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## DEVELOPMENT OF FUZZY SLIDING MODE CONTROLLER FOR DECOUPLED INDUCTION MOTOR DRIVE

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*Abstract :* Robust control of a decoupled induction motor drive is attempted in this paper. Sliding Mode Controller and Fuzzy Sliding Mode Controller are designed for the speed loop of the drive. The design steps for both the controllers are laid down clearly. Only four fuzzy rules, and Center-of-Sums defuzzification technique are used in the fuzzy controller for simplicity reasons. The performance of the fuzzy sliding mode controller has been evaluated, through simulation studies, with respect to the sliding mode controller in order to establish its suitability for induction motor drive.

### 1. INTRODUCTION

Induction Motors (IM) are most suitable for industrial drives, because of their simple and robust structure, higher torque-to-weight ratio, higher reliability and ability to operate in hazardous environment. However, their control is a challenging task, because the rotor current, responsible for the torque production, is induced from the stator current and also contributes to net air-gap flux resulting in coupling between torque and flux. The decoupling control or vector control of IM as proposed by Blaschke [1], leads to decoupling between the flux and torque, thus, resulting in improved dynamic torque and speed responses. For the systems, where model imprecision, parameter fluctuations and noise exist, for them *sliding mode control* is an appropriate robust control method. The sliding mode control is especially appropriate for the tracking control of robot manipulators and also for motors whose mechanical load change over a wide range. The induction motor drive as a plant is non-linear with imprecise model. Therefore, sliding mode controller

is expected to be a better choice. Benchaib et al. [2] have presented the comparative performance of a sliding mode and an input-output linearizing control scheme for a field oriented induction motor drive. Lin et al. [3] have developed a robust P-I control scheme with an observer based on model reference adaptive system for a speed-sensorless induction motor drive under direct field oriented control. Shieh and Shyu [4] have applied the Sliding Mode Control philosophy for the torque control with adaptive back stepping. In another interesting application, Park and Lee [5] have combined the theory of input-output linearization and sliding mode control to develop an integrated controller for an induction motor drive under field-oriented control. It has also been proved that in principle, a *fuzzy logic controller* (FLC) works like a modified sliding mode controller [6].

This paper investigates the applicability of fuzzy sliding mode controller to a field oriented induction motor drive. Systematic procedure is developed to design sliding mode controller and fuzzy sliding mode controller, and a comparative study is carried out.

## 2. SLIDING MODE CONTROLLER DESIGN

The dynamic equations of the induction motor in the arbitrary rotating d-q reference frame, with stator current and rotor flux components as variables is considered. The classical vector control or decoupling control requires that:

$$\Psi_{qr} = 0 \quad \text{and} \quad \dot{\Psi}_{qr} = 0 \quad (1)$$

Equation (1) is satisfied and decoupling obtained, when [7] :

$$\omega_e = P \omega_r + a_5 i_{qs} / \Psi_{dr} \quad (2)$$

When eqn. (2) is satisfied, the dynamic behavior of the induction motor is:

$$\dot{i}_{ds} = -a_1 i_{ds} + a_2 \Psi_{dr} + \omega_e i_{qs} + c v_{ds} \quad (3)$$

$$\dot{i}_{qs} = -\omega_e i_{ds} - a_1 i_{qs} - P a_3 \omega_r \Psi_{dr} + c v_{qs} \quad (4)$$

$$\dot{\Psi}_{dr} = -a_4 \Psi_{dr} + a_5 i_{ds} \quad (5)$$

$$T_e = K_t \Psi_{dr} i_{qs} \quad (6)$$

where,  $c = L_r / (L_s L_r - L_m^2)$ ,

$$a_1 = c R_s + c R_r L_m^2 / L_r^2, \quad a_2 = c R_r L_m / L_r^2,$$

$$a_3 = c L_m / L_r, \quad a_4 = R_r / L_r, \quad a_5 = R_r L_m / L_r$$

Equations (1-6) are considered for the design of sliding mode controller (SMC) and fuzzy sliding mode controller (FSMC) to control the torque and hence speed of the induction motor drive.

In sliding mode control, the system is controlled in such a way that the error in the system state (say, speed) always moves towards a sliding surface. The sliding surface (s) is defined with the tracking error (e) of the state and its rate of change ( $\dot{e}$ ) as variables.

$$s = \dot{e} + \lambda e \quad (7)$$

The distance of the error trajectory from the sliding surface and its rate of convergence are used to decide the control input. The sign of the control input must change at the intersection of tracking error trajectory with the sliding surface. In this way the error trajectory is forced to move always towards the sliding surface. Once it reaches the sliding surface, the system is constrained to slide along this surface to the equilibrium point. The condition of sliding mode [8] is:

$$\dot{s} \cdot \text{sgn}(s) \leq -\eta \quad (8)$$

To design a sliding mode speed controller for the decoupled drive system, the steps are as follows. The speed dynamic equations are given by:

$$\dot{\omega}_r = g_1 + (T_L / J) \quad (9)$$

$$\text{and, } \ddot{\omega}_r = G + u + d \quad (10)$$

where, u is the control input given by:

$$u = K_T \Psi_{dr}^* c v_{qs} / J \quad (11)$$

G is a function, which can be estimated from measured values of currents and speed.

$$G = (-\beta g_1 + K_T \Psi_{dr}^* g_2) / J \quad (12)$$

$$g_1 = (-\beta \omega_r + K_T \Psi_{dr}^* i_{qs}) / J$$

$$g_2 = -(a_1 + a_4) i_{qs} - P \omega_r (1 + a_3 L_m) i_{ds}$$

In eqn. (10), d is the disturbance due to the load torque, and error in estimation of G, which may occur due to measurement inaccuracies.

Substituting (7), and (10) in (8) and simplifying

$$(G + d + \lambda \dot{e} - \ddot{\omega}_r^*) \text{sgn}(s) + u \cdot \text{sgn}(s) \leq -\eta \quad (13)$$

To achieve the sliding mode of (8), u is chosen as [8]

$$u = (-\hat{G} - \lambda \dot{e}) - K \cdot \text{sgn}(s) \quad (14)$$

The first term in (14),  $(-\hat{G} - \lambda \dot{e})$  is a compensation term and the second term is the controller. The compensation term is continuous and reflects knowledge of the system dynamics. The controller term is discontinuous and ensures the sliding to occur. From eqns. (13-14), the controller gain, K is derived as [8]

$$K_{\max} \geq (|\Delta G_{\max}| + |d_{\max}| + \eta + v) \quad (15)$$

The controller gain, K is determined using (15) and considering various conditions such as:

- (i) increase in stator and rotor resistance due to temperature rise
- (ii) change in load torque
- (iii) variation in the reference speed

For the induction motor whose rating and parameters are given in Table-1, taking a typical case as (i) 50% increase in stator and rotor resistance, (ii) change in load torque by 10 N·m in 50 ms (rated torque is 5 N·m), (iii) 50% change in reference (base) speed in 50 ms, the controller gain,  $K_{\max}$  is obtained as

$$K_{\max} = 56000 \text{ rad/s}^3$$

In a system, where modeling imperfection, parameter variations and amount of noise are more, the value of K must be large to obtain a satisfactory tracking performance. But larger value of K leads to more chattering of the control variable and system states. To reduce chattering, a boundary layer of

width  $\varphi$  is introduced on both sides of the switching line. Then the control law of (14) is modified as:

$$u = -\hat{G} - \lambda \dot{e} - K \cdot \text{sat}(s/\varphi) \quad (16)$$

$$\text{where, } \text{sat}(s/\varphi) = \begin{cases} s/\varphi & \text{if } |s| \leq \varphi \\ \text{sgn}(s) & \text{if } |s| > \varphi \end{cases}$$

This amounts to a reduction of the control gain inside the boundary layer and results in a smooth control signal.

$$\text{The tracking precision is given by: } \theta = \varphi / \lambda \quad (17)$$

To have a tracking precision,  $\theta = 1$  rad/s,  $\varphi = \theta \lambda = \lambda$ .

$$K_{\max} = \varphi \lambda = \lambda^2 \quad (18)$$

$$\lambda = \sqrt{K_{\max}} = \sqrt{56.0 \times 10^3} = 236.6 \text{ rad/s}$$

$$\text{and } \varphi = \theta \lambda = 236.6 \text{ rad/s}^2$$

Table – 1 Rating and Parameters of the Induction Motor

Three phase, 50 Hz, 0.75 kW, 220V, 3A, 1440 rpm
Stator and rotor resistances: $R_s = 6.37 \Omega$ , $R_r = 4.3 \Omega$
Stator and rotor self inductances: $L_s = L_r = 0.26 \text{ H}$
Mutual inductance between stator and rotor: $L_m = 0.24 \text{ H}$
Moment of Inertia of rotor and load: $J = 0.0088 \text{ Kg} \cdot \text{m}^2$
Viscous friction coefficient: $\beta = 0.003 \text{ N} \cdot \text{m} \cdot \text{s/rad}$

### 3. FUZZY SLIDING MODE CONTROLLER DESIGN

The fuzzy sliding mode controller (FSMC) explained here is a modification of the sliding mode controller (eqn. (14)), where the switching controller term,  $-K \cdot \text{sgn}(s)$ , has been replaced by a fuzzy control input as given below.

$$u = (-\hat{G} - \lambda \dot{e}) + u_{\text{Fuzz}} \quad (19)$$

$$\text{and } u_{\text{Fuzz}} = -K_{\text{Fuzz}}(e, \dot{e}, \lambda) \text{sgn}(s) \quad (20)$$

The gain,  $K_{\text{Fuzz}}$  of the controller is determined from fuzzy rules. The qualitative rules of the fuzzy sliding mode controller are as follows.

- The normalized fuzzy output,  $u_{\text{Fuzz|N}}$  should be negative above the switching line, and positive below it.

- $|u_{\text{Fuzz|N}}|$  should increase as the distance,  $d_1$  between the actual state and the switching line,  $s = 0$ , increases. The distance,  $d_1$  is given by

$$d_1 = \frac{|s|}{\sqrt{1+\lambda^2}} = \frac{|\lambda e + \dot{e}|}{\sqrt{1+\lambda^2}} \quad (21)$$

- $|u_{\text{Fuzz|N}}|$  should increase as the distance,  $d_2$  between the actual state and the line perpendicular to the switching line increases. The distance,  $d_2$  between the actual state and the line perpendicular to the switching line, is:

$$d_2 = \sqrt{e^2 + \dot{e}^2 - d_1^2} \quad (22)$$

The reasons for this rule to be followed are:

- (a) the discontinuities at the boundaries of the phase plane are avoided.
- (b) the central domain of the phase plane is arrived at very quickly.

- Normalized states,  $e_N, \dot{e}_N$  that fall out of the phase plane should be covered by the maximum values,  $|u_{\text{Fuzz|N}}|_{\max}$  with the respective sign of  $|u_{\text{Fuzz|N}}|$ .

The normalized distances,  $d_{1N}$  and  $d_{2N}$  are:

$$d_{1N} = N_1 d_1 \quad \text{and} \quad d_{2N} = N_2 d_2$$

These normalized inputs ( $d_{1N}$  and  $d_{2N}$ ) to the fuzzy controller are fuzzified by two-member fuzzy set :

$$\{ \mathbf{Z}: \text{Zero}, \quad \mathbf{P}: \text{Positive} \}$$

The fuzzy set for normalized controller gain (output of the fuzzy controller),  $K_{\text{Fuzz|N}}$  (also denoted as  $K_N$  for brevity) is :

$$\{ \mathbf{Z}: \text{Zero}, \quad \mathbf{P}: \text{Positive}, \quad \mathbf{LP}: \text{Large Positive} \}$$

The membership functions for the normalized inputs are shown in Fig. 1-a, and those for the normalized output are shown in Fig. 1-b. Linear and symmetrical membership functions are used for ease of realization. Only two-member input sets and three-member output set are chosen, based on engineering experience, so as to have a simple fuzzy controller consisting of four fuzzy rules. The rules are listed in Table 2.

Table – 2 Fuzzy Rules

Rules	$d_{1N}$	$d_{2N}$	$K_{\text{Fuzz N}}$
1	Z	Z	Z
2	Z	P	P
3	P	Z	P
4	P	P	LP

The inference engine performs fuzzy implications, and computes the degree of membership of the output (normalized controller gain) in each fuzzy set using Zadeh AND and OR operations. Then defuzzification is carried out by the Center-of-Sums method as given in eqn (23).

$$\bar{K}_{\text{Fuzz|N}} = \frac{\sum_i^{K_{2,i}} \int_{K_{1,i}} \mu_{\text{out}} \cdot K_N \cdot dK_N}{\sum_i^{K_{2,i}} \int_{K_{1,i}} \mu_{\text{out}} \cdot dK_N} \quad (23)$$

The defuzzified value,  $\bar{K}_{\text{Fuzz|N}}$  is denormalized with respect to the corresponding physical domain,  $K_{\text{Fuzz}}$  by the denormalization factor,  $N_u$ .

$$N_u = \frac{K_{\text{Fuzz|max}}}{\bar{K}_{\text{Fuzz|N|max}}} \quad (24)$$

where,  $\bar{K}_{\text{Fuzz|N|max}}$  is the maximum value of defuzzified (but normalized) controller gain, and  $K_{\text{Fuzz|max}}$  is the maximum value of the controller gain,  $K_{\text{Fuzz}}$ .

Since the sliding mode controller and the fuzzy sliding mode controller described in this paper, are structurally similar, the maximum gain  $K_{\text{Fuzz|max}}$  is taken equal to the gain of the sliding mode controller,  $K_{\text{max}}$ , so that comparison of both can be made under similar conditions.

$$K_{\text{Fuzz|max}} = 56000 \text{ rad/s}^3$$

For  $N_1 = N_2 = 0.1$ , and the above value of  $K_{\text{Fuzz|max}}$ , the denormalization factor is obtained as,  $N_u = 70000$ .

#### 4. RESULTS AND DISCUSSIONS

The 3-phase induction motor drive system whose rating and parameters given in Table – 1, is subjected to various simulation tests with both the above controllers.

The simulation study is carried out with a ramp (linear) change in reference speed. The reference speed is linearly increased from 1000 r/min to 1500 r/min in 100 ms, i.e., at a rate 5 (r/min)/ms. The reference d-axis rotor flux linkage is kept at 0.45 V·s, and load torque is kept at zero. The simulation responses of the drive system with sliding mode controller (SMC) are shown in Fig. 2 and those with fuzzy sliding mode controller (FSMC) are shown in Fig. 3. Though the responses with FSMC are

generally similar to those with SMC, the q-axis stator voltage increases from initial steady state value of 104 V to final steady state value of 156 V with a peak value of 203 V in SMC and 194 V in FSMC during the transient period. The responses of q-axis stator voltage and current, and control input ( $u_c$ ) have chattering in SMC, but are free of chattering in FSMC. The q-axis component of stator voltage and current are only affected as they control the torque and hence speed. The decoupling action by both the controllers is apparent, as the d-axis stator current and rotor flux remain constant.

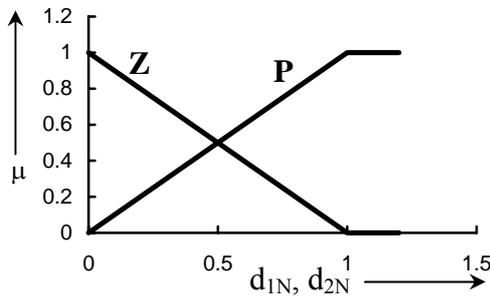
To see the chattering-free robust responses of FSMC, the load torque is suddenly increased from 0 to 5 N·m (rated torque) and then the load is removed after 1 sec. With both SMC (Fig. 4) and FSMC (Fig. 5), there is an instantaneous speed change of 13 r/min during the change of load. But the drive system recovers to the reference speed of 1000 r/min almost instantaneously. With SMC, the response of q-axis stator current ( $i_{qs}$ ), the q-axis stator input voltage ( $v_{qs}$ ), and the control input ( $u$ ) have chattering, during the load period. But no such chattering is present in case of FSMC.

#### 5. CONCLUSIONS

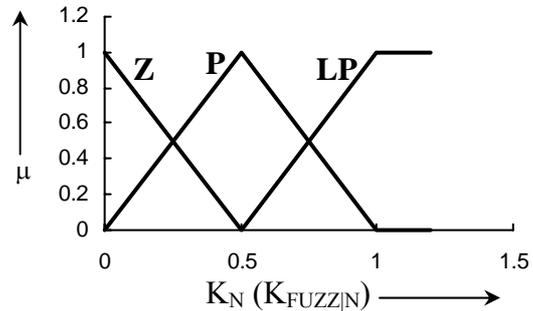
Sliding mode and fuzzy sliding mode controllers are designed for a decoupled induction motor drive, to have the same maximum controller gain. From the simulation study, it is observed that the control input, the stator input voltage, and some of the states like speed and stator current components have chattering with sliding mode controller, whereas these are free of chattering with fuzzy sliding mode controller. For the same maximum gain with both the controllers, the speed response is also the same, but the stator input voltage is less in case of FSMC compared to SMC. In other words, with fuzzy sliding mode controller the maximum gain can be increased at the cost of increased stator input voltage leading to better speed response. So, for chattering free, robust control of decoupled induction motor drive, fuzzy sliding mode controller is a better choice than sliding mode controller.

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(a) Input membership functions



(b) Output membership functions

Fig. 1 Fuzzy set membership functions for normalized inputs and output

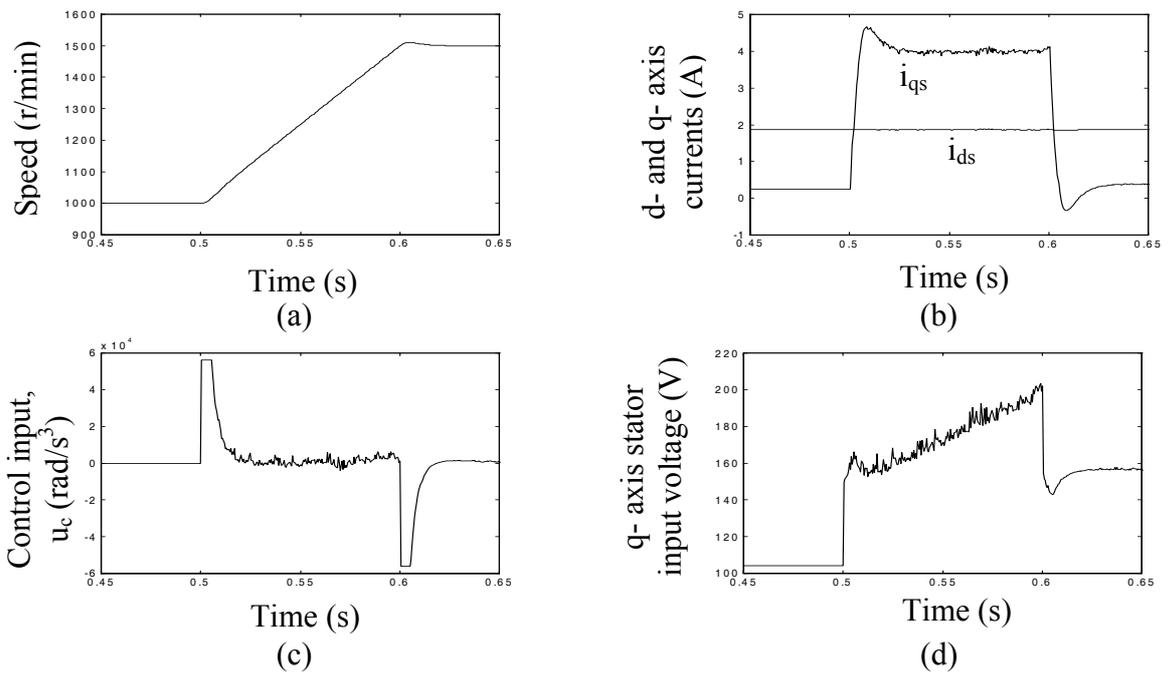


Fig. 2 Ramp (linear) change in reference speed with SMC: (a) Speed, (b) d- and q- axis stator currents, (c) Control input, (d) q- axis stator input voltage

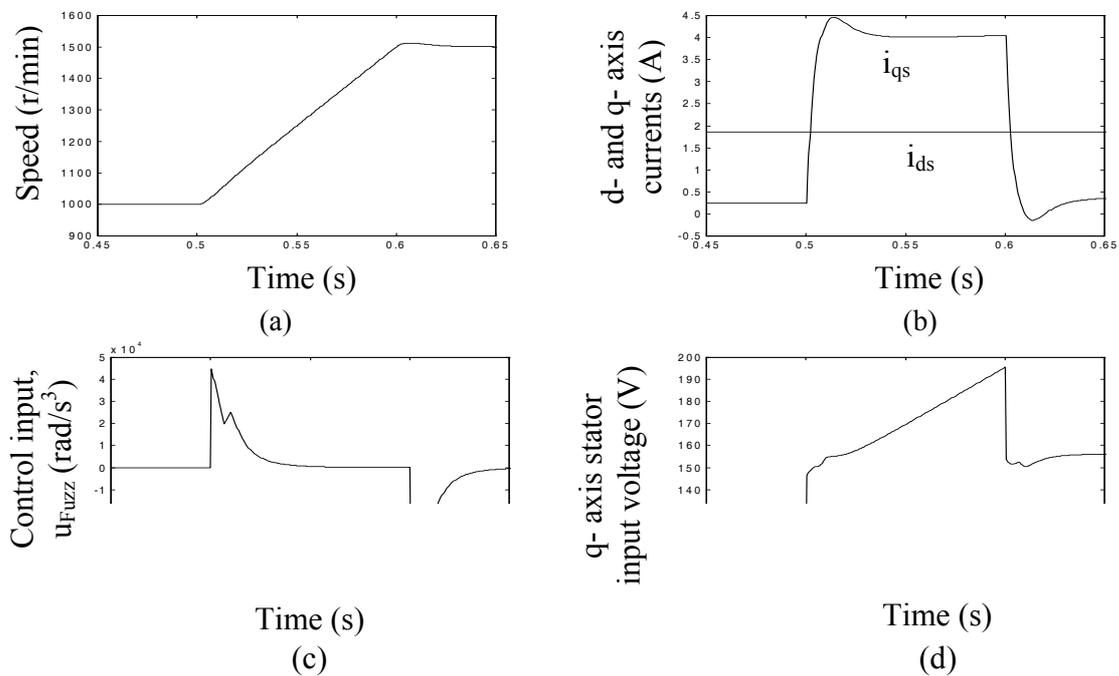


Fig. 3 Ramp (linear) change in reference speed with FSMC: (a) Speed, (b) d- and q- axis stator currents, (c) Control input, (d) q- axis stator input voltage

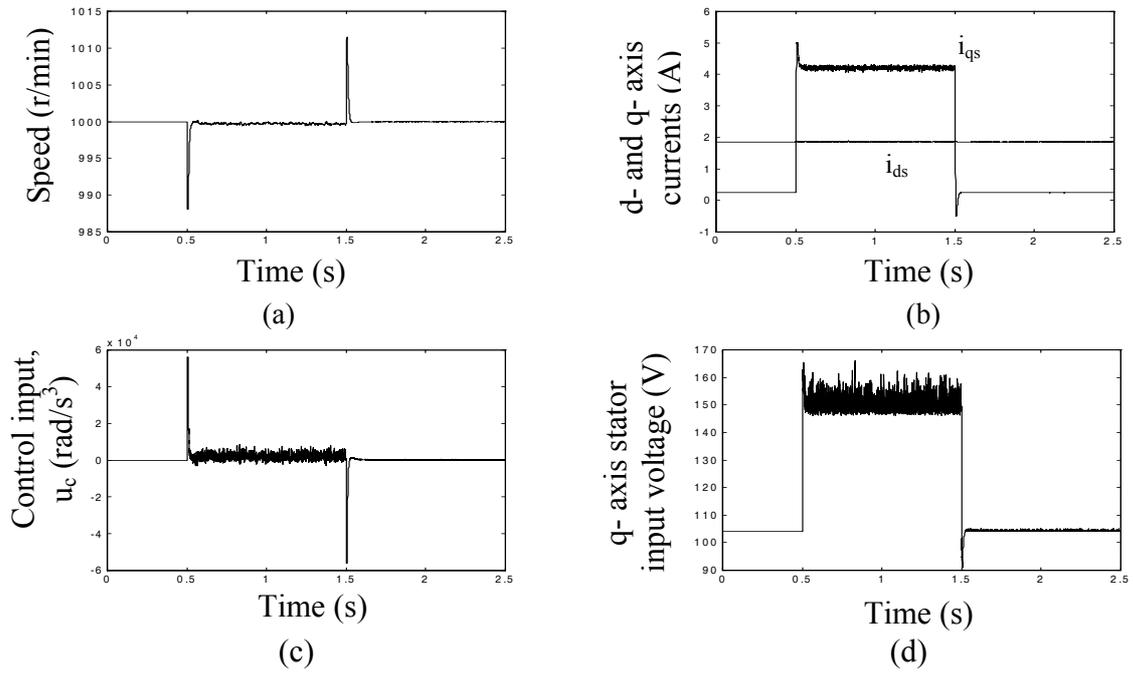


Fig. 4 Step changes in load torque with SMC: (a) Speed, (b) d- and q- axis stator currents, (c) Control input, (d) q- axis stator input voltage

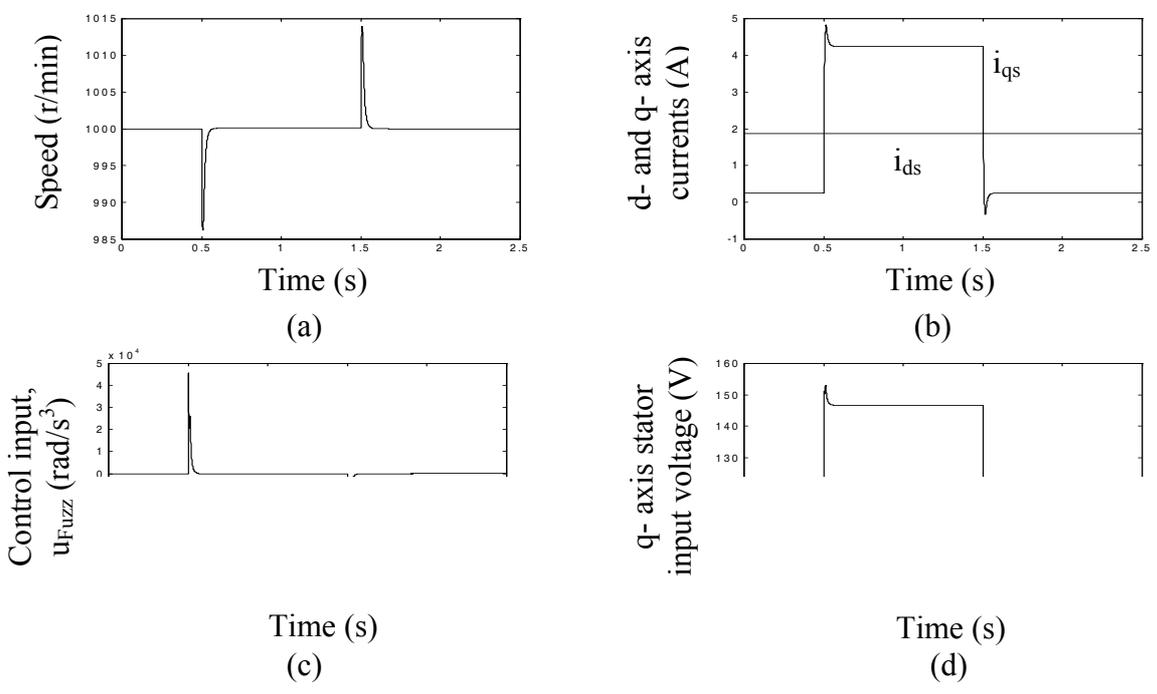


Fig. 5 Step changes in load torque with FSMC: (a) Speed, (b) d- and q- axis stator currents, (c) Control input, (d) q- axis stator input voltage