SINGLE-BIT CLOSED-LOOP QUASI-ORTHOGONAL SPACE-TIME CODES FOR MIMO SYSTEMS

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ABSTRACT

This paper proposes a closed-loop beamforming algorithm for MIMO systems employing quasi-orthogonal space-time block codes (QOSTCB) and low rate feedback channels over slowly fading channels. A recursion is formulated using a stochastic signed gradient approximation, where the receiver employs a single feedback bit to command the transmit antenna weights adaptation, at a frequency much lower than that of the data rate. The algorithm tracks the weight vector that optimizes a link performance measure, which takes into account both the self-interference properties of the space-time code and the channel conditions. Simulations show that the proposed algorithm can effectively exploit the channel temporal correlation, and outperform existing single-bit feedback schemes in low mobility scenarios.

1. INTRODUCTION

Space-time block codes (STBC) have been extensively studied in MIMO systems due to their ability to exploit diversity, thus enhancing the link reliability, see, e.g., [1]. Among space-time block codes, orthogonal designs are specially appealing, because maximum likelihood (ML) detection can be implemented with linear processing. However, it is known that full-rate orthogonal STBC (OSTBC) employing complex-valued symbol constellations only exist for the special case of two transmit antennas. Full-rate quasi-orthogonal space-time codes (QOSTBC), on the other hand, sacrifice orthogonality to retain full-rate, see, e.g., [2, 3, 4]. Due to their inherent self-interference, however, they do not fully exploit the available diversity. This can be alleviated by using symbol constellation rotations (see, e.g., [5]), thereby restoring the full diversity when operating under the ML detector, which is non-linear for QOSTBC. In contrast, the availability of some channel state information at the transmitter can enable both full diversity and array gain under a linear detector. In frequency division duplex (FDD) systems such as the one considered here, a feedback channel is required.

The use of beamforming in conjunction with OSTBC has been studied in [6] and extended to QOSTBC in [7]. These algorithms employ multiple-direction beamformers based on the channel mean and covariance, as conveyed to the transmitter through the feedback channel. In contrast, the schemes presented here exploit a single feedback bit to track the antenna weight vector that optimizes a link performance metric, given the current channel conditions.

Other works relating beamforming and STBC found in literature are as follows: phase rotation of a group of antennas has been considered in [8, 9]. These works compute the exact phase rotation necessary to eliminate the self interference of some specific QOSTBC. However, it is not known how the quantization associated to the feedback channel affects their performance. Another study is given in [10], where a beamformer is derived assuming that the transmitter has an estimate of the antenna correlations, either from an unspecified feedback mechanism, or from uplink measurements. In [11], the authors propose a pseudo random phase rotation that alters the statistics of the self-interference term, and provides gain in a channel-coded system without feedback. The use of matrix codebooks and the necessary conditions to guarantee full diversity has been studied in [12]. These codebooks, however, have larger feedback requirements and a computational complexity that grows exponentially with the number of feedback bits.

On the other hand, closed-loop antenna and code selection have also been studied, see, e.g., [13, 14, 15]. In [16], the authors present a closed-loop design based on OSTBC, which specializes to a single feedback bit algorithm, for the case of four transmit antennas. These group-coherent codes have been analyzed in [17]. Further references can be found in the extensive review of limited feedback techniques [18].

This paper presents a formulation for transmit weight recursion in temporally correlated channels. Given a QOSTBC and a linear receiver structure, the recursion utilizes a single feedback bit, in order to track the antenna weights that optimize the uncoded BEP. This can be viewed as a closed-loop beamforming system with a single beam where the metric is a link performance measure instead of the received power, and the QOSTBC encoder is taken into account explicitly. This contrasts with [6, 7], which employ beamforming matrices derived from transmit-side estimates of the channel mean and covariance, and with [8, 9], where only phase adjustments are performed, as opposed to per-antenna power and phase control used in the proposed algorithm. The single feedback bit is computed as a signed stochastic gradient approximation, a method which has been applied in closed-loop eigenbeamforming systems for both single and multiple beams, see for example [19, 20, 21].

2. SYSTEM MODEL

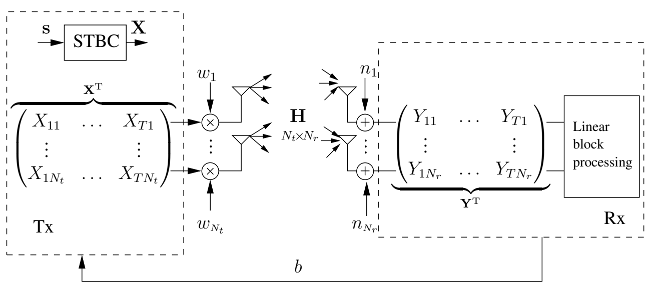

Fig. 1. System model. A space-time codeword $\mathbf{X} \in \mathbb{C}^{T \times N_t}$ is transmitted over T symbol periods, during each of which the elements of a row of $\mathbf{X}$ are transmitted through the corresponding antennas and their weighting coefficients. The operator $(\cdot)^{\mathrm{T}}$ represents matrix transpose.

The discrete time MIMO system under consideration is illustrated in Fig. 1, and consists of a fixed transmitter with N_t antennas and a mobile receiver with N_r antennas. The wireless channel is assumed to be frequency-flat and fade slowly. The transmission is arranged into slots or groups of $L \gg 1$ symbol periods, and during each period all the antennas transmit one symbol. Let $\mathbf{s} \in \mathbb{C}^{T \times 1}$ be the T non-redundant information symbols encoded into the ST codeword $\mathbf{X} \in \mathbb{C}^{T \times N_t}$, which is transmitted over $T \ll L$ symbol periods. At the end of each slot, a single feedback bit $b \in \{-1, 1\}$ is sent by the mobile to the transmitter and processed without delay, so that the changes to the transmit antenna weights are effective at the beginning of the next slot, and the weights remain the same during the whole slot. The feedback message frequency is f_b Hz, and the symbol and block periods are $1/(Lf_b)$ and $T/(Lf_b)$, respectively.

Assuming that the channel is constant over the T periods of the transmission, the received codeword $\mathbf{Y}(k) \in \mathbb{C}^{T \times N_r}$ is

$$\begin{aligned}\mathbf{Y}(k) =& \mathbf{X}(k)\mathbf{W}(l)\mathbf{H}(k) + \mathbf{N}(k)\\ & k = (l-1)(L/T) + m, m = 0, \ldots, L/T - 1\end{aligned} \tag{1}$$

where for the kth codeword, $\mathbf{H}(k) \in \mathbb{C}^{N_t \times N_r}$ is the channel matrix assumed to have a coherence time of more than one slot, $\mathbf{W}(l) \in \mathbb{C}^{N_t \times N_t}$ is a diagonal matrix containing the antenna transmit weights $w_1(l), \ldots, w_{N_t}(l)$, $l > 0$ is the slot index, and $\mathbf{N}(k) \in \mathbb{C}^{T \times N_r}$ is a matrix containing white Gaussian noise with i.i.d. entries drawn from a circularly symmetric Gaussian distribution of variance σ^2. In this article, we restrict our attention to full-rate QOSTBCs, and therefore consider $T = N_t$. Furthermore, only one set of N_t weights will be used, and it will be denoted $\mathbf{w} = [w_1 \ \ldots \ w_{N_t}]^T$, with $||\mathbf{w}|| = 1$.

Depending on the QOSTBC of choice, one can rearrange the entries of $\mathbf{Y}$ to write an equivalent linear system in terms of non-redundant information symbols $\mathbf{s}(k) \in \mathbb{C}^{T \times 1}$:

$$\tilde{\mathbf{y}}(k) = \mathcal{H}(k,l)\mathbf{s}(k) + \tilde{\mathbf{n}}(k) \tag{2}$$

where $\tilde{\mathbf{y}}(k), \tilde{\mathbf{n}}(k) \in \mathbb{C}^{N_t N_r \times 1}$ are the vectorized versions of $\mathbf{Y}(k), \mathbf{N}(k)$ that can include conjugation of some entries [1]. Note that the N_t transmit antenna weights $w_1(l), \ldots, w_{N_t}(l)$ are included inside the equivalent channel matrix $\mathcal{H}(k,l)$.

A linear receiver $\mathbf{\Omega}$ is assumed, so that the detection of the block is based on the vector

$$\begin{aligned}\mathbf{z}(k) &= \mathbf{\Omega}^{\dagger}(k,l)\tilde{\mathbf{y}}(k)\\ &= \mathbf{\Omega}^{\dagger}(k,l)\mathcal{H}(k,l)\mathbf{s}(k) + \mathbf{\Omega}^{\dagger}\tilde{\mathbf{n}}(k)\\ &:= \mathbf{G}(k,l)\mathbf{s}(k) + \mathbf{n}'(k)\end{aligned} \tag{3}$$

where † denotes Hermitian transpose.

3. CLOSED-LOOP QOSTBC

Equation (3) defines an equivalent linear system for the T symbols encoded into the codeword $\mathbf{X}$, given the QOSTBC encoder function and the linear receiver structure. The total gain matrix $\mathbf{G} := \mathbf{\Omega}^{\dagger}\mathcal{H}$ and the covariance matrix of the filtered noise $\mathbf{n}'$ determine the conditional uncoded BEP of the system. It is straightforward to write the signal-to-interference ratio of each of the T symbols from (3) as

$$\gamma_i = \frac{|G_{ii}|^2}{||\mathbf{G}_{i\cdot}||^2 - |G_{ii}^2| + \mathbf{\Omega}_{\cdot i}^{\dagger}\mathbf{Q}\mathbf{\Omega}_{\cdot i}} \quad i = 1, \ldots, T \tag{4}$$

where $\mathbf{G}_{i\cdot}$ is the ith row of $\mathbf{G}$, $\mathbf{\Omega}_{\cdot i}$ is the ith column of $\mathbf{\Omega}$, G_{mn} is the element (m,n) of $\mathbf{G}$ and $\mathbf{Q} = \mathrm{E}\left\{\tilde{\mathbf{n}}(k)\tilde{\mathbf{n}}^{\dagger}(k)\right\} = \sigma^2\mathbf{I}$ is the covariance matrix of the additive white Gaussian noise at the receive antennas.

The combined conditional BEP across the T symbols is a function of the current weights and channel $\mathbf{w}$, $\mathbf{H}$, and reads

$$P(\mathbf{w}|\mathbf{H},\mathbf{Q}) = \frac{1}{T}\sum_{i=1}^{T}\mathcal{P}(\gamma_i/\mathcal{B}) \tag{5}$$

where $\mathcal{P}(\cdot)$ represents the conditional BEP expression for the symbol constellation employed, and $\mathcal{B}$ is the number of bits per symbol in that constellation. The conditional BEP function $\mathcal{P}(\cdot)$ can be any suitable SINR to BEP mapping, such as laboratory measurements of a particular receiver implementation.

3.1. Signed gradient approximations

In this section, we consider recursions based on a single feedback bit, which can be used to adapt $\mathbf{w}$ to optimize the uncoded BEP (5). At a given update instance, the receiver evaluates two candidates for the update of $\mathbf{w}$, namely $\mathbf{w}_-$ and $\mathbf{w}_+$, and chooses the one that offers a smaller BEP, as given in (5). The feedback bit $b(l)$ then informs the transmitter which candidate was chosen, and the transmitter updates its transmit weights. The candidates are generated as perturbations around the current weights $\mathbf{w}(l)$ through the use of a common random seed, which allows the transmitter and the receiver to generate the same candidates synchronously. Different recursion types are possible when generating $\mathbf{w}_\pm$. In general, all the recursions are governed by a step size parameter μ and are based on the idea of probing the cost function in diametrical directions. More specifically, given the two candidates for the update $\mathbf{w}_\pm$, the feedback bit is computed as

$$b(l) = \begin{cases} 1 & \text{if } P(\mathbf{w}_+|\mathbf{H},\mathbf{Q}) < P(\mathbf{w}_-|\mathbf{H},\mathbf{Q}) \\ -1 & \text{otherwise} \end{cases} \tag{6}$$

where $P(\mathbf{w}_\pm|\mathbf{H},\mathbf{Q})$ is the total uncoded BEP from (5) and $\mathbf{H} \equiv \mathbf{H}(lL/T-1)$ is the channel associated to the last block of slot l. The step size reflects a trade-off between accuracy in the BEP optimization and tracking performance given the feedback frequency f_b and the fading rate of the channel. Upon the arrival of b, the update at the transmitter is done by

$$\mathbf{w}(l+1) = \begin{cases} \mathbf{w}_+(l) & \text{if } b(l) = 1 \\ \mathbf{w}_-(l) & \text{otherwise} \end{cases} \tag{7}$$

Note that if more than one feedback bit per slot were available, then a vector codebook approach would be feasible, where the vector giving the smallest BEP would be selected, its index transmitted through the feedback channel. This approach, however, has the disadvantage that the computational complexity of the update grows exponentially with the number of feedback bits. Alternatively, more directions could be exploited following the approach in (6).

We consider two different update recursions in the following sections, which define how the candidates $\mathbf{w}_\pm$ are built. Other recursions are possible as well, such as the use of premultiplication by unitary matrices and their Hermitian transposes. This approaches, however, will not be considered.

3.1.1. STBC-SCGAS

The first recursion under consideration uses a parameterization of complex-valued vectors in terms of real-valued angles, through a cascade of complex-valued Givens rotors. Parameterizations involving Givens rotors have been used in the context of closed-loop eigenbeamforming, see for example [22, 23, 21]. Here, we consider the mapping employed in the SCGAS algorithm [21], restricted to a single beam.

One mapping of $\boldsymbol{\theta} \in \mathbb{R}^{2(N_t-1)\times 1}$ angles to a norm-one vector in $\mathbb{C}^{N_t \times 1}$ can be written as

$$\mathcal{M}(\boldsymbol{\theta}) = \left[\prod_{m=2}^{N_t} \mathbf{J}^{m1}(\theta_{2m-3}, \theta_{2m-2})\right] \mathbf{w}_0 \tag{8}$$

where $\mathbf{w}_0$ is the first column of the identity matrix of size N_t, θ_m is the element m of $\boldsymbol{\theta}$ and $\mathbf{J}^{pq}(\alpha,\beta) \in \mathbb{C}^{N_t\times N_t}$ is a complex-valued Givens rotor [24], which is a unitary matrix built from an identity by replacing the entries (p,p), (q,p), (p,q), (q,q) with terms $\cos(\alpha)$ and $\sin(\alpha)e^{j\beta}$. This mapping can represent any norm-one complex-valued vector, up to scaling by a unit-modulus scalar. This ambiguity does not affect the performance of the algorithm, as such a scaling represents a phase rotation of the same amount to all the antennas, which cannot alter the relative time of arrival of the signals to the receive antenna array.

The generation of the update candidates takes the form of adding a real-valued perturbation to the angles defining the current weights. That is, if $\mathbf{w}(l) = \mathcal{M}(\boldsymbol{\theta}(l))$ with $\mathcal{M}(\cdot)$ defined in (8), then the candidate weights are

$$\mathbf{w}_{\pm,SCGAS}(l) = \mathcal{M}\{\boldsymbol{\theta}(l) \pm \mu\mathbf{p}(l)\} \tag{9}$$

with $\mathbf{p}(l) \in \mathbb{R}^{2(N_t-1)\times 1}$ generated as a Gaussian vector with i.i.d. entries of zero mean and unit variance. Since the mapping $\mathcal{M}(\cdot)$ is a cascade of unitary matrices, the resulting vector has always unit norm. After the update, the parameters are restored to their nominal ranges by the inverse mapping:

$$\boldsymbol{\theta}(l+1) = \mathcal{M}^{-1}\{\mathbf{w}(l+1)\} \tag{10}$$

where $\mathcal{M}^{-1}(\cdot)$ involves $N_t - 1$ rotors $\mathbf{J}^{m1}$, $m = N_t, N_t - 1, \ldots, 2$ that sequentially null the lowermost $N_t - 1$ elements of $\mathbf{w}(l+1)$ [24]. This is used to restore the angles to their nominal ranges, in order to keep the search procedure within a bounded space.

3.1.2. STBC-BZ

Another recursion under consideration is obtained by addition of complex-valued vector perturbations, followed by normalization of the perturbed vector. This can be considered an extension of the stochastic sign gradient feedback algorithm by Banister and Zeidler (BZ) [19], where the original cost function is replaced by (5).

The candidates are generated as

$$\mathbf{w}_{\pm,BZ}(l) = \frac{\mathbf{w}(l) \pm \mu\mathbf{q}(l)}{||\mathbf{w}(l) \pm \mu\mathbf{q}(l)||} \tag{11}$$

with $\mathbf{q} \in \mathbb{C}^{N_t\times 1}$ generated with i.i.d. circular Gaussian entries of unit variance.

3.2. Closed-loop ABBA

In this section, we illustrate the proposed method with the "ABBA" QOSTBC [2], for $N_r = 1$. The equivalent MIMO matrix $\mathcal{H}$ and the linear minimum mean square error (LMMSE) combiner read [1]

$$\mathcal{H}_{ABBA} = \begin{bmatrix} w_1h_1 & w_2h_2 & w_3h_3 & w_4h_4 \\ w_2^*h_2^* & -w_1^*h_1^* & w_4^*h_4^* & -w_3^*h_3^* \\ w_3h_3 & w_4h_4 & w_1h_1 & w_2h_2 \\ w_4^*h_4^* & -w_3^*h_3^* & w_2^*h_2^* & -w_1^*h_1^* \end{bmatrix}$$
$$\tilde{\mathbf{y}} = \begin{bmatrix} Y_{1,1} \\ Y_{2,1}^* \\ Y_{3,1} \\ Y_{4,1}^* \end{bmatrix} \qquad (12)$$
$$\mathbf{\Omega}^\dagger = (\mathcal{H}^\dagger\mathcal{H} + \sigma^2\mathbf{I})^{-1}\mathcal{H}^\dagger$$

The algorithms presented in Section 3.1 work with an equivalent system (3) defined by inserting (12) in (2), which enables to compute the uncoded BEP (5) in order to command the weight adaptation through the single-bit feedback channel.

4. SIMULATIONS

The BEP performance of the proposed closed-loop QOSTBC is simulated in a system with $N_t = 4$, $N_r = 1$ antennas, a receiver moving at 3 km/h and a carrier frequency of 2.1 GHz. The slot contains $L = 160$ symbol periods and the feedback frequency is $f_b = 1500$ Hz. The ST encoder of choice is the "ABBA" scheme [2] with an LMMSE detector as given in Section 3.2, using $T = 4$ QPSK independent symbols per space-time word. The wireless channels are modeled as spatially uncorrelated Rayleigh channels sampled every symbol period and filtered with a fourth order digital Butterworth filter adjusted to the maximum Doppler frequency. The average channel over each block is used in the ST decoder.

For the purpose of comparison, the group-coherent codes proposed in [16] are also simulated. Here, the Alamouti code is used as the building block, and the code selection has two possibilities, according to the single feedback bit. QPSK symbols are used over a period $T = 2$, which gives two non-redundant data bits per channel use, the same as in the proposed closed-loop QOSTBC. As reference, we plot the performance curves for the original ABBA QOSTBC scheme [2] and a fourth-order diversity OSTBC performance bound.

In order to assess the maximum performance of the BEP formulation (5), a "genie-aided" scheme is considered, where the receiver optimizes $\mathbf{w}$ for each slot, and makes it available to the transmitter without any distortion due to the feedback mechanism. This is done by employing a numerical optimization technique, the random walk with direction exploitation [25], as a search procedure over the angle space parameterizing $\mathbf{w}$ through the mapping $\mathcal{M}(\boldsymbol{\theta})$ described in Section 3.1.1.

Figure 2 shows the simulated average bit error rate (BER) curves. It can be seen that both the group-coherent codes and the proposed closed-loop ABBA outperform the pure fourth order diversity. However, the proposed scheme provides additional array gain, compared to the group-coherent codes, and a shift of about 0.5 dB can be observed. Furthermore, the "genie aided" ABBA-SCGAS curve shows that additional array gain can still be obtained, but this would increase the computational complexity and the feedback requirements. The convergence step μ has been optimized for both recursions under consideration, and the effect of the step size choice is shown in Fig. 3.

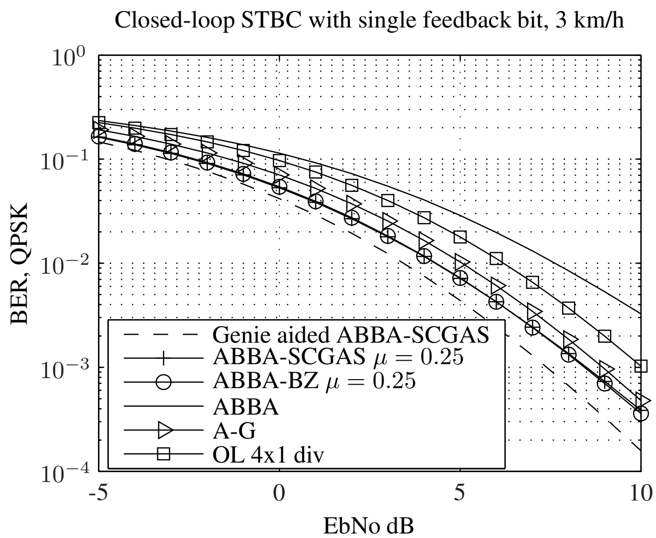

Fig. 2. Average uncoded BER performance of the proposed single-bit closed-loop recursion for ABBA, at pedestrian speeds. A closed-loop assisted orthogonal design by Akhtar and Gesbert (A-G) [16] featuring the same data and feedback rate is shown. The "genie aided" performance refers to a system that can optimize (5) at every update without feedback limitations.

5. CONCLUSIONS

This work proposed a closed-loop enhancement for quasi-orthogonal space-time block codes in temporally correlated channels. The proposed scheme uses a single feedback bit to enable a recursion, by which the transmitter adjusts its antenna transmit weights. An uncoded conditional bit error rate formulation is employed as a cost function, and the feedback bit is interpreted as a stochastic signed gradient. Simulations show that the proposed scheme can outperform existing closed-loop space-time block codes operating at the same data and feedback rate, at the cost of a slightly larger computational complexity.

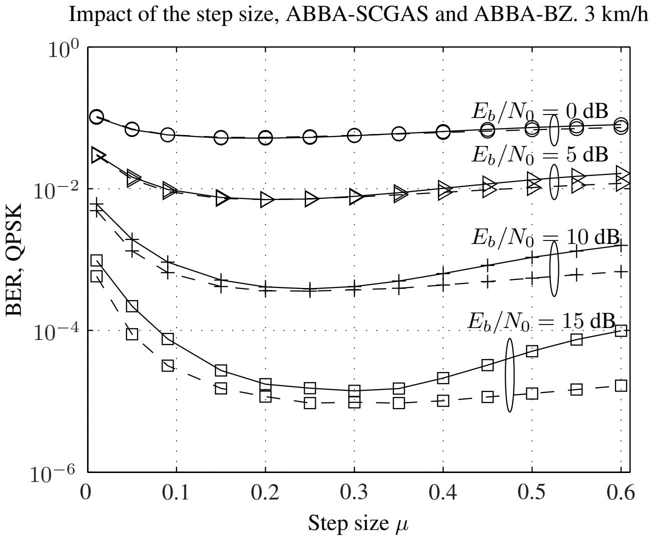

Fig. 3. Average uncoded BER performance for different values of the step size μ, receiver moving at 3 km/h.

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