African Journal of Mathematics and Computer Science Research Vol. 2(3), pp. 030-037, April, 2009
Available online at http://www.academicjournals.org/AJMCSR

Full Length Research Paper

Profile of conjugate gradient method algorithm on the performance appraisal for a fuzzy system

J.O. Omolehin[1], A.O. Enikuomehin[1], R.G. Jimoh[2] and K. Rauf[1]*

[1]Mathematics Department, University of Ilorin, P. M. B.1515, Ilorin, Kwara State, Nigeria.
[2]Computer Science Department, University of Ilorin, P.M.B.1515, Ilorin, Kwara State, Nigeria.

Accepted 24 February, 2009

The formalism of Minkwosky's inequality in Omolehin (2007a) is used in this work fundamentally to generate the fuzzy model for system performance appraisal in two public examinations in Nigeria. The Conjugate Gradient Method Algorithm (CGM) is employed to evaluate the validity of our results from the large system fuzzy performance. Results showed that our methods are credible and reliable.

Key words: Minkowski's inequality, conjugate, gradient method, fuzzy system, operator, quadratic cost functional.

INTRODUCTION

Most of the real-world problems are characterized by varied degrees of intricacy and the conventional procedures are not capable of dealing with these intricacies very efficiently. Soft computing is a computational method that is tolerant to sub-optimality, impreciseness, vagueness etc giving quick, simple and sufficient good solutions (Chen and Chen, 1994). Introduction of Soft Computing techniques (Zadeh, 1997), which include artificial neural network; fuzzy logic, genetic algorithm, and rough set theory have opened new avenues to the complex system research. Potential of such method in dealing with various real world problems is well documented in literature (Chattopadhyay and Chattopadhyay, 2008a, b; Chattopadhyay, 2006; Zadeh, 1994; Berenji, 1991 and many others). Fuzzy logic is an area of research, which provides solutions to the problems of vagueness which departs from the all or nothing logic. It logically redefines yes or no ideas in proper form (Berenji and Khedkar, 1992). Fuzzy sets were proposed to deal with vagueness related to the way people sense things (e.g. tall versus short, big versus small). A set is defined by its elements and the membership of each element in the set (Sugeno, 1985). Fuzzy logic constitutes the basis for linguistic approach. Under this approach, variables can assume linguistic values. Each linguistic value is characterized by a label and a meaning. This label is a sentence of a language. The meaning is a fuzzy subset of a universe of discourse. Models, based on this approach, can be constructed to stimulate approximate reasoning. The implementation of these presents two major problems namely: how to associate a label with an unlabelled fuzzy set on the basis of semantic similarity (linguistic approximation) and how to perform arithmetic operation with fuzzy numbers.

Two main directions in fuzzy logic have to be distinguished; one is older, better known, heavily applied but does not ask deep logical questions and serves mainly as apparatus for fuzzy control, analysis of vagueness in natural language, control machine, fuzzy traffic controller, fuzzy aggregator etc. It is one of the techniques of computing.

Fuzzy logic in the narrow sense is symbolic with comparative notion of truth developed fully in spirit of classical logic (syntax, semantics, truth preserving deduction, completeness etc) both prepositional and predicate logic. It is a branch of many- valued logic based on the paradigm of inference under vagueness. This fuzzy logic is a relatively young discipline both serving as a foundation for fuzzy logic in broad sense and of independent logical interest since it turns out that strict logical calculations can go with any fuzzy operation using fuzzy system in the prepositional and predicate forms which are both aspects of fuzzy logic in the narrow sense (Zadeh, 1965). For further applications, see Zimmermann (1987b), Altrock (1995), Omolehin et al. (2007b), Zadeh (2006) and Satish et al. (2002).

Fuzzy set operation: The following rules which are common in classical set theory are also applicable in fuzzy set theory: Sum of two sets, product of two sets, intersection, complement, containment, equality, associativity, commutativity and distributivity an De Morgan's law

*Corresponding author. E-mail: balk_r@yahoo.com.

(Zadeh, 1965).

A more detailed discussion of these and other notions may be found in Zimmermann (1987a) and Zimmermann et al. (1993).

Fuzzy systems input undergo three transformations viz: Fuzzification, Rulebase and Defuzzification process.

Fuzzification

This is a process that uses predefined membership functions that map each system input into one or more degree of membership(s).

Rulebase

Rule (Predefined) is evaluated by combining degrees of membership to form output strengths.

Defuzzification

This is a process that computes system outputs based on strengths and membership functions. The two most popular Defuzzification methods are the Mean-Of-Maximum (MOM) and the Centre of Area (COA) methods. For MOM, the crisp output Δq is the mean value of all points ωi which membership values $\mu c(\omega i)$ are maximum. In the case of discrete universal set W, MOM is defined by,

$$X = \sum_{i=1}^{n} \frac{\omega_i}{n}$$

Where

$$\{\omega_i \mid \mu_c(\omega_i) \leq \mu_c(\omega_j), \omega_i, \omega_j \in W, \omega_i \neq \omega_j \}$$

and n is the number of such support values. As for COA, the crisp output Δq is the centre of gravity of distribution of membership function μc. In the case of the discrete universal set W, COA is defined as in Zimmermann (1987b):

$$X = \frac{\sum_{i=1}^{n} (\mu_c(\omega_i) x \omega_i)}{\sum_{i=1}^{n} \mu_c(\omega_i)}$$

Where n is the number of elements of the fuzzy set C, and $\omega \; \varepsilon \; W$. In this model, the COA method is used for Defuzzification.

Fuzzy logic is not just restricted to just two categories as illustrated above; it can be applied to any number of the categories. For example, an element x can belong to set A with membership function a, to set B with membership function b, to set C and membership function c and so on. However, it is important to keep it in mind that the sums a, b, c etc should equal unity.

MAIN RESULTS

The main objective of this paper is to use the theory of fuzzy logic to evaluate students' performance in more than the usual one rule-based system, which is frequently used in fuzzy logic for performance comparison. This work is aimed at converting series of linguistic rules such as performance being considered as low, poor, high, excellent etc. to fuzzy value in analyzing student performance. The whole concept is aimed at comparing students' performance in some public examinations, particularly WAEC and NECO (West Africa Examination Council and National Examination Council). The case study of this work involved two states namely: Kwara and Yobe. The two principal subjects considered are Mathematics and English Language for the entire period, 2002-2004.

We know that decision situation and its subsequent making is a knowledge based on the discovery of fuzzy logic in application. Therefore, this research work performs series of operations using the knowledge of fuzzy logic to compare the known performance of students in two states for the past three years and using the result to predict the outcome of subsequent examinations.

Such comparisons are necessary in improving the standard of learning. In this case, English and Mathematics were singled out as testers in the two examinations and with fuzzy logic operation we were able to know the states that lack good performance in each subject. Such result if presented to the ruling administration could provide useful information in updating the knowledge of the teachers, in which case the teaching skill of the teachers could be enhanced.

Fuzzy logic is used in this sense to analyze all the individual result obtained in the two states. The application was done as written below.

We choose the following years- 2002, 2003 and 2004 for our case study. We further obtained the result (the harmonized result for each state) for only English Language and Mathematics. Though, this research work can be extended to include some other subjects but for the fact that we want a broad situation, not considering science students or Art students. It is good to limit it to these two subjects; the result of this work could then further be used in future research work. Tools used in obtaining the required information are number of students that have at least a credit in English Language (credit and above), number of students that have at least credit in Mathematics (credit and above), a decision table is developed and fuzzy logic tools were also used such as fuzzification and defuzification techniques. The result obtained for the period for each year is then converted into percentage. The percentage table for each state in each examination is given. Let us define the variables used for our work in Table 1.

The above representation is for each of the states being considered. The results were transformed into fuzzy systems. The Yobe and Kwara States analyses for English and Mathematics for a period of three years

Table 1. Variables analysis.

Variable	Analysis
WE2	WAEC ENGLISH FOR 2002
WE3	WAEC ENGLISH FOR 2003
WE4	WAEC ENGLISH FOR 2004
WM2	WAEC MATHEMATICS FOR 2002
WM3	WAEC MATHEMATICS FOR 2003
WM4	WAEC MATHEMATICS FOR 2004
NE2	NECO ENGLISH FOR 2002
NE3	NECO ENGLISH FOR 2003
NE4	NECO ENGLISH FOR 2004
NM2	NECO MATHEMATICS FOR 2002
NM3	NECO MATHEMATICS FOR 2003
NM4	NECO MATHEMATICS FOR 2004

Table 2. WAEC analysis for Yobe.

WE2	26%	WM2	30%
WE3	45%	WM3	28%
WE4	51%	WM4	48%

Table 3. NECO analysis for Yobe State.

NE2	68%	NM2	74%
NE3	81%	NM3	90%
NE4	94%	NM4	92%

Table 4. WAEC analysis for Kwara State.

WE2	46%	WM2	63%
WE3	49%	WM3	43%
WE4	48%	WM4	51%

Table 5. NECO analysis for Kwara State.

NE2	63%	NM2	60%
NE3	87%	NM3	80%
NE4	87%	NM4	48%

(2002 – 2004) represented in percentage in Tables 2 - 5.

After converting the crisp into percentages and apply the fuzzy logic for the analysis of the problem then, the model developed serves the purpose of wanting to fuzzify the result of students with credit pass and above for the three consecutive years for the two subjects in the two states.

Model formulation

A model was developed as a result of a combination of series of linguistic rules. These rules include the following:

- If a student from Kwara state scores 25% or less in Mathematics in 2003, then assign very poor.
- if a student from Yobe state scores 75% or less in English in 2004 then assign very good, to mention but a few. For similar rules, in tabular form, see Omolehin et al. (2005).

More than 50 rules were generated for the purpose of this work. They were converted from the linguistic form to variable form by processing the values from our data table in accordance with the performance level. Model is developed, using the formalism of Minkowski's inequality generated in Omolehin (2007a). Having this inequality in mind, we develop a model with an upper bound for each year under consideration. The upper bound allows for individual examination analysis. That is, in performing fuzzy operation for each year and each examination body, we associate an upper bound. Factors used in developing our model include the following:

a. The highest possible score is 100. Thus, we represent our data in percentage.
b. The range [0, 1] is the membership result of fuzzy consideration of the vague nature of linguistic term. For example, in very poor and poor, we establish the difference by their respective membership function. 20% may be assigned poor in English (2002, NECO) and the same 20% may be said to be very poor in Mathematics (2002, WAEC). In fuzzy arithmetic, all these linguistic representtations have their own mathematical representation.
c. The ranges [0, 1] also exists as a definition that shows that all expected values for experimentation should lie between [0.0, 0.1].
d. The body of the model is as a result of translating the linguistic IF-THEN rules having in mind the general Minkowsky's inequality concept for our model development.
e. The decision table considered for the model development was based on fuzzy max

$$[s_{(x)}].$$

We involved a parametric method in developing a model that assigned a credit and above for pass marks in English and Mathematics and below credit for failure.

We choose the following representations: lower boundary as L, x as any member of the data and p is defined as $p = 100 - x$ where R is the nearest upper integer of the data.

We thus develop the model f(x) as:

Table 6. WAEC analysis for Kwara State.

No of student registered	Credit and above		
Year	**Number**	**WAEC English (WEn)**	**WAEC Mathematics (WMn)**
2002	51,966	26,503	24,125
2003	46,590	20,965	13,045
2004	53,753	13,976	16,125

Table 7. NECO analysis for Kwara State.

No of student registered	Credit and above		
Year	**Number**	**NECO English (NEn)**	**NECO Mathematics (NMn)**
2002	53,966	51,118	49,780
2003	48,100	39,200	43,200
2004	53,758	36,575	39,805

Table 8. WAEC analysis for Yobe State.

No of student registered	Credit and above		
Year	**Number**	**WAEC English (WEn)**	**WAEC Mathematics (WMn)**
2002	41,192	21,011	15,312
2003	35,615	17,499	15,312
2004	45,920	21,011	28,862

Table 9. NECO analysis for Yobe State.

No of student registered	Credit and above		
Year	**Number**	**NECO English (NEn)**	**NECO Mathematics (NMn)**
2002	38,150	33,272	31,364
2003	41,210	36,210	33,138
2004	49,320	31,002	29,647

$$\begin{cases} 0, & x < L \\ \frac{1}{2}\left[\frac{1}{100-x} + \frac{1}{x} + \frac{x}{R}\right], & L \le x \le R \\ 1, & x > R \end{cases}$$

Table 10. Joint table for Kwara State WAEC AND NECO.

WE1	26%	WM1	30%
WE2	45%	WM2	28%
WE3	51%	WM3	48%
NE1	68%	NM1	74%
NE2	81%	NM2	90%
NE3	94%	NM3	92%

Table 11. Joint table for Yobe State WAEC AND NECO.

WE1	46%	WM1	63%
WE2	49%	WM2	43%
WE3	48%	WM3	51%
NE1	63%	NM1	60%
NE2	87%	NM2	80%
NE3	87%	NM3	82%

The methodology of this is that the percentages were individually, for each value of x, put into the model and the results were used in computing a fuzzified (F) table for both examination councils i.e. F(WM2), F(WM3), F(NM2), F(NM3) F(NM4), F(WE2), F(WE3), F(WN2), F(WN3) and F(WN4) for the two states.

We introduced a comparison of NECO and WAEC Mathematics for each state and NECO and WAEC English for the same state as the first stage of our operation and after getting the table, we applied the model as a given factor again to know the nature of the result.

This was further put into the matrix form in order to analyze the general performance in Mathematics in both examinations for each state as compared to that of English. Then in conclusion for the model operation of this work, the results were again modeled and the resulting data were used as a tool for complete fuzzification. With our new data which have 144 entries in a 12×12 matrix, the fuzzified value that we once talked about in the literature review has now been achieved. It was got as a result of using our model to convert the “crisp” values into fuzzy numbers. The model developed was used to transfer some crisp model into fuzzy numbers, a process called fuzzification and we say the data have been fuzzified.

Corresponding tables and their equivalent in percentage

Tables 6-9 show the result of students in the mentioned states in English and Mathematics in the year under review for WAEC and NECO.

The percentages for each state for both subjects are combined (Tables 10 and 11) to form a joint table for the purpose of this work.

Table 12. Kwara State: F(WE).

0.2759	0.4529	0.5105
0.4529	0.5104	0.2759
0.5104	0.2759	0.4529

Table 13. Yobe State: F(WE).

0.4801	0.5100	0.5000
0.5100	0.5000	0.4801
0.5000	0.4801	0.5100

Table 14. Kwara State joint WAEC-NECO English fuzzified result F(WNE).

0.1628	0.2571	0.2868	0.3809	0.4588	0.5834
0.2571	0.2868	0.3809	0.4588	0.5834	0.1028
0.2868	0.3809	0.4588	0.5834	0.1028	0.2571
0.3809	0.4588	0.5834	0.1028	0.2571	0.2868
0.4588	0.5834	0.1028	0.2571	0.2868	0.3809
0.5834	0.1028	0.2571	0.2868	0.3809	0.4588

Table 15. Kwara State joint WAEC-NECO Maths fuzzified result F(WNM).

0.1850	0.1753	0.2781	0.1658	0.5394	0.7778
0.1753	0.2781	0.2781	0.5394	0.7778	0.1850
0.2781	0.1658	0.5394	0.7778	0.1850	0.1753
0.1658	0.5394	0.7778	0.1850	0.1753	0.2781
0.5394	0.7778	0.1850	0.1753	0.2781	0.1658
0.7778	0.1850	0.1753	0.1658	0.1658	0.5394

The fuzzified matrices from the above tables are given in Tables 12 and 13 below, ranging from their single analyzed form to the generalized joint form. The Decision Tables that is, the fuzzified values which were derived from the application of the model are shown below by matrices (Tables 14 -17).

The generalized matrices

The generalized matrices are the matrices that combine the performance of students in both states for the two courses for all the years under review. They represent Decision tables in fuzzy system and they are given by the matrices below. These results form 12 x 12 matrices which can be solved by the CGM algorithm.

The generalized matrices, which serve as the total joint results for both NECO and WAEC in both subjects are as shown in Tables 18 and 19, respectively and are used as the control operator in our algorithm.

CGM algorithm

At this stage, we are going to employ our CGM algorithm to test whether our system is working or not. The fuzzified values of our results are transformed into matrices and these matrices will be used as the entries for our control operator A. Note that this matrix operator is associated with the CGM algorithm. The CGM has a well worked out theory with an elegant convergence profile. It has been proved that the algorithm converges, at most, in n iterations in a well posed problem and the convergence rate is given as:

$$E(x_n) = \left\{ \frac{1 - \frac{m}{M}}{1 + \frac{m}{M}} \right\}^{2n} E(x_0)$$

Where m and M are smallest and largest spectrums of matrix A in the CGM algorithm, respectively. See, Ibiejugba (1980), Ibiejugba (1985) and Omolehin et al. (2006). That is, for an n dimensional problem, the algorithm will converge in at most n iterations. The Conventional CGM algorithm is due to Hestene and Stiefel (1952).

In its original form, CGM was designed to handle quadratic functional of the form,

$$f(x) = f_0 + < a, x >_H + \frac{1}{2} < x, Ax >_H$$

Where f_0, a constant in H, x is a vector in H. A is a positive definite, symmetric and constant matrix operator. When A is no longer a constant matrix operator, the situation becomes difficult and the CGM algorithm can no longer hold. This is what motivated Ibiejugba (1980) to construct a control operator to handle quadratic cost functional of the form,

$$\int_0^T \{ av^2(t) + bu^2(t) \} dt$$

Subject to,

$$\dot{x}(t) = cx(t) + du(t)$$

Steps involved in conjugate gradient method

Consider descent with a functional F on a Hilbert space H in which F has a Taylor series expansion truncated after the second order terms namely:

$$F(x) = F_o + < a, x >_H + \frac{1}{2} < x, Ax, >_H$$

Where $< a, b >= ab^T$.

Table 16. Yobe State joint WAEC- NECO English fuzzified result F(WNE).

0.2815	0.2984	0.3096	0.3794	0.5385	0.5385
0.2984	0.3096	0.3794	0.5385	0.5385	0.2815
0.3096	0.3794	0.5385	0.5385	0.2815	0.2984
0.3794	0.5385	0.5385	0.2815	0.2984	0.3096
0.5385	0.5385	0.2815	0.2984	0.3096	0.3794
0.5385	0.2815	0.2984	0.3096	0.3794	0.5385

Table 17. Yobe State joint WAEC-NECO English fuzzified result F(WNE).

0.2815	0.2984	0.3096	0.3794	0.5385	0.5385
0.2984	0.3096	0.3794	0.5385	0.5385	0.2815
0.3096	0.3794	0.5385	0.5385	0.2815	0.2984
0.3794	0.5385	0.5385	0.2815	0.2984	0.3096
0.5385	0.5385	0.2815	0.2984	0.3096	0.3794
0.5385	0.2815	0.2984	0.3096	0.3794	0.5385

Table 18. Fuzzified table for English for both states.

0.2571	0.2868	0.1628	0.3809	0.4588	0.5834	0.2815	0.2984	0.3096	0.3794	0.5385	0.5385
0.2868	0.1628	0.3809	0.4588	0.5834	0.2815	0.2984	0.3096	0.3794	0.5385	0.5385	0.2571
0.1628	0.3809	0.4588	0.5834	0.2815	0.2984	0.3096	0.3794	0.5385	0.5385	0.2571	0.2868
0.3809	0.4588	0.5834	0.2815	0.2984	0.3096	0.3794	0.5385	0.5385	0.2571	0.2868	0.1628
0.4588	0.5834	0.2815	0.2984	0.3096	0.3794	0.5385	0.5385	0.2571	0.2868	0.1628	0.3809
0.5834	0.2815	0.2984	0.3096	0.3794	0.5385	0.5385	0.2571	0.2868	0.1628	0.3809	0.4588
0.2815	0.2984	0.3096	0.3794	0.5385	0.5385	0.2571	0.2868	0.1628	0.3809	0.4588	0.5834
0.2984	0.3096	0.3794	0.5385	0.5385	0.2571	0.2868	0.1628	0.3809	0.4588	0.5834	0.2815
0.3096	0.3794	0.5385	0.5385	0.2571	0.2868	0.1628	0.3809	0.4588	0.5834	0.2815	0.2984
0.3794	0.5385	0.5385	0.2571	0.2868	0.1628	0.3809	0.4588	0.5834	0.2815	0.2984	0.3096
0.5385	0.5385	0.2571	0.2868	0.1628	0.3809	0.4588	0.5834	0.2815	0.2984	0.3096	0.3794
0.5385	0.2571	0.2868	0.1628	0.3809	0.4588	0.5834	0.2815	0.2984	0.3096	0.3794	0.5385

Let us first consider what is termed conjugate descent with F. With conjugate descent, it is assumed that a sequence,

$$\{p_i\} = p_0, p_1, ..., p_k, ...$$

is available with the members of the sequence conjugate with respect to the positive definite linear operator A.

By conjugate with respect to A we mean that,

$$< p_i, Ap_j >_H \begin{cases} \neq 0, & if \quad i \neq j \\ = 0, & if \quad i = j \end{cases}$$

In the case here, A is assumed positive definite so $< p_i, Ap_j >_H > 0.$

The conventional Conjugate Gradient Method (CGM) is used for the minimization of a quadratic objective functional of the form:

$$F(x) = F_o + < a, x >_H + \frac{1}{2} < x, Ax >_H,$$

Where A is an nxn symmetric positive definite matrix operator on the Hilbert space $H. a,$ is a vector in H and F_o is a constant term.

These are the steps involved in CGM algorithm (if $H \equiv R^n$):

Step 1: The first element $x_o \in H$ of the descent sequence is guessed while the remaining members of the sequence are computed with the aid of the following formulae:

Step 2: $p_o = -g_o = -(a + Ax_o)$

(p_o is the descent direction and g_o is the gradient of

Table 19. Fuzzified table for Mathematics for both states.

0.1850	0.1753	0.2781	0.1658	0.5394	0.7778	0.4010	0.2795	0.3273	0.3827	0.5132	0.5279
0.1753	0.2781	0.1658	0.5394	0.7778	0.4010	0.2795	0.3273	0.3823	0.5132	0.5279	0.1850
0.2781	0.1658	0.5394	0.7778	0.4010	0.2795	0.3273	0.3823	0.5132	0.5279	0.1850	0.1753
0.1658	0.5394	0.7778	0.4010	0.2795	0.3273	0.3823	0.5132	0.5279	0.1850	0.1753	0.2781
0.5394	0.7778	0.4010	0.2795	0.3273	0.3823	0.5132	0.5279	0.1850	0.1753	0.2781	0.1658
0.7778	0.4010	0.2795	0.3273	0.3823	0.5132	0.5279	0.1850	0.1753	0.2781	0.1658	0.5394
0.4010	0.2795	0.3273	0.3823	0.5132	0.5279	0.1850	0.1753	0.2781	0.1658	0.5394	0.7778
0.2795	0.3273	0.3823	0.5132	0.5279	0.1850	0.1753	0.2781	0.1658	0.5394	0.7778	0.4010
0.3273	0.3823	0.5132	0.5279	0.1850	0.1753	0.2781	0.1658	0.5394	0.7778	0.4010	0.2795
0.3823	0.5132	0.5279	0.1850	0.1753	0.2781	0.1658	0.5394	0.7778	0.4010	0.2795	0.3273
0.5132	0.5279	0.1850	0.1753	0.2781	0.1658	0.5394	0.7778	0.4010	0.2795	0.3273	0.3823
0.5279	0.1850	0.1753	0.2781	0.1658	0.5394	0.7778	0.4010	0.2795	0.3273	0.3823	0.5123

Table 20. Convergence rate of CGM for Table 19.

Iteration	Gradient	Minimizing vector
11	0.649E-07	X1=-0.2234 X2=-0.2234 X3=-0.2234 X4=-0.2234 X5=-0.2234 X6=-0.2234 X7=-0.2234 X8=-0.2234 X9=-0.2234 X10=-0.2234

Table 21. Convergence rate of CGM for Table 20.

Iteration	Gradient	Minimizing vector
11	0.583E-03	X1=-0.220 X2=-0.2194 X3=-0.2197 X4=-0.2200 X5=-0.2192 X6=-0.2197 X7=-0.2199 X8=-0.2194 X9=-0.2203 X10=-0.2192

$F(x)$ when $x = x_o$)

Step3:

$$X_{i+1} = x_i + a_i p_i; \quad a_i =< g_i, g_i >_H / < p_i; Ap_i >_H$$

$g_{i+1} = g_i + a_i Ap_i$; a is the step length

$$p_{i+1} = -g_{i+1} + \beta_i p_i; \quad \beta_i =< g_{i+1}, g_{i+1} >_H / < g_i, g_i >_H$$

Step 4: if $g_i = 0,$ for some $i,$ terminate the sequence else, set $i = i + 1$ and go to step 3.

Numerical results

The summary of the results obtained from CGM algorithm are given in Tables 18 and 19.

Conclusion

In Tables 20 and 21, the minimizing vector, MV, is at the minimum and it can be seen from the two tables that the CGM algorithm converges at the eleventh iteration. This shows that our method is admissible for a better performance appraisal system. If not, the result will not converge, based on CGM convergence profile. We noted that the results perform better and more reliable as N approaches infinity.

REFERENCES

Altrock CV (1995). Fuzzy logic and Neuro fuzzy applications explained. Prentice-Hill, Inc. Upper Saddle River, NJ, US.

Berenji HR, Khedkar P (1992). ‘Learning and Tuning Fuzzy logic controllers through reinforcement’, IEEE Trans. Neural Networks, 3(5): 724-740.

Berenji HR (1991). Fuzzy logic controllers. In An Introduction to Fuzzy Logic Applications in intelligent System.. Kluwer Academic Publishers, Boston; 69-96.

Chen CL, Chen WC (1994). ‘Fuzzy Controller design by using Neural Network Techniques’, IEEE Transactions on fuzzy systems, 2(3): 235-244.

Chattopadhyay G, Chattopadhyay S (2008a). A probe into the chaotic nature of total ozone time series by correlation dimension method, Soft Computing, 2008. 12(10): 1007-1012.

Chattopadhyay S, Chattopadhyay G (2008b). Comparative study among different neural net learning algorithms applied to rainfall time series, Meteorological applications. 15(2): 273-280.

Chattopadhyay S (2006). Soft Computing Techniques in combating the complexity of the atmosphere-a review, Arxiv preprint nlin/0608052.

Hestenes MA, Stiefel E (1952). Method of Conjugate Gradient for solving linear systems, J. Res. Nat. Bur. Standards, 49: 409-445.

Ibiejugba MA (1980). Computing methods in optimal control. PhD thesis, University of Leeds, England.

Ibiejugba MA (1985). Conjugate Gradient Descent of a Smooth Func-

Functional, Advances in Modeling and Simulation.(4) 3:37-43.
Omolehin JO, Oyelade JO, Ojeniyi OO, Rauf K (2005). Application of Fuzzy Logic in decision making on Students' Academic Performance, Bull. of Pure and Appl. Sci. (24)2: 381-387.
Omolehin JO, Rauf K (2006). Computational capability of CGM algorithm, CREATIVE MATH. & INF. 5:93 – 106.
Omolehin JO, Enikuomehin AO, Rauf K, Jimoh RG (2007a). Minkwosky inequality to fuzzy model in performance appraisal, Pan-African Journal Series.(In Press).
Omolehin JO, Oyelade JO, Ojeniyi OO, Rauf K (2007b). A Comparison of Fuzzy and Deterministic Models for Assessing Students' Performance, Nig. J. Pure and Appl. Sci. 22:1-8.
Satish B, Srinivas S, Sectha H (2002). A study on Neuro-Fuzzy Systems and applications: Bulletin of Pure and Applied Sciences. 21(1): 227-239.
Sugeno M (1985). Industrial Applications of Fuzzy Control, North-Holland, Amsterdam, London, New York 1985.
Zadeh LA (1965). Fuzzy Sets, Information and Control. 8: 338-353.
Zadeh LA (1994). Fuzzy logic, neural networks, and soft computing, Communications of the ACM. (37) 3: 77 - 84
Zadeh LA (2006). Generalized theory of uncertainty (GTU – principal concepts and ideas. Computational Statistics & Data Analysis 51: 15 – 46.
Zadeh LA (1997). What is soft computing? SOFT COMPUTING. (1)1: 1.
Zimmermann HJ (1987a). Fuzzy Sets in Pattern Recognition, Pattern Recognition Theory and Applications. Springer Verlag, Berlin, Heidelberg, New York. 383-391.
Zimmermann HJ (1987b). Fuzzy Sets, Decision Making, and Expert Systems, Kluwer Academic Publishers, Boston, Dordrecht, Lancaster.
Zimmermann HJ, Becker K, Kamahi H, Juffernbruch K, Rau G, Kalff G (1993). An Intelligent Alarm System for Decision-Support in Cardioanaesthesia, Knowledge Base and User Interface. First European Congress on Fuzzy and Intelligent Technologies, Aachen, September 7-10: 1023-1026.