The Rich Domain of Risk

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Abstract

We report on two experiments challenging the common assumption that events with objective probabilities constitute a unique source of uncertainty. We find that, similar to the domain of ambiguity (Abdellaoui et al. 2011), the domain of risk is rich in the sense that behavior is systematically different when subjects face risky bets based on simple or more complex events. Further, we find a tight association between attitudes toward complex risky bets and attitudes toward both ambiguity and compound lotteries. These results raise questions about the characterization of ambiguity aversion and the modeling of decisions under uncertainty.

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The question then is not "Are there uncertainties that are not risks?", as posed by Ellsberg, but "Are there risks that are not risks?" — V. Smith (1969, p 329)

1. Introduction

The literature on uncertainty generally distinguishes decisions made under risk (with objective probabilities) from decisions made under ambiguity (with unknown probabilities).[1] Recently, Abdellaoui, Baillon, Placido and Wakker (2011) (ABPW hereafter) showed that the domain of ambiguity is rich, in the sense that a decision maker may have different attitudes toward different sources of ambiguity. As is usual, however, events with objective probabilities are assumed to constitute a unique source. This assumption is important as ambiguity aversion is typically measured in contrast with attitude toward risk.

In this paper, we conduct two within subjects experiments to test the common assumption of a unique source of risk. The first is an urn experiment which replicates ABPW's Ellsberg experiment under risk and ambiguity. The second is a dice experiment with simple and compound risks. In both experiments, we add a new risky treatment with less trivial events whose objective probabilities are arguably more difficult to calculate.

We find that the domain of risk is rich in the sense that subjects have systematically different attitudes toward risks depending on whether the events are simple or more complex. Further, we find that aversion to nontrivial risky events is tightly related to aversion to ambiguity and compound lotteries.

These results have implications for the characterization and the modeling of ambiguity attitudes. In particular, ambiguity attitudes cannot be unequivocally characterized in contrast to attitudes toward risk. Further, most models of choices under risk and ambiguity are incomplete as they fail to explain at-

[1] In this paper, the term uncertainty captures both risk and ambiguity.

titudes toward complex risk. In contrast, our results are consistent with a modified version of ABPW's "source method" under which the many kinds of uncertainties are differentiated by their degree of subjective complexity.

2. The Source Method — The Binary Case

ABPW's source method combines Chew and Sagi's (2008) concept of source preference with Tversky and Kahneman's (1992) (cumulative) prospect theory. A decision maker (DM hereafter) faces a binary bet $\overline{x}_E\underline{x}$, i.e. wins $\overline{x}$ when event E realizes and $\underline{x} \leq \overline{x}$ otherwise. A source S is loosely defined as "a group of events that is generated by a common mechanism of uncertainty." Formally, ABPW assume that sources are algebras of events. A source is said to be "uniform" if probabilistic sophistication holds within the source. If S is uniform, the DM's utility can be written

$$U\left(\overline{x}_E\underline{x}\right) = w_S\left(p\right) u\left(\overline{x}\right) + \left(1 - w_S\left(p\right)\right) u\left(\underline{x}\right) \tag{2.1}$$

where p is the subjective probability of E and w_S is a weighting function, called "source function," associated with S. The utility function u is assumed to be the same regardless of the source S.

In the case of risk, E has an objective probability P and $p = P$. Further, every source of risk S is assumed to have the same source function: $w_S(P) \equiv w_R(P)$. In contrast, w_S can differ depending on the source of ambiguity S. Hence, the DM has the same attitude toward every source of risk, but he may have different attitudes toward different sources of ambiguity. The difference between w_R and w_S characterizes the DM's ambiguity attitude toward S.

3. Experiment 1: The Urns Experiment

3.1. The Design

The within subject experiment (described in Appendix A) consists of three treatments, the two treatments in ABPW's Ellsberg experiment and a new treatment. Subjects face a series of binary bets $\overline{x}_E\underline{x}$ for which they report a certainty equivalent using ABPW's computerized iterative choice list method.

In the first risky (known) treatment (denoted $\mathtt{K}$), the bet is settled by drawing a ball from a transparent urn containing eight balls of different colors. Elementary events are thus equally likely with probability 1/8. In treatment $\mathtt{K}$, the bets are based on simple events (e.g. "the ball is red").

In the ambiguous treatment (denoted $\mathtt{U}$), the bet is settled by drawing a ball from an opaque urn containing eight balls. The balls' possible colors are the same as in treatment $\mathtt{K}$, but the composition of the opaque urn is unknown. The bets in treatment $\mathtt{U}$ involve the same events as in treatment $\mathtt{K}$.

For the new risky treatment (denoted $\mathtt{K}_2$), there are two transparent urns each containing eight balls of different colors (as in treatment $\mathtt{K}$). The bet is settled by the simultaneous draw of two balls, one from each urn. Elementary events (i.e. a pair of colored balls) are equally likely (as in treatment $\mathtt{K}$) but with probability 1/64. The bets in treatment $\mathtt{K}_2$ are based on what may be considered more complex events than in treatment $\mathtt{K}$ (e.g. "the two balls are of different colors").[2] To simplify, we will often refer to the risky bets in treatments $\mathtt{K}$ and $\mathtt{K}_2$ as "simple" and "complex," respectively.

Following ABPW, subjects face 13, 19 and 19 bets in treatments $\mathtt{K}$, $\mathtt{U}$ and $\mathtt{K}_2$, respectively. Within each treatment, the bets are presented in the same

[2] The events in treatment $\mathtt{K}_2$ were specifically constructed to avoid systematic judgement biases due to e.g. anchoring (Bar-Hillel 1973), framing (Tversky and Kahneman 1981), the use of ratios (Pacini and Epstein 1999) and frequencies (Gigerenzer 1991).

order as ABPW. In particular, in treatments $\mathtt{K}$ and $\mathtt{U}$, the first bets have events that combine from 1 to 7 colors while $\{\underline{x}, \overline{x}\}$ is fixed at $\{0, 25\}$; the last six bets are based on a 4 colors event, while $\underline{x}$ and $\overline{x}$ vary from 0 to 25. Subjects could take as long as they wanted to complete the experiment and they did not have access to calculators. At the beginning of the experiment, each subject is told that one of his choices will be randomly selected for payment.

The implementation of the experiment is similar to ABPW with five notable exceptions: i) Bets are settled by having the subject draw ball(s) from physical urn(s); ii) the experiment was not conducted individually but in sessions with 17 to 21 subjects; iii) a show-up fee of €5 was paid to every subject; iv) the 77 subjects in our experiment were university students in Toulouse (France), not students at elite graduate engineering schools; v) treatment $\mathtt{K}_2$ was conducted between treatments $\mathtt{K}$ and $\mathtt{U}$.

3.2. Raw Results

Figure 1 shows that the average certainty equivalents (divided by 25) for the bets 25_P0 are close to the diagonal for any P in treatment $\mathtt{K}$. In contrast, they are below the diagonal for $P > 1/4$ in treatments $\mathtt{U}$ and $\mathtt{K}_2$.[3] As shown in Table $E1$,[4] a series of Wilcoxon signed-rank tests confirms statistically (at the 5% level) that, for any $P > 1/4$, simple risky bets in treatment $\mathtt{K}$ are valued differently than corresponding bets in treatments $\mathtt{U}$ and $\mathtt{K}_2$. In contrast, the valuation of ambiguous and complex risky bets cannot be differentiated at standard significance levels for any P. These raw results suggest that i) risky bets are valued differently depending on whether they are based on simple or

[3] As shown in Appendix B, we fail to reject the hypothesis of a uniform source in treatments $\mathtt{U}$ and $\mathtt{K}_2$ at any usual significance level. Thus, we follow ABPW and assume that objective and subjective probabilities are equal (i.e. $p = P$) in each treatment.

[4] Tables and figures with numbers preceded by "E" can be found in Appendix E.

more complex events, ii) the value of an ambiguous bet relative to a risky bet depends on the complexity of the events on which the risky bet is based on.

3.3. Structural Econometric Approach

We now estimate the subjects' source and utility functions. Following ABPW, we assume that subject i in treatment $t \in \{\mathtt{K}, \mathtt{K}_2, \mathtt{U}\}$ has a power utility function $u_{it}(x) = x^{r_{it}}$ and a source function $w_{it}(p) = \exp\left(\ln(a_{it}) \left[\ln(p) / \ln(a_{it})\right]^{b_{it}}\right)$ where $a_{it} \in (0,1)$ and $b_{it} > 0$. Observe that under the parametrization $\{\alpha_{it} = b_{it}, \beta_{it} = (-\ln a_{it})^{1-b_{it}}\}$ we get the source function proposed by Prelec (1998) and used by ABPW: $w_{it}(p) = \exp\left(-\beta_{it}\left[-\ln p\right]^{\alpha_{it}}\right)$. The specification in $\{a_{it}, b_{it}\}$ was preferred to Prelec's because the parameters are easier to interpret. Indeed, a_{it} is the fixed point of w_{it}, while b_{it} is the slope of w_{it} at this fixed point ($w'_{it}(a_{it}) = b_{it}$). As a result, in the spirit of ABPW, b_{it} can be interpreted as a likelihood sensitivity index and a_{it} as a pessimism (optimism) index when w_{it} is (inverse) S-shaped, that is when $b_{it} > 1$ ($b_{it} < 1$).

Under the source method, the indifference of subject i between bet j and his elicited certainty equivalent CE_{ijt} implies:

$$(CE_{ijt})^{r_{it}} = \exp\left(\ln(a_{it}) \left[\ln(P_j) / \ln(a_{it})\right]^{b_{it}}\right) \left[(\overline{x}_j)^{r_{it}} - \left(\underline{x}_j\right)^{r_{it}}\right] + \left(\underline{x}_j\right)^{r_{it}} \quad (3.1)$$

As explained in Appendix C, we prefer to adopt a different econometric approach than ABPW. Using (3.1), we estimate the structural parameters $\{r_{it}, a_{it}, b_{it}\}$ jointly by NLLS with all the data collected for subject i in treatment t. To test for differences across treatments we define:

$$r_{it} = 1 + \mathrm{r}_{i\mathtt{K}} + \mathrm{r}_{i\mathtt{K}_2}\mathtt{K}_2 + \mathrm{r}_{i\mathtt{U}}(\mathtt{K}_2 + \mathtt{U}),\ a_{it} = a_{i\mathtt{K}} + \mathrm{a}_{i\mathtt{K}_2}\mathtt{K}_2 + \mathrm{a}_{i\mathtt{U}}(\mathtt{K}_2 + \mathtt{U}),\ b_{it} = b_{i\mathtt{K}} + \mathrm{b}_{i\mathtt{K}_2}\mathtt{K}_2 + \mathrm{b}_{i\mathtt{U}}(\mathtt{K}_2 + \mathtt{U})$$

where $\mathtt{K}_2$ ($\mathtt{U}$) is a dummy variable equal to 1 when $t = \mathtt{K}_2$ ($t = \mathtt{U}$). As a result, $\mathrm{r}_{i\mathtt{U}}$ ($\mathrm{r}_{i\mathtt{K}_2}$) captures the difference in the utility function of subject i when he faces ambiguous instead of simple (complex) risky bets.[5]

To assess the robustness of the results we estimate both a fully heterogenous model as in ABPW (i.e. $\{r_{it}, a_{it}, b_{it}\}$ differs across subjects) and a homogenous model in which $\{r_{it}, a_{it}, b_{it}\} = \{r_t, a_t, b_t\}$. Each approach has advantages and drawbacks. Estimates from the homogenous model are severely constrained but easy to interpret. Estimates from the heterogenous model are unrestricted across subjects but rely on small samples (for subject i in treatment t, $\{r_{it}, a_{it}, b_{it}\}$ is estimated with 13 or 19 observations depending on t). Finally, to account for the small sample size, the standard deviations of estimates and the p-values of tests are calculated by nonparametric bootstrap.

3.4. Results from the Structural Estimation

The estimation results reported in Table 1 indicate that the subjects' utility functions in treatment K are nearly linear on average (as in ABPW) but highly heterogenous across subjects. Indeed, $\mathrm{r}_{\mathtt{K}}$ is not significantly different from 0 in the homogenous model, and the average estimated $\mathrm{r}_{i\mathtt{K}}$ is close to 0 in the heterogenous model. Further, the large standard deviation of the estimated $\mathrm{r}_{i\mathtt{K}}$ (0.496) indicates important differences in utility across subjects. In fact, we can reject the linearity of u_i for about half of the subjects in treatment K. The parameter $\mathrm{r}_{\mathtt{U}}$ ($\mathrm{r}_{\mathtt{K}_2}$) is insignificant in the homogenous model, and $\mathrm{r}_{i\mathtt{U}}$ ($\mathrm{r}_{i\mathtt{K}_2}$) is significant for only 14% (9%) of the subjects in the heterogenous model. Thus, a subject generally exhibits the same utility in treatments K, $\mathtt{K}_2$ and U. Similarly, ABPW found the same u_i in treatment K and U.

[5] Note that the nature of the results presented next does not change when we use for the left hand side of (3.1) $(CE_{ijt})^{r_{i\mathtt{K}}}$ instead of $(CE_{ijt})^{r_{it}}$ when $t \in \{\mathtt{K}_2, \mathtt{U}\}$.

We now turn to the estimation of the source functions. The homogenous model in Table 1 reveals that, similar to ABPW, the optimism and likelihood sensitivity indexes are close to 0.5 and 0.8 in treatment $\mathtt{K}$, and significantly larger in treatment $\mathtt{K}$ than in treatment $\mathtt{U}$ (since $\mathrm{a}_{\mathtt{U}}$ and $\mathrm{b}_{\mathtt{U}}$ are both significantly lower than 0). The heterogenous model confirms these results and shows that we can reject the equality of the likelihood sensitivity index (optimism index) in treatment $\mathtt{K}$ and $\mathtt{U}$ for 57% (31%) of the subjects. In contrast, $\mathrm{a}_{\mathtt{K}_2}$ and $\mathrm{b}_{\mathtt{K}_2}$ are negative but insignificant in the homogenous model and insignificant for most subjects in the heterogenous model. Thus, as seen in Figure 2, the optimism and likelihood sensitivity indexes in treatment $\mathtt{K}_2$ are significantly lower than in treatment $\mathtt{K}$, but statistically indistinguishable from those in treatment $\mathtt{U}$.

To sum up, the structural estimations suggest that subjects have i) similar utility functions across treatments, ii) different source functions when facing simple and complex risky bets, iii) similar source functions when facing ambiguous and complex risky bets. Naturally, one may wonder whether result ii) is due to calibration errors subjects made when calculating objective probabilities for complex events. In Appendix D, we estimate a structural model accounting for treatment specific calibration errors and find that, while subjects make larger calibration errors for complex risky bets, there is still a significant difference between the source functions in treatment $\mathtt{K}$ and $\mathtt{K}_2$.

3.5. Aversion to Ambiguity and Aversion to Complex Risky Events

Is there a link between aversion to ambiguity and aversion to complex risky events? To address this question, we follow Abdellaoui et al. (2013) and define for each subject i and each P=1/8, ..., 7/8 the ambiguity and "complex risk" premium as $CE_{ip\mathtt{K}} - CE_{ip\mathtt{U}}$ and $CE_{ip\mathtt{K}} - CE_{ip\mathtt{K}_2}$, respectively. These individual premia (averaged across P=1/8, ..., 7/8) are plotted in Figure 3. Observe that there is a strong positive correlation ($\rho = 0.714$) between a subject's ambiguity and complex risk premia (as indicated by the positive trend line).[6]

To explore the link between neutrality to ambiguity and neutrality to complex risky bets, we calculate for each subject the absolute value of the ambiguity and complex risk premia averaged across P=1/8, ..., 7/8. If a subject is perfectly neutral to ambiguity (complex risk), then his average absolute ambiguity (complex risk) premium is 0. As shown in Figure 3, no subject was perfectly neutral to ambiguity or complex risk for every P=1/8, ..., 7/8. If we define a subject with an average absolute premium lower than €1 as "nearly neutral," then, out of the 77 subjects, 10 are nearly neutral to both ambiguity and complex risk, 4 are nearly neutral to ambiguity only, and 3 are nearly neutral to complex risk only. Thus, 77% of the subjects with similar attitudes toward simple and complex risky bets are ambiguity nearly neutral. A Fisher exact test confirms (p-value=2.2 10^{-7}) that near neutrality to ambiguity and near neutrality to complex risk can be considered tightly associated.

4. Experiment 2: The Dice Experiment

We conduct a second experiment to confirm that the domain of risk is rich and to study the link between attitudes toward compound and complex risks. To assess the robustness of the results, the second experiment differs from the first in several dimensions: Uncertainty is generated with dice, the acts are generated with a quadratic scoring rule (as in Andersen et al. 2009), the

[6] As shown in Figure E1, this positive relationship holds for each P=1/8, ..., 7/8. It also holds when the ambiguity and complex risk premia are calculated with respect to expected value (computed with P) instead of simple risk ($\rho = 0.698$), and when we compare the source functions $w_{i\mathrm{U}}(P)$ and $w_{i\mathrm{K}_2}(P)$ for P=1/8, ..., 7/8 ($\rho = 0.659$).

experiment is conducted with pen and paper, the subject pool comes from a developing country, and incentives are substantially higher.

4.1. The Design

The within subject experiment (described in Appendix F) also consists of three treatments. In each treatment of experiment 2, subjects predict the probability of 10 risky events for which they are rewarded with a quadratic scoring rule. Each event describes the outcome of the roll of two 10-sided dice (one black, one red). For comparison, the 10 events have the same objective probabilities in each treatment (3, 5, 15, 25, 35, 45, 61, 70, 80, and 90%).

In treatment `K`, the red (black) die determines the first (second) digit of a number between 1 and 100. Elementary events thus follow a uniform distribution. The events in treatment `K` have objective probabilities that are simple to calculate (e.g. the 25% probability event is described as "the number drawn is between 1 (included) and 25 (included)").[7]

In treatment $\mathtt{K}_2$, the two dice are added to form a number between 0 and 18. Elementary events thus follow a triangular shaped distribution. The events in treatment $\mathtt{K}_2$ have objective probabilities that are arguably more difficult to calculate than those in treatment `K` (e.g. the 25% probability event is described as "the sum is between 2 (included) and 6 (included)"). Note, however, that the same mechanism (i.e., the roll of two 10-sided dice) generates uncertainty in both treatments. Only the construction of the events differs.

In treatment `C`, the subjects face compound lotteries, i.e. lotteries whose prizes are other lotteries. Specifically, they have to make a single prediction not for one but for two possible events. After predictions are made, a fair coin

[7] This treatment comes from Armantier and Treich (2013).

determines which of the two possible events matters for payments. The events in treatment `C` are similar to those in treatment `K`, i.e. the roll of the dice produces a number between 1 and 100.

Experiment 2 took place in Ouagadougou, the capital of Burkina Faso. The subjects were recruited by a local recruiting firm (Opty-RH) by placing fliers around the city. To be eligible, subjects had to be at least 18 years old and be current or former university students. Two sessions with 21 and 22 subjects were conducted, each taking around 90 minutes to complete. Subjects were familiar with probabilities. In particular, 65% reported having taken a college level course in probability or statistics. Subjects earned 3,000 FCFA on average which corresponds to a 3-day wage for a university graduate.

4.2. Experimental Results

It is well known that the quadratic scoring rule is incentive compatible only when subjects maximize expected payoffs. Under model (2.1), the relationship between objective and reported probabilities is expected to be inverse S-Shaped (Armantier and Treich 2013). Nevertheless, if the source of risk is unique, a subject's should report the same probabilities for the three treatments. As in experiment 1, we find evidence against this hypothesis. Indeed, Figure 4 displays systematic differences: Treatment `K` (with trivial events) yields the smallest biases for virtually all objective probabilities (i.e. reported probabilities are consistently closest to the diagonal), while treatment $\mathtt{K}_2$ (with complex events) generates the largest biases. This ranking across treatments is confirmed statistically by nonparametric Friedman tests (see Table $E2$).

As in experiment 1, we estimate the structural parameters $\{r_{it}, a_{it}, b_{it}\}$ characterizing subject i utility and source functions in treatment $t \in \{\mathtt{K},\mathtt{K}_2,\mathtt{C}\}$.[8]

[8] The parameters are estimated by NLLS by comparing for every event j subject i's

The results are presented in the right panel of Table 1. Similar to experiment 1, we cannot reject the hypothesis that the subjects have the same utility function across treatments. In contrast to experiment 1 however, the subjects' average utility function is concave (which may be due to the higher earnings at stakes). With respect to the source functions, we find no statistical difference across treatments in the pessimism index a_{it}. In contrast, $b_{\mathtt{C}}$ and $b_{\mathtt{K}_2}$ are both significantly smaller than 0 in the homogenous model. Thus, as shown in Figure 5, the curvature of the source function is more pronounced for compound than simple events, and the most pronounced for complex risky events.

Finally, we calculate the complex (compound) risk premium as $\pi_{ip\mathtt{K}} - \pi_{ip\mathtt{K}_2}$ $(\pi_{ip\mathtt{K}} - \pi_{ip\mathtt{C}})$, where π_{ipt} is the expected payoff corresponding to subject i prediction for the event with probability P in treatment $t \in \{\mathtt{K},\mathtt{K}_2,\mathtt{C}\}$. Figure 6, where the individual premia averaged across all P are plotted, reveals a strong positive correlation ($\rho = 0.688$) between complex and compound risk premia. Further, out of the 43 subjects in experiment 2, 8 are nearly neutral to both compound and complex risks, 3 are nearly neutral to compound risk only, and 2 are nearly neutral to complex risk only.[9] Thus, 80% of the subjects with similar attitudes toward simple and complex risks are nearly neutral to compound risk. A Fisher exact test confirms (p-value=4.4 10^{-5}) that near neutrality to compound and complex risks can be considered tightly associated.

To sum up, experiment 2 provides further evidence that the domain of risk is rich in the sense that subjects have different attitudes toward risks based on simple, complex or compound events. Further, aversions to complex and compound risks are found to be highly correlated.

reported probability $\widehat{P}_{ijt}$ with $P_j^*(r_{it}, a_{it}, b_{it})$ the optimal report under a quadratic scoring rule by an agent with utility and source functions characterized by $\{r_{it}, a_{it}, b_{it}\}$.

[9]Near neutrality is defined as having an average absolute premium lower than 75 FCFA ($1/40^{th}$ of the 3,000 FCFA earned on average). Alternative definitions yield similar results.

5. Discussion

Summary: We conducted two experiments to test the common assumption of a unique source of risk. We find evidence against this hypothesis as subjects display significantly different attitudes when facing risks based on simple events and risks based on more complex events. Further, these differences appear to be systematic (i.e. the source function deviates more from linearity for complex risky events) and not driven by calibration errors. Thus, we find that, similar to the domain of ambiguity, the domain of risk is rich. We also identify a tight link between attitudes toward complex risky bets and attitudes toward ambiguity and compound risk. In particular, subjects are essentially neutral to ambiguity and compound risk when those are measured in contrast to complex risk. Finally, our experiment shows that complexity aversion can be empirically relevant as it affected behavior as much as ambiguity and compound risk aversions. We now discuss possible implications of these results.

Characterization of ambiguity aversion: Finding that the domain of risk is rich raises questions about the characterization of ambiguity attitudes in theory and in practice. In particular, an agent's attitude toward an ambiguous source S is defined under ABPW's source method as the difference between the source function for S and the source function for risk. Thus, ambiguity attitudes cannot be characterized uniquely if there are many sources of risk. Moreover, the central "uncertainty aversion" axiom of Gilboa and Schmeidler (1989) maxmin model implicitly relies on the assumption of a unique source of risk. In practice, ambiguity aversion is almost exclusively measured in Ellsberg like experiments by comparing attitudes toward known and unknown probabilities (Trautmann and van de Kuilen 2013). Experimental measures of ambiguity aversion are thus contingent on the source of risk considered.

Modeling of ambiguity aversion: A variety of models have been proposed to capture attitudes toward both risk and ambiguity. While the class of multiple prior models considers that beliefs cannot be represented by a unique distribution (Wald 1950, Gilboa and Schmeidler 1989), the now popular multi-stage approach induces ambiguity aversion through a failure to reduce compound lotteries (Segal 1987, Klibanoff et al. 2005, Seo 2009). The latter approach recently found support in experiments showing an association between the inability to reduce compound objective probabilities and ambiguity aversion (Halevy 2007, Abdellaoui et al. 2013). Our results suggest that these ambiguity (aversion) models are incomplete as they fail to capture attitudes toward complex risk. Further, the tight association between attitudes toward complex risk (with no obvious interpretation as compound lotteries), and attitudes toward both ambiguity and compound risk suggests that ambiguity and compound risk aversions may be special cases of complexity aversion.

Accounting for complexity: How complexity affects decision making has been modelled in various fields of economics.[10] Our results suggest that a comprehensive model of choices under uncertainty should also account for complexity aversion. To do so, one may follow one of the many bounded rationality approaches proposed in economics. Alternatively, one may consider ABPW's source method under the assumption that the whole domain of uncertainty is rich, regardless of whether probabilities are known or not. Consistent with our results, the source function can be considered to reflect the subjective degree of "complexity" of the source: A source is more complex to an agent when its source function deviates more from linearity. This descriptive approach offers several advantages: it is rooted in decision theory, it is based on an empirically

[10] E.g. macroeconomics (Sims 2006), game theory (Gale and Sabourian 2005), industrial organization (Ellison and Ellison 2009), or finance (Caballero and Simsek 2013).

validated model (prospect theory), and changes in complexity (aversion) can be studied by varying the shape of the source function.

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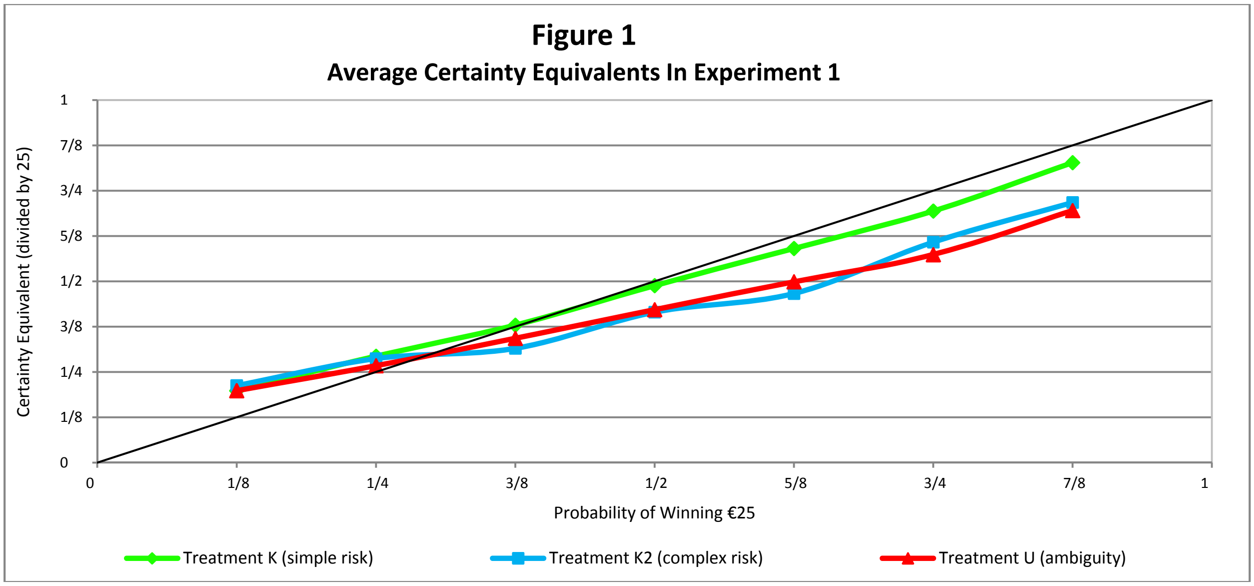
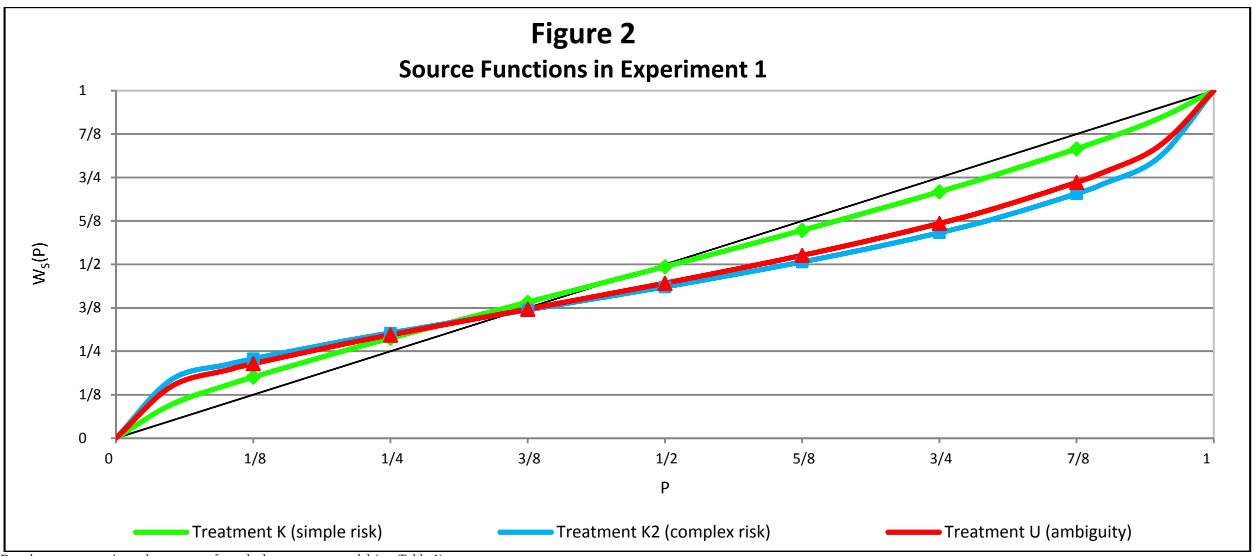
Table 1
Structural Estimation of Utility and Source Functions

	Experiment 1 (Urn)					Experiment 2 (Dice)			
		Heterogenous					Heterogenous		
	Homo-genous	Average across Subjects	Std across Subjects	% of subjects with significant parameter at 5%		Homo-genous	Average across Subjects	Std across Subjects	% of subjects with significant parameter at 5%
r_K	-0.012 (0.025)	0.081	0.496	49.4%	r_K	0.181** (0.082)	0.090	0.643	58.1%
r_U	-0.031 (0.039)	-0.056	0.534	14.3%	r_C	0.056 (0.088)	0.008	0.495	9.3%
r_{K_2}	0.013 (0.045)	0.009	0.481	9.1%	r_{K_2}	-0.099 (0.113)	-0.023	0.501	7.0%
a_K	0.491*** (0.035)	0.462	0.336	—	a_K	0.469*** (0.098)	0.536	0.167	—
a_U	-0.084** (0.042)	-0.096	0.351	31.2%	a_C	-0.029 (0.106)	-0.043	0.228	16.3%
a_{K_2}	-0.022 (0.039)	-0.004	0.356	16.9%	a_{K_2}	-0.004 (0.047)	0.003	0.224	4.7%
b_K	0.789*** (0.019)	0.819	0.208	—	b_K	0.783*** (0.186)	0.725	0.280	—
b_U	-0.205*** (0.057)	-0.231	0.388	57.1%	b_C	-0.184*** (0.043)	-0.209	0.461	48.8%
b_{K_2}	-0.069 (0.065)	-0.066	0.254	14.3%	b_{K_2}	-0.083** (0.036)	-0.105	0.374	39.5%

Estimates' standard errors and test statistics' distributions are estimated by bootstrap (size=10,000) to account for the finiteness of the sample.
***, **, * represent parameters significant at respectively the 1%, 5% and 10% significance level.
As a comparison, $\{r_K, a_K, b_K\}$={0.05,0.54,0.85} and $\{r_U, a_U, b_U\}=\{0.04, -0.20, -0.21\}$ in ABPW.





Based on average estimated parameters from the heterogenous model (see Table 1).

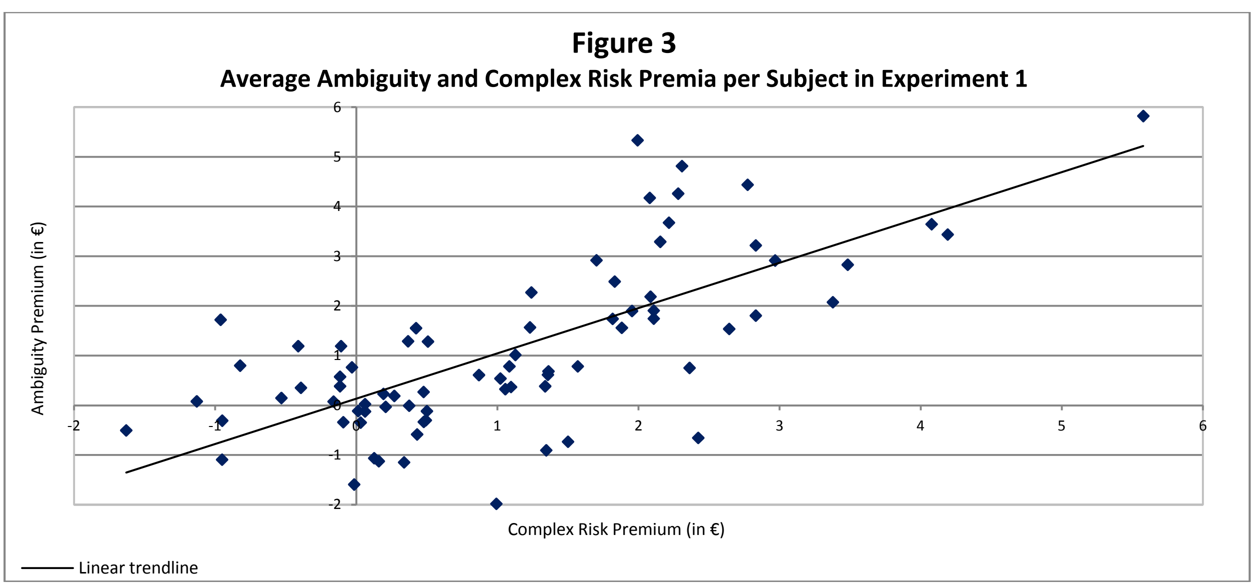


Figure 4

Reported Probabilities in Experiment 2

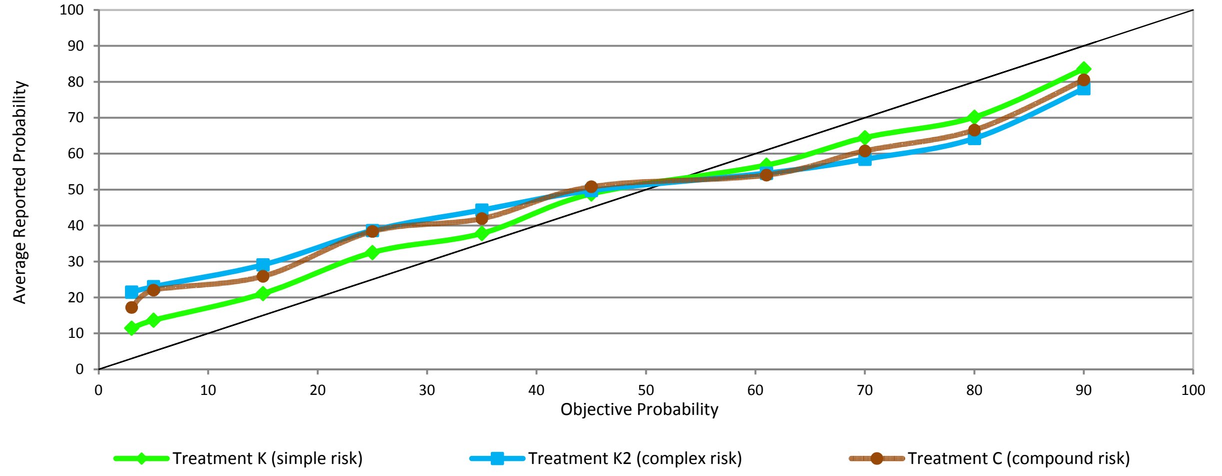
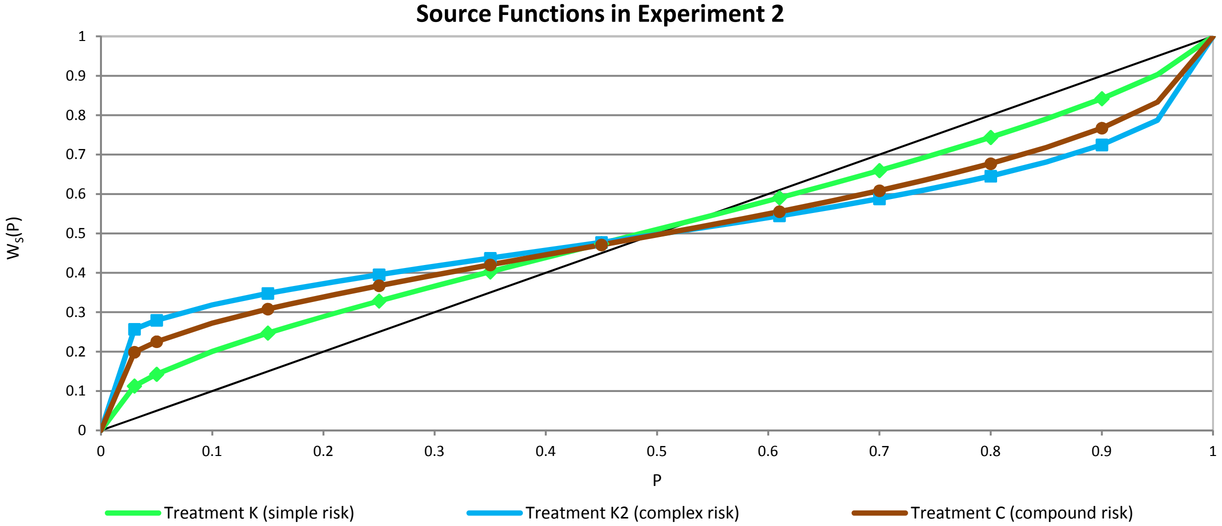


Figure 5

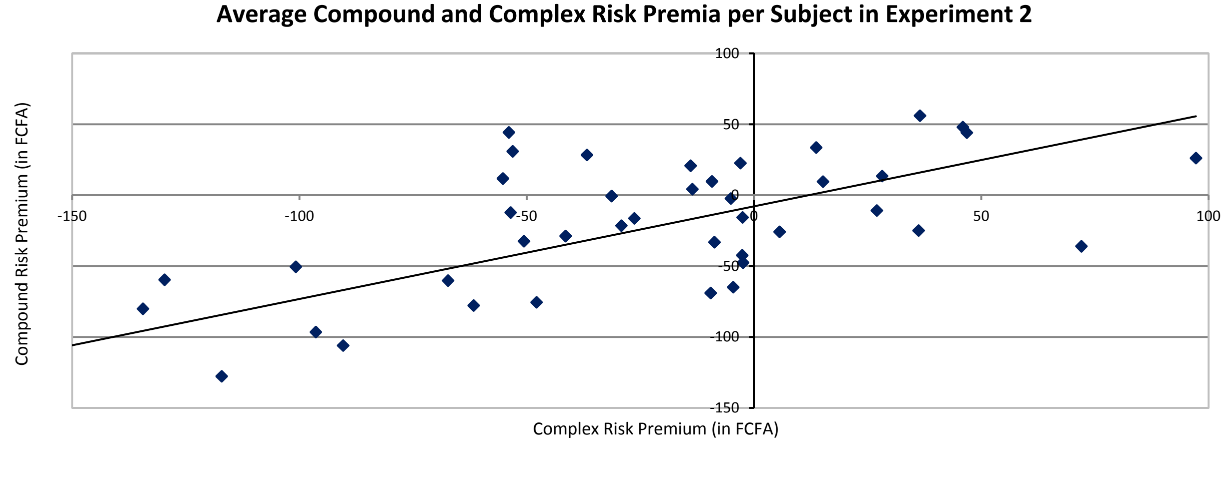
Source Functions in Experiment 2



Based on average estimated parameters from the heterogenous model (see Table 1).

Figure 6

Average Compound and Complex Risk Premia per Subject in Experiment 2



Appendices

(<u>Not for Publication</u>)

- Appendix A: The Design of Experiment 1
- Appendix B: Uniform Source Test
- Appendix C: ABPW Empirical Approach
- Appendix D: Structural Model with Calibration Errors
- Appendix E: Additional Figures and Tables
- Appendix F: Instructions for Experiment 2

Appendix A: The design of Experiment 1

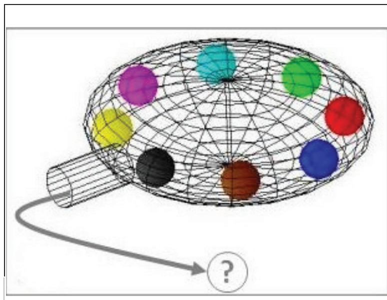
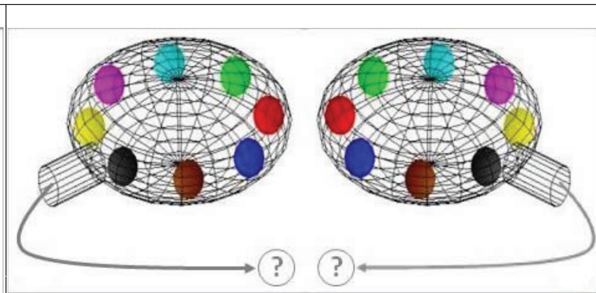
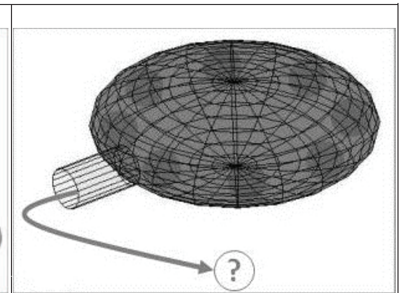
Description of the Events in Each Treatment of Experiment 1		
	Treatment K [and U]	Treatment K_2
	An urn contains the following 8 balls: 🔴 ⚪ 🔵 🟡 🟣 🟠 🟢 ⚫ We draw one ball at random from the urn. [Treatment U: Identical except that the number of balls of each color in the urn is unknown]	There are 2 urns. In each urn there are the following 8 balls: 🔴 ⚪ 🔵 🟡 🟣 🟠 🟢 ⚫ We draw one ball at random from each of the 2 urns. The order in which the balls are drawn does not matter. Only the color of the balls drawn matters. We denote with brackets the color of the balls drawn. For instance, {🔵,🔴} means that we drew one blue ball and one red ball.
Probability	**The subject wins if**	**The subject wins if**
1/8	The ball drawn is ⚪.	One ball is 🟢 or 🟠, the other ball is 🔴 or 🔵. For instance, you win if we draw {🟢,🔵}, you lose if we draw {⚪,🔵} or {⚫,🟣}.
2/8	The ball drawn is ⚪ or 🟠.	There is no 🟣 ball, no 🟢 ball, no ⚫ ball, no 🟡 ball. For instance, you win if we draw {🔵,🔴}, you lose if we draw {🔵,🟡}.
3/8	The ball drawn is ⚪ or 🟠 or 🔴.	One ball is 🔵 or ⚪, the other ball is neither 🔵 nor ⚪. For instance, you win if we draw {🔵,🔴}, you lose if we draw {🟠,🟡} or {🔵,🔵}
4/8	The ball drawn is ⚪ or 🟠 or 🔴 or 🔵.	One ball is ⚪ or 🟠 or 🔴 or 🔵, the other ball is 🟣 or 🟡 or ⚫ or 🟢. For instance, you win if we draw {🟠,🟣}, you lose if we draw {🟠,🔵}.
5/8	The ball drawn is ⚪ or 🟠 or 🔴 or 🔵 or 🟣.	There is no ⚪ ball and no 🔴 ball, or the 2 balls are ⚪ and/or 🔴. For instance, you win if we draw {🟠,🟣} or {⚪,⚪} or {⚪,🔴}, you lose if we draw {⚪,🟢}.
6/8	The ball drawn is ⚪ or 🟠 or 🔴 or 🔵 or 🟣 or 🟡.	There is no 🔴 ball, and the 2 balls are not both 🟡. For instance, you win if we draw {⚪,🟢} or {🟡,🟣}, you lose if we draw {🔴,🟠} or {🟡,🟡}.
7/8	The ball drawn is ⚪ or 🟠 or 🔴 or 🔵 or 🟣 or 🟡 or 🟢.	The 2 balls drawn are not of the same color. For instance, you win if we draw {🟡,🟣}, you lose if we draw {🔵,🔵}.

Instructions (screen shots translated from french)

Hello and thanks for your participation in this experiment!

During this experiment you will see several series of questions. In these questions, you will see different urns. Each urn contains 8 balls.
The balls can be of the following colors :

The urns you will face are as follows :

		
In this urn, we know that there is one ball of each color. One ball is drawn at random from this urn.	**There are 2 urns. In each urn, we know that there is one ball of each color. One ball is drawn at random from each of the 2 urns. The order in which the balls are drawn does not matter. Only the color of the balls drawn matters. For instance, { , } indicates that a red ball and a blue ball have been drawn.**	**In this urn, the proportion of balls of each color is not known. One ball is drawn at random from this urn.**

In each question you have to choose between two options :
- Option 1 indicates with which urn(s) and under which conditions you can win an amount between 0 à 25 €.
- Option 2 is a list of sure amounts.
For each question you have to choose between Option 1 and each of the amounts listed under Option 2.

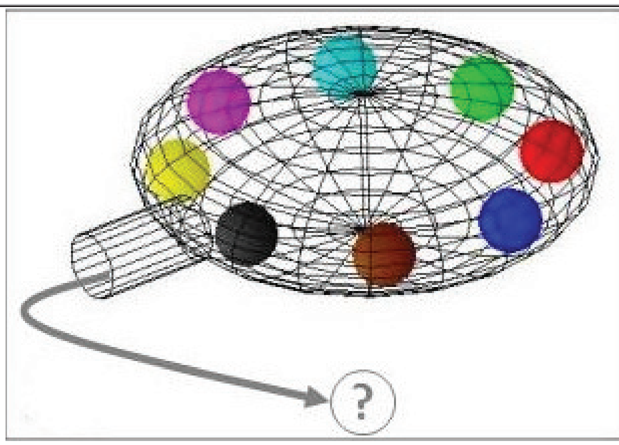
Start the Examples

In the next screens, the subject went through two examples illustrating the three phases of the computerized iterative choice list method. In **Phase 1**, the range of payments is divided into 5 categories. In **Phase 2**, the range of payments corresponding to the choices in Phase 1 is subdivided into 10 categories. In the **Confirmation Phase** the subject sees the choices she has made in Phase 1 and 2. She can then confirm or change her choices. The three phases are illustrated with screen shots from the experiment over the next four pages. See ABPW for further details on the design.

Phase 1

Question 1

Which of these two options do you prefer?

<table>
<tr><th>Option 1
Play the lottery below</th><th>1</th><th>2</th><th>Option 2
Receive the amount below for sure</th></tr>
<tr><td rowspan="6">

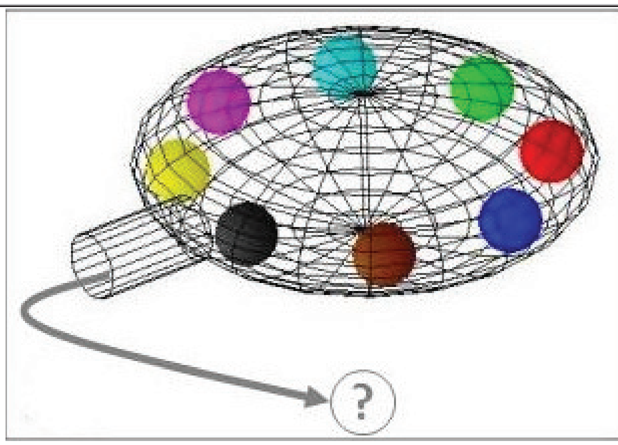
Earn 25 € if ■
Earn 0 € if ■ or ■ or ■ or ■ or ■ or ■ or ■</td><td>◉</td><td>○</td><td>0 €</td></tr>
<tr><td>◉</td><td>○</td><td>5 €</td></tr>
<tr><td>◉</td><td>○</td><td>10 €</td></tr>
<tr><td>◉</td><td>○</td><td>15 €</td></tr>
<tr><td>◉</td><td>○</td><td>20 €</td></tr>
<tr><td>○</td><td>◉</td><td>25 €</td></tr>
</table>

Continue

Phase 2

Question 1

Which of these two options do you prefer?

Option 1 *Play the lottery below*	1	2	Option 2 *Receive the amount below for sure*
	◉	○	20 €
	◉	○	20.5 €
	◉	○	21 €
	◉	○	21.5 €
	◉	○	22 €
	◉	○	22.5 €
	◉	○	23 €
	○	◉	23.5 €
Earn 25 € if	○	◉	24 €
Earn 0 € if or or or or or or	○	◉	24.5 €
	○	◉	25 €

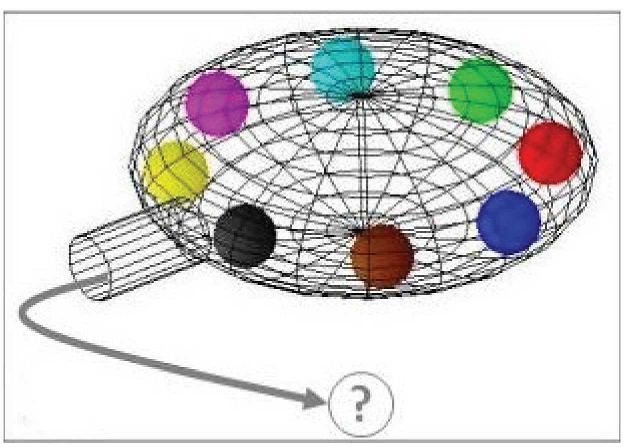
Back **Continue**

Confirmation Phase

Question 1

Which of these two options do you prefer?

Option 1 *Play the lottery below*	1	2	Option 2 *Receive the amount below for sure*
	◉	○	0 €
	◉	○	0.5 €
	◉	○	1 €
	◉	○	1.5 €
	◉	○	2 €
	◉	○	2.5 €
	◉	○	3 €
	◉	○	3.5 €
	◉	○	4 €
	◉	○	4.5 €
	◉	○	5 €
	◉	○	5.5 €
	◉	○	6 €
	◉	○	6.5 €
	◉	○	7 €
	◉	○	7.5 €
	◉	○	8 €
	◉	○	8.5 €
	◉	○	9 €
	◉	○	9.5 €
	◉	○	10 €
	◉	○	10.5 €
	◉	○	11 €
	◉	○	11.5 €
	◉	○	12 €
	◉	○	12.5 €
	◉	○	13 €
	◉	○	13.5 €
	◉	○	14 €



Earn 25 € if

Earn 0 € if or or or or

or or

	◉	○	14.5 €
	◉	○	15 €
	◉	○	15.5 €
	◉	○	16 €
	◉	○	16.5 €
	◉	○	17 €
	◉	○	17.5 €
	◉	○	18 €
	◉	○	18.5 €
	◉	○	19 €
	◉	○	19.5 €
	◉	○	20 €
	◉	○	20.5 €
	◉	○	21 €
	◉	○	21.5 €
	◉	○	22 €
	◉	○	22.5 €
	◉	○	23 €
	○	◉	23.5 €
	○	◉	24 €
	○	◉	24.5 €
	○	◉	25 €

Back **Validate**

Appendix B: Uniform Source Test

Following ABPW, we verify that the sources in treatments U and K_2 are uniform. That is, we test whether the bet $25_E 0$ is valued equally when event E is expressed using different colors. For instance, as explained in ABPW, if the source in treatment U is uniform, then a subject should report the same certainty equivalent for the events "the ball is blue" and "the ball is red," thereby indicating that he perceives the two events as equiprobable. Using a nonparametric Friedman test, we fail to reject the hypothesis of a uniform source in treatments U and K_2 at any usual significance level. Specifically, in treatment U the *P-values* are 0.51, 0.37 and 0.66 for the events with 1, 2 and 3 colors, respectively, while in treatment K_2 the *P-values* are 0.39, 0.45 and 0.28 for the events with probability 1/8, 2/8 and 3/8.

Appendix C: ABPW Empirical Approach

ABPW's econometric approach to test whether the source function under risk differs from the source function under ambiguity consists of three steps.

In step 1, r_{it} and an auxiliary parameters δ_{it} are estimated for subject i in treatment t by NLLS using the certainty equivalents CE_{ijt} elicited for the six bets j in which $\underline{x}_j$ and $\overline{x}_j$ vary from 0 to 25 while P_j is fixed at 1/2. The equality used to implement the NLLS is based on the cumulative prospect theory value function:

$$(CE_{ijt})^{r_{it}} = \delta_{it}\left[(\overline{x}_j)^{r_{it}} - \left(\underline{x}_j\right)^{r_{it}}\right] + \left(\underline{x}_j\right)^{r_{it}}$$

where $\delta_{it} = w_{it}(1/2)$.

In step 2, using the parameter $\widehat{r}_{it}$ estimated in step 1, the parameters $\{\alpha_{it}, \beta_{it}\}$ from the source function proposed by Prelec (1998) are estimated by NLLS using the remaining 7 or 13 certainty equivalents elicited for the bets in which P_j vary from 1/8 to 7/8 while $\left\{\underline{x}_j, \overline{x}_j\right\}$ remain fixed at $\{0, 25\}$. The equality used to implement the NLLS is based on subject i's indifference under cumulative prospect theory between a prospect j and his elicited certainty equivalent

$$(CE_{ijt})^{\widehat{r}_{it}} = \exp\left(-\beta_{it}\left[-\ln P_j\right]^{\alpha_{it}}\right)\left[(\overline{x}_j)^{\widehat{r}_{it}} - \left(\underline{x}_j\right)^{\widehat{r}_{it}}\right] + \left(\underline{x}_j\right)^{\widehat{r}_{it}}$$

In step 3, the estimated parameters $\left\{\widehat{a}_{it}, \widehat{b}_{it}\right\}$ are used to calculate subject i's estimated source function value $\widehat{w}_{it}(j/8) = \exp\left(-\widehat{\beta}_{it}\left[-\ln(j/8)\right]^{\widehat{\alpha}_{it}}\right)$ for

$j = 1, ..., 7$. To test for treatment effects for each $j = 1, ..., 7$ the distributions of $\widehat{w}_{it}(j/8)$ are compared across treatments using a t test. Additionally, a likelihood sensitivity index and a pessimism index are calculated for each subject and compared across treatments.

We believe ABPW's empirical approach may be improved in three ways. First, the t tests conducted in step 3 to compare the distributions of $\widehat{w}_{it}(j/8)$ across treatments are valid if one treats the $\widehat{w}_{it}(j/8)$ as (recoded) data, but they are not valid if one treats the $\widehat{w}_{it}(j/8)$ as econometric estimates, i.e. random variables whose standard deviations depend on the sampling error from the estimation of $\left\{\widehat{r}_{it}, \widehat{a}_{it}, \widehat{b}_{it}\right\}$ in steps 1 and 2. Consistent with the econometric literature, our empirical approach treats $\left\{\widehat{r}_{it}, \widehat{a}_{it}, \widehat{b}_{it}\right\}$ as econometric estimates. Second, ABPW estimate four parameters $\left\{\widehat{\delta}_{it}, \widehat{r}_{it}, \widehat{a}_{it}, \widehat{b}_{it}\right\}$ when only three are necessary: $\left\{\widehat{r}_{it}, \widehat{a}_{it}, \widehat{b}_{it}\right\}$. Given the small sample size with which the parameters are estimated (13 or 19 observations depending on the treatment) adding an auxiliary parameter reduces the efficiency of the estimates. In our empirical approach, we therefore estimate only the parameters of interest $\left\{\widehat{r}_{it}, \widehat{a}_{it}, \widehat{b}_{it}\right\}$. Third, by segmenting the data between step 1 and step 2, some information is ignored when estimating the parameters. Indeed, $\left\{\widehat{\delta}_{it}, \widehat{r}_{it}\right\}$ are estimated with 6 observations, while $\left\{\widehat{a}_{it}, \widehat{b}_{it}\right\}$ are estimated with the remaining 7 or 13 observations (depending on the treatment). Instead, we estimate $\left\{\widehat{r}_{it}, \widehat{a}_{it}, \widehat{b}_{it}\right\}$ jointly with all the 13 or 19 observations collected for subject i in treatment t. In general, such a joint estimation should provide more precise estimates.

Appendix D: Structural Model with Calibration Errors

Did subjects report different certainty equivalents for risky bets in treatment `K` and $\texttt{K}_2$ because of calibration errors they made when calculating objective probabilities? To address this question, we assume that the subjective probability of subject i for prospect j in treatment t is equal to the actual probability plus some noise: $p_{ijt} = P_j + \varepsilon_{ijt}$ where ε_{ijt} follows a truncated normal $N_{\left[-P_j, 1-P_j\right]}(0, \sigma_t^2)$ so that $p_{ijt} \in [0, 1]$. It is easy to show that if $\sigma_{\texttt{K}_2} > \sigma_{\texttt{K}}$, then the certainty equivalents will be higher (respectively lower) on average in $\texttt{K}_2$ than in `K` when $P_j < 1/2$ (respectively $P_j > 1/2$).[10] In other words, differences

[10] Although similar in spirit, this model in which calibration errors are added to the objective probability is different from random utility models (see e.g. Blavatsky 2007, "Stochastic

in σ_t could explain (at least in part) why subjects generally select different certainty equivalents across treatments.

Under the source method, the indifference of subject i between prospect j and his elicited certainty equivalent implies:

$$(CE_{ijt})^{r_{it}} = \exp\left(\ln(a_{it})\left[\ln p_{ijt}/\ln(a_{it})\right]^{b_{it}}\right)\left[(\overline{x}_j)^{r_{it}} - \left(\underline{x}_j\right)^{r_{it}}\right] + \left(\underline{x}_j\right)^{r_{it}} \quad (5.1)$$

or equivalently

$$p_{ijt} = \exp\left\{\left[\ln\left(a_{it}\right)\right]^{1-1/b_{it}}\left[\ln\left(\frac{(CE_{ijt})^{r_{it}} - \left(\underline{x}_j\right)^{r_{it}}}{(\overline{x}_j)^{r_{it}} - \left(\underline{x}_j\right)^{r_{it}}}\right)\right]^{1/b_{it}}\right\}$$

with $p_{ijt} \sim N_{[0,1]}\left(P_{_j}, \sigma_t^2\right)$. The structural parameters $\{r_{it}, a_{it}, b_{it}, \sigma_t\}$ can then be estimated by maximum likelihood. Similar to the other parameters we write $\sigma_t = \sigma_{\mathtt{K}} + \sigma_{\mathtt{K}_2}\mathtt{K}_2 + \sigma_{\mathtt{U}}(\mathtt{K}_2{+}\mathtt{U})$.

As shown in Table $E3$, $\sigma_{\mathtt{U}}$ and $\sigma_{\mathtt{K}_2}$ are both positive and significant. Thus, subjects facing an ambiguous bet appear to make larger calibration errors than when they face a risky bet with simple events. Further, subjects make the largest calibration errors when they face risky bets with complex events (which are arguably more difficult to evaluate than bets with simple events).

These differences in calibration errors, however, are not sufficient to explain the differences in certainty equivalents across treatments. Indeed, comparing the results in Table $E1$ with those in the left panel of Table 1 shows that the sign, magnitude and significance of the other parameters remain essentially unchanged when we account for calibration errors. In other words, while there are differences in the calibration errors subjects make when calculating probabilities for simple, complex and ambiguous bets, we still find evidence that a subject's source function is statistically different for risky bets based on simple events and for risky bets based on more complex events.

Expected Utility Theory," Journal of Risk and Uncertainty) in which an error is added to the expected utility. In particular, it is easy to show that random utility models do not necessarily produce the pattern just described, i.e. higher (respectively lower) certainty equivalents on average when $P_{_j} < 1/2$ (respectively $P_{_j} > 1/2$).

Appendix E: Additional Figures and Tables

Table E1
Comparison of Certainty Equivalents across Treatments in Experiment 1

Treatments	P=1/8	P=2/8	P=3/8	P=4/8	P=5/8	P=6/8	P=7/8
K vs. U	*Z*=0.229 *P*=0.819	*Z*=1.362 *P*=0.173	*Z*=1.873 *P*=0.061	*Z*=3.578 *P*=3.5E-4	*Z*=5.127 *P*=2.9E-7	*Z*=6.399 *P*=1.6E-10	*Z*=5.881 *P*=4.1E-9
K vs. K_2	*Z*=-0.577 *P*=0.564	*Z*=0.249 *P*=0.803	*Z*=3.615 *P*=3.0E-4	*Z*=4.542 *P*=5.6E-6	*Z*=6.362 *P*=2.0E-10	*Z*=5.410 *P*=6.3E-8	*Z*=5.969 *P*=2.4E-9
U vs. K_2	*Z*=1.181 *P*=0.238	*Z*=1.842 *P*=0.065	*Z*=-1.884 *P*=0.059	*Z*=-0.582 *P*=0.560	*Z*=-1.200 *P*=0.230	*Z*=1.697 *P*=0.090	*Z*=1.013 *P*=0.311

Wilcoxon Sign-Ranked tests are conducted to compare subjects' certainty equivalents across treatments. Under the null hypothesis, the distributions of subjects' certainty equivalents are the same across treatments. Shaded cells indicate tests significant at the 5% level.

Table E2
Comparison of Reported Probabilities across Treatments in Experiment 2

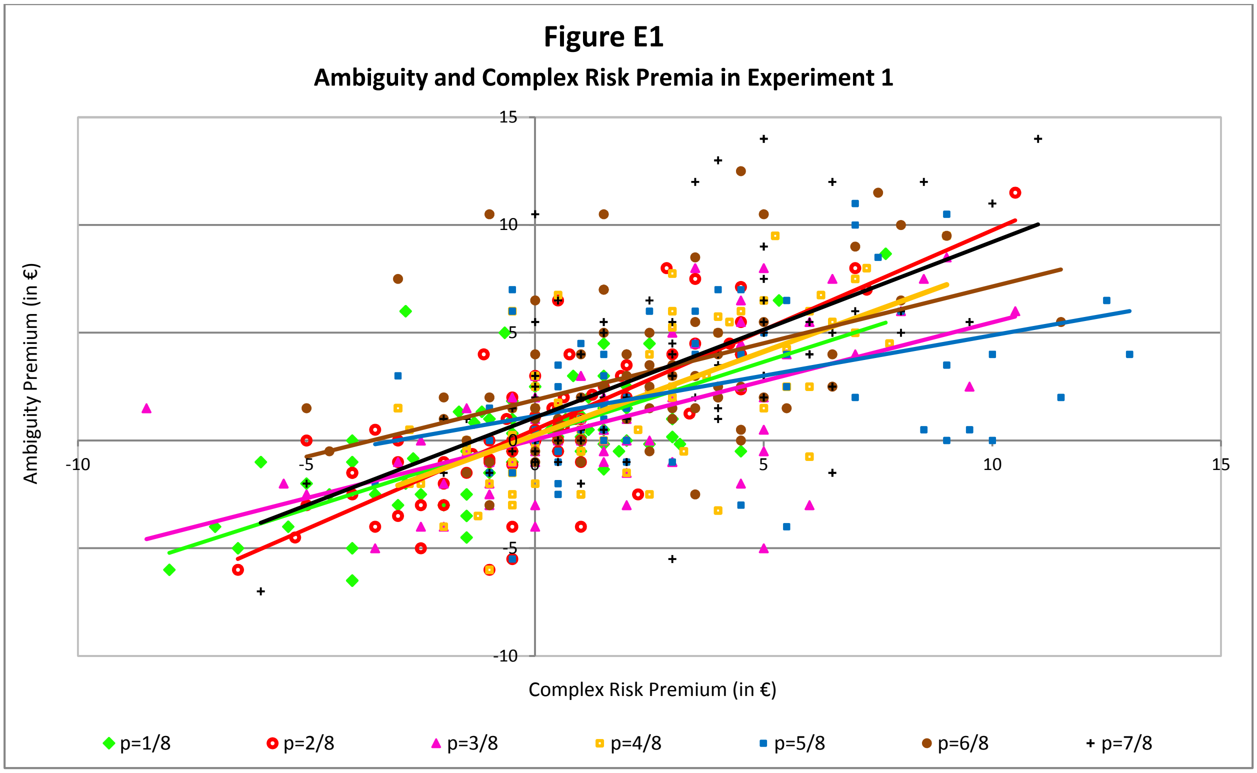
Objective Probability	Rank Sum for each Treatment			Friedman Statistic	p-value	Objective Probability	Rank Sum for each Treatment			Friedman Statistic	p-value
	K	K_2	C				K	K_2	C		
3%	53.5	108	96.5	40.26	1.81E-9	45%	70	95	94	10.92	0.004
Objective Probability	Rank Sum for each Treatment			Friedman Statistic	p-value	Objective Probability	Rank Sum for each Treatment			Friedman Statistic	p-value
	K	K_2	C				K	K_2	C		
5%	58	103	97	28.43	6.71E-7	61%	102	78	79	9.86	0.007
Objective Probability	Rank Sum for each Treatment			Friedman Statistic	p-value	Objective Probability	Rank Sum for each Treatment			Friedman Statistic	p-value
	K	K_2	C				K	K_2	C		
15%	60	103	96	25.02	3.68E-6	70%	109	66	84	23.45	8.10E-6
Objective Probability	Rank Sum for each Treatment			Friedman Statistic	p-value	Objective Probability	Rank Sum for each Treatment			Friedman Statistic	p-value
	K	K_2	C				K	K_2	C		
25%	67	98 94		13.49	0.001	80%	110	60	89	32.19	1.02E-7
Objective Probability	Rank Sum for each Treatment			Friedman Statistic	p-value	Objective Probability	Rank Sum for each Treatment			Friedman Statistic	p-value
	K	K_2	C				K	K_2	C		
35%	59	113	87	35.57	1.89E-8	90%	105	70	84	15.99	1.74E-3

Friedman tests are conducted to compare a subject's reported probabilities across treatments. Under the null hypothesis, the distributions of a subject's reported probabilities are the same across treatments. Shaded cells indicate tests significant at the 5% level.

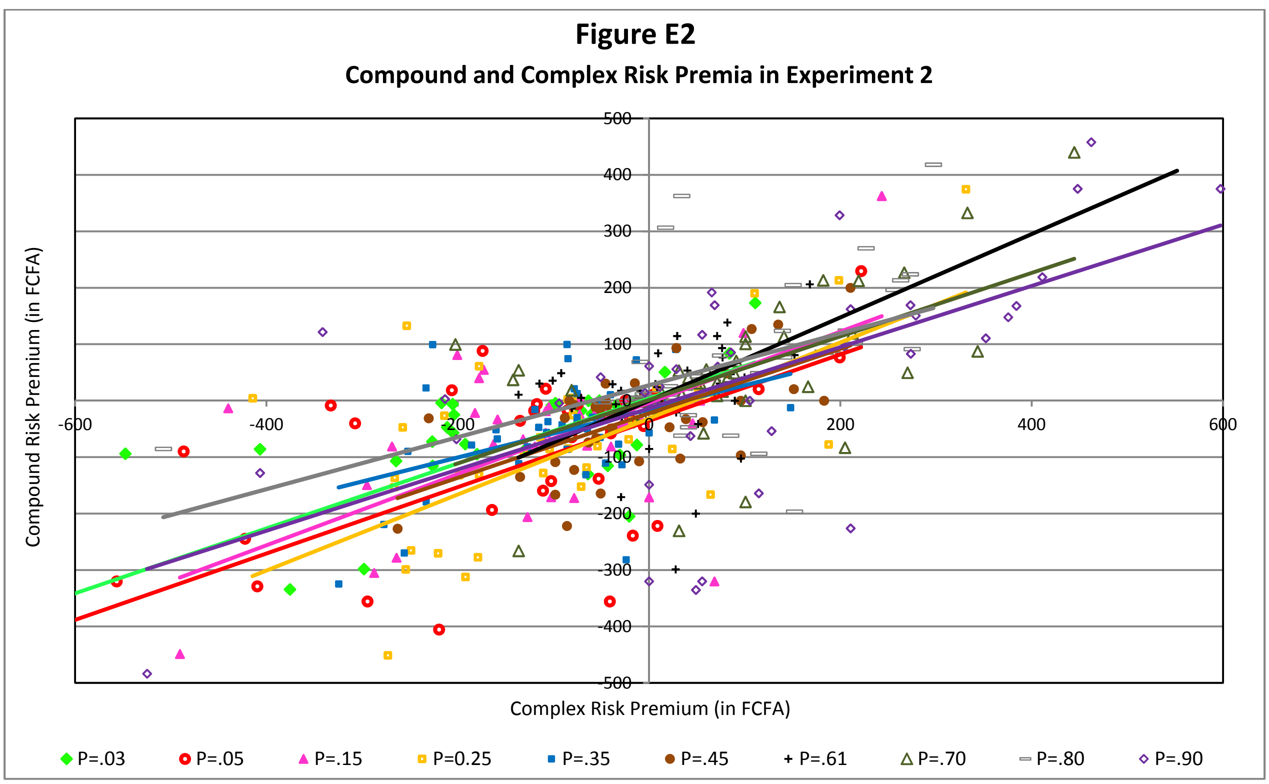
Table E3
Structural Estimation of Utility and Source Functions
Models with Calibration Errors

	Homo-genous	Heterogenous				Homo-genous	Heterogenous		
		Average across Subjects	Std across Subjects	% of subjects with significant parameter at 5%			Average across Subjects	Std across Subjects	% of subjects with significant parameter at 5%
r_K	0.039 (0.050)	0.095	0.452	51.9%	b_K	0.817*** (0.043)	1.002	0.218	—
r_U	-0.030 (0.071)	-0.018	0.413	11.7%	b_U	-0.201*** (0.053)	-0.242	0.413	51.9%
r_{K_2}	0.023 (0.095)	0.006	0.596	10.4%	b_{K_2}	-0.054 (0.058)	-0.068	0.377	18.2%
a_K	0.512*** (0.032)	0.478	0.385	—	σ_K	0.098*** (0.003)	0.060	0.033	—
a_U	-0.113** (0.049)	-0.085	0.423	33.8%	σ_U	0.039*** (0.008)	0.027	0.045	37.7%
a_{K_2}	-0.047 (0.041)	-0.044	0.417	14.3%	σ_{K_2}	0.018** (0.009)	0.026	0.053	27.3%

Estimates' standard errors and test statistics' distributions are estimated by bootstrap (size=10,000) to account for the finiteness of the sample.
***, **, * represent parameters significant at respectively the 1%, 5% and 10% significance level.



Each dot represents a subject. Each color represents a probability P=1/8,…,7/8. Each line represents the linear trendline for the corresponding color/probability. The figures shows that the positive relationship between ambiguity and complex risk premia holds for each P=1/8,…,7/8.



Each dot represents a subject. Each color represents a probability P=3%,…,90%. Each line represents the linear trendline for the corresponding color/probability. The figures shows that the positive relationship between compound and complex risk premia holds for each P=3%,…,90%.

Appendix F: Instructions Experiment 2 (translated from French)

Your Identification Code: _______

You are about to take part in an experiment aimed at better understanding decisions made under uncertainty. In the experiment you will earn an amount of money. This amount of money will be paid to you at the end of the experiment, outside the lab, in private, and in cash. The amount of money you will earn may be larger if :

1. You read the instructions below carefully.
2. You follow these instructions precisely.
3. You make thoughtful decisions during the experiment.

If you have any questions while we read the instructions or during the experiment, then call us **by raising your hand.** Any form of communication between participants is absolutely forbidden. If you do not follow this rule, then we will have to exclude you from the experiment without any payment.

The Task

You will be given 30 different «events», divided into 3 series of 10. Each of these events describes the possible outcome produced by the roll of 2 dice. One of the die is red, the other die is black. Each die has 10 sides numbered from 0 to 9. Each die is fair, which means that any of the 10 sides has an equal chance to come up when the die is rolled. Consider now two examples of events we could give you:

- Event 1: ***«The red die equals 5 and the black die equals 3»***.
- Event 2: ***«The red die produces a number strictly greater than the black die»***.

As explained below, 1 out of the 30 events will be randomly selected for payment at the end of the experiment. We will then roll the 2 dice once in order to determine whether the event occurs or whether the event does not occur. For instance, if Event 1 above is randomly selected for payment, then we will say that Event 1 occurred when the outcome of the roll of the 2 die is such that the red die produces a 5 and the black die produces a 3. For any other number produced by either the black or the red die, we will say that Event 1 did not occur. Likewise, if Event 2 is randomly selected for payment, then we will say that Event 2 occurred when the outcome of the roll of the 2 dice is such that the red die produces a number strictly greater than the black die. Otherwise, we will say that Event 2 did not occur.

Your Choices:

For each of the 30 events, you will be asked to make a choice. One of these choices will determine the amount of money you will earn both when the event randomly selected for payment occurs and when it does not occur. Each of your choices consists in selecting a number between 1 and 149 in the table we gave you separately. We will now explain how your choice for the event randomly selected for payment affects the amount of money you will earn.

If you look at the table, you can see that there are two amounts associated with each of the 149 possible choice numbers. The first is the amount of money you receive if the event occurs. The second is the amount of money you receive if the event does not occur. For instance, you can see in the table that the amounts associated with the choice number “1” are 53 and 4,000. This means that the amount of money you earn would be 53FCFA if the event occurs or 4,000FCFA if the event does not occur. As you can see, when the choice number increases from 1 to 149, the amounts in the first columns increase, while the amounts in the second column decrease. For instance, the amounts associated with the choice number “90” are 3,360FCA and 2,560FCFA. In other words, if you choose the number “90” instead of the number “1” then you would earn more if the event occurs (3,360FCFA instead of 53FCFA), but you would earn less if the event does not occur (2,560FCFA instead of 4,000FCFA). Note also, that the highest choice numbers (those closer to 149) produce the largest amounts of money when the event occurs, but the smallest amounts of money when the event does not occur. For instance, the choice number “140” produces 3,982FCFA if the event occurs, but only 516FCFA when the event does not occur.

For each of the 30 events, you are free to select any choice number you want. Note that there is no correct or incorrect choice. The choice numbers selected may differ from one individual to the next. In general however, you may find it profitable to choose a higher choice number when you think the chances that the event occurs are higher. Indeed, as we just explained, such a choice number will produce a larger amount if the event occurs. Conversely, you may find it profitable to choose a smaller number when you think the chances that the event occurs are lower.

Your Payment

The amount of money you receive today will be determined in 3 steps. In a first step, we will randomly select one of the 30 events for payment. In a second step, we will roll the 2 dice once to determine whether the event selected for payment occurs or does not occur. Finally, in a third step, we will look at the choice number you chose for the event selected for payment in order to determine the amount of money you will receive.

We will proceed as follows to select one of the 30 events for payment. At the beginning of the experiment, we will ask you to write your identification code on a piece of paper that you will then fold. Your identification code is located on the top right hand corner on the first page of the instructions. At the end of the experiment, we will draw at random one of the pieces of paper. The person whose identification number has been drawn will randomly choose 1 out of 30 numbered tokens from a bag. The number written on the token selected indicates the event that will be considered for the payment of each person in the room.

We will then draw at random a second piece of paper. The person whose identification code has been drawn will roll the 2 dice once to determine whether the event selected occurs or not. This single roll will be used to determine the payment of each person in the room.

If you do not wish to be one of the persons rolling the dice or drawing the token, then simply leave your piece of paper blank. Just fold it without writing your identification code.

Comprehension Test:

Understanding the instructions well is important if you want to improve your chances to earn a larger amount of money during the experiment. In order to make sure you understand the instructions well, we will now conduct a quick test without monetary consequences. Imagine first that Event 1: ***«The red die equals 5 and the black die equals 3»*** has been selected for payment. In addition, imagine that an individual selected the choice number **98** for this event, while a different individual selected the choice number **139**. Please, write in the table below the amount of money each of these 2 individuals would receive if the roll of the dice produces the following outcomes:

Outcome produced by the roll of the 2 dice	Payment to the individual with	
	A choice number of **98**	A choice number of **139**
The red die equals 6 and the black die equals 4	________FCFA	________FCFA
The red die equals 5 and the black die equals 4	________FCFA	________FCFA
The red die equals 5 and the black die equals 3	________FCFA	________FCFA

Imagine now that Event 2: ***«The red die produces a number strictly greater than the black die»*** has been selected for payment. In addition, imagine that an individual selected the choice number **6** for this event, while a different individual selected the choice number **71**. Please, write in the table below the amount of money each of these 2 individuals would receive if the roll of the dice produce the following outcomes:

Outcome produced by the roll of the 2 dice	Payment to the individual with	
	A choice number of **6**	A choice number of **71**
The red die equals 3 and the black die equals 9	________FCFA	________FCFA
The red die equals 5 and the black die equals 2	________FCFA	________FCFA
The red die equals 0 and the black die equals 5	________FCFA	________FCFA

Please, do not hesitate to raise your hand now if the instructions we just read were not perfectly clear. Once the experiment starts you can still call us to answer any question by raising your hand.

Note that the amount of money you will receive today may be larger or smaller depending on your choices and on the outcome produced by the roll of the 2 dice. By accepting to participate in the experiment, you accept the consequences associated with your choices and with the roll of the dice. If you do not wish to participate in the experiment you are free to leave now, in which case you will receive a flat fee of 500FCFA.

Series 1 :

For the first series of 10 events, we will consider that the red die determines the first digit (meaning 0, 10, 20, 30, 40, 50, 60, 70, 80, or 90) and the black die determines the second digit (meaning 0, 1, 2, 3, 4, 5, 6, 7, 8, or 9) of a number between 1 and 100 (both dice equal to zero corresponds to the number 100). As a result, every number between 1 and 100 has an equal chance to come out from the roll of the 2 dice.

Event	Description	Your Choice Number
1	***« the number is between 1 (included) and 25 (included) »***	
2	***« the number is between 62 (included) and 66 (included) »***	
3	***« the number is between 16 (included) and 76 (included) »***	
4	***« the number is between 3 (included) and 92 (included) »***	
5	***« the number is between 52 (included) and 96 (included) »***	
6	***« the number is between 9 (included) and 88 (included) »***	
7	***« the number is between 44 (included) and 58 (included) »***	
8	***« the number is between 23 (included) and 25 (included) »***	
9	***« the number is between 37 (included) and 71 (included) »***	
10	***« the number is between 28 (included) and 97 (included) »***	

Series 2 :

For the next series of 10 events, we will sum the outcome of the red die to the outcome of the black die. Since each die can only produce a number between 0 and 9, the sum obtained can only be a number between 0 and 18. Observe that some of these sums (for instance 0) can only be obtained from a unique combination of the 2 dice, while other sums (for instance 6) can be obtained from multiple combinations of the 2 dice. As a result, some of the 19 possible sums have more chances to come out than other sums.

Event	Description	Your Choice Number
11	***« The sum is between 0 (included) and 4 (included) »***	
12	***« The sum is between 2 (included) and 10 (included) »***	
13	***« The sum is equal to 16 »***	
14	***« The sum is between 4 (included) and 14 (included) »***	
15	***« The sum is between 5 (included) and 13 (included) »***	
16	***« The sum is between 0 (included) and 14 (included) »***	
17	***« The sum is between 10 (included) and 18 (included) »***	
18	***« The sum is equal to 4 »***	
19	***« The sum is between 11 (included) and 17 (included) »***	
20	***« The sum is between 2 (included) and 6 (included) »***	

Series 3 :

The last series of 10 events is similar to the first series. The red die determines the first digit and the black die determines the second digit of a number between 1 and 100. The difference with the first series is that, when you select your choice number, you are not facing 1, but 2 possible events. For instance, the 1st of the 2 possible events could be ***«the number is between 1 (included) and 25 (included)»*** and the 2nd of the 2 possible events could be ***«the number is between 55 (included) and 59 (included)»***. You are asked to select a single choice number without knowing which of the 2 possible events will be used to determine your payment. It is only at the end of the experiment that we will toss a coin to identify which of the 2 possible events will be used for payment. If the coin lands on **Heads**, then your payment will be determined using the 1st event. If the coin lands on **Tails**, then your payment will be determined using the 2nd event. As with Series 1, we will then roll the 2 dice to determine whether the event identified by the coin toss occurs or not. Here is an example :

♦ If the coin lands on **Heads**, then the event is :
«the number is between 1 (included) and 25 (included)».
♦ Or, if the coin lands on **Tails**, then the event is :
«the number is between 55 (included) and 59 (included)».

You must select a unique choice number before you know which of the possible 2 events will be used for payment. Imagine for instance that an individual selects the choice number 70. We have to distinguish between different 2 situations to determine how much the individual will be paid:

- ♦ Either the coin tossed at the end of the experiment lands on **Heads**. In this case, the event used for payment is ***«the number is between 1 (included) and 25 (included)»***. Then, the event occurs if the 2 dice produce a number that is indeed between 1 (included) and 25 (included), and the individual in our example is paid 2,862FCFA. On the other hand, if the 2 dice produce a number that is not between 1 (included) and 25 (included), then the event does not occur and the individual in our example is paid 3,129FCFA.
- ♦ Or the coin tossed at the end of the experiment lands on **Tails**. In this case, the event identified is ***«the number is between 55 (included) and 59 (included)»***. Then, the event occurs if the 2 dice produce a number that is indeed between 55 (included) and 59 (included), and the individual in our example is paid 2,862FCFA. On the other hand, if the 2 dice produce a number that is between 55 (included) and 59 (included), then the event does not occur and the individual in our example is paid 3,129FCFA.

To summarize, there are only 2 cases under which the event occurs : 1) The coin lands on **Heads** and the 2 dice produce a number between 1 (included) and 25 (included), or 2) the coin lands on **Tails** and the 2 dice produce a number between 55 (included) and 59 (included). In all other cases, the event does not occur. Thus, when you select your choice number, you might want to imagine the different cases under which the event occurs and does not occur.

If these explanations are not sufficiently clear, please call us by raising your hand. We will then come to your desk to answer any questions you may have. We would like to remind you that it is important for you to understand the instructions well so that you can make the decisions that suit you the best.

Event	Description	Your Choice Number
21	♦ If the coin lands on **<u>Heads</u>**, then the event is : ***«the number is between 48 (included) and 82 (included)»***. ♦ <u>Or</u>, if the coin lands on **<u>Tails</u>**, then the event is : ***«the number is between 14 (included) and 48 (included)».***	
22	♦ If the coin lands on **<u>Heads</u>**, then the event is : ***«the number is between 21 (included) and 35 (included)»***. ♦ <u>Or</u>, if the coin lands on **<u>Tails</u>**, then the event is : ***«the number is between 30 (included) and 44 (included)».***	
23	♦ If the coin lands on **<u>Heads</u>**, then the event is : ***«the number is between 25 (included) and 89 (included)»***. ♦ <u>Or</u>, if the coin lands on **<u>Tails</u>**, then the event is : ***«the number is between 2 (included) and 96 (included)».***	
24	♦ If the coin lands on **<u>Heads</u>**, then the event is : ***«the number is between 66 (included) and 97 (included)»***. ♦ <u>Or</u>, if the coin lands on **<u>Tails</u>**, then the event is : ***«the number is between 13 (included) and 70 (included)».***	
25	♦ If the coin lands on **<u>Heads</u>**, then the event is : ***«the number is between 56 (included) and 58 (included)»***. ♦ <u>Or</u>, if the coin lands on **<u>Tails</u>**, then the event is : ***«the number is between 78 (included) and 80 (included)».***	
26	♦ If the coin lands on **<u>Heads</u>**, then the event is : ***«the number is between 82 (included) and 89 (included)»***. ♦ <u>Or</u>, if the coin lands on **<u>Tails</u>**, then the event is : ***«the number is between 25 (included) and 66 (included)».***	
27	♦ If the coin lands on **<u>Heads</u>**, then the event is : ***«the number is between 7 (included) and 88 (included)»***. ♦ <u>Or</u>, if the coin lands on **<u>Tails</u>**, then the event is : ***«the number is between 3 (included) and 100 (included)».***	
28	♦ If the coin lands on **<u>Heads</u>**, then the event is : ***«the number is equal to 12»***. ♦ <u>Or</u>, if the coin lands on **<u>Tails</u>**, then the event is : ***«the number is between 49 (included) and 57 (included)».***	
29	♦ If the coin lands on **<u>Heads</u>**, then the event is : ***«the number is between 26 (included) and 86 (included)»***. ♦ <u>Or</u>, if the coin lands on **<u>Tails</u>**, then the event is : ***«the number is between 14 (included) and 74 (included)».***	
30	♦ If the coin lands on **<u>Heads</u>**, then the event is : ***«the number is between 1 (included) and 83 (included)»***. ♦ <u>Or</u>, if the coin lands on **<u>Tails</u>**, then the event is : ***«the number is between 36 (included) and 91 (included)».***	