# Micro-grid State Estimation Using Belief Propagation on Factor Graphs

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***Abstract*—Smart grid envisions the potential to manage diverse energy resources and enable a future self-dispatch and self-healing grid. This would first require the micro-grid visibility of node behavior (i.e. electrical parameters). In this paper we propose a novel approach to construct a stochastic model that makes global inference on every node at the micro-grid level. The micro-grid system can be modeled as a factor graph addressing proper correlation functions including distributed renewable generation correlation. We conduct statistical inference on the factor graph using Belief Propagation (BP) algorithm. The purpose is that given incomplete measurements, marginal probability distribution for unmetered node behavior can be derived. Simulation of the BP algorithm is performed on a simplified micro-grid model with linear local correlations. The results demonstrate that loopy BP can converge to optimal state estimates efficiently.**

## I. Introduction

The existing power grid has been serving the public for several decades without significant technical improvements. The power grid will undergo transformations in the next several years as demands for electricity increase, renewable energy generation become more prevalent (with intermittent sources such as wind and solar) due to environmental and energy sustainability concerns, variable pricing, sophisticated communication and sensing of parameters of the power grid, and security concerns (privacy, anomaly detection, and encryption). The future smarter and greener power grid is often labeled "smart grid"—the so called "grid of the future" and has drawn increasing attention worldwide. The smart grid should have the potential to manage diverse energy resources, enable failure prediction and assessment, distributed decision-making and control as well as end user participation.

A *micro-grid* is defined as a subsystem of the power grid that incorporates both associated loads and distributed generation in order to realize the emerging potential of distributed renewable generation [1]. Current utilities have limited monitoring and control over the micro-grid, especially the distribution feeders. The reliability of the micro-grid can be achieved with widely deployed system monitoring and predictable condition-based maintenance. In addition, with better visibility of the grid, supply and demand can be balanced more efficiently through intelligent control and dispatch algorithm with the lowest possible intervention by utility workers. The ability to monitor and control every node at the micro-grid level will become a big challenge since wide deployment of smart metering devices such as Advanced Metering Infrastructure (AMI) and Phase Measurement Unit (PMU) requires huge investment. In addition, distribution system automation and optimization involve with extensive data processing. The utility needs the validation of the effectiveness of this data-driven approach so that further investments can be made with deploying sensing and metering devices.

In order to bridge the gap, stochastic formulation can be used to estimate key parameters of all the nodes in the micro-grid. There are two reasons for the stochastic approach. Firstly, for large distribution feeders the pattern of demand tends to become stable and daily load curve can be derived using statistical data. These patterns are used for regulating the generation of power plants. The closer it gets to individual customer, the more randomness the load behavior will be. Secondly, the integration of distributed renewable generation would increase the randomness of load behavior at the micro-grid since renewable energy generation is subject to weather conditions (solar and wind). For example, in Hawaii the cloud movements have significant effects on the stability of solar generation.

Currently utility companies have limited real-time visibility of distributed renewable generation. Even now some sites are equipped with sensing devices such as pyranometers, anemometers and sodars [2], the extensive resource data is seldom processed and utilized by state estimators or distribution system automation software. Conversely, the renewable generation at customer side is often viewed as negative load. However, the increasing penetration of renewable energy generation brings reliability issues to the existing grid and calls for the real-time estimation of both the intermittent renewable generation and circuit load variability. This would provide insights on the influence of renewable generation on circuit load behavior and enable further intelligent control and optimization.

Our purpose is to assess the first and second order statistics of node behavior through observations from partially deployed smart metering devices. For a micro-grid which has scattered observations, we propose an approach to construct a probablistic model for the system and use a statistical inference approach to estimate the node behavior for all the nodes. The probability estimates are derived for those nodes that do not have metering devices. The first step for our approach is to model the micro-grid as a factor graph. Then we use the Belief Propagation algorithm to conduct statistical inference and derive posterior estimates for state variables.

This paper is organized as follows. Section II introduces the factor graph and discusses how to model the micro-grid as a factor graph. The BP algorithm and its computation rules are presented in Section III. In Section IV different scheduling schemes for the BP algorithm are discussed. After comparison with conventional state estimation approach, we apply the parallel BP algorithm on the micro-grid state estimation and address its ability to work on many loopy graphs. Section VI discusses our simulation of the loopy BP algorithm on a simplified micro-grid system. The results show that optimal marginals can be approximated with the loopy BP algorithm very efficiently.

## II. Model Micro-grid as a factor graph

The micro-grid system is the portion of the power grid extending from the last substation to buildings and individual residences. It represents the distribution system downstream from the last substation including distribution feeders. The major micro-grid components may include substation transformer, feeder transformer, voltage regulator, shunt capacitor banks, distribution line, customer load, etc. As we model only key electrical parameters of the grid at the steady-state, such as bus voltage, line current and power flow, the simplified approach is to model several key components such as feeder transformer, distribution line and customer load. We assume that voltage and active power are specified at the load side of substation transformer as given parameters.

The system map at the physical level needs to be first transformed to a bus/branch model with proper numeric labeling of every line segment between two connected buses or between the bus and the load. The network topology, circuit breaker status and the switching-device status are stored in Network Topology Processor (NTP) [3]. The next step is to map the bus/branch model to a factor graph.

A factor graph is a graphical representation of probabilities, aimed at capturing factorizations. The factor graph is constructed of both variable nodes and factor nodes. One or more variable nodes are connected with the factor node which represents the correlation among neighboring variables [4]. Factor graph representation constructs the framework for the statistical inference approach since the local correlations represented by factor functions enable the distributed message passing algorithm.

In a regular factor graph, variable nodes and factor nodes are represented by circles and squares respectively and they constitute a bipartite graph. In the context of electrical engineering, a Forney-style factor graph, also known as “Normal factor graph” [5], is discussed more often since it captures two-way flow of information. In a Forney-style factor graph, a variable node is represented by an edge or half edge while a factor node is still represented by a square. It has the following properties:

1) The node representing certain factor $f$ is connected with the edge or half-edge representing variable $x$ if and only if $f$ is a function of $x$;
2) The variable has maximum degree of two, which means no variable appears in more than two factors [4].

As a comparison, in a bipartite factor graph a variable node can be connected to any number of factor nodes. However it can be easily transformed to a Forney-style factor graph by adding the factor that represents equality equation. In this paper we mainly discuss the Forney-style factor graph.

A variable node in the factor graph may represent a conductor in the micro-grid, that is a branch in the bus/branch model. The state of the variable is defined by a vector constructed of several different measurements. As a full range of measurements, the following could be included in the vector: voltage, current, frequency, phase, mechanical power, solar power, wind power, etc. For different nodes in a micro-grid, different kinds of measurements need to be involved. For example, the component of solar power is only useful when the node is associated with solar generation unit. When distributed renewable generation is integrated in the grid, it is essential to explicitly model the power flow at every generation unit. For this consideration, we view every distributed generation unit as a separate power injection connected directly to the bus. If in reality the distributed generation is connected to the grid through a customer load, it would be straightforward to derive the combined behavior at the customer branch to the bus.

The definition of state variables varies at different approaches. We define the state vector as follows:

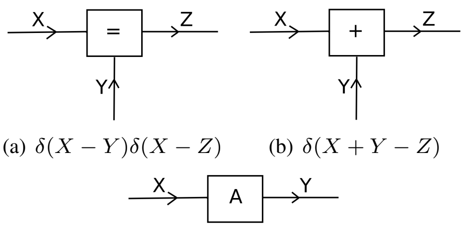
1) Regular variable node
   - voltage magnitude $V_k$
   - voltage angle $\theta_k$
   - current magnitude $I_k$
   - current angle $\varphi_k$
2) Variable node representing distributed renewable generation:
   - voltage magnitude $V_k$
   - voltage angle $\theta_k$
   - injected real power $P_k$

Note that for regular variable node power flow can be calculated from the complex voltage and current. For variable node associated with renewable generation, line current can be derived from the above state variables too.

Factor nodes should be able to represent local electrical relationships, such as Kirchhoff’s current law (KCL), Kirchhoff’s voltage law (KVL), Ohm’s law, and the conservation of power law. For example, the voltage drop on distribution line can be simply modeled using Ohm’s law because the distribution line is commonly within a short (i.e., 5 miles) distance. In comparison, a proper model for long distance transmission line would be a $\pi$ model [6]. The feeder transformer usually comes with specification in forms of turns ratio magnitude and phase shift angle. The factor function can then be represented by a linear equation with these two parameters.

In addition, since the smart grid may include distributed renewable generation such as solar and wind resource, the corresponding factor nodes may represent the solar/wind correlation due to the geographic proximity of neighboring nodes [7]. The correlation functions should be both time-correlated and location-sensitive. Cloud movements need to be taken into consideration as we derive the sequential and spatial correlation for solar generation. Some interesting research on wind power variability and wind correlation among different sites is presented in [8].

In a linear factor graph where factor functions involve only linear equations, we can use three factors as building blocks: addition, equality and multiplication. Fig. 1 presents the three factors in the Forney-style factor graph. Impulse functions are used to represent local relationships among variable X, Y and Z. One example of the application of sub-figure (b) on micro-grid local correlation is KCL at a power system bus. Sub-figure (c) can be used to model a simple resistive load function or the feeder transformer.




(a) $\delta(X-Y)\delta(X-Z)$ (b) $\delta(X+Y-Z)$

(c) $\delta(Y-AX)$

Fig. 1. Three factors representing linear building blocks

## III. Computation Rules of BP Algorithm

The BP algorithm is a distributed message passing algorithm to achieve global inference, i.e., marginalization over all global variables in graphical models. It is also known as sum-product algorithm and has applications in communications, signal and image processing, especially for communication tasks such as equalization and decoding [9]–[12]. There has been very little research in applying the BP algorithm to energy systems and power grids. Reference [13] presents research on optimizing the power grid with the message passing algorithm through adding interconnections to the distribution system.

The marginal distributions for unobserved nodes are posterior probabilities conditioned on measurements from observed nodes. Because of the scale and complexity of a micro-grid system, the computation could be high to derive the marginal probability of one particular node by summing/integrating the probabilities of all the other nodes. Consider a factor graph including $n$ variable nodes with binary values. To get the marginal distribution, one needs to sum over $2^{n-1}$ values.

The BP algorithm captures the factor graph factorization and performs summation/integration locally. The computational complexity can be greatly reduced compared with the brute force integration. As the number of variable nodes grows larger, this efficiency becomes more prominent. In the following discussion we elaborate on the local computation rules of the algorithm.

### A. Sum-product algorithm

Here we use messages to refer to the probability density functions passed along edges, which should meet specific computation rules defined by factor nodes. Evidence potential $\psi_i(x_i)$ is defined as the conditional probability to capture conditioning information given observations [14]. It reflects the local belief of variable $i$'s current state.

Since every variable is involved in at most two factors, the message $m_{is}$, flowing from variable $i$ to factor $s$, is the product of its own evidence potential $\psi_i(x_i)$ with the message that variable $i$ receives from its other neighbor, if it exists. Otherwise, when variable $i$ is a half edge, the message $m_{is}$ is simply $\psi_i(x_i)$.

The message flowing from factor $s$ to variable $j$ is computed as follows, also shown in Fig. 2:

$$m_{sj}(x_j) = \alpha \sum_{x_{N(s)\setminus j}} \left( f_s(x_{N(s)}) \prod_{i \in N(s)\setminus j} m_{is}(x_i) \right) \quad (1)$$

$\alpha$ is a normalization factor for probability representation. From (1) it is clear that the product is taken over all incoming messages $m$ to factor $s$, except for the message from variable j that is the recipient of the message $m$. Then the product of function representing factor $s$ and $\prod_{i\in N(s)\setminus j} m_{is}(x_i)$ is summed up over all neighboring variables of factor $s$ except for variable $j$.

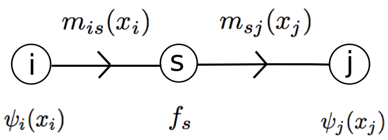



Fig. 2. Message update rules on a simplified graph

It is easily verified that the marginal probability distribution $f(x_i)$ of variable $x_i$ is the product of all the incoming messages flowing into $x_i$ with $\psi_i(x_i)$. Thus in order to derive the marginal probabilities of all the variables, we need to compute all the messages flowing along every edge/half edge in both directions.

### B. Gaussian Belief Propagation

Gaussian Belief Propagation is the BP algorithm where all probability distributions are Gaussian. Since the mean and covariance of a Gaussian random vector characterize its probability density function, the message only needs to include the mean vector $m$, covariance matrix $V$ or weight matrix $W = V^{-1}$. We are considering two-way flow of information on the edge, which makes it necessary to further divide the messages flowing along edges into forward message and backward message. This differentiation also prepares for the discussion of Kalman filtering and smoothing algorithm which include both forward message passing and backward message passing. Fig. 3 shows the parameters for Gaussian Belief Propagation. The computation rules for these parameters in Gaussian BP algorithm are derived in [4].

| | Forward Message | Backward Message |
|---|---|---|
| Mean vector | $\overrightarrow{m}$ | $\overleftarrow{m}$ |
| Covariance matrix | $\overrightarrow{V}$ | $\overleftarrow{V}$ |
| Weight matrix | $\overrightarrow{W} = \overrightarrow{V}^{-1}$ | $\overleftarrow{W} = \overleftarrow{V}^{-1}$ |

Fig. 3. Parameters for Gaussian Belief Propagation

## IV. Belief Propagation Scheduling

The order of passing messages is referred to as the schedule. Under the computation rules, local beliefs are propagated to the entire factor graph according to several different schedules. Reference [14] discusses in detail the message passing algorithm with a specific schedule, which is a variation of the Elimination algorithm. Exact inference can be performed by repeated elimination of variables on a radial graph. The graph should be a tree-like structure so that bounded length of elimination order can be derived.

Another way to derive marginal probabilities is the BP algorithm with flooding schedule, which means all the messages are updated simultaneously instead of passing messages according to some specific order [15]. It is widely applied since it can often work on general graphs that may contain loops. There is both a serial and a parallel approach for the flooding Belief Propagation. We derive the serial Belief Propagation from the Kalman Filter algorithm and then move forward to the discussion of parallel Belief Propagation which is more computationally efficient.

### A. Kalman Filter and Serial BP

In a micro-grid, observations are provided by measurement units, such as AMI, PMU or relay units. We want to derive the posterior probability of every variable given these observations taking into account the uncertainty caused by noise. The purpose of Kalman filter is to use noisy observations to estimate the true states. The Kalman filter is defined by a set of mathematical equations that provides an efficient recursive mean to estimate the state of a process, in a way that minimizes mean squared error [16]–[18]. In a factor graph representing a micro-grid, every variable is viewed as a hidden state. Given the initial probability distribution, we can perform the Kalman filter algorithm to estimate the distribution of node behavior based on noisy observations from the grid.

With a broad definition, the Kalman filter algorithm includes Kalman filtering and Kalman smoothing. Kalman filtering is the forward belief propagation recursion which provides posterior probability distribution of the state $X[t]$ given the observation sequence $Y$ up to time $t$. By computing also the backward messages, Kalman smoothing calculates the posterior probability of all the variables given the whole observation sequence $Y$. Here we apply the Kalman filter on a linear State-Space Model (SSM), which assumes that the true state at time $t+1$ evolves from the state at time t according to

$$x_{t+1} = Ax_t + Gw_t \tag{2}$$

where $A$ is the state transition model and $w_t$ is the process noise, $w_t \sim N(0, Q)$. $N(0, Q)$ is a Gaussian random variable with mean zero and variance $Q$. This corresponds to the time update.

At time $t$ an observation $y_t$ of the true state $x_t$ is made according to

$$y_t = Cx_t + z_t \tag{3}$$

where $C$ is the observation model which maps the true state space to the observed space. $z_t$ is the observation noise, $z_t \sim N(0, R)$. This corresponds to the measurement update.

Fig. 4 is the factor graph of linear State-Space Model representing the above state equations. This model can be extended in a time/spatial manner.

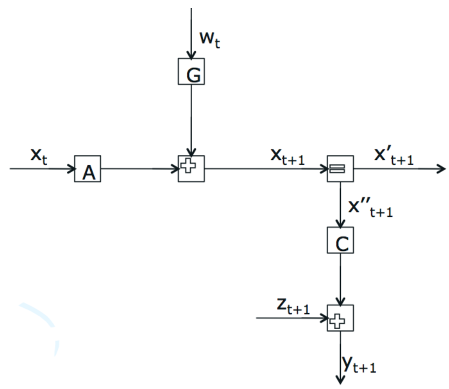



Fig. 4. Factor graph of linear State-Space model

For a system with known state transition matrix $A$, we can directly use the Kalman filter algorithm to obtain state estimates. However, as the system grows larger and more complex, the dimension of $A$ grows and so does the computational complexity of this approach. Thus we turn to the discussion of serial Belief Propagation, which applies the idea of the Kalman filter while avoiding the computation of transition matrix $A$.

Serial BP algorithm is derived directly from the Kalman filter. For a Kalman filter system the transition between subsequent states takes place when a new observation is available for the later state. When BP is used for state estimation in the grid, spatially connected nodes are viewed as different state variables. The propagation process includes several iterations of measurement updates and time updates. Observations are taken in a sequential manner to perform measurement update in subsequent iterations. At every time update, all the messages are passed and updated according to the computation rules discussed in Section III. This process corresponds to the Kalman filtering and smoothing. After all the observations are processed, global inference for all variables can be derived.

### B. Parallel Belief Propagation

We refer to the BP with parallel computation as parallel BP. It has applications in many different areas, e.g. LDPC decoding. Parallel BP improves the efficiency of state estimation since it involves less iterations of message passing.

Instead of fetching one observation at each time step as in serial BP, parallel BP does the measurement update for all the variables at one time once a new set of observations is available. The frequency of data collection can be determined according to the distributed control and dispatch ability of the grid. For example, if the grid requires five minutes ahead of time control, the sampling period of metering data should be less than that to allow for data processing and control actions. The new iteration of measurement update can take place only after the convergence of the previous iterations of the time update. The accuracy of state estimation can be affected by the concurrency of measurement collection because the BP algorithm captures a snapshot of system states. Thus the metering devices equipped with GPS that provides time-stamped data would be preferable for this data-driven application.

Once all the variables have their local beliefs based on the given set of observations, their beliefs are propagated in the graph for certain time units until every variable receives beliefs from all the other variables. This process corresponds to a time update in the Kalman filter. The time update process involves simultaneous update of all the messages from variables to factors, followed by the update of messages from factors to variables. The above two steps together make one iteration. Section III discusses the computation rules for these two kinds of messages.

Suppose $E$ is the set of all the variables and $F$ is the set of all the factors. We summarize the parallel BP algorithm in the following steps:

1) For every variable $i$, initialize the evidence potential $\psi_i(x_i)$ given possible observation.
2) While the stop criteria is not met,
   - For every $i \in E$, update messages $m_{is}^{t+1}$ flowing from variable $i$ to all of its neighboring factors $s$ at time $t+1$ according to $m_{ki}^{t}$.
   - For every $s \in F$, update messages $m_{sj}^{t+1}$ flowing from factor $s$ to all of its neighboring variables $j$ at time $t+1$ according to $m_{is}^{t+1}$.
3) Compute marginals $P(x_i)$ according to the steady state messages $m_{si}$.

The stop criteria used can be either when all the posterior estimates converge or when a maximum number of iterations are reached. If the factor graph has a cycle-free structure, we can envision that the iteration time would be $R/2$ where $R$ is the diameter of the tree. After $R/2$ iterations BP will necessarily converge to the true marginals because every local belief is propagated to the entire graph and the global inference for every variable node can be derived based on all the observations.

## V. State Estimation

### A. Conventional State Estimation

Conventional state estimation techniques have been applied in the power grid, mostly in the bulk transmission system, to estimate the true states of the overdetermined system. State estimation algorithm estimates the voltages (magnitude and angle) at all the system buses given both network impedance and measurements received from the substations [6]. In most practical cases the number of measurements is more than the number of states. That is to say, more data is available than it is utilized for state estimation [19]. The purpose of the traditional state estimation approach is to eliminate incorrect measurements and determine the power flow in the unmetered parts of the grid given enough redundancy.

The algorithm is based on several assumptions:

- All the loads can be modeled as constant variables and are known to the system.
- System topology is correct and analog bad data can be identified whenever redundancy allows it [3].

Most state estimation programs formulate the system as nonlinear equations and use recursive algorithm, i.e, Weighted Least Square (WLS) algorithm, to solve for the true system states [20]. This approach can not be directly applied on the distribution system because there are not as many real-time measurements [21]. Reference [22] resolves this problem by using the load modeling approach to generate pseudo-measurements from historical load data.

The distributed state estimation (DSE) models the distribution system by viewing the entire distribution feeder as a load without modeling the individual customer in the feeder circuit. However, the smartness of the future grid requires the modeling and control ability at the customer level. Thus an efficient distributed algorithm is needed to perform state estimation at the micro-grid level, rather than the centralized recursive algorithm utilized in current state estimators. Parallel BP is such a distributed algorithm that iteratively propagates belief and converges at global inference.

### B. Micro-grid State Estimation

In the micro-grid state estimation, we assume certain amount of smart metering devices are deployed that provide real-time measurements. However, currently there are very limited real-time load data provided downstream of the distribution feeders. We can generate pseudo-measurements, as in the DSE approach, from a proper prior distribution of the loads. The proper model for the prior distribution is crucial for the Bayesian approach and affect the estimation performance. The load modeling algorithm for nodes in the micro-grid is different from the common approach for a state estimator that models the load as constant power, impedance or current [23].

As discussed in Section I, the load behavior at micro-grid level is stochastic and should be modeled as a random distribution. The loads are commonly modeled as Gaussian distributions, which is not justified because the statistical distribution of load variation may not follow any specific probability distribution function [24]. Reference [24] further presents the approach of representing the load pdf as a Gaussian mixture model. The mixture components can be reduced to form a single Gaussian approximation. In our model, the pdf for both electrical parameters and renewable generation power flow is modeled as Gaussian distribution.

In the micro-grid state estimation, we use Gaussian BP to derive global inference. BP is only applicable when the variables are discrete-valued or follow multivariate Gaussian distribution. If the factor function is nonlinear, the variable distribution will not remain Gaussian after message passing. When injected power flow from renewable generation is considered as a state variable, its corresponding correlation function is nonlinear. In order to still utilize the Gaussian BP approach, linear approximation can be performed around the most recent voltage and current values.

The BP algorithm will always work on a tree-like structure. Inference on loopy graphs is known to be NP-hard. However, for general graphs which may contain loops, BP can often converge to the optimal state estimates when a proper stop condition is specified. This approach has shown experimental success in many applications. It is also called *loopy Belief Propagation* to address the fact that it can often work on a general graph which might include loops.

Reference [25] derives the analytical relationship between the approximate marginals computed with loopy BP and the true marginals on a graph with single loop. Graphs exist where loopy BP fails to converge. Reference [26] describes three techniques for analyzing the behavior of loopy BP.

The structure of the existing power grid is mostly radial. There are two cases that may result in loopy graphs representing the grid.

1) Possible loops may be needed to address ancillary backup power lines.
2) Time and spatial correlations of solar/wind generation would result in loops in the micro-grid model.

For the first case, we can adopt the assumption that the distribution systems are built with pre-installed on/off switches and known status of circuit breakers to construct a particular configuration. This configuration includes tree-like subgraphs of the full loopy distribution graph. For a given time it allows for currents flowing over trees. As for the second case, the exact behavior of the loopy BP algorithm on a micro-grid with high renewable penetration is still under research. Despite of this, the BP scheme provides heuristic tools for graphic models on finite sparse graphs and can be used for algorithmic optimization and control of the distribution system [13].

## VI. Simulation

The actual micro-grid structure can be quite large and complex while the number of nodes involved may vary from tens to thousands. The study on the many details of the distribution system is quite complex and requires simplifications. The simulation of the statistical inference approach is conducted on a simplified micro-grid system which incorporates distributed renewable energy generation. The assumption is made that all the local relationships are linear including the electrical relationship and correlation among renewable generation. In this case the optimum linear minimum mean squared error estimate is the optimum minimum mean squared error estimate.

We simulate the loopy BP algorithm on a factor graph with 210 variables, which represents a micro-grid incorporating distributed renewable generation. The micro-grid is comprised of 10 clusters which are interconnected at the boundary, i.e. variable $x_{16}$ has a connection with variable $x_1$ of the neighboring cluster. Every local graph has the same structure as shown in Fig. 5 [7]. The micro-grid has certain percentage of scattered smart metering devices being deployed such as AMIs and relay units. The power supply comes both from the substation and renewable generation. The system on the graph should enable two-way flow of information. Please refer to [7] for detailed explanation of this micro-grid model.

The corresponding factor graph for the cluster is shown as Fig. 6 [7]. Note that in the factor graph within every cluster five more variable nodes are added to address generation and load variables corresponding to the factor function $g$ and $f_L$. $f_E$ represents the electrical relationship on the system bus, which includes KVL, KCL and conservation of power law. $g_S$ and $g_W$ represent solar and wind correlation among neighboring generation units corresponding to the dotted lines in Fig. 5.

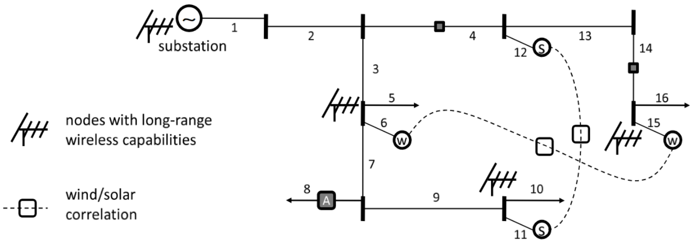



Fig. 5. One cluster of the smart micro-grid model

As a proof-of-concept study we make several assumptions in the simulation listed as follows. In the realistic power grid, these assumptions would not be necessarily true, but they provide basics for future study. The future improvements on the modeling will be discussed in the last section.

1) All the factors involve only linear operations, i.e., correlations among variables are linear and can be expressed with the three linear building blocks, as described in Fig. 1.
2) The initial probability distributions of all the variables are assumed to be multivariate Gaussian distributions.
3) The sampling period of new observations is longer than the convergence speed of BP algorithm.

The parameters in this simulation are set as follows:

- The state of every variable is defined by four components, i.e. active power, reactive power, voltage magnitude and angle. They take on real analog values.
- We use equality factor, as shown in Fig. 1(a) to represent correlation function $g$, addition factor, as shown in Fig. 1(b) to represent $f_E$ and multiplication factor as shown

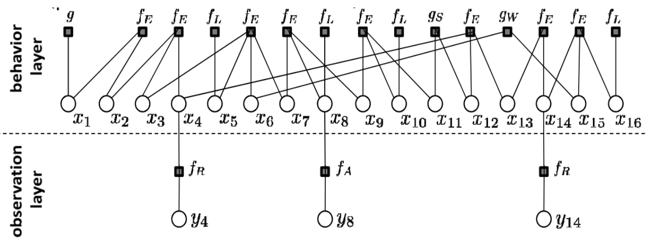



Fig. 6. Factor graph representation of Fig. 5

in Fig. 1(c) to represent $f_L$, $g_S$, $g_W$. Note that for load function and solar/wind correlation function we assume $A$ is a diagonal matrix $0.5I$.

- The maximum iteration times are set to be $1000$.
- The error distance for convergence $\delta$ is chosen to be $0.001$. This means when total squared error of mean estimates is smaller than $\delta$, we consider steady state has been reached.

Fig. 7 shows the estimate error versus iteration times. From the figure we can see that the loopy BP algorithm converges after 10 iterations in this simulation. The first and second order statistics of node behavior reach the steady state which is considered to be the optimal state estimate. The precision of state estimation will rely on both the prior model of node behavior and number of observations provided.

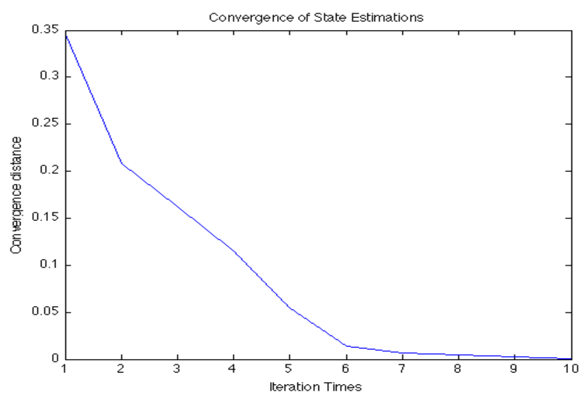



Fig. 7. Convergence of state estimations

## VII. Conclusion and Future Research

This paper presents a novel approach to estimate the node behavior in a smart micro-grid. The first step is to model the micro-grid as a factor graph through defining the correlations among correlated nodes as factor functions. Then we conduct statistical inference by using the loopy BP algorithm. The algorithm derives true marginals on radial graph and approximates marginals well on loopy graphs.

Before utilities and industries make large investments on deploying smart metering devices, it is essential to conduct the studies in determining how accurate statistical inference procedures are when only a few of these metering devices are deployed. There are many issues to be considered from where to place the metering devices to how many of the devices to deploy. This paper proposes a statistical model where we can begin to answer these questions. In formulating simulation models we must be careful that we use realistic random models that fit real micro-grid systems. By conducting extensive simulation studies we can easily vary the number of metering devices used and their locations. By understanding the models and observing simulation results we can then develop strategies for placement of AMIs and PMUs in future micro-grids.

Future work should also include the following directions:

1) Deal with nonlinear message passing using the approach of Extended Kalman Filter (EKF) [27].
2) Conduct statistical inference for variables with non-Gaussian continuous distribution.
3) Derive proper model for solar and wind correlations. Before we could derive the model that accounts for general situation, the first step should be to gather massive environmental information and renewable generation data from different sites which are geographically related.
4) Properly model generation function as well as load function which can be demand-responsive.

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