_{X,i}$ will be the eigenvalues of the matrix $\boldsymbol{R}_X$.

If no CSI is available, the reasonable choice is to allot the available power equally over the transmit antennas, that is, to take equal $\lambda_{X,i}$. In this way, $R_X = \frac{P_T}{M_T}\boldsymbol{I}$ with $\boldsymbol{I}$ is the identity matrix. With this choice, (24) reduces to the expression of (4). Notice that this choice of $\boldsymbol{R}_X$ implies independent Gaussian signals on transmit antennas.

If CSI is provided at the transmitter, we have,

$$\det(\boldsymbol{B}) = (\sigma^2)^{|M_T - M_R|} \prod_{i=1}^{M} (a^2 \lambda_{X,i}\, \lambda_{H,i}^2 + \sigma^2)\,, \tag{27}$$

where $|\cdot|$ denotes the absolute value operator, and $M = \min(M_T, M_R)$. Now to further maximize $\det(\boldsymbol{B})$, we can apply the method of Lagrange multipliers to the right side of (27).

$$f = \prod_{i=1}^{M} (a^2 \lambda_{X,i} \lambda_{H,i}^2 + \sigma^2) + \theta \left(\sum_{i=1}^{M_T} \lambda_{X,i} - P_T \right)\,; \quad \frac{\partial f}{\partial \lambda_{X,i}} = 0\,.$$

It can be easily shown that this maximization results in the WF solution given by (6) and (7).

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