Fundamentals of Coding for Broadcast OFDM

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Abstract

The same information is often broadcast to many receivers over different frequency selective channels. Digital audio signals and digital video signals are commonly distributed by such broadcasts. A coded modulation technique for orthogonal frequency division multiplexing (OFDM) is presented that provides consistent performance over a variety of frequency selective channels. The range of performance for an example of the new scheme over three different frequency selective channels is 0.75 dB at $P_e = 10^{-6}$ as compared to 4 dB for a commonly used scheme.

The new technique employs relatively dense constellations combined with low rate codes designed to allow the receiver to take advantage of reliable symbols to compensate for unreliable symbols. Frequency hopped systems and flat fading systems with interleaving can also benefit from the new technique.

1 Introduction

Multicarrier modulation, or orthogonal frequency division multiplexing (OFDM), is an alternative to equalized single carrier modulation. Multicarrier is especially attractive in environments where severe intersymbol interference (ISI) makes equalization difficult. Terrestrial wireless broadcast from multiple towers is an example of a severe ISI environment where multicarrier is attractive [1, 2].

When multicarrier is used for point to point communication, the coded modulation can be adapted to the channel, distributing information and power optimally among the subcarriers [3]. However a broadcast transmitter must use a single coded modulation that will provide consistent performance on a variety of channels.

In Section 2, a review of multicarrier is presented. The theoretical limits of broadcast transmission are discussed in Section 3. Section 4 presents criteria for the design of broadcast coded modulation with periodic symbol interleaving. Section 5 uses these criteria to design an example coded modulation. Section 6 presents simulation results comparing the designed coded modulation to a commonly used COFDM system for three different frequency selective channels. Section 7 concludes the paper.

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2 Multicarrier modulation

A multicarrier transmission system that creates N subcarriers in the frequency domain using the discrete Fourier transform (DFT) is shown in Figure 1. As shown in Figure 2, the multicarrier transmission system is equivalent to N parallel subchannels each consisting of a complex scalar gain a_i and additive white Gaussain noise (AWGN) n_i.

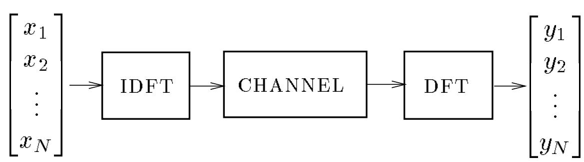

Figure 1: Multicarrier modulation system

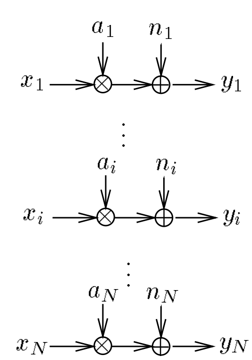

Figure 2: Parallel subchannels in frequency

The scale factors a_i are exactly the values of the DFT of the channel impulse response. A cyclic prefix (guard interval) approximately the length of the channel impulse response must be inserted between each

DFT block for the transformation to parallel channels to be exact. It is assumed that $E[{n_i}^2] = \sigma^2$ for all i.

3 Theoretical Limits

For a fixed set of transmitter powers $\{E[{x_i}^2]\}$, the highest data rate possible on a set of subchannels described by Figure 2 is the mutual information I defined as

$$I = \sum_{i=1}^{N} \log_2 \left(1 + \frac{{a_i}^2 E[{x_i}^2]}{\sigma^2}\right). \quad (1)$$

The value of I above is less than the channel capacity achieved by choosing the values of $\{E[{x_i}^2]\}$ to have the optimal water–pouring distribution [4]. Since the subcarrier SNRs are not known at a broadcast transmitter, the optimal distribution of power is not possible. Furthermore, each channel over which data is being broadcast has a different optimal power distribution.

Shannon's fundamental coding theorem ensures that for each AWGN–ISI channel there is a code that will permit reliable transmission at rate I in Eq. (1). Can one code work for all channels with rate I? Root and Varaiya's 1968 result [5] on the Gaussian compound channel implies that the answer is yes. Specifically, given data rate R and transmitter powers $\{E[{x_i}^2]\}$ there exists a "universal" code that will reliably transmit at rate R over all AWGN–ISI channels for which $I > R$.

To achieve I on the overall channel, the mutual information I_i of each subchannel must be achieved where

$$I_i = \log_2 \left(1 + \frac{{a_i}^2 E[{x_i}^2]}{\sigma^2}\right). \quad (2)$$

Thus consistent performance can be achieved only if high capacity subcarriers are able to provide the receiver with their full potential of information.

A subchannel cannot provide more than n bits of information to the receiver where n is the base 2 log of the constellation size. Thus the constellation should be as large as possible to allow the highest capacity subcarriers to be fully utilized. This can be contrasted with the AWGN case where n is should always be $k+1$ for a rate k/n code [6]. Practical considerations such as precision limit how large n can be in practice. Still, n can be made sufficiently large to allow consistent performance on typical ISI channels.

4 Code design for periodic interleaving

These large constellations are combined with low rate codes that allow the receiver to use the information obtained from the high capacity subchannels to compensate for information unavailable from the low capacity subchannels. This section presents criteria useful in the design of these codes when periodic symbol interleaving is used.

Subsection 4.1 reviews periodic interleaving. Subsection 4.2 introduces the periodic distance vector. Subsections 4.3 and 4.4 introduce periodic effective code length and periodic product distance which are computed from the periodic distance vectors. Subsection 4.5 presents a procedure for coded modulation design based on these two criteria.

4.1 Periodic interleaving

Recall from Section 2 that the subchannel gains a_i are the values of the DFT of the channel impulse response. Since the frequency response of a typical channel is continuous, subchannel gains adjacent in frequency will have similar values. A long sequence of small subchannel gains for consecutive received symbols will typically cause a decoding error. To avoid poor performance, consecutive code symbols should not be mapped to subchannels adjacent in frequency.

Interleaving is used to implement a different mapping of code symbols to subchannels. There are various types of interleaving, and the choice of an interleaver affects how well a particular coded modulation will perform. Periodic symbol interleaving is used in this paper, and the coded modulation is designed specifically to give good performance when combined with periodic symbol interleaving.

Periodic symbol interleaving can be implemented by writing the constellation points into a matrix column by column and then reading them out row by row. In a multicarrier system with 512 subchannels, periodic interleaving with period $P = 8$ is accomplished by writing the coded modulation constellation points c_i into an 8×64 matrix column by column and then reading the subchannel inputs x_i row by row. This matrix is shown below with both write and read labeling.

$$\begin{bmatrix} c_1 & c_9 & \dots & c_{505} \\ c_2 & c_{10} & & c_{506} \\ \vdots & & \ddots & \vdots \\ c_8 & c_{16} & \dots & c_{512} \end{bmatrix} \quad \begin{bmatrix} x_1 & x_2 & \dots & x_{64} \\ x_{65} & x_{66} & & x_{128} \\ \vdots & & \ddots & \vdots \\ x_{449} & x_{450} & \dots & x_{512} \end{bmatrix}$$

A periodic interleaver with period P provides the maximum possible separation in frequency of any P consecutive code symbols $(c_i, c_{i+1}, \dots c_{i+P-1})$. However, codewords c_i and c_{i+P} are transmitted on subchannels adjacent in frequency.

4.2 Periodic distance vectors

Suppose that $\{c_i\}$ and $\{\hat{c}_i\}$ are two sequences of constellation points that are valid outputs of a particular coded modulation. Define distances $\{d_i\}$ such that $d_i = c_i - \hat{c}_i$. Assuming a maximum likelihood decision between $\{c_i\}$ and $\{\hat{c}_i\}$, define $P_{c\hat{c}}$ be the probability of mistaking $\{\hat{c}_i\}$ for $\{c_i\}$. When $\{c_i\}$ is periodically interleaved so that $x_{\pi(i)} = c_i$ is transmitted over subchannels as in Figure 2, $P_{c\hat{c}}$ decreases as $\sum a_{\pi(i)}^2 d_i^2$ increases.

Periodic interleaving maps adjacent subchannel gains to codewords with indexes separated by P. Since adjacent subchannel gains have similar values,

$$a_{\pi(i)} \approx a_{\pi(i+P)} \cdots \approx a_{\pi(i+jP)} \cdots \tag{3}$$

where j is an integer. The summation $\sum a_{\pi(i)}^2 d_i^2$ can be approximated by summing up all the squared distances d_i^2 which have the same index modulo P and then multiplying these P different distance sums by an appropriate a_i^2 and summing as shown below.

$$\sum_{i=1}^{\infty} a_{\pi(i)}^2 d_i^2 \approx \sum_{i=1}^{P} a_{\pi(i)}^2 \tilde{d}_i^2 \tag{4}$$

$$\text{where} \qquad \tilde{d}_i^2 = \sum_{j=0}^{\infty} d_{i+jP}^2. \tag{5}$$

The vector $[\tilde{d}_1, \tilde{d}_2, \ldots, \tilde{d}_P]$ will be referred to as the periodic distance vector. To the extent that the above approximations are valid, the periodic distance vector represents all the information available about what values of $\sum a_{\pi(i)}^2 d_i^2$ are possible. Recall that larger values of $\sum a_{\pi(i)}^2 d_i^2$ imply smaller values of $P_{c\hat{c}}$.

4.3 Periodic effective code length

For a given coded modulation, the smallest number of nonzero elements in a periodic distance vector for that coded modulation will be referred to as the periodic effective code length (PECL) of the coded modulation. Larger values of PECL allow more small values of a_i to be tolerated since at least some nonzero elements see large values of a_i.

In [7], Lapidoth suggested designing codes to transmit error free for the largest possible number of erasures in the periodic erasure pattern resulting from a block erasure channel combined with periodic interleaving. This is equivalent to maximizing the PECL.

The PECL is essentially a periodic version of the effective code length discussed by Wilson and Leung [8], Divsalar and Simon [9], and Sundberg and Seshadri [10]. These papers show that the effective code length indicates the diversity provided by the code. The PECL indicates the diversity provided by a code combined with periodic symbol interleaving.

4.4 Periodic product distance

A coded modulation is desired which performs well for all channels having $I \geq R$ where I is defined in Eq. (1) and R is the desired per symbol information rate. For any particular periodic distance vector there is a particular channel with $I \geq R$ which produces the smallest value of $\sum a_{\pi(i)}^2 \tilde{d}_i^2$.

Without loss of generality, assume $E[x_i^2]/\sigma^2 = 1$. Neglecting the 1 in Eq. (1), the set of channels with $I \geq R$ is approximately the set of channels satisfying $\sum \log_2(a_i^2) \geq R$. When $\tilde{d}_i^2 > 0$ for $i \in \{1, 2, \ldots P\}$, the smallest sum $\sum a_{\pi(i)}^2 \tilde{d}_i^2$ over this set of channels is

$$P \cdot (e^R \prod_{i=1}^{P} \tilde{d}_i^2)^{1/P}. \tag{6}$$

The above minimum value of $\sum a_{\pi(i)}^2 \tilde{d}_i^2$ is monotonic in the product of the elements of the periodic distance vector. Thus this product is a good metric for code design. For a given convolutional code, the periodic product distance of order i (PPD_i) is defined as the smallest product of the nonzero elements of a periodic distance vector having exactly i nonzero elements. As with the PECL, the periodic product distances are essentially periodic versions of the product distances discussed in [9, 10].

Maximizing the product distance implies spreading the available Euclidean distance as evenly as possible over the periodic distance vector. Intuitively, this is desirable to achieve good performance on a variety of channels having different values of $\{a_i\}$.

4.5 Design procedure

A coded modulation scheme using an M state rate k/n convolutional code and a signal mapper using a 2^n point constellation is to be designed to give good performance in a broadcast multicarrier system with N subcarriers using periodic symbol interleaving with period P. Typically M, k, and N are specified before the coded modulation is designed. As discussed in Section 2, n should be made as large as possible.

The remaining issues are the choice of P, the labeling of the constellation, and the design of the convolutional code polynomial. P should be a factor of N. Small values of P make it more likely that subcarriers in one period will be independent by making their separation in frequency (N/P) larger. However, increasing P can allow larger values of PECL up to a point. A good initial choice of P is to increase P until the PECL stops increasing.

The constellation labeling should be chosen so that the set of minimum distances associated with error patterns is as large as possible. Set partitioning is not necessarily required since maximizing PECL precludes transmission of an uncoded bit. Gray coding is not necessarily required, and is sometimes not even possible as with the 32 cross constellation.

The convolutional code should be chosen so that it provides the maximum possible PECL. Typically, many convolutional codes will satisfy this criterion. From these maximum PECL codes, one code is selected which has large values of PPD_i for values of i ranging from PECL to P. Since there are no periodic distance vectors with fewer than PECL nonzero elements, PPD_i is not of interest for $i < \text{PECL}$.

5 Code design example

As an example, a coded modulation is designed for OFDM with 512 subcarriers. The parameters $k = 1$ and $M = 64$ are used in the following example to allow a fair comparison with the commonly used coded modulation discussed in [1], which combines the 64 state rate 1/2 "industry standard" convolutional code (having polynomials 133 171) with 4 PSK.

As discussed in Section 2, n should be chosen to be as large as possible. In this example n is chosen to be 4. This is large enough to give a clear improvement over the commonly used code with $n = 2$, but still small enough to be considered practical for multicarrier broadcast [2, 11].

A 16 point constellation is implied by $n = 4$. For 16–QAM with a rate 1/4 convolutional code, the codes achieved with gray labeling are isomorphic to those achieved with Ungerboeck labeling. The QAM constellation with the gray labeling shown below in hexadecimal was used in this example.

$$\begin{array}{cccc} 3 & 1 & 5 & 7 \\ 2 & 0 & 4 & 6 \\ a & 8 & c & e \\ b & 9 & d & f \end{array} \tag{7}$$

The PECL can be no larger than the constraint length of the convolutional code. Another upper bound on PECL follows from the discussion in [7] of convolutional codes which are "matched" to a periodic interleaver. Specifically,

$$\text{PECL} \le \frac{n-k}{n} P + 1. \tag{8}$$

A 64 state rate 1/4 convolutional code has a constraint length of 7. Thus no choice of P will provide a PECL larger than 7 in this example. The smallest value of P for which the bound in (8) allows a PECL of 7 is $P = 8$. There are many 64 state rate 1/4 convolutional codes that achieve PECL = 7 when $P = 8$, so P is chosen to be 8. The algorithm proposed by Lapidoth in [7] for computing the bit error rate of convolutional codes using periodic interleaving on an erasure channel can be adapted to compute PECL.

The final step is to select a code having the maximum possible PECL which also has large values of PPD_i for values of i from $i = \text{PECL} = 7$ to $i = P = 8$. It is not possible to simultaneously maximize both PPD_7 and PPD_8. In this example, codes with maximum PECL were ordered according to the sum of the logs of PPD_7 and PPD_8. Values of PPD_i were computed by direct trellis search.

The five codes with the largest values of this sum are listed in Table 1 along with their relevant properties. The top code was selected because it had the largest sum and both PPD_7 and PPD_8 were relatively large. Note that a code further down on the list would have been chosen if either PPD_7 or PPD_8 had been particularly small for the top code.

(octal)				$\log_2(\cdot)$		
code polynomial				PPD_7	PPD_8	Sum
43	176	171	45	11.068	10.461	21.53
73	105	131	147	9.769	11.764	21.53
71	103	137	145	9.939	11.501	21.44
73	105	131	147	9.876	11.447	21.32
43	175	155	103	10.671	10.617	21.29

Table 1: Codes with large product distances

6 Simulation Results

Bit error rate (BER) simulations were carried out on three different frequency selective channels. Figure 3 shows the frequency responses in terms of subchannel SNR. Channel 1 is an AWGN channel. Channel 2 is the notch channel studied by Sari in [1]. Channel 3 is the slope distortion channel used by Cioffi in [12].

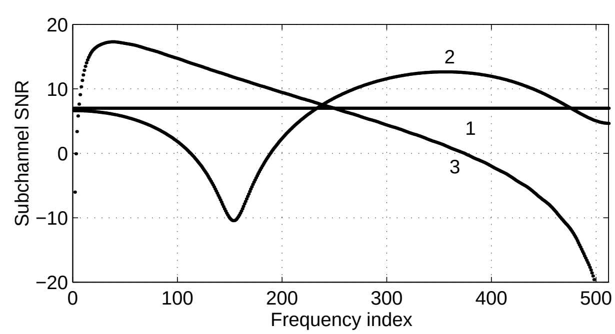

Figure 3: Channel frequency responses

Figure 4 compares performance of the the rate 1/2 4–PSK scheme used in [1] with code designed above (the top code in Table 1).

Performance comparisons across different channels should compare performance of channels that have the same mutual information I as defined in (1). In Figure 4 every BER plotted at a given SNR is for a channel with I equal to the AWGN channel capacity at that SNR. Thus the AWGN channel is plotted in the standard way, and the other channels are compared fairly.

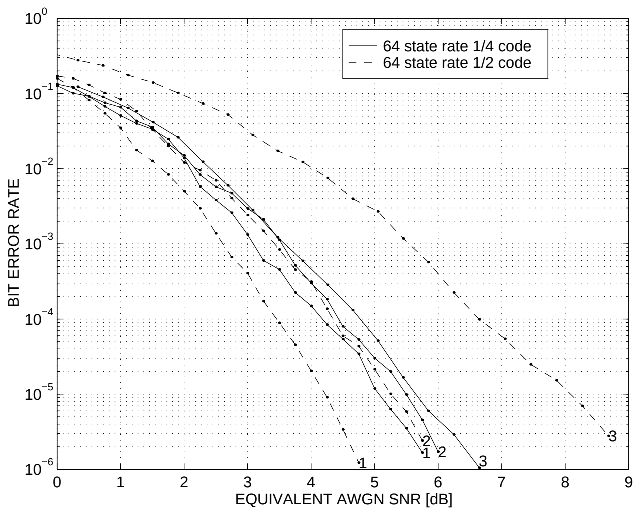

Figure 4: BER performance comparison

Consistent BER performance across the three channels studied is obtained at the cost of poorer performance on the AWGN channel. At 3×10^{-6} BER, the range of performance on all three channels is reduced to less than .75 dB from more than 4 dB for the commonly used rate 1/2 code. A gain of about 2.5 dB is obtained on channel 3, and about 1 dB is lost on the AWGN channel (channel 1) compared with the commonly used code.

7 Conclusions

With properly chosen coded modulation, multicarrier transmission systems can provide consistent performance on a variety of frequency selective channels. To achieve this consistent performance, dense constellations combined with low rate codes are required. This combination allows high capacity subchannels to carry their full potential of information.

When periodic symbol interleaving is used, the periodic effective code length and periodic product distances are good criteria for coded modulation design. This is true for frequency hopped spread spectrum systems and flat fading channels as well as the multicarrier scenario which has been the focus of this paper.

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8 An Additional Channel

This section presents an additional simulation which demonstrates that the newly designed coded modulation performs well even when more than half the channel is severely attenuated. It is not part of the published Asilomar paper.

More than half of the subchannels in channel 4 illustrated in Figure 5 have a capacity that is essentially zero for the SNRs of interest. For this channel, the rate 1/2 code can transfer at most 2 bits of information per subchannel on less than half of the subchannels. Thus it cannot reliably send the attempted average of 1 bit per subchannel.

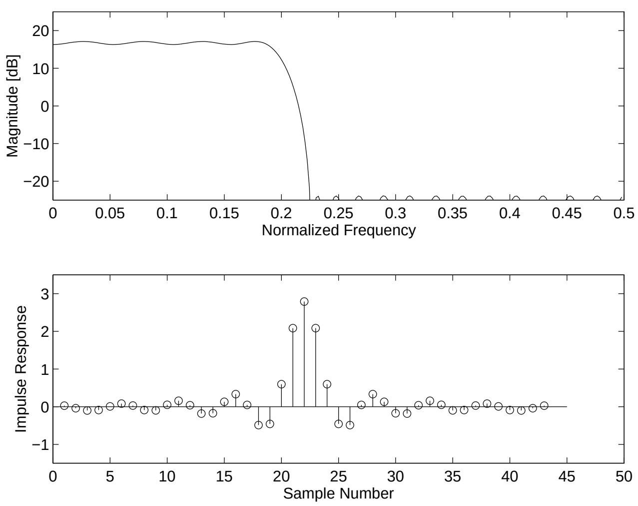

Figure 5: Channel 4

As shown in Figure 7, the rate 1/2 code has a high BER on channel 4 for the entire range of SNR studied.

A 1/4 code can transfer up to 4 bits of information per subchannel and is thus not precluded from reliably sending an average of 1 bit per subchannel when half the band is severely attenuated. As shown in Figure 7, the newly designed rate 1/4 code performs about as well on this channel as it did on the three channels discussed earlier.

Thus, depending on the channels encountered, improvements provided by the code design technique discussed in this paper can be dramatic.

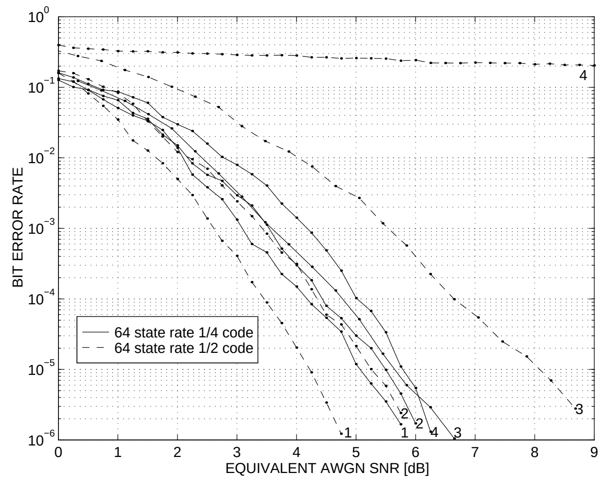

Figure 6: BER performance comparison