Performance Analysis of Maximum Likelihood Detection in a MIMO Antenna System

Xu Zhu and Ross D. Murch, *Senior Member, IEEE*

***Abstract*—In this letter, we provide an analysis of the performance of maximum likelihood detection (MLD) over flat fading channels in a wireless multiple input–multiple output (MIMO) antenna system. A tight union bound with an asymptotic form on the probability of symbol error rate (SER) for MIMO MLD systems with two-dimensional signal constellations (such as QAM and PSK) is introduced. Using this analytic bound, performance of the MIMO antenna system is demonstrated quantitatively with respect to channel estimation, constellation size, and antenna configuration.**

***Index Terms*—Constellation, MIMO, MLD, symbol error rate.**

I. Introduction

Wireless multiple input–multiple output (MIMO) systems promise improved performance compared to conventional systems. Techniques for achieving these advantages [1]–[3] include zero-forcing (ZF), minimum mean square error (MMSE), maximum likelihood detection (MLD) and Vertical Bell Laboratories Layered Space–Time (V-BLAST). Among these techniques, MLD is the optimum in terms of minimizing the overall error probability and, with small numbers of transmit antennas and low-order constellations, the complexity of MLD is not overwhelming [4]. In [4], an upper bound of MLD for a MIMO system was derived for two-dimensional (2-D) constellations like QAM, however, it is loose and assumes perfect channel estimation. Results for joint detection in a multi-user detector were provided in [5] and a tight union bound on the symbol error rate (SER) with imperfect channel estimation was derived. A more explicit form of the bound was demonstrated in [6]. However, these bounds are only valid for PSK modulation.

In this letter, we provide a performance analysis of MLD over flat fading channels. A tight union bound and an asymptotic bound on the SER are developed, by applying and extending the work in [5] and [6] to the MIMO configuration, with 2-D constellations. These bounds are then utilized to demonstrate the performance of MLD quantitatively. Our approach of deriving the pairwise symbol error probability might be extended to evaluate the pairwise block error probability of the Viterbi-based MLD for a coded system.

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The authors are with the Center for Wireless Information Technology, Department of Electrical & Electronic Engineering, The Hong Kong University of Science & Technology, Clear Water Bay, Kowloon, Hong Kong (e-mail: eezhuxu@ee.ust.hk; eermurch@ee.ust.hk).

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II. System Model

We consider a MIMO system with K transmit and L receive antennas, where the transmitted signals are assumed to be independent in time as well as space. The transmitted signal vector at a particular time instant is written as $\boldsymbol{d}$ and consists of K QAM or PSK symbols each with a constellation size of M and average symbol energy E_s. The received signal vector $\boldsymbol{y}$ is given by $\boldsymbol{y} = \boldsymbol{Hd} + \boldsymbol{n}$ where $\boldsymbol{H}$ is an $L \times K$ channel gain matrix for the flat fading channel, whose elements are independent zero-mean complex Gaussian random variables with unit variance, and the L elements of vector $\boldsymbol{n}$ are samples of independent complex additive white Gaussian noise (AWGN) processes with single-sided power spectral density N_0.

Channel estimation is determined by channel state information (CSI) and, following [5], we assume that the estimate of true channel gain matrix $\boldsymbol{H}$ is denoted by $\boldsymbol{V}$ which also consists of independent zero-mean complex Gaussian random variables, with variance σ_v^2. Let ρ_{hv} denote the correlation coefficient between corresponding elements of $\boldsymbol{H}$ and $\boldsymbol{V}$ and, since they are jointly Gaussian distributed with independent components, we can write

$$\boldsymbol{H} = \beta_{hv}\boldsymbol{V} + \boldsymbol{E} \tag{1}$$

where $\beta_{hv} = \rho_{hv}/\sigma_v$ is the coefficient for MMSE estimation of $\boldsymbol{V}$ and $\boldsymbol{H}$, and $\boldsymbol{E}$ is a zero mean Gaussian distributed error matrix with the variance $(1 - |\rho_{hv}|^2)$. It is assumed that $|\beta_{hv}| = 1$ and note that, with perfect CSI, $\rho_{hv} = 1$ and $\beta_{hv} = 1$. The conditional probability density function (pdf) of the received $\boldsymbol{y}$, given the channel estimate $\boldsymbol{V}$ and the candidate data vector $\boldsymbol{d}$, is given by

$$p_{\boldsymbol{y}}(\boldsymbol{y}|\boldsymbol{d}, \boldsymbol{V}) = \frac{1}{(2\pi)^L \left(\sigma_y^2\right)^L} \exp\left(\frac{-\mu}{2\sigma_y^2}\right) \tag{2}$$

where $\sigma_y^2 = (1 - |\rho_{hv}|^2)||\boldsymbol{d}||^2 + N_0$, and the Euclidean distance metric μ can be expressed [5] as

$$\mu = ||\boldsymbol{y} - \beta_{hv}\boldsymbol{V}\boldsymbol{d}||^2 = \sum_{l=1}^{L} |y_l - \beta_{hv}\boldsymbol{v}_l\boldsymbol{d}|^2 \tag{3}$$

where y_l is the lth received signal and $\boldsymbol{v}_l$ denotes the lth row of $\boldsymbol{V}$. Neglecting hypothesis-independent terms, the ML metric to be minimized is given by

$$\Lambda = L \ln\left(\sigma_y^2\right) + \frac{\mu}{2\sigma_y^2}. \tag{4}$$

Note that the ML metric Λ reduces to the Euclidean metric μ with perfect CSI and any signal constellation, or imperfect CSI and constant symbol energy (e.g., PSK). With imperfect CSI and nonconstant symbol energy (e.g., 16QAM), Λ can be approximated by μ. Our analysis to follow is based on the Euclidean metric, hence, it can be regarded as "approximate MLD" when 16QAM with imperfect CSI is investigated in Section V-A.

III. Union Bound on SER for MIMO MLD

A tight union bound on the SER of the kth $(k = 1, \ldots, K)$ transmitted signal stream can be found by applying the results in [5] and [6] to the MIMO configuration for 2-D constellations under the channel estimate (1). It is assumed that all the possible symbols are equally probable. We define $\{s_m\}$ as the set of all M possible symbols transmitted at a particular antenna, and $\{\boldsymbol{d}\}$ as the set of all M^K possible symbol vectors from the K transmit antennas. We also let $\{\boldsymbol{d}_j\}$ denote a subset of $\{\boldsymbol{d}\}$ in which vectors have s_m as their kth element so that in total there are M^{K-1} vectors in $\{\boldsymbol{d}_j\}$. We also define $\{\boldsymbol{d}_i\}$ as the set of transmission vectors that differ in their kth position from $\{\boldsymbol{d}_j\}$ so that there are a total of $(M^K - M^{K-1})$ such vectors. The distance metrics of $\boldsymbol{d}_i$ and $\boldsymbol{d}_j$ are denoted by μ_i and μ_j, respectively, and a pairwise error occurs when the detector chooses the erroneous $\boldsymbol{d}_i$ over $\boldsymbol{d}_j$ if $D_{ij} = \mu_i - \mu_j < 0$. Hence, the union bound on the SER of the signal stream transmitted by the kth antenna is

$$P_s \leq M^{-K} \sum_m \sum_j \left(\sum_i P_{s_m,ij} \right) \tag{5}$$

where $P_{s_m,ij} = P(D_{ij} < 0 \,|\, s_m, \boldsymbol{d}_j)$ denotes the pairwise error probability between $\boldsymbol{d}_i$ and $\boldsymbol{d}_j$, given that s_m is transmitted by the kth antenna. There can be up to $M^K(M^K - M^{K-1})$ pairwise error probabilities but symmetry in the constellation allows simplifications. For the case of PSK, (5) reduces to $P_s \leq \sum_i P_{s_m,ij}$ for all $s_m, \boldsymbol{d}_j$ and provides the same result as in [5]. For higher order QAM $(M > 4)$ with standard square constellation, elements of $\{s_m\}$ have $q = (M + 2\sqrt{M})/8$ different symbol energies. For each particular $\boldsymbol{d}_j$, there are only $Q = (q^{K-1} + q)/2$ different energies. Hence, at most $qQ(M^K - M^{K-1})$ instead of $M^K(M^K - M^{K-1})$ pairwise error probabilities need to be found.

The pairwise error probability $P_{s_m,ij}$ is determined by

$$P_{s_m,ij} = P(D_{ij} < 0 \,|\, s_m, \boldsymbol{d}_j) = \int_{-\infty}^{0} p(D_{ij})\, dD_{ij} \tag{6}$$

where $p(D_{ij})$ is the pdf of D_{ij}, and its two-sided Laplace transform $\Phi_{D_{ij}}(s)$ is expressed as [5]

$$\Phi_{D_{ij}}(s) = \left[\frac{p_{ij1} p_{ij2}}{(s - p_{ij1})(s - p_{ij2})} \right]^L \tag{7}$$

where p_{ij1} and p_{ij2} denote the poles in the left and right half-plane, respectively. Letting $r_{s_m,ij} = -p_{ij2}/p_{ij1}$ and following the derivation in [7] yield a closed-form expression of $P_{s_m,ij}$

$$P_{s_m,ij} = \frac{1}{(1 + r_{s_m,ij})^{2L-1}} \sum_{l=0}^{L-1} \binom{2L-1}{l} r_{s_m,ij}^l. \tag{8}$$

Further derivation in Appendix A yields a fully analytic form as

$$r_{s_m,ij} = a_{s_m,ij}\Gamma_{s_m,j} + \sqrt{(a_{s_m ij}\Gamma_{s_m,j})^2 + 2\,(a_{s_m,ij}\Gamma_{s_m,j})} + 1 \tag{9}$$

where

$$a_{s_m,ij} = ||\boldsymbol{d}_i - \boldsymbol{d}_j||^2 / 2E_s$$

and

$$\Gamma_{s_m,j} = \frac{\gamma_c |\rho_{hv}|^2}{\frac{\gamma_c(1 - |\rho_{hv}|^2)}{E_s} ||\boldsymbol{d}_j||^2 + 1} \tag{10}$$

with $\gamma_c = E_s/N_0$ denoting the average symbol SNR per diversity branch since variance of the channel gain has been normalized to be unity. Note that, with perfect CSI, $\Gamma_{s_m,j} = \gamma_c$.

Note that our approach of deriving the pairwise symbol error probability might be extended to evaluate the pairwise block error probability of the Viterbi-based MLD for a coded system.

IV. Asymptotic Union Bound

When SNR γ_c becomes high, the asymptotic form of $P_{s_m,j}$ can be expressed as

$$P_{s_m,ij,\text{asymp}} = r_{s_m,ij}^{-L} \binom{2L-1}{L-1} \tag{11}$$

where $r_{s_m,ij} \approx 2a_{s_m,ij}\Gamma_{s_m,j}$ and is an extension of the results in [6]. Substituting (11) into (5) and approximating further, the asymptotic bound for SER of the kth transmitted signal stream is

$$P_{s,\text{asymp}} = \alpha \left(\frac{1}{2\bar{\Gamma}} \right)^L \binom{2L-1}{L-1} \tag{12}$$

where $\alpha = M^{-K} \sum_m \sum_j (\sum_i a_{s_m,ij}^{-L})$ and $\bar{\Gamma} = (\gamma_c |\rho_{hv}|^2) / (K\gamma_c(1 - |\rho_{hv}|^2) + 1)$. In a model of channel estimation based on the pilot symbol assisted modulation (PSAM), channel estimation correlation coefficient ρ_{hv} varies with SNR and can be expressed [5] as $|\rho_{hv}(\gamma_c)| = \sqrt{1 - b/\gamma_c}$. Therefore, (12) becomes

$$P_{s,\text{asymp}} = \alpha \left(\frac{Kb + 1}{2\gamma_c} \right)^L \binom{2L-1}{L-1}. \tag{13}$$

V. Performance Analysis

In this section, we present a set of performance analyses based on analytic and numerical results. The results that are given are in terms of averaged E_b/N_0 as defined in [1] and [4]

$$\gamma_b = \frac{E_b}{N_0} = \frac{L}{n} \cdot \frac{E_s}{N_0} = \frac{L}{n} \gamma_c \tag{14}$$

which can be regarded as the total received E_b/N_0 per transmit branch, where $n = \log_2 M$ is the bits per symbol. Equation (14) is also useful in that, for two systems with the same performance

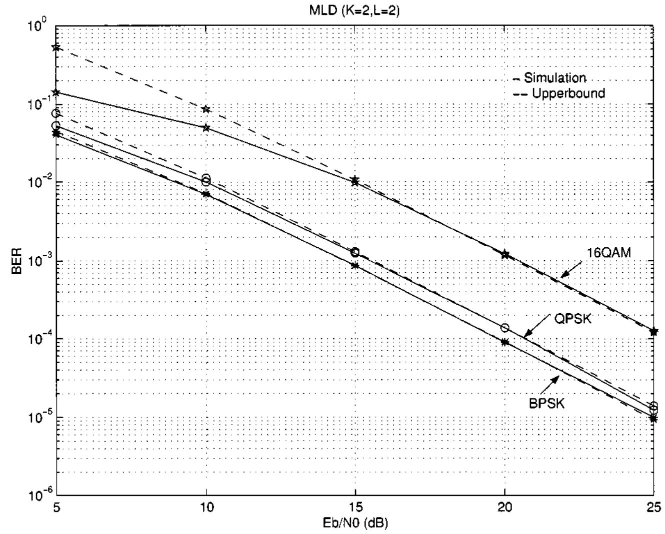

Fig. 1. Illustration of tightness of the analytic union bound.

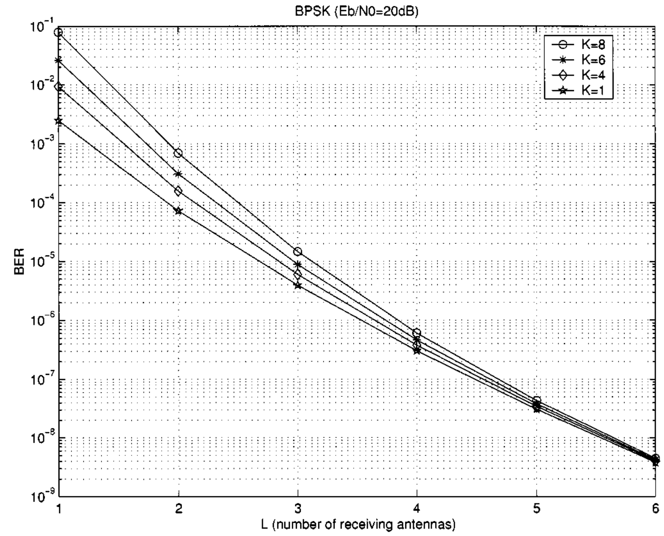

Fig. 2. Performance of BPSK with SNR defined in (14) and set to 20 dB.

in terms of γ_b, the system with more receive antennas requires less total transmit power [4].

By assuming Gray coding, an approximate bit error rate (BER) can be obtained from the union bound on SER to give [7] $P_b \approx P_s/n$. Using this asymptotic value, the BER becomes

$$P_{b,\text{asymp}} = \frac{\alpha}{n}\left(\frac{L(Kb+1)}{2n\gamma_b}\right)^L \binom{2L-1}{L-1}. \qquad (15)$$

Comparing our union bound given by (5) and (8) to simulation results (see Fig. 1) for BPSK, QPSK, and 16QAM with two transmit, two receive antennas, and perfect CSI $(b=0)$, we observe that, when the true BER is below about 0.01, the maximum relative error of our bound is only about 5%.

A. Effect of Imperfect CSI on Performance

It has been shown in (15) approximately that imperfect CSI degrades the SNR by an asymptotic factor of $(Kb+1)$, independent of the number of receive antennas. Using our union bound given by (5) and (8), the effect of imperfect CSI on the performance has been investigated with $b = 0.199$ (implying $|\rho_{hv}| = 0.9990$ when $\gamma_c = 20$ dB). We found that with two transmit antennas this leads to an SNR penalty of about 1.4 dB for both BPSK and 16QAM and matches our conclusion from the asymptotic form of the union bound.

B. Diversity Order

From (13), it can be deduced that with a relatively high SNR (i.e., BER is below a specific level such as 0.01), the error probability is proportional to the inverse of the SNR to the power of L [4], [6]. This implies that the diversity order of MLD is equal to the number of receive antennas, independent of the number of transmit antennas. Furthermore, in this case the SNR penalty due to the increased number of transmit antennas plays a major role in the performance change.

For BPSK, without loss of generality, we assume that the elements of $\boldsymbol{d}_j$ are all ones, so $\alpha_{\text{BPSK}} = \sum_i [\sum_{p=1}^K (1 - \text{Re}(d_{ip}))]^{-L}$ where $\text{Re}(d_{ip})$ denotes the real part of the pth element of $\boldsymbol{d}_i$. Assuming $\boldsymbol{d}_i$ differs from $\boldsymbol{d}_j$ in c $(c = 1, \ldots, K)$ symbols, there are $\binom{K-1}{c-1}$ possible $\boldsymbol{d}_i$'s, and $\sum_{p=1}^K (1 - \text{Re}(d_{ip})) = 2c$. Therefore,

$$\begin{aligned}\alpha_{\text{BPSK}} &= 2^{-L}\left[\sum_{c=1}^{K} \binom{K-1}{c-1} c^{-L}\right] \\ &= \begin{cases} 2^{-L}, & K=1 \\ 2^{-L}\left[1 + \sum_{c=2}^{K} \binom{K-1}{c-1} c^{-L}\right], & K>1. \end{cases}\end{aligned} \qquad (16)$$

With a large L, the value of α_{BPSK} approaches 2^{-L}, and with perfect CSI (15) becomes

$$P_{b,\text{asymp}} = \left(\frac{L}{4\gamma_b}\right)^L \binom{2L-1}{L-1} \qquad (17)$$

which is equivalent to the single transmitter situation [7, eq. (14-4-18)]. From (17), we can deduce that, with a large number of receive antennas, the SNR penalty due to increased number of transmit antennas approaches 0 dB. This is demonstrated by Fig. 2 using our explicit union bound, where γ_b is fixed to be 20 dB, and the horizontal axis denotes the number of receive antennas. It is deduced that, with large numbers of receive antennas (e.g., $L = 6$), the number of transmit antennas has little effect on the system performance. This implies that (and without regard to complexity) we can achieve an arbitrary high data rate with a low SNR penalty when the number of receive antennas is sufficiently large.

C. Performance Comparison Among 2-D Constellation Systems

In [4], numerical results were used to show the tradeoffs on performance and constellation size with a given data rate. We now give the theoretic analysis using our asymptotic bound.

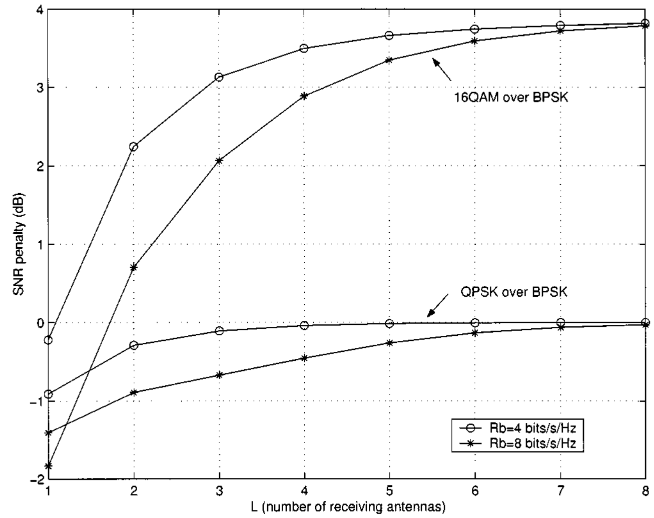

Fig. 3. SNR penalty of 16QAM and QPSK compared to BPSK for various numbers of receive antennas L and fixed data rate R_b.

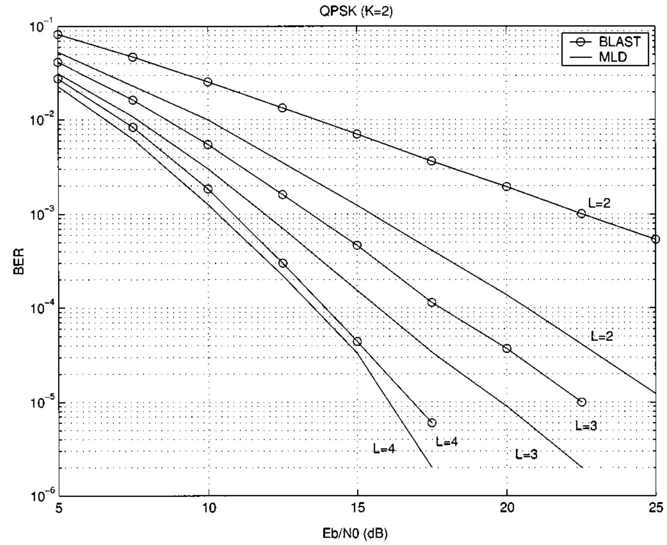

Fig. 4. Performance comparison between MLD and BLAST with two transmit antennas $(K = 2)$.

Given perfect CSI and a fixed number of receive antennas L, the SNR penalty of M-ary PSK relative to BPSK, to maintain the same data rate and BER obtained from (15), is given by

$$R_{\frac{\mathrm{MPSK}}{\mathrm{BPSK}}} = \frac{\gamma_{b,\mathrm{MPSK}}}{\gamma_{b,\mathrm{BPSK}}} = \frac{1}{n}\left(\frac{\alpha_{\mathrm{MPSK}}}{n\alpha_{\mathrm{BPSK}}}\right)^{1/L}. \tag{18}$$

Similar expressions can be found for M-ary QAM. With BPSK as a reference, Fig. 3 illustrates the SNR penalty of QPSK and 16QAM versus the number of receive antennas, where R_b denotes the total data rate (e.g., for QPSK $n = 2, K = 4, R_b = nK = 8$ bit/s/Hz). We observe that with multiple receive antennas, QPSK outperforms the other two-dimensional signal constellations, with a small SNR gain of less than 1 dB over BPSK, and a greater gain over 16QAM. With increasing numbers of receive antennas, the SNR penalty approaches a constant which is about 0 dB and 3.8 dB for QPSK and 16QAM, respectively. This constant is also independent of data rate R_b. Given a fixed number of receive antennas, when the data rate increases QPSK has more SNR gain over BPSK and 16QAM has less SNR penalty over BPSK. For instance, with $L = 2$, when R_b increases from 4 bit/s/Hz to 8 bit/s/Hz, the SNR penalty of 16QAM over BPSK decreases by about 1.5 dB, and QPSK obtains an SNR gain over BPSK increases by about 0.6 dB. Similar results hold for other higher order constellations. In summary, with a given data rate R_b and receive antennas L, QPSK outperforms other 2-D signal constellations. When L increases, the SNR gain of QPSK over another constellation approaches a certain constant independent of R_b. With a given L, when R_b increases, QPSK has more SNR gain over BPSK, but less SNR gain over 16QAM.

D. *Performance Comparison Between MLD and V-BLAST*

The diversity order of a conventional detection technique like MMSE and ZF [1]–[4] is limited to $L - K + 1$ [8]. The newly developed V-BLAST technique [2], [3] improves that diversity order by layered space–time detection. Unlike MLD, however, that diversity order is still constrained by the number of transmit antennas, and BLAST does not work when the number of transmit antennas is greater than that of receive antennas $(K > L)$, due to properties of MMSE and ZF.

Fig. 4 illustrates the performance comparison between MLD and BLAST with two transmit antennas, QPSK modulation, and perfect CSI, where ZF criterion is employed in BLAST. It shows that the performance of BLAST approaches that of MLD at the cost of an increased number of receive antennas. When the number of receive antennas is similar to or less than that of transmit antennas, MLD has a significant advantage of performance over BLAST.

Performance comparison between space–time trellis coding and BLAST with layered codes was demonstrated in [9], with Viterbi-based MLD used for decoding. It was shown that the latter is inferior in performance to the former due to the loss of diversity. The performance improvement of BLAST by using space–time block coding and Turbo decoding was shown in [10].

VI. Conclusion

In this letter, we have introduced a tight union bound and an asymptotic form on the SER for a MIMO MLD antenna system with 2-D signal constellations. It is shown that a very high data rate can be achieved with little SNR penalty when the number of receive antennas L becomes large and that the diversity order of MLD is equal to L. We also present a performance comparison among two-dimensional constellations and also between MLD and V-BLAST.

Appendix A

In this appendix, we derive the value of $r_{s_m,ij}$ and use superscript H to denote conjugate transpose. Starting from (3), the Euclidean distance metric of vector $\boldsymbol{d}_j$ can be expressed as

$$\mu_j = \sum_{l=1}^{L} \boldsymbol{z}_l^H \tilde{\boldsymbol{d}}_j^* \tilde{\boldsymbol{d}}_j^T \boldsymbol{z}_l \tag{A1}$$

$$\boldsymbol{A} = \boldsymbol{R}\boldsymbol{F}_{ij} = \begin{pmatrix} -|\rho_{hv}|^2(E_{ji} - E_{jj}) & |\rho_{hv}|^2 \left(E_{ji}\boldsymbol{d}_i^T - E_{jj}\boldsymbol{d}_j^T\right) - (E_{jj} + N_0)\left(\boldsymbol{d}_i^T - \boldsymbol{d}_j^T\right) \\ -|\rho_{hv}|^2(\boldsymbol{d}_i^* - \boldsymbol{d}_j^*) & |\rho_{hv}|^2(\boldsymbol{d}_i^* - \boldsymbol{d}_j^*)\boldsymbol{d}_i^T \end{pmatrix} \tag{A5}$$

where $\boldsymbol{z}_l = [y_l \, \beta_{hv} \boldsymbol{v}_l]^T \; \tilde{\boldsymbol{d}}_j = [1 \; -\boldsymbol{d}_j^T]^T$ are vectors of $(K+1)$ elements, and μ_i is defined similarly. The difference between distance metrics of signal vectors $\boldsymbol{d}_i$ and $\boldsymbol{d}_j$ is given by

$$D_{ij} = \mu_i - \mu_j = \sum_{l=1}^{L} \boldsymbol{z}_l^H \boldsymbol{F}_{ij} \boldsymbol{z}_l \tag{A2}$$

where

$$\boldsymbol{F}_{ij} = \tilde{\boldsymbol{d}}_i^* \tilde{\boldsymbol{d}}_i^T - \tilde{\boldsymbol{d}}_j^* \tilde{\boldsymbol{d}}_j^T = \begin{pmatrix} 0 & -\left(\boldsymbol{d}_i^T - \boldsymbol{d}_j^T\right) \\ -(\boldsymbol{d}_i^* - \boldsymbol{d}_j^*) & \left(\boldsymbol{d}_i^* \boldsymbol{d}_i^T - \boldsymbol{d}_j^* \boldsymbol{d}_j^T\right) \end{pmatrix}. \tag{A3}$$

Let $\boldsymbol{R}$ denote the covariance matrix of $\boldsymbol{z}_l$, and it is given by

$$\boldsymbol{R} = E\left[\boldsymbol{z}_l \boldsymbol{z}_l^H \mid \boldsymbol{d}_j\right] = \begin{pmatrix} \|\boldsymbol{d}_j\|^2 + N_0 & |\rho_{hv}|^2 \boldsymbol{d}_j^T \\ |\rho_{hv}|^2 \boldsymbol{d}_j^* & |\rho_{hv}|^2 \boldsymbol{I} \end{pmatrix} \tag{A4}$$

where $\boldsymbol{I}$ is a $K \times K$ identity matrix. Letting $E_{ii} = \|\boldsymbol{d}_i\|^2, E_{jj} = \|\boldsymbol{d}_j\|^2$ and $E_{ij} = E_{ji}^* = \boldsymbol{d}_i^T \boldsymbol{d}_i^*$, we define (A5) as shown at the top of the page. It has been proven in [5] that the rank of $\boldsymbol{A}$ is only two. Letting λ_{ij1} (positive) and λ_{ij2} (negative) denote the two nonzero eigenvalues of $\boldsymbol{A}$, it can be shown that

$$\Phi_{D_{ij}}(s) = \left[\frac{1}{\det(\boldsymbol{I} + s\boldsymbol{A})}\right]^L = \left[\frac{1}{(1+\lambda_{ij1}s)(1+\lambda_{ij2}s)}\right]^L. \tag{A6}$$

Defining $p_{ij1} = -1/\lambda_{ij1}$ and $p_{ij2} = -1/\lambda_{ij2}$ yields (7).

Following the method of [6], $r_{s_m,ij}$ is given by

$$r_{s_m,ij} = -p_{ij2}/p_{ij1} = -\lambda_{ij1}/\lambda_{ij2} = -\frac{T_1 + \sqrt{2T_2 - T_1^2}}{T_1 - \sqrt{2T_2 - T_1^2}} \tag{A7}$$

where $T_1 = \mathrm{trace}[\boldsymbol{A}]$ and $T_2 = \mathrm{trace}[\boldsymbol{A}^2]$. It can be shown that

$$T_1 = |\rho_{hv}|^2(E_{ii} - 2\mathrm{Re}[E_{ij}] + E_{jj}) = |\rho_{hv}|^2 \|\boldsymbol{d}_i - \boldsymbol{d}_j\|^2 \tag{A8}$$

and $T_2 = T_1^2 + 2\eta T_1$ where

$$\eta = (1 - |\rho_{hv}|^2)E_{jj} + N_0 = (1 - |\rho_{hv}|^2)\|\boldsymbol{d}_j\|^2 + \frac{E_s}{\gamma_c}. \tag{A9}$$

Further derivation of (A7) yields

$$r_{s_m,ij} = \frac{T_1}{2\eta} + \sqrt{\left(\frac{T_1}{2\eta}\right)^2 + \frac{T_1}{\eta}} + 1. \tag{A10}$$

It is easy to show that (A10) is equivalent to (9) with $T_1/2\eta = a_{s_m,ij}\Gamma_{s_m,j}$.

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