

MONETARY MECHANISMS*

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September 4, 2015

Abstract

We provide a series of results for economies where exchange is facilitated by liquid assets. Compared to past work, minimal structure is imposed on the mechanism determining the terms of trade. Four simple axioms lead to a class of mechanisms encompassing standard bargaining theories, competitive price taking and other solution concepts. Using only properties implied by the axioms, we prove existence and uniqueness of nondegenerate steady state. We also show how to support desirable outcomes using creatively designed mechanisms. Special cases include pure currency economies, but we also consider extensions to incorporate real assets and credit.

JEL Classification Nos: E42, E51

Keywords: Money, Mechanisms, Liquidity, Credit

*The paper emerged out of a bigger project with several others, including Fabrizio Mattesini, Cyril Monnet, Yu Zhu and Han Han. In particular, there is some overlap with Gu et al. (2015), which uses several of results in this paper, but these results are developed and proved only in this paper. Gu thanks the Economic and Policy Analysis Research Center at the University of Missouri for research support. Wright thanks the Ray Zemon Chair in Liquid Assets at the Wisconsin School of Business. The usual disclaimers apply.

1 Introduction

This note provides a series of useful technical results for economies where exchange is facilitated by liquid assets. The starting point is the environment in Lagos and Wright (2005), with some centralized and some decentralized trade, except we impose minimal structure on the mechanism determining the terms of trade. Instead of assuming, e.g., Nash bargaining or Walrasian pricing, we show how four simple axioms lead to a class of mechanisms that encompass standard bargaining theories, competitive price taking, and other interesting solution concepts. Using only the properties implied by our axioms, we characterize existence, and prove generic uniqueness of stationary monetary equilibrium. The uniqueness argument is related to one used in Wright (2010) for Nash bargaining, although a few issues in that presentation are corrected, and moreover we establish results for *any* mechanism in a broad class. We also show how to support desirable outcomes using a variant of the approach in Hu et al. (2009). Further, while pure-currency economies constitute a special case, we also consider extensions to incorporate real assets and credit.

The results extend and consolidate much previous work. The original Lagos and Wright (2005) setup has bilateral random matching and Nash bargaining, while extensions by Lagos and Rocheteau (2005), Rocheteau and Wright (2005) and others consider price taking and price posting with directed search. Alternative axiomatic and strategic bargaining solutions are studied by Aruoba et al. (2007), Zhu (2015) and others. Hu et al. (2009) use pure mechanism design, Galenianos and Kircher (2008) and Dutu et al. (2012) use auctions, while Silva (2015) considers monopolistic competition. See Nosal and Rocheteau (2011) or Lagos et al. (2015) for surveys of the literature; suffice it to say here the framework has become a workhorse in monetary economics. The relevance of this is that the model is not merely one of many chosen at random from the journals; it is being used extensively in theory and applied work. Hence, while our analysis is motivated by technical rather than substantive issues, the results should be of wide interest to those working on applications.

2 Environment

Time is discrete and continues forever. In each period two markets convene sequentially: first there is a decentralized market, or DM, with frictions detailed below; then there is a frictionless centralized market, or CM. There are two types of agents, buyers denoted b , and sellers denoted s . Types are permanent, although the results are basically the same when types are determined randomly each period. For now, the DM involves bilateral trade: a buyer meets a seller with probability α , and a seller meets a buyer with probability ζ . In the DM, sellers can produce but do not want to consume, while buyers want to consume but cannot produce, which precludes direct barter. For now, there is no record keeping (monitoring or communication) for DM activity, which precludes credit. Therefore DM transactions require some payment instrument, which will be cash in the benchmark specification. In the frictionless CM all agents work, consume, adjust their portfolios and settle their debts, depending on the version of the model under consideration.

Preferences between the CM and DM are separable, and the utility function in the CM is quasi-linear. Thus, the period utility functions of buyers and sellers are

$$\mathcal{U}^b(q, x, \ell) = u(q) + U(x) - \ell \text{ and } \mathcal{U}^s(q, x, \ell) = -c(q) + U(x) - \ell,$$

where q is the DM good, x is the CM good, and ℓ is labor. One unit of ℓ produces one unit of x in the CM, so the real wage is one (this is easily relaxed). The constraints $x \geq 0$ and $\ell \in [0, 1]$ are assumed not to bind, as can be guaranteed in the usual way. Assume U , u and c are twice continuously differentiable, with $U', u', c' > 0$, $U'', u'' < 0$ and $c'' \geq 0$. Also, $u(0) = c(0) = 0$. Agents discount between the CM and DM according to $\beta = 1/(1+r)$, with $r > 0$.¹

¹Quasi-linear CM utility is common in this literature, and is convenient because it implies all agents of a given type have the same continuation value entering the DM (see below). However, it can be relaxed as in Wong (2015a): at the cost of simplicity, our results hold for any CM utility function $\tilde{U}(x, 1 - \ell)$ with $\tilde{U}_{11}\tilde{U}_{22} = \tilde{U}_{12}^2$, including, e.g., $\tilde{U} = x^a(1 - \ell)^{1-a}$ and $\tilde{U} = [x^a + (1 - \ell)^a]^{1/a}$. See Wong (2015a) and Gu et al. (2015) for applications using this more general specification. Alternatively, one can use *any* monotone and concave \tilde{U} , and get the same results, by assuming indivisible labor, $\ell \in \{0, 1\}$, as in Rogerson (1988) (see Rocheteau et al. 2008). Separability between the CM and DM can also be relaxed (see Rocheteau et al. 2007).

Goods q and x are nonstorable. There is a storable asset interpreted for now as money, with supply per buyer M . Assume $M_{+1} = (1 + \pi)M$, where the subscript $+1$ on a variable indicates its value next period. Changes in M are accomplished by lump sum transfers if $\pi > 0$ or taxes if $\pi < 0$ (the results also hold if instead government uses new money to buy CM goods). For convenience, only buyers pay taxes or get transfers. Let ϕ be the price of money in terms of the CM numeraire x . We focus on stationary outcomes where real variables are constant, including real balances $z = \phi M$. Hence, inflation is $\phi_{+1}/\phi = 1 + \pi$, a version of the quantity equation. We also make use of the Fisher equation, $1 + i = (1 + \pi)(1 + r)$.² We restrict attention to $\pi > \beta - 1$, or the limit $\pi \rightarrow \beta - 1$, which means $i = 0$ and is called the Friedman rule; there are no monetary equilibria with $\pi < \beta - 1$.

3 Baseline Model

The state of an agent in the baseline model is real balances in terms of CM numeraire, $z = \phi m$. The value functions in the CM and DM are $W(z)$ and $V(z)$, which are time invariant in stationary equilibrium.

3.1 The CM Problem

The CM problem for an agent of type $j = b, s$ is

$$W_j(z) = \max_{x, \ell, \hat{z}} \{U(x) - \ell + \beta V_j(\hat{z})\} \text{ st } z + \ell = x + (1 + \pi)\hat{z} + T.$$

where \hat{z} denotes the money carried to the following DM and T is the lump sum tax.

The FOC's are

$$U'(x) - 1 = 0 \tag{1}$$

$$z + \ell - (1 + \pi)\hat{z} - x - T = 0 \tag{2}$$

$$-(1 + \pi)U'(x) + \beta V'_j(\hat{z}) \leq 0, = \text{ if } \hat{z} > 0. \tag{3}$$

²One can interpret $1 + i$ as the return in dollars that agents would require in the next CM to give up a dollar in the current CM. As usual, we can price such trades, whether or not they occur in equilibrium.

From (1), $x = x^*$ where x^* solves $U'(x^*) = 1$. The envelope condition is $W_j'(z) = 1$. From (3), all buyers choose the same \hat{z} , which is $\hat{z}_b > 0$ in monetary equilibrium. All sellers choose $\hat{z}_s = 0$, since they have no use for cash in the DM.

3.2 The DM Problem

When a buyer and seller meet in the DM they choose a quantity q and payment p according to some *general trading mechanism* Γ mapping z into (p, q) , which depends on z because of the feasibility constraint $p \leq z$. A trade (p, q) generates surpluses

$$\begin{aligned} S_b &= u(q) + W_b(z - p) - W_b(z) = u(q) - p \\ S_s &= -c(q) + W_s(p) - W_s(0) = p - c(q), \end{aligned}$$

by virtue of the envelope condition $W_j' = 1$. To guarantee gains from trade are positive but finite, assume $u'(0) > c'(0)$ and $\exists \bar{q} > 0$ such that $u(\bar{q}) = c(\bar{q})$. Then define the unconstrained efficient quantity q^* by $u'(q^*) = c'(q^*)$, and let $p^* = \inf \{z : \Gamma_q(L) = q^*\}$ be the minimum payment required for a buyer to get q^* .

We focus on mechanisms of the form

$$\Gamma_p(z) = \begin{cases} z & \text{if } z < p^* \\ p^* & \text{otherwise} \end{cases} \quad \text{and} \quad \Gamma_q(z) = \begin{cases} v^{-1}(z) & \text{if } z < p^* \\ q^* & \text{otherwise} \end{cases} \quad (4)$$

where v is a strictly increasing function with $v(0) = 0$ and $v(q^*) = p^*$. Section 4 presents axioms that imply Γ must take the form in (4) and discusses examples. In terms of economic content, (4) says that a buyer gets the efficient quantity q^* and pays some amount $p^* = v(q^*)$, determined by the mechanism, if he can afford it in the sense that $p^* \leq z$; and if he cannot afford it, he pays $p = z$ and gets some $q = v^{-1}(z) < q^*$ determined by the mechanism. Thus, $v^{-1}(z)$ is the quantity a constrained buyer gets, while $v(q)$ is how much it costs. For convenience, let us also assume v is twice continuously differentiable almost everywhere.

In the case of a pure-currency economy, even though sellers carry no money to the DM, we still generally write

$$V_s(z) = W_s(z) + \zeta \mathbb{E}[p - c(q)].$$

This sets the DM payoff to the continuation value from not trading plus the expected surplus from trading. The surplus could be random, in principle, because $p = \Gamma_p(\tilde{z})$ and $q = \Gamma_q(\tilde{z})$ depend on the real balances of the random buyer the seller meets, although in equilibrium we know that \tilde{z} is the same for all buyers.

The value function for a buyer is

$$V_b(z) = W_b(z) + \alpha [u(q) - p],$$

where $p = \Gamma_p(z)$ and $q = \Gamma_q(z)$ depend on his own real balances. It is easy to show that in stationary monetary equilibrium buyers are constrained, $p < z$ and $q < q^*$. Substituting this into V_b , then V_b into W_b , after simplifying we get

$$W_b(z) = \bar{W} + \beta \{-i\hat{z} + \alpha [u(q_{+1}) - v(q_{+1})]\},$$

where i comes from the Fisher equation, and $\bar{W} = z + U(x^*) - x^* - T + \beta W_b(0)$ is irrelevant for the choice of \hat{z} . Thus, $W_b(z) = \bar{W} + \alpha\beta J(q_{+1}; i)$ where

$$J(q_{+1}; i) = u(q_{+1}) - (1 + i/\alpha)v(q_{+1}).$$

This change of variable replaces the choice of \hat{z} with the direct choice of q_{+1} . Without loss of generality, impose $q \in [0, q^*]$ and represent the problem by

$$q_i = \arg \max J(q; i) \text{ st } q \in [0, q^*]. \quad (5)$$

An interior solution $q_i > 0$ satisfies the FOC

$$u'(q) - (1 + i/\alpha)v'(q) = 0.$$

3.3 Equilibrium

A stationary monetary equilibrium is a CM allocation (x, ℓ) , a DM outcome (p, q) and real balances z such that x and ℓ solve (1)-(2), q solves (5) and $p = z = v(q) > 0$. A nonmonetary equilibrium is similar except $p = q = z = 0$. Given $\mathcal{U}^b(q, x, 1 - \ell)$ is separable between the CM and DM, (p, q) is independent of (x, ℓ) , so we can discuss

some properties of the former without reference to the latter. Then, to proceed, assume $\exists q > 0$ such that $v(q) < u(q)$. This means that there would be trade absent liquidity considerations, and it holds automatically for any reasonable mechanism.

Proposition 1 *For generic parameters the solution q_i to (5) exists, is unique, and $q \in (0, q_0) \Rightarrow \partial q / \partial i < 0$.*

Proof. The buyers' problem is $\max_q J(q; i)$ st $q \in [0, q^*]$, where $J(q; i)$ is twice continuously differentiable and $J(0; i) = 0$. Clearly a solution exists. Since $v(q) < u(q)$ for some $q > 0$, 0 is not a solution when $i \approx 0$. Hence, for i not too big, $q_i > 0$ maximizes $J(q; i)$, although there might be multiple local maximizers, as shown in Figure 1a, where the higher curve is for $i = i_1$ and the lower one is for $i_2 > i_1$. At any local maximum $J_q(q; i) = u' - (1 + i/\alpha)v' = 0$ and $J_{qq} = u'' - (1 + i/\alpha)v'' < 0$. We claim the global maximizer is unique for generic i . To see this, suppose $J(q_1^*; i) = J(q_2^*; i) = \max_q J(q; i)$ with $q_2^* > q_1^*$. Since $J(q_1^*; i)$ is the lowest global maximizer, $J(q_1^*; i) > J(q; i) \forall q < q_1^*$. Increase i to $i + \varepsilon$. For any $q < q_1^*$, by continuity, $J(q_1^*; i + \varepsilon) > J(q; i + \varepsilon)$ if ε is small enough. Since $J_i(q; i) = -v(q) < 0$ and $v(q)$ is increasing, $J_i(q_2^*; i) < J_i(q_1^*; i)$. Thus $J(q_1^*; i + \varepsilon) > J(q_2^*; i + \varepsilon)$, and now the global maximizer is unique at a q near q_1^* .

As shown in Figure 1a, increasing i_1 to i_2 shifts J down and shifts each local maximizer to the left. In particular, the global maximizer shifts to the left, and thus $\partial q_i / \partial i < 0$ when q_i is single valued. If we increase i from $i \approx 0$, we may reach a nongeneric point \tilde{i} where there are multiple global maximizers, say q_1^* and $q_2^* > q_1^*$. By the argument used above, for $\tilde{i} + \varepsilon$ the unique global maximizer is close to q_1^* and for $\tilde{i} - \varepsilon$ the unique global maximizer is close to q_2^* . So q_i is continuously decreasing in i and single-valued function except possibly for i in a set of measure 0, where it has multiples values, and then jumps to the left as i increases. Since for generic i there is a unique q_i , there cannot be multiple equilibria. ■

Corollary 1 *A stationary monetary equilibrium exists if $q_i > 0$, which is true as long as i is not too big.*

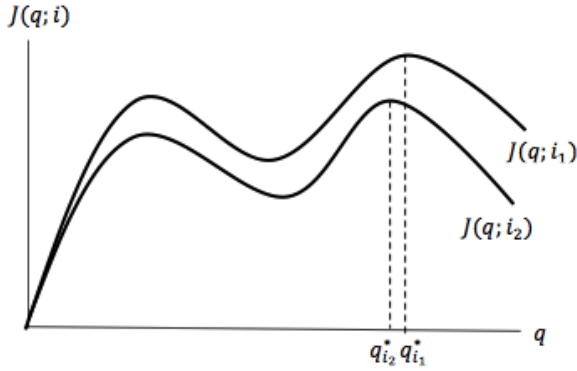


Figure 1a: The Objective Function

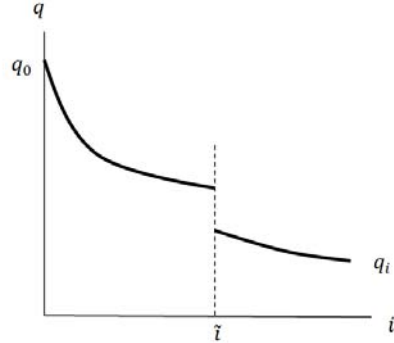


Figure 1b: The Solution q_i vs i

The key step in the above argument involves turning an equilibrium problem into a decision problem. This is not the same as the usual technique of characterizing competitive equilibrium by solving a planner's problem, of course, since equilibrium here is not generally efficient. Still, the idea is that buyers can take $q = v^{-1}(z)$ as given, similar to the way they take price as given in Walrasian markets, even though it can be determined here as the outcome of, e.g., bargaining. Figure 1b shows the solution q_i , which is strictly decreasing, and happens in this case to have a jump at $i = \tilde{i}$. One can interpret $z_i = v(q_i)$ as the demand for liquidity, the cost of which is fixed by policy at i . At $i = \tilde{i}$, buyers are indifferent between the two values on the demand correspondence, and so we can assign a fraction to each; changing i to $\tilde{i} + \varepsilon$, however, implies a unique q_i . Also note that the intercept is $q_0 \leq q^*$ (e.g., $q_0 = q^*$ with Kalai bargaining, while $q_0 < q^*$ with Nash bargaining).

The main result, we suggest, concerns generic uniqueness, and to some extent the monotone comparative statics, since existence is relatively standard, although note that the requirement in Corollary 1, that i is not too big, may or may not bind (e.g., with Kalai bargaining one can compute the upper bound for i , while with Nash monetary equilibrium exists for all i). To be clear, we emphasize that while stationary monetary equilibrium is generically unique, with fiat currency there is always a nonmonetary equilibrium, as well as nonstationary monetary equilibria.

Still, since many applications concentrate on stationary monetary equilibrium, the results should be very useful to practitioners.³

4 Mechanisms

We do not want the results to depend on a particular way of determining the terms of trade. How general is our class of mechanisms? Consider the following:

Axiom 1 (*Feasibility*): $\forall z, 0 \leq \Gamma_p(z) \leq z, 0 \leq \Gamma_q(z)$.

Axiom 2 (*Individual Rationality*): $\forall z, u \circ \Gamma_q(z) \geq \Gamma_p(z)$ and $\Gamma_p(z) \geq c \circ \Gamma_q(z)$.

Axiom 3 (*Monotonicity*): $\Gamma_p(z_2) > \Gamma_p(z_1) \Leftrightarrow \Gamma_q(z_2) > \Gamma_q(z_1)$.

Axiom 4 (*Bilateral Efficiency*): $\forall L, \exists(p', q')$ with $p' \leq z$ such that $u(q') - p' > u \circ \Gamma_q(z) - \Gamma_p(z)$ and $p' - c(q') > \Gamma_p(z) - c \circ \Gamma_q(z)$.

Note Axiom 3 does not say S_b and S_s are increasing in z , only that one must pay more p to get more q , so that Nash bargaining, e.g., satisfies this even though as is well known S_b may not be increasing in z . Also, while Axiom 4 seems reasonable, it is not critical for many results (e.g., they hold for the monopolistically competitive and monopsony mechanisms mentioned below). Also, Axiom 4 is an ex post condition in the DM saying that we cannot make the parties better off conditional on z ; it does not say the ex ante choice of z in the CM is efficient.

Proposition 2 *Any Γ satisfying Axioms 1-4 takes the form given in (4).*

Proof. First consider $z < p^*$. By Axiom 3 and the definition of p^* , we have $q = \Gamma_q(z) < q^*$. We prove $p = z$ by contradiction. Suppose $p \neq z$. We cannot have

³See the surveys by Nosal and Rocheteau (2011) and Lagos et al. (2015) for literally dozens of applications. Some of these integrate this monetary formulation with other models – e.g., Berentsen et al. (2011) combine it with a Pissarides (2000) labor market. That specification can have multiple monetary steady states, because there is feedback from the goods market to the labor market and vice versa. Our result is still useful in this context, however, since it at least guarantees the goods market outcome is unique given the labor market outcome.

$p > z$, by Axiom 1, so $p < z$. Consider $p' = p + \varepsilon_p < z$ and $q' = q + \varepsilon_q < q^*$, which is feasible for small $(\varepsilon_p, \varepsilon_q)$. If $\varepsilon_p = u(q') - u(q)$, one can easily check that the buyer's surplus S_b does not change, while for the seller

$$dS_s = p' - c(q') - p + c(q) = u(q') - c(q') - u(q) + c(q). \quad (6)$$

Since $q^* > q' > q$, $u(q) - c(q)$ is increasing in q . Therefore S_s increases, contradicting Axiom 4. Hence, $z < p^* \Rightarrow p = z$.

Next, consider $z \geq p^*$. We prove $q = q^*$ by contradiction. Suppose $q = \Gamma_q(z) < q^*$. We know $p = \Gamma_p(z) < p^*$ by Axiom 3. Let $p' = p + \varepsilon_p$ and $q' = q + \varepsilon_q$. As in the previous step, one can check (p', q') dominates (p, q) , contradicting Axiom 4. Suppose instead $q > q^*$. Let $p' = p - \varepsilon_p$ and $q' = q - \varepsilon_q > q^*$, where $\varepsilon_p = u(q) - u(q')$, which satisfies Axiom 1 and Axiom 3 for small $(\varepsilon_p, \varepsilon_q)$. One can check that S_b does not change while the change in S_s is the same as (6). Since $q > q' > q^*$, $u(q) - c(q)$ is decreasing in q . Therefore S_s increases, contradicting Axiom 4. Hence, $z \geq p^*$ implies $q = q^*$, which implies $p = p^*$ by the definition of p^* and Axiom 3. Hence, $v^{-1}(p^*) = q^*$. By Axiom 3, v^{-1} and v are strictly increasing. By Axiom 2, $v^{-1}(0) = 0$. By definition, $v^{-1}(p^*) = q^*$. This shows Γ is as described by (4). ■

A simple example is provided by Kalai's proportional bargaining solution, with θ the buyers' bargaining power. In this context, Kalai bargaining maximizes S_b wrt (p, q) subject to $S_b = \theta(S_b + S_s)$ and $p \leq z$. Let

$$v(q) = \theta c(q) + (1 - \theta) u(q) \quad (7)$$

and let $p^* = v(q^*)$. Then Kalai's solution is: if $p^* \leq z$ then $(p, q) = (p^*, q^*)$; and if $p^* > z$ then $p = z$ and $q = v^{-1}(z)$ with v given by (7). Another example is generalized Nash bargaining, which maximizes $S_b^\theta S_s^{1-\theta}$ wrt (p, q) subject to $p \leq z$. This gives a similar qualitative outcome, except

$$v(q) = \frac{\theta u'(q) c(q) + (1 - \theta) c'(q) u(q)}{\theta u'(q) + (1 - \theta) c'(q)}.$$

The Nash and Kalai solutions are the same if $\theta = 1$; if $\theta \in (0, 1)$, given $u'' < 0$ or $c'' > 0$, they are different when $p \leq z$ binds. Yet both take the form in (4).

Zhu (2015) shows that a simple strategic bargaining game also satisfies our axioms. This game has the seller make an initial offer (p, q) . If the buyer agrees, they trade; otherwise, a coin flip determines which one of them makes a final offer. If the final offer is accepted, they trade; otherwise, they part with no trade. This game is nice, for our applications, for several reasons discussed in Zhu (2015).⁴ He also shows a different game, where the buyer instead of the seller makes the initial offer, does not satisfy our axioms. In particular, it violates strict monotonicity, because the buyer might pay a higher p for the same q .

We can also use Walrasian pricing, motivated by saying agents trade in large groups, not bilaterally (see Rocheteau and Wright 2005 for details). This gives a similar qualitative outcome, except now $v(q) = \bar{p}q$, where agents take \bar{p} as given, even though in equilibrium $\bar{p} = c'(q)$. Silva (2015) provides a related example using monopolistic competition. Another example that violates Axiom 4 is a monopsonist taking as given $c'(q)$ rather than price,

$$\max_q [u(q) - qc'(q)] \text{ st } qc'(q) \leq z.$$

Let \tilde{q} be a solution without the liquidity constraint (the standard monopsonist outcome) and let $\tilde{p} = \tilde{q}c'(\tilde{q})$. This mechanism looks like (4), except the critical value is $\tilde{p} = v(\tilde{q})$ rather than $p^* = v(q^*)$. Still, it can be used in the monetary model.

Next, consider trying to construct $v(q)$ so to support a desirable q^o , which could be $q^o = q^*$ or something else, as in Hu et al. (2009) and Wong (2015b).

Proposition 3 *Let \hat{q} solve $u(\hat{q}) = (1 + i/\alpha)c(\hat{q})$. Then there exists a mechanism to support any $q^o \leq \min\{q^*, \hat{q}\}$. In particular, if $q^* \leq \hat{q}$ we can achieve $q^o = q^*$, even if $i > 0$. There is no way to support $q^o > q^*$.*

⁴One reason is that axiomatic bargaining is problematic for nonstationary equilibria (Coles and Wright 1998; Coles and Muthoo 2003). Another is that the game ends for sure, on or off the equilibrium path, in finite time; standard bargaining (Rubinstein 1982; Binmore et al. 1986) can in principle go on forever, which is awkward when the DM closes and the CM opens in finite time. It is also better than simply flipping a coin to see who makes a take-it-or-leave-it offer, as in some search models with linear utility (Gale 1987; Mortensen and Wright 2001): since our agents are risk averse, it is Pareto superior to use the initial offer and avoid the coin flip.

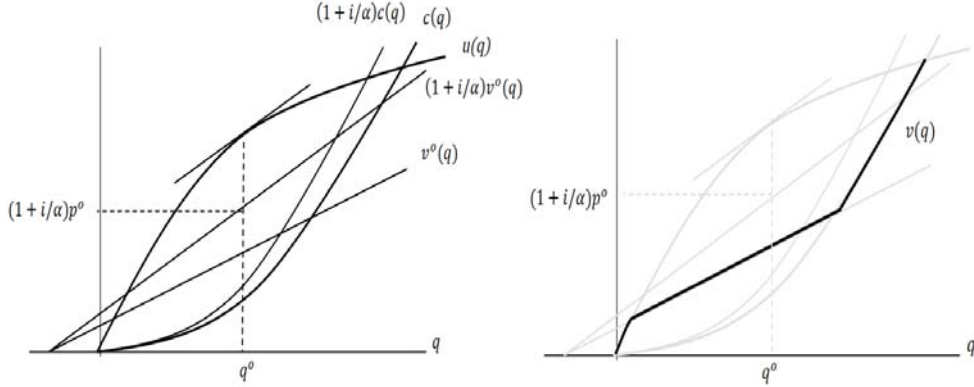


Figure 2: HKW Style Mechanism

Proof. Consider $q^o \leq \min\{q^*, \hat{q}\}$. Figure 2 shows $u(q)$ and $(1+i/\alpha)c(q)$. Pick $(1+i/\alpha)p^o$ such that $(1+i/\alpha)c(q^o) < (1+i/\alpha)p^o < u(q^o)$, which is possible since $q^o \in [0, \hat{q}]$. Draw a line through $(q^o, (1+i/\alpha)p^o)$ with slope $u'(q^o)$, labelled $(1+i/\alpha)v^o(q)$ in the graph. Now define $v(q)$ on $[0, \bar{q}]$ as follows: first rotate $(1+i/\alpha)v^o(q)$ to get $v^o(q)$; then truncate it above by $u(q)$ or $c(q)$, whichever v^o meets first from the left; and truncate it below by $u(q)$ or $c(q)$, whichever v^o meets first from the right. If v^o is truncated by u then J is negative on that truncation. If it is truncated by c then J is concave and the maximizer is not on the truncation. Since $v(\cdot)$ is strictly increasing, $v^{-1}(\cdot)$ is well defined, and a mechanism Γ is given by (4). This mechanism is consistent with trading q^o and $p^o = v(q^o)$ ex post, in the DM, because $c(q^o) \leq p^o \leq u(q^o)$. And it is consistent with the ex ante decision to bring enough z from the CM, because q^o is the global maximizer of $J(q; i) = u(q) - (1+i/\alpha)v(q)$. Hence we support q^o .

Now consider $q^o > q^*$. We claim this cannot be supported. Although a buyer may have the ex ante incentive to bring enough \hat{z} from the CM to pay p^o and get q^o , ex post in the DM there is an alternative (p, q) that Pareto dominates (p^o, q^o) , involving a reduction in q from q^o toward q^* combined with a reduction in p^o . Given Axiom 4, we cannot support $q^o > q^*$. Note this is not an issue for $q^o \leq q^*$, as when a buyer only brings enough to get q^o , renegotiation towards $q^* > q^o$ violates Axiom

1. So we can construct mechanisms that deliver any $q^o \leq q^*$ in the case with $\hat{q} > q^*$. If $\hat{q} < q^*$ we cannot support q^* ; \hat{q} is the highest incentive feasible q . ■

Intuitively, the idea in the proof is that $v(q)$ should give buyers the incentive in the CM to choose the right \hat{z} . This implies $u'(q^o) = (1 + i/\alpha)v'(q^o)$, which tells us something about $v(q)$, but we also have to ensure agents want to trade q^o after they meet in the DM. Another observation is that $q > \hat{q}$ cannot be supported, and i big makes \hat{q} small. So when i is high, we cannot achieve q^* even with this type of mechanism. Having said that, we can support q^* for $i > 0$ as long as i is not too big (i.e., we may not need the Friedman rule to get q^*). Finally, note that there is more than one way construct such mechanisms, and while our approach is similar in spirit to Hu et al. (2009), the details are rather different. In particular, that mechanism is linear over the range where the DM incentive conditions are slack.

5 Extensions

5.1 Real Assets

Consider replacing fiat money by an asset a , say a Lucas tree, giving off dividend $\rho > 0$ in terms of numeraire in each CM.⁵ The supply is fixed at A , and its price in terms of x is ψ . Then the CM constraint becomes $x = \ell + (\psi + \rho)a - \psi\hat{a}$, and the DM constraint $v(q) \leq (\psi + \rho)\hat{a}$. Also, to avoid a minor technicality, assume $q_0 = q^*$.⁶ Then, if the DM constraint does not bind, we get $\psi = \psi^* \equiv \rho/r$ and $q = q^*$. But if it does bind, which it will iff ρA is low, we get $\psi > \psi^*$ and $q < q^*$.

Let us assume ρA is low – the interesting case – and consider

$$J(q; \psi) = u(q) - \left[1 + \frac{r\psi - \rho}{\alpha(\psi + \rho)} \right] v(q).$$

⁵While we assume $\rho > 0$, it is possible to have equilibrium with assets valued, for their liquidity, even if $\rho < 0$, but that is more complicated (see Han et al. 2015 for an analysis in a related model).

⁶This is always true for Walrasian pricing or Kalai bargaining, e.g., but not necessarily Nash bargaining. In any case, the technicality is that in principle buyers may want to hold some real assets that they do not bring to the DM – something that never happens with fiat money. See Geromichalos et al. (2007) or Lagos and Rocheteau (2008) for more discussion.

The FOC is

$$u'(q) = \left[1 + \frac{r\psi - \rho}{\alpha(\psi + \rho)} \right] v'(q).$$

It is not hard to show, similar to Proposition 1, that the solution q_ψ exists, is generically unique and strictly decreases with ψ , with $q_\psi \rightarrow q^*$ as $\psi \rightarrow \psi^*$ and $q_\psi \rightarrow 0$ as $\psi \rightarrow \infty$. This can be interpreted as the demand for liquidity. The (inverse) supply function is given by $(\psi + \rho)A$, which is endogenous because it depends on ψ , and is strictly increasing with ψ . Hence, there is a unique equilibrium at $\psi > \psi^*$.⁷

5.2 Credit

Now assume that a buyer can make a promise of payment in numeraire at the next CM, subject to $p \leq D$. Here the debt limit D is exogenous, but it can be endogenized along the lines of Kehoe and Levine (1993) (see Gu et al. 2005 for details). The CM state variable is now wealth $z - d$, where d is debt from the previous DM. Then

$$W_j(z - d) = \max_{x, \ell, \hat{z}} \{U(x) - \ell + \beta V_j(\hat{z})\} \text{ st } z - d + \ell = x + (1 + \pi)\hat{z} + T$$

again implies $W_j' = 1$. A mechanism now maps a buyer's *liquidity position*, $L \equiv z + D$, into (p, q) . All of the above results go through, including the form of the mechanism in (4), with L replacing z .

One can easily show that $D \geq v(q^*)$ implies there is no monetary equilibrium, as buyers can get $q = q^*$ on credit. If $D < v(q^*)$, their choice reduces to

$$\tilde{q}_i = \arg \max J(q; i) \text{ st } q \in [q_D, q^*],$$

where $q_D = v^{-1}(D)$. If $\tilde{q}_i > q_D$, there is a monetary equilibrium where buyers use cash plus credit, while if $\tilde{q}_i = q_D$ there is no monetary equilibrium. One can show

⁷This is stronger than the result for fiat money, which establishes only generic uniqueness, because with $\rho = 0$ the analog of supply is perfectly elastic and could be like \tilde{i} in Figure 1b. Another difference from fiat money is that $\rho > 0$ rules out a stationary equilibria with $\psi = 0$, as well as nonstationary equilibria where $\psi \rightarrow 0$, but not necessarily other nonstationary equilibria, including cyclic, chaotic and stochastic outcomes (see, e.g., Rocheteau and Wright 2013). Also, one can study the model with both real assets and fiat money, which exists when ρA and i are both low; we leave that as an exercise.

\tilde{q}_i is unique and $\partial\tilde{q}_i/\partial i < 0$ in monetary equilibrium by an argument similar to the one for Proposition 1. Further, suppose for the sake of illustration that J is single peaked at, say, \bar{q}_i . Then $q_D < \bar{q}_i$ implies $\tilde{q}_i = \bar{q}_i$, while $q_D \geq \bar{q}_i$ implies $\tilde{q}_i = q_D$. Thus, monetary equilibrium exists iff D is not too big.⁸

Paralleling the discussion of Hu et al. (2009), one can also construct a mechanism to support a desirable q^o when credit is available up to limit D . Define $q_L \equiv u^{-1}(D)$ and $q_H = c^{-1}(D)$. Let \hat{q}_c solve $u(q) - (1 + i/\alpha)c(q) + iD/\alpha = 0$. This is the maximum amount of q that the buyer will accept ex ante given the best terms of trade. The next result is proved the Appendix.

Proposition 4 *If $D \geq u(q^*)$ then only q^* can be supported. If $D < u(q^*)$ then we have the following: any $q^o \in [q_L, \min\{q_H, q^*\}]$ can be supported by a nonmonetary equilibrium; any $q^o \in [q_L, \min\{\hat{q}_c, q^*\}]$ can be supported by a monetary equilibrium. No other q^o can be supported.*

5.3 Costly Credit

Now assume that a buyer can make a promise of any payment $p > z$ at a cost described by $\gamma(p - z)$, where $\gamma', \gamma'' > 0$ and $\gamma(0) = 0$.⁹ A trading mechanism Γ now delivers an outcome in the constrained core, constructed as follows. First, solve

$$\max_{p,q} S_b = [u(q) - p - \gamma(p - z) \mathbb{I}_{\{p > z\}}] \quad \text{st } p - c(q) = S_s$$

The FOC implies

$$u'(q) = \{1 + \gamma'[S_s + c(q) - z]\} c'(q), \quad (8)$$

if $z < S_s + c(q^*)$, and $q = q^*$ otherwise. Then impose $S_b, S_s \geq 0$ to get

$$\mathcal{C} \equiv \{(p, q) \mid q \text{ solves (8), } p = c(q) + S_s, S_s \geq 0, \text{ and } S_b \geq 0\}.$$

⁸As an aside, note that since q_i is pinned down by $J_q(q; i) = 0$ independent of D , the allocation does not depend on credit conditions in monetary equilibrium – a special case of the neutrality theorems in Gu et al. (2015).

⁹See Lester et al. (2012), Liu et al. (2014) or Lotz and Zhang (2014) for recent papers taking this approach; see Nosal and Rocheteau (2011) for a review of an earlier literature on costly credit. Note that sometimes the cost is imposed on sellers rather than buyers, which would not matter here, and sometimes there is a fixed cost, which matters because it introduces nonconvexities. In any case, costly credit is interesting because it avoids the neutrality results mentioned in fn. 8.

We now revise Axiom 1 by dropping the constraint $\Gamma_p \leq z$ and Axiom 4 by dropping $p' \leq z$. The next result is also proved in the Appendix:

Proposition 5 *Any Γ satisfying (the revised) Axioms 1-4 takes the form*

$$\Gamma(z) = \begin{cases} (p, v^{-1}(p)) \in \mathcal{C}, p > z & \text{if } z < p^* \\ (p^*, q^*) & \text{otherwise} \end{cases} \quad (9)$$

where v is some strictly increasing function with $v(0) = 0$ and $v(q^*) = p^*$.

To describe the outcome in more detail, first rewrite (8) as

$$u'(q) = \{1 + \gamma' [v(q) - z]\} c'(q). \quad (10)$$

Then differentiate to get

$$\frac{\partial q}{\partial z} = \frac{-\gamma'' c'}{u'' - c''(1 + \gamma') - c' \gamma'' v'} > 0$$

Hence, q is monotone, and we write (10) as $z = f(q)$, where $f' > 0$. As p increases with q , it also increases with z . Then (10) pins down the use of credit as $v(q) - z = g(q)$, where $g(q) \equiv \gamma'^{-1} [u'(q) / c'(q) - 1]$ with $g'(q) < 0$. Given this,

$$V(z) = \begin{cases} W(z) + \alpha [u(q) - v(q) - \gamma \circ g(q)] & \text{if } z < p^* \\ W(z) + \alpha [u(q^*) - v(q^*)] & \text{otherwise} \end{cases}$$

It is easy to see that $z \geq p^*$ is not a solution to the CM problem, which we now write as

$$\tilde{J}(q; i) = u(q) - \gamma \circ g(q) - g(q) - (1 + i/\alpha) f(q).$$

Let q_c solve $g(q) = v(q)$, which is the amount of trade without money, and notice a buyer's ex ante surplus is equal to his ex post surplus in this case. Also, $\tilde{J}(q^c; i)$ is constant wrt i , and so if i changes, \tilde{J} rotates around q^c . Then

$$\tilde{q}_i = \arg \max_{q \in [0, q^*]} \tilde{J}(q; i)$$

As $u - \gamma - g$ and f are strictly increasing, the argument used in the proof of Proposition 1 implies \tilde{q}_i is generically unique. There are two cases: (i) $\tilde{q}_i = q^c$ and

buyers use credit only; (ii) $\tilde{q}_i > q^c$ and they use cash plus credit. In the second case, as in Proposition 1, $\partial \tilde{q}_i / \partial i < 0$.

Again paralleling Hu et al. (2009), we can design a mechanism of the form (9) to support q^o , although the construction is more complicated. Let q_L^c solve $g(q_L^c) = u(q_L^c) - \gamma \circ g(q_L^c)$ and let q_H^c solve $g(q_H^c) = c(q_H^c)$. With credit only, q_L^c is the lowest incentive compatible q for a buyer, and q_H^c is the highest for a seller. Define $h(q) = u(q) - \gamma \circ g(q) + ig(q) / \alpha$. By monotonicity of $u - \gamma$ and g , there is a threshold i , say \hat{i} , below which h is strictly increasing on $[0, q^*]$. Let $Q = \{q | h(q) - (1 + i/\alpha)c(q) \geq 0\}$ be the set of ex ante incentive compatible q for a buyer in monetary equilibrium. The following proposition is also proved in the Appendix.

Proposition 6 *Suppose $i < \hat{i}$. If $q_L^c < q_H^c$, any $q^o \in [q_L^c, q_H^c]$ can be supported as a nonmonetary equilibrium. Any $q^o \in Q \cap [q_L^c, q^*]$ can be supported as a monetary equilibrium. No other q^o can be supported.*

6 Conclusion

We provided a series of results for economies where liquid assets facilitate decentralized exchange, with or without allowing some credit or some real assets to serve in a similar capacity. A main innovation is that we impose minimal structure on the mechanism determining the terms of trade. Simple axioms generated a class of mechanisms that encompass many interesting solution concepts in this environment. While the approach may not work in all settings (e.g., it does not work in the numerical analysis of Molico 2006), it provides strong results for what is now a benchmark model in monetary economics. In particular, we established uniqueness and monotone comparative statics for any mechanism in the class. We also showed how to support desirable outcomes using some creatively designed mechanisms, for pure-currency economies as well as those with credit. These results should be of interest to the many people using versions of this framework in applications.

Appendix

Proof of Proposition 4 First consider $D < u(q^*)$. As $q^o > q^*$ violates Axiom 4, it cannot be supported. Figure 3 shows the construction of a v that supports $q^o \in [q_L, \min\{q_H, q^*\}]$ as a nonmonetary equilibrium. Draw a line with slope $u'(q^o)$ through (q^o, D) , labeled $v^o(q)$ in the graph. Truncate it above by $u(q)$ or $c(q)$, whichever v^o meets first from the left, and below by $u(q)$ or $c(q)$, whichever v^o meets first from the right. In Figure 3, v^o is truncated by c on both sides. Given D and $v(q)$, buyers pay $v(q^o)$ using only credit if $z = 0$. Suppose they pick $z > 0$ to get $q > q^o$. As u is concave, $u(q) - v(q) < u(q^o) - v(q^o)$. Ignoring common terms, ex ante expected utility is $u(q) - v(q) - i/\alpha [v(q) - D]$ in monetary equilibrium and $u(q^o) - v(q^o)$ in nonmonetary equilibrium. Since $v(q) > D$, monetary equilibrium yields lower expected utility. Therefore, buyers set $z = 0$ and get q^o .

To deliver $q^o \in (q_L, \min\{\hat{q}_c, q^*\}]$, we construct $v(q)$ as in the benchmark case. Pick p^o such that $(1 + i/\alpha) \min\{c(q^o), D\} < (1 + i/\alpha)p^o < u(q^o) + iD/\alpha$. Draw a line through $(q^o, (1 + i/\alpha)p^o)$ with slope $u'(q^o)$, labeled $(1 + i/\alpha)v^o$. Rotate the line to get $v^o(q)$ then truncate it above by $c(q)$, and below by the y axis and replace $v(0) = 0$. Then ex ante utility is represented by $J + iD/\alpha$. To show $J(q^o; i) > J(q_D; i)$, note that if q_D is on the linear part of v or the truncated $c(q)$, by convexity, $J(q^o; i) > J(q_D; i)$. This mechanism is consistent the ex ante decision to bring z , as illustrated in Figure 4.

To get $q^o < q_L$, buyers are willing to pay $v(q^o) < D \leq L$, as otherwise they get negative surplus. However, this violates bilateral efficiency as $q^o < q^*$ but $v(q^o) < L$. Therefore, $q^o < q_L$ cannot be supported. A nonmonetary equilibrium cannot support $q^o > q_H$, as the seller would get a negative surplus. It cannot support $q^o > q^*$, as it violates Axiom 4. By the argument in the proof of Proposition 3, $q^o > \min\{\hat{q}_c, q^*\}$ cannot be supported in monetary equilibrium. Next, consider the case $D \geq u(q^*)$. If $q^o < q^*$ then $v(q^o) = D$. However, this yields a negative surplus for the buyer. If $q^o = q^*$, we can pick any $v(q^*) \in [c(q^*), u(q^*)]$ and construct a nonmonetary equilibrium as in the case of $D < u(q^*)$. ■

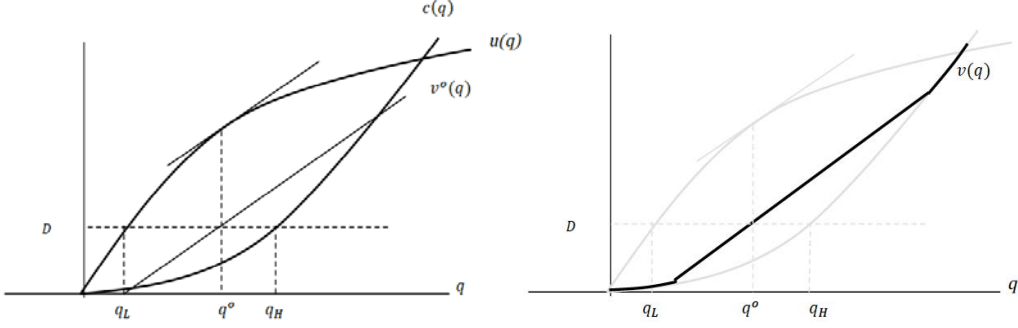


Figure 3: HKW Style Mechanism with Free Credit – Nonmonetary Equilibrium

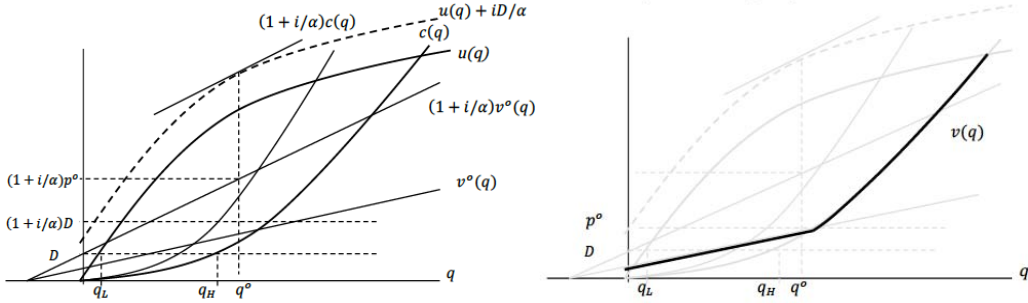


Figure 4: HKW Style Mechanism with Free Credit – Monetary Equilibrium

Proof of Proposition 5 First consider $z < p^*$. We show $(p, q) \in \mathcal{C}$ by contradiction. Suppose $(p, q) \notin \mathcal{C}$. Now, suppose $u'(q) > \{1 + \gamma' [S_s + c(q) - z]\} c'(q)$. Let $q' = q + \varepsilon_q$ and $p' = p + \varepsilon_p$, where $\varepsilon_p = c(q') - c(q)$. The sellers' surplus does not change, but S_b increases since it is concave in q given S_s , contradicting Axiom 4. By the same reasoning, if $u'(q) < \{1 + \gamma' [S_s + c(q) - z]\} c'(q)$ then $p' = p - \varepsilon_p$ and $q' = q - \varepsilon_q$, with $\varepsilon_p = c(q) - c(q')$, improves S_b and leaves S_s unchanged, contradicting Axiom 4. We now show $p > z$ if $z < p^*$. Suppose $p \leq z$. Then $\gamma = 0$, and $(p, q) \in \mathcal{C}$ iff $u'(q) = c'(q)$, which implies $q = q^*$. By Axiom 3 and the definition of p^* , we must have $p = p^*$, which is a contradiction.

Next consider $z \geq p^*$. Use the argument in the proof of Proposition 2, one can show $q = \Gamma_q(z) < q^*$ violates Axiom 4. Suppose $q > q^*$. Let $p' = p - \varepsilon_p$ and $q' = q - \varepsilon_q > q^*$, where $\varepsilon_p = u(q) - u(q')$. One can check that S_b weakly increases

as buyers save on costly credit, while S_s increases because $u(q) - c(q)$ falls with q for $q > q^*$. This contradicts Axiom 4. Hence, $z \geq p^*$ implies $q = q^*$, and hence $p = p^*$ by the definition of p^* and Axiom 3. This implies $v^{-1}(p^*) = q^*$. By Axiom 3, v^{-1} is strictly increasing. By Axiom 2, $v^{-1}(0) = 0$. ■

Proof of Proposition 6 First, define the set of incentive feasible trades \mathcal{I} . For $S_b, S_s \geq 0$ this requires

$$u(q) - \gamma \circ g(q) - p \geq 0 \quad (11)$$

$$p - c(q) \geq 0 \quad (12)$$

Any payments in the core must be at least equal to $g(q)$. That is,

$$p - g(q) \geq 0. \quad (13)$$

As usual, $q \leq q^*$, and therefore, $\mathcal{I} \equiv \{(p, q) \mid (p, q) \text{ satisfies (11)-(13) and } q \leq q^*\}$. This implies an ex post feasible q is in $[q_L^c, q^*]$. The buyer's CM objective function is $\tilde{J}(q; i) = h(q) - (1 + i/\alpha)v(q)$. Note that as we do not require h to be concave, \mathcal{Q} is not necessarily convex.

We seek $v(q)$ such that the distance between h and $(1 + i/\alpha)v(q)$ is maximized at some q^o . In Figure 5, the area surrounded by the curves $(1 + i/\alpha)(u - \gamma)$, $(1 + i/\alpha)c$, $(1 + i/\alpha)g$ and $q = q^*$ is ex post feasible. Pick $(1 + i/\alpha)v(q^o)$ in the area surrounded by the dashed curves. Consider first nonmonetary equilibrium, with payment $p^o = g(q^o)$. Draw a line with slope $\varepsilon > 0$ that goes through $(1 + i/\alpha)g(q^o)$, labeled $(1 + i/\alpha)v^o$. Denote the intersection of h and $(1 + i/\alpha)v^o$ by q^2 . The slope ε is chosen to be small so that the distance between the line and h is maximized on $[q^2, q^o]$. This can be done as h is strictly increasing for small i . Denote the intersection of h and $(1 + i/\alpha)c$ by q^1 . The constructed $(1 + i/\alpha)v$ is as follows.

$$(1 + i/\alpha)v = \begin{cases} (1 + i/\alpha)c & \text{if } q \leq q^1 \\ (1 + i/\alpha)v^o & \text{if } q^2 < q \leq q^o \\ h & \text{if } q > q^o \text{ and } \max\{q^1, 0\} < q \leq q^2 \end{cases} \quad (14)$$

Now v is strictly increasing and $h - (1 + i/\alpha)v$ is maximized at q^o . Ex ante, buyers will not bring $z > 0$ as $p^o = g(q^o)$ does not require cash.

Next, consider monetary equilibrium. For $q^\circ \in \mathcal{Q} \cap [q_L^c, q^*]$, pick $p^\circ > g(q^\circ)$ in \mathcal{I} . Draw a line with slope $\varepsilon > 0$ through $(1 + i/\alpha)p^\circ$, labeled $(1 + i/\alpha)v^\circ$. Denote the intersection of h and $(1 + i/\alpha)v^\circ$ by q^2 . Let $(1 + i/\alpha)v$ take the form in (14). Now, v is strictly increasing and $h - (1 + i/\alpha)v$ is maximized at q° . See Figure 6. For $q^\circ \notin \mathcal{Q}$, any incentive compatible p° results in $z = 0$. If $q^\circ \in [q_L^c, q_H^c]$, a nonmonetary equilibrium can support q° , but monetary equilibrium is not feasible. For $q^\circ \notin [q_L^c, q^*]$, the allocation is also not feasible. ■

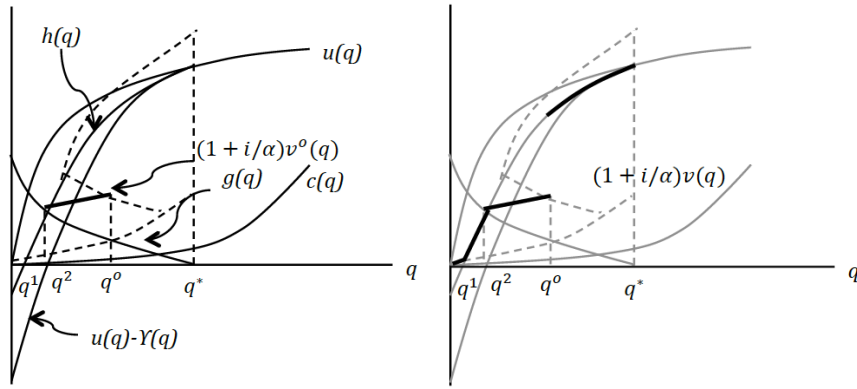


Figure 5: HKW Style Mechanism with Costly Credit – Nonmonetary Equilibrium

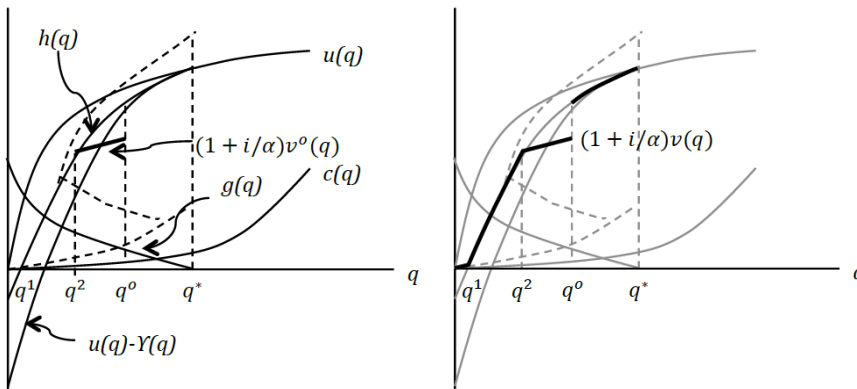


Figure 6: HKW Style Mechanism with Costly Credit – Monetary Equilibrium

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