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Performance Analysis of Directional Beacon based Position Location Algorithm for UWB Systems

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Abstract— **We propose an enhanced directional beacon based position location procedure for UWB systems. Unlike previous work that relies on strongest return and therefore prone to errors in obstructed line of sight (OLOS) environments, our procedure identifies the LOS return by detecting the earliest arrival. To overcome synchronization problems, we propose a correlation based window algorithm to detect the earliest arrival and hence the LOS component across a 360° rotation of directional beacon. We performed a detailed and exact analysis of the algorithm that has not yet reported in the literature. The analysis quantifies the dependence of the algorithm on the variance of noise and the beam pattern. The analysis also motivated us to mitigate the effect of noise. We, therefore, propose an improved algorithm and implement it in a computationally efficient way. To improve position estimation we perform a three point search around the estimated time index n and selects the point that minimizes l_2 norm of a performance measure vector. From simulation results we claim position location accuracy of around 30 cm with a pragmatic 4-element antenna array using 2.4GHz UWB pulses and 4-bit A/D converter at 28dB SNR.**

I. INTRODUCTION

Wireless sensor networks – characterized by small, and low power devices (nodes) equipped with limited sensing, computing, and communication capabilities – is an emerging paradigm in wireless communications [1]. In a number of application domains, the sensor nodes have to be aware of their spatial location. For instance, in environmental monitoring applications such as air or water quality monitoring, sensing the data without knowing the sensor location is meaningless. Similarly in military applications, motion detection is of no use without the spatial location of the sensing node. Therefore position location is central to several of wireless sensor network applications.

During the past three years there has been an increasing interest in ultra wideband (UWB) based communication systems. In particular, UWB systems have potentially low complexity and low cost; are resistant to multipath propagation; and have very good time domain resolution, allowing for location and tracking applications. These inherent properties of UWB systems make them a promising choice for wireless sensor networks [2].

The position of a desired node can be determined in a variety of ways, such as Angle of Arrival (AOA), Time of Arrival (TOA), Time Difference of Arrival (TDOA), or Received Signal Strength (RSS) [3]. The fine time resolution of UWB signals makes it more appropriate for position location systems. UWB signals have been employed in position location systems [4], [5], [6], [7]. Lee and Scholtz [4] proposed time of arrival based ranging technique using a UWB radio link. The ranging scheme utilizes generalized maximum likelihood and implements a search algorithm for the detection of direct path. In [5], a UWB based ranging method that utilizes TOA of the strongest path is investigated. We showed in our previous work [8] that ranging based on strongest path incurs severe errors in obstructed LOS environments. Use of directional beacons with UWB for position location was first evaluated by Chung *et. al* [6]. Multipath returns are poorly handled in the scheme and could lead to erroneous position location in realistic UWB channels.

Our work is different from [6] in several aspects. Firstly, our scheme looks for the earliest arrival in order to determine the LOS component and does not rely on strongest arrival that deteriorates the performance, in general. Secondly, we perform a detailed mathematical analysis of the algorithm that has not been reported in the literature earlier. The analysis of the algorithm reveals some important aspects of the algorithm and suggests ways to improve the algorithm. Thirdly, based on the analysis, we propose a computationally efficient enhancement to the algorithm that leads to improve position location estimation.

The paper is organized as follows: Section II gives system details and outlines the localization principle. The proposed algorithm is discussed in Section III followed by its detailed mathematical analysis in Section IV. An improved algorithm is discussed in Section V. Finally, Section VI provides a numerical example and the simulation results. We conclude the paper in Section VII.

II. SYSTEM MODEL AND LOCALIZATION PRINCIPLE

A. The System Model

This section provides the details of the system model and localization principle used for UWB signals. Consider a situation where we need to obtain the position of node Q surrounded by three reference nodes RN-1, RN-2 and RN-3. In this paper, we are interested in determining the coordinates of Q, (x, y), relative to a specified origin. The reference nodes are capable of generating a UWB pulse periodically

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that will be transmitted using a directional antenna. The signal from reference nodes can be considered as *beacon* in radar terminology. We consider UWB pulses in the form of Gaussian

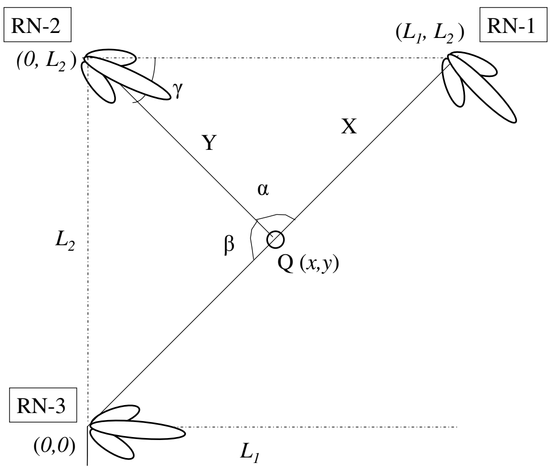

Fig. 1. Arrangement of three beacon nodes

monocycles and implement directional beam using M-element uniform linear antenna array as this is the simplest array structure to produce directional beams.

B. Localization Principle

The beacon signals are designed to enable node Q or any other node to determine its angular bearing with respect to the reference nodes. For this purpose, the reference nodes need to change their beam direction such that the beam from each reference node will be pointed to node Q once in a complete rotation. The localization principle is based on observing the times when node Q receives beacon signals from different reference nodes, and evaluating its angular bearings and location with respect to the reference nodes by triangulation [9]. If the times at which Q receives beacons from RN-1, RN-2 and RN-3 are t_1, t_2 and t_3, respectively, the bearings can be obtained as:

$$\begin{aligned} \alpha &= \phi - \omega(t_2 - t_1) \\ \beta &= \phi - \omega(t_3 - t_2) \end{aligned} \tag{1}$$

where ω is the angular speed of the rotating directional beam in degrees/s and ϕ is a constant angular separation between the reference nodes. Using simple trigonometry, it can be shown that the coordinates (x, y) of the node Q can be computed as

$$\begin{aligned} x &= \frac{L_2 \cos\gamma}{\sin\beta}\cos(\beta - \gamma) \\ y &= \frac{L_2 \cos\gamma}{\sin\beta}\sin(\beta - \gamma) \end{aligned} \tag{2}$$

where

$$\gamma = \tan^{-1}\left(\frac{L_2 \cot\beta - L_1}{L_1 \cot\alpha - L_2}\right)$$

III. Proposed Algorithm

The proposed algorithm requires the detection of direct path component to mark the instant when the transmitting beam is aligned with the receiver. Due to the very nature of indoor multipath propagation environment, the direct path component gets either deteriorated or completely blocked [3]. The former situation is commonly termed as obstructed LOS (OLOS) and is the focus of our interest in this paper. The latter case with the proposed algorithm was considered by the authors in their earlier work [8].

To detect the direct path component in OLOS environment - where the strongest arrival is not necessarily direct path - we rely on earliest arrival in case of perfect synchronization. To perform synchronization, we propose a correlation based window algorithm to detect the earliest arrival. Let's consider each case individually.

A. With perfect synchronization

The algorithm details for the system in Fig. 1 are as follows:

Step 1. Reference node RN-1 will transmit a UWB pulse s_{φ_i} while it's main beam is at angle φ_i measured from some reference.

Step 2. The receiver (node Q) receives a train of pulses due to multipath propagation environment and uses a matched filter to detect the transmitted pulse. The matched filter output at discrete time index k is given by:

$$z_{\varphi_i}(k) = h_i\, s_{\varphi_i}(k) + \eta_i(k)$$

where h_i is the channel gain for the transmitted pulse s_{φ_i} and η denotes additive white Gaussian noise (AWGN).

Step 3. With perfect synchronization, the earliest arrival r_i from RN-1 at angle φ_i is the first detected peak, *i.e.*,

$$r_i := z_{\varphi_i}(k)\big|_{k=1} \tag{3}$$

Step 4. Rotate the beam of RN-1 by $\Delta\varphi$ and repeat *Steps 2-3*. The rotation step $\Delta\varphi = \varphi_i - \varphi_{i-1}$ is so called angular resolution of the directional beacon.

Step 5. Store the time and amplitude information of earliest arrivals across 360° rotation of beam of RN-1. This will generate a sequence $\{r_i\}_{i=1}^{N}$ where N is the total steps needed by RN-1 to make one complete rotation.

Step 6. Determine the maximum absolute value of r_k that represents the instant when the beacon from RN-1 is aligned with the receiver, *i.e.*,

$$n = \underset{k}{\arg\max}\,|r_k| \quad \text{and hence} \quad t_1 = nT_r$$

where $T_r = \Delta\varphi/\omega$ is the time taken by beam to rotate by angle $\Delta\varphi$.

Step 7. Repeat the above steps for reference nodes RN-2 and RN-3 to obtain t_2 and t_3, respectively. Finally, use (1) and (2) to obtain the coordinates of node Q.

B. With imperfect synchronization

The receiver is said to be synchronized if the first arrival is received at $n = 1$. In order to synchronize, we need to find the length of time-slot that corresponds to the reception of all the returns for a single pulse transmission from RN-1. Let W denotes the length of the time slot that has support in discrete time $n \in [n_a, n_b]$ such that $W = n_b - n_a$. Once synchronized the receiver will receive the first arrival at n_a. Here we describe the steps to get W, n_a and n_b.

Step 1. Store a few (say five) detected peaks $z_{\varphi\, i}(k)$ for $i = 1$ to 5, and form a concatenated vector $Z = [\underline{z}^T_{\varphi\,1}, \underline{z}^T_{\varphi\,2}, \cdots, \underline{z}^T_{\varphi\,5}]$

Step 2. To get W, calculate the autocorrelation of Z. The autocorrelation will be a periodic sequence with period W. Thus, the separation of the first two consecutive peaks of the autocorrelation gives the window size (W).

Step 3. To mark the start of the window, n_a, we will make use of the rms delay spread (τ) of the channel[1]. For $n > n_a + \tau$ the received signal strength is almost zero. With this condition we can properly mark the start of the window at $n = n_a$ and hence achieve the synchronization. It is important to note that the transmission from the transmitting beacon that occurs in discrete time intervals needs to be adjusted appropriately so that $W \gg \tau$.

Step 4. After synchronization, repeat *Step 1-7* of Sec III-A to estimate the position.

IV. Mathematical Analysis of the Position Location Algorithm

To perform mathematical analysis of the above algorithm consider a generalized form of (3) in matrix form

$$\mathbf{r} = H\mathbf{s} + \boldsymbol{\eta} \tag{4}$$

where $\mathbf{r} = [r_1, r_2, \cdots, r_N]^T$ is an $N \times 1$ vector of received earliest arrivals, H is $N \times N$ matrix of channel coefficient such that $H = \text{diag}(h_1, h_2, \cdots, h_N)$ and $\mathbf{s}$ and $\boldsymbol{\eta}$ are $N \times 1$ vectors of peaks of transmitted UWB pulses and samples noise, respectively. Assuming that the noise samples are iid Gaussian with zero mean and variance σ^2, the kth element of $\mathbf{r}$ follows Gaussian distribution with $r_k \sim \mathcal{N}(r_k, \sigma^2)$.

The algorithm searches for the time index n such that

$$n = \arg\max |r_k| \tag{5}$$

and the desired time t_1 is given by

$$t_1 = \frac{n\Delta\varphi}{\omega} \tag{6}$$

The presence of noise can cause a drift in the value of n by an amount Δn such that $r_{n+\Delta n}$ will appear as maximum instead of r_n. This drift causes an error in time estimation and hence lead to an erroneous measurement of position.

To quantify how Δn affects the position location estimation, we start by considering the error in time estimation. If Δt represents the average value of error in t_1 then it is related to Δn by

$$\Delta t = \frac{\Delta\varphi E\left[\Delta n\right]}{\omega} \tag{7}$$

where $E[.]$ represents the expected value and can be computed as

$$E\left[\Delta n\right] = \sum_{i=-\infty}^{\infty} |\, i\,|\; \Pr[\Delta n = i] \tag{8}$$

The probability in (8) represents the probability mass function (PMF) of Δn and can be evaluated as

$$\begin{aligned}\Pr\left[\Delta n = i\right] & \\ &= \Pr\left[\max|r_k| = r_{n+i}\right] \\ &= \Pr\left[r_{n+i} > r_1, r_{n+i} > r_2, \cdots, r_{n+i} > r_N\right] \\ &= \int_{-\infty}^{\infty} \Pr\left[r_{n+i} > r_1, \cdots, r_{n+i} > r_N | r_{n+i} = \rho\right] \\ &\qquad\qquad \times\; p(\rho)\, d\rho\end{aligned} \tag{9}$$

where $p(\rho)$ is the probability density function of $\rho = r_{n+i}$ that is Gaussian with $\rho \sim \mathcal{N}(r_{n+i}, \sigma^2)$. Since the $\{r_k\}$ are statistically independent, the joint probability factors into a product of $N-1$ marginal probabilities of the form

$$\begin{aligned}\Pr\left[r_k < r_{n+i} | r_{n+i} = \rho\right] &= \int_{-\infty}^{r_{n+i}} p_{r_k}(r_k) dr_k \\ &= 1 - \frac{1}{2}\text{erfc}\left(\frac{r_{n+i} - r_k}{\sqrt{2}\sigma}\right)\end{aligned}$$

where p_{r_k} is the probability density function of r_k with $r_k \sim \mathcal{N}(r_k, \sigma^2)$ and erfc(.) is the complementary error function. With this substitution, (9) becomes

$$\begin{aligned}&\Pr[\Delta n = i] \\ &\quad = \int_{-\infty}^{\infty} \prod_{\substack{k=1 \\ k \neq n+i}}^{N-1} \left[1 - \frac{1}{2}\text{erfc}\left(\frac{r_{n+i} - r_k}{\sqrt{2}\sigma}\right)\right] p(\rho)\, d\rho\end{aligned} \tag{10}$$

The integration in (10) has no closed form and should be evaluated by numerical techniques. Assuming that the error in t_1, t_2 and t_3 has same average value then average error in the bearings is given by combining (1), (7) and (8)

$$\begin{aligned}\Delta\alpha = \Delta\beta &= 2\omega\,\Delta t \\ &= 2\Delta\varphi \sum_{i=-\infty}^{\infty} |\, i\,|\; \Pr[\Delta n = i]\end{aligned}$$

The error in bearings leads to an erroneous estimate of position that can be computed by obtaining the total differentials of the coordinates of node $Q(x, y)$ as

$$\begin{aligned}\Delta x &= \frac{\partial x}{\partial \alpha}\,\Delta\alpha + \frac{\partial x}{\partial \beta}\,\Delta\beta \\ \Delta y &= \frac{\partial y}{\partial \alpha}\,\Delta\alpha + \frac{\partial y}{\partial \beta}\,\Delta\beta\end{aligned} \tag{11}$$

Finally, the error in position location or ranging error is related to these differentials as

$$\epsilon = \sqrt{(\Delta x)^2 + (\Delta y)^2} \tag{12}$$

[1] for UWB it's of the order of 50ns in indoor environment

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A. Discussions

The analysis we performed reveals some important aspects of the algorithm. A careful observation of the analysis reveals that the performance of the position location algorithm solely depends on the PMF of Δn. From (10), the two important factors that affect the PMF of Δn are : 1) Variance of Noise and 2) Antenna beam pattern. Large values of noise variance increase the probability of Δn being non-zero and causes more errors in position location. Similarly, if the beam pattern of antenna exhibits narrow beam then $r_{n+i} - r_k$ will be large and the variance of Δn will be comparatively small. For small variance of Δn, its PMF decays sharply and introduces less error in position location.

V. Improved Algorithm

Based on the discussions in the previous section, we strive to improve the position location algorithm. In this work, we look for some means to alleviate the effect of noise. The motivation behind this improvement stems right from the analysis of (10). We will show later in the next section that about 50% of errors in time of arrival estimation are due to a drift of $\Delta n = \pm 1$. This means that the position location algorithm can be significantly improved if we can avoid these errors.

A simple solution to this degradation is to search for a possibly better estimate of n around the value obtained from (5). To minimize the complexity, we limit the search space to three points $\{n-1, n, n+1\}$. Assuming that the desired node has information about the beam pattern and the channel gain, we can define a vector of performance measure ($\boldsymbol{\mu}_{n+i}$) in the search space as :

$$\boldsymbol{\mu}_{n+i} = \begin{bmatrix} r_{n-1+i} \\ r_{n+i} \\ r_{n+1+i} \end{bmatrix} - \begin{bmatrix} h_{n-1} \\ h_n \\ h_{n+1} \end{bmatrix} s \quad \text{for} \quad i = -1, 0, 1 \qquad (13)$$

where the symbols have same definition as in (4). The performance measure $\boldsymbol{\mu}_{n+i}$ essentially represents the mismatch due to noise. To find a better estimate $n^{'}$, we need to minimize the vector $\boldsymbol{\mu}_{n+i}$ under some criterion. Since the noise has Gaussian distribution, an optimal choice is to minimize the l_2 norm of $\boldsymbol{\mu}_{n+i}$, *i.e.*,

$$n^{'} = \arg\min ||\boldsymbol{\mu}_{n+i}||^2 \qquad (14)$$

where $||.||$ denotes l_2 norm of a vector.

VI. Numerical Example and Simulation Results

In this section, we present the simulation results of the proposed UWB position location system using directional beacon. To assess the validity of analytical expressions we compare the simulation results with the analysis performed in Section IV. Consider a scenario depicted in Fig. 1 with $L_1 = L_2 = \sqrt{32}$ m. For simplicity, we set the nodes RN-1, RN-2 and RN-3 at coordinates (L_1, L_2), $(0, L_2)$ and $(0, 0)$, respectively, and apply the proposed algorithm to estimate the coordinates of node $Q(x, y)$. Directional beacons are produced by employing 4 element ($M = 4$) uniformly spaced linear antenna array. The beam pattern of such an antenna array is shown in Fig. 2. For simulations, we used Gaussian monopulses with bandwidth 2.4 GHz and 4-bit A/D converter. The channel model used in simulations is the standard UWB channel model proposed for IEEE 802.15.3A [10].

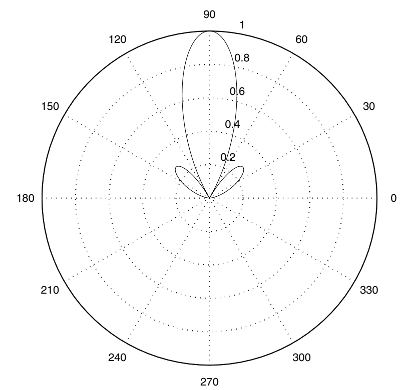

Fig. 2. A typical beam pattern for uniform linear antenna array (M=4)

To start with, we perform simulations to obtain the probability mass function (PMF) of Δn that represents the drift in the value of n due to additive noise such that $r_{n+\Delta n}$ will appear as maximum instead of r_n. The resulting PMF at 20dB SNR is plotted in Fig. 3. The figure also compares simulation results with those obtained analytically from (10) and it is obvious that the results match closely. In Fig. 3, Pr $[\Delta n = 0]$ corresponds to the event of absolutely correct position location estimation and it can be seen from the figure that the proposed algorithm can detect the position absolutely correctly for 32% of times. As demonstrated by (10), the pattern of PMF is closely related with the beam pattern, *i.e.*, the narrower the beam pattern, the smaller the variance of Δn.

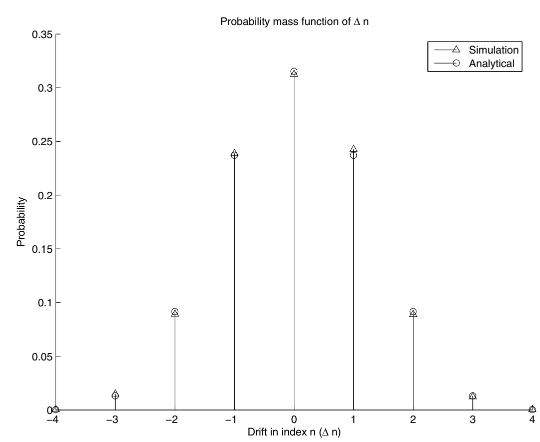

Fig. 3. Probability mass function of Δn *i.e.* $\Pr[\Delta n = i]$

Next, we evaluate the effect of noise on the estimation of time of arrivals that directly depends on the PMF of Δn. The average error in time, Δt, for different values of SNR is plotted in Fig. 4. In the figure, the simulation results are obtained for the system mentioned above and averaged over 10000 realizations. The analytical results are provided by (7) where

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the integration in (10) is performed numerically using Matlab routines. The figure shows a close match between simulation and analytical results.

Now, we consider the error in position location incurred by the proposed algorithm for different values of SNR. Using simulations, we obtained the coordinates of node Q and then computed the error in position location using (12). Fig. 5 presents the simulation as well as analytical results. It can be seen from the figure that for high SNR ($\sim$ 30dB) the error in position (ϵ) is as small as 30 cm. It is important to mention here that the use of directional beacons does not employ the product of propagation speed and time to compute distance, and hence the error in time estimation does not amplify the error in ranging severely.

Finally, we performed simulations to investigate the proposed improvement. In Fig. 6, we compare the error in time estimation by using (5) to get n and by using (14) to get an improved estimate of n. The figure shows significant improvement in time estimation that will consequently lead to improved position location.

VII. Conclusion

We proposed a directional beacons based position location scheme for UWB systems in OLOS environments. Different from the previous works, we proposed LOS detection through earliest arrival. Correlation based window algorithm to detect the earliest arrival and hence the LOS component is devised to overcome synchronization problems. To quantify the errors in position location, we performed detailed and exact analysis of the algorithm. The discussion of analytical results motivated us to reduce the effect of noise and we proposed a computational efficient search scheme to improve the estimation of position. Analytical results closely match with the simulation results and demonstrate the robustness of the algorithm for OLOS. The accuracy of the proposed algorithm depends on beamwidth, though considerable accuracy (≈30 cm at 28dB SNR) in position location can be achieved with 4-element antenna array. Our future study will focus on the effect of beam pattern on the accuracy of position estimates.

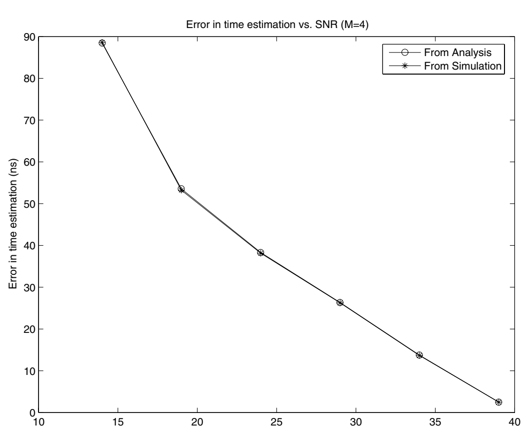

Fig. 4. Average estimation error in time vs. SNR ($M = 4$)

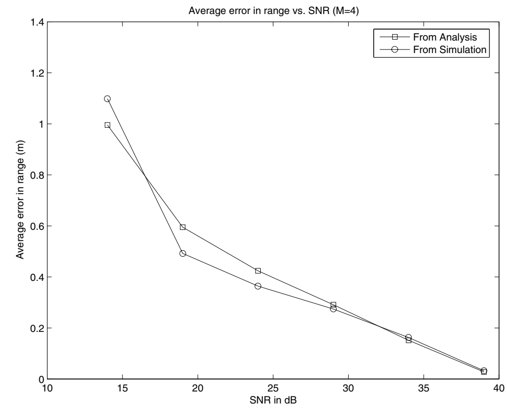

Fig. 5. Average estimation error in range vs. SNR($M = 4$)

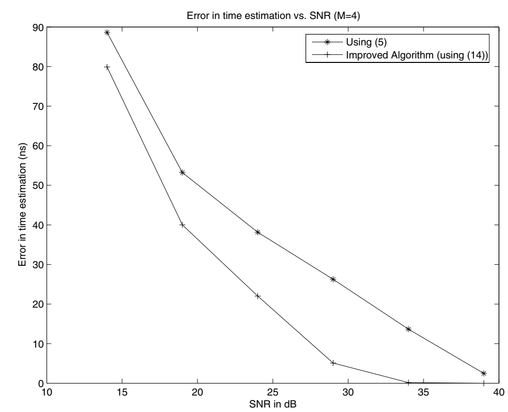

Fig. 6. Error in time estimation using improved algorithm

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