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On the Degrees of Freedom of the Three-User MIMO Interference Channel with Delayed CSIT

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***Abstract*—The three-user multiple-input multiple-output interference channel under i.i.d. fading is studied, where the transmitters have the delayed channel state information. The case where all transmitters and all receivers are equipped with M and N antennas, respectively, is considered. For this case, a new transmission scheme is proposed that achieves a number of degrees of freedom higher than previously reported for the range of $3/4 < M/N < 1$, where the parameters of the scheme are determined as functions of the ratio M/N. The degrees of freedom gains compared to the previous approaches are due to the more effective use of transmit and receive antennas.**

I. Introduction

The number of degrees of freedom (DoF) is a performance measure which characterizes the capacity behaviour of a communication system in high signal-to-noise ratio (SNR) regime. For the single-input single-output (SISO) interference channel (IC), the number of DoF has been achieved using a technique named interference alignment (IA) [1]. However, IA requires perfect channel state information at transmitters (CSIT) for current and future time slots, which is an unrealistic assumption.

In absence of instantaneous CSIT and under i.i.d. fading, the number of DoF of the SISO IC as well as of the multiple-input single-output (MISO) broadcast channel (BC) is one for any network size [2]. However, [3] has shown that for the MISO BC the number of DoF is greater than one if the transmitters obtain the delayed CSIT through feedback from the receivers. The DoF gains are achieved by splitting the transmission into multiple phases, where in each phase the delayed CSIT of the previous phase is employed for transmission. The approach proposed in [3] has been shown to be also applicable to SISO IC in [4] and [5], where achievable numbers of DoF greater than one have been reported. For the three-user SISO IC, [4] has shown that the number of DoF of 9/8 is achievable. This result has been later improved to 36/31 DoF in [5].

Employing multiple antennas at transmitters and receivers is known to increase the number of DoF of the IC as compared to the SISO case when the instantaneous CSIT is available [6]. A similar DoF analysis has been performed for the case when the CSIT is delayed in [7]. In [7], the three-user multiple-input multiple-output (MIMO) IC scenario is considered, where the transmitters and receivers are equipped with M and N antennas, respectively. [7] shows that in such a network, different transmission schemes provide higher number of DoF for different antenna setups characterized by the ratio M/N. For the regions of $1/2 < M/N \leq 31/32$ and $31/32 < M/N \leq 18/13$, Torrellas et al. [7] propose two MIMO transmission schemes, which are based on the transmission schemes for the three-user SISO IC described in [4] and [5], respectively. For the scheme based on the scheme described in [5], a limitation for the transmitters and receivers to use only $\min\{M, N\}$ antennas is used.

In this paper, we consider the three-user MIMO IC scenario, which is identical to the one considered in [7]. We propose a new transmission scheme which achieves the number of DoF greater than reported in [7] in the region of $3/4 < M/N < 1$. The proposed transmission scheme is based on the SISO transmission scheme described in [5]. However, in contrast to [7], we omit the assumption of using only $\min\{M, N\}$ antennas at the transmitters and receivers and derive the parameters of the transmission scheme as functions of the ratio M/N, which allows to achieve higher number of DoF.

The the rest of the paper is organized as follows. Section II describes the system model. Section III describes the proposed transmission scheme and gives performance results.

II. System Model

We consider a three-user MIMO IC scenario as depicted in Fig. 1. Each transmitter Tx_i has M antennas and each receiver Rx_i has N antennas, $i \in \{1, 2, 3\}$. The communication period spans T time slots, during which each transmitter Tx_i intends to communicate a data vector comprised of b_1 symbols $\mathbf{u}_i \in \mathbb{C}^{b_1 \times 1}$ to its corresponding receiver Rx_i.

Let $\mathbf{H}_{ji}(t) \in \mathbb{C}^{N \times M}$ be the channel matrix between Tx_i and Rx_j, in time slot t, $1 \leq t \leq T$, $\forall i, j \in \{1, 2, 3\}$. All channel entries are randomly drawn from a continuous complex distribution and are identically and independent distributed (i.i.d.) across antennas and time, as well as across different transmitter and receiver pairs. It is supposed that each receiver has the instantaneous global channel knowledge, i.e. in time slot t, $1 \leq t \leq T$, each receiver has access to the set of channel matrices $\{\mathbf{H}_{ji}(\tau)\}_{\tau=1}^{t}$, $\forall i, j \in \{1, 2, 3\}$. Each transmitter obtains the global channel knowledge with a single time slot delay, i.e. in time slot t, $2 \leq t \leq T$ it has access to the set of channel matrices $\{\mathbf{H}_{ji}(\tau)\}_{\tau=1}^{t-1}$, $\forall i, j \in \{1, 2, 3\}$. We further refer to these assumptions as delayed CSIT.

The transmission is split into three phases, where two types of processing are applied for the transmission. Firsly, the

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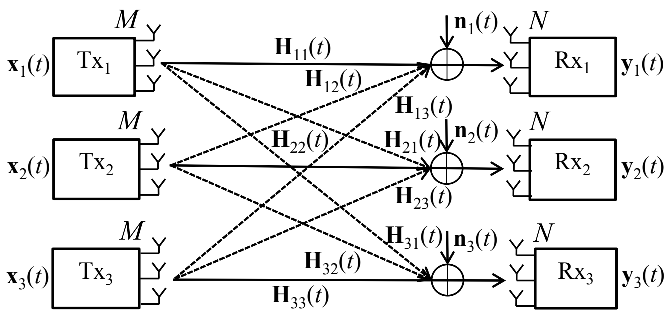

Fig. 1. The three-user MIMO IC

signals to be transmitted are generated recursively using the delayed CSIT of the previous phases. Secondly, additional linear precoding is applied, which does not depend on CSIT and is done randomly. The random precoding is performed jointly over multiple time slots, where a jointly encoded transmission is called a transmission block. Each phase is comprised of multiple transmission blocks, which have identical transmission parameters, but correspond to different transmitted data.

In each transmission block of phase i, $i \in \{1,2,3\}$, a subset of the transmitters is scheduled for transmission, where according to the methodology of [5], in phases 1 and 3 all transmitters transmit simultaneously and in phase 2 only two transmitters are scheduled for transmission. Per transmission block, each of the scheduled transmitters transmits b'_i symbols in T'_i time slots, where in total $b'_{i\Sigma}$ symbols are transmitted by all transmitters. After transmission in phase i, $i \in \{1,2\}$, terms to be transmitted in phase $i+1$ are generated using the delayed CSIT of phase i, where in total $q'_{i\Sigma}$ terms are generated per transmission block at all transmitters.

The number k_j of the transmission blocks of phase j, $j \in \{1,2,3\}$, is to be chosen such that the total number $k_i q'_{i\Sigma}$ of the terms generated after phase i is equal to the total number $k_{i+1} b'_{i+1\Sigma}$ of the terms transmitted phase $(i+1)$, $i \in \{1,2\}$, i.e. the values k_i are to be chosen, such that the equalities

$$k_i q'_{i\Sigma} = k_{i+1} b'_{i+1\Sigma}, i \in \{1,2\} \tag{1}$$

hold. In such a case, each transmitter transmits $b_i = k_i b'_{i\Sigma}/3$ terms in the i-th phase, $i \in \{1,2,3\}$. The overall duration of the i-th phase is $T_i = k_i T'_i$ time slots, with the total duration of the transmission $T = \sum_{i=1}^{3} T_i$ time slots. We describe each phase by only specifying the structure of a single transmission block and the number of the transmission blocks.

Let us consider the k-th transmission block of phase 1, $1 \leq k \leq k_1$. The transmission of the block spans the T'_1 time slots $(k-1)T'_1 + 1 \leq t \leq kT'_1$. Let $\mathbf{u}_{[i]}^{(k)} \in \mathbb{C}^{b'_1 \times 1}$ be the data vector to be transmitted during the k-th transmission block of phase 1 by Tx_i, $i \in \{1,2,3\}$. Let $\mathbf{x}_i(t) \in \mathbb{C}^{M \times 1}$ be the signal transmitted by Tx_i from its M antennas in time slot t. The precoding of the signal $\mathbf{u}_{[i]}^{(k)}$ in time slot t is described by the matrix multiplication $\mathbf{x}_i(t) = \mathbf{C}_{[i]}(t)\,\mathbf{u}_{[i]}^{(k)}$, where $\mathbf{C}_{[i]}(t) \in \mathbb{C}^{M \times b'_1}$ is the precoding matrix in time slot t. The elements of the precoding matrix are randomly taken from a continuous distribution and are mutually independent.

Let us denote the overall precoding matrix used by Tx_i for the k-th transmission block of phase 1 as

$$\mathbf{C}_{[i]}^{(k)} = \left[\mathbf{C}_{[i]}\left((k-1)T'_1+1\right)^{\mathrm{T}}, ..., \mathbf{C}_{[i]}\left(kT'_1\right)^{\mathrm{T}}\right]^{\mathrm{T}} \in \mathbb{C}^{MT'_1 \times b'_1}. \tag{2}$$

Here, for decodability of the transmitted data the inequality

$$b'_i \leq MT'_i \tag{3}$$

is to be fulfilled. Let us denote by $\mathbf{x}_i^{(l,\kappa)}$ the concatenation of the signal vectors transmitted by Tx_i during the κ-th transmission block of phase l, $1 \leq \kappa \leq k_l$, $l \in \{1,2,3\}$. For the k-th transmission block of phase 1, the concatenation is described as

$$\mathbf{x}_i^{(1,k)} = \left[\mathbf{x}_i\left((k-1)T'_1+1\right)^{\mathrm{T}}, ..., \mathbf{x}_i\left(kT'_1\right)^{\mathrm{T}}\right]^{\mathrm{T}} \in \mathbb{C}^{MT'_1 \times 1}, \tag{4}$$

which is calculated as

$$\mathbf{x}_i^{(1,k)} = \mathbf{C}_{[i]}^{(k)} \mathbf{u}_{[i]}^{(k)}. \tag{5}$$

The transmitted vector $\mathbf{x}_i^{(1,k)}$ is subject to the average power constraint $\frac{1}{T'_1}\mathrm{E}\left[\mathbf{x}_i^{(1,k)\mathrm{H}}\mathbf{x}_i^{(1,k)}\right] \leq P$, where P is the maximum transmit power.

Let $\mathbf{y}_j^{(l,\kappa)}$ be the concatenation of the signal vectors received in the κ-th transmission block of phase l by Rx_j, $1 \leq \kappa \leq k_l$, $j, l \in \{1,2,3\}$. We define the concatenation of the signal vectors received in the k-th transmission block of phase 1 as

$$\mathbf{y}_j^{(1,k)} = \left[\mathbf{y}_j\left((k-1)T'_1+1\right)^{\mathrm{T}}, ..., \mathbf{y}_j\left(kT'_1\right)^{\mathrm{T}}\right]^{\mathrm{T}} \in \mathbb{C}^{NT'_1 \times 1}, \tag{6}$$

where $\mathbf{y}_j(t)$ corresponds to the signal received in time slot t. Let $\mathbf{H}_{ji}^{(l,\kappa)} \in \mathbb{C}^{NT'_1 \times MT'_1}$ be the channel matrix between Tx_i and Rx_j in the κ-th transmission block of phase l, $1 \leq \kappa \leq k_l$, $i, j, l \in \{1,2,3\}$. The channel matrix corresponding to the k-th transmission block of phase 1 has the following block diagonal structure:

$$\mathbf{H}_{ji}^{(1,k)} = \begin{bmatrix} \mathbf{H}_{ji}\left((k-1)T'_1+1\right) & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \mathbf{H}_{ji}\left(kT'_1\right) \end{bmatrix}. \tag{7}$$

Let $\mathbf{n}_j^{(l,\kappa)} \sim \mathcal{CN}\left(0, \mathbf{I}_{NT'_1}\right)$ be the additive white Gaussian noise vector at Rx_j in the κ-th transmission block of phase l, $1 \leq \kappa \leq k_l$, $l \in \{1,2,3\}$. The channel input-output relationship for the k-th transmission block of phase 1 is

$$\mathbf{y}_j^{(1,k)} = \sum_{i=1}^{3} \mathbf{H}_{ji}^{(1,k)} \mathbf{C}_{[i]}^{(k)} \mathbf{u}_{[i]}^{(k)} + \mathbf{n}_j^{(1,k)}. \tag{8}$$

Due to the focus of the paper on the DoF analysis, the noise term will be further omitted throughout the paper. Precoding and transmission of the transmission blocks of phases 2 and 3 are similar to the ones described by (5) and (8).

We say that the number of DoF $d = 3b_1/T$ is achievable in the interference channel if b_1 symbols transmitted by each transmitter Tx_i to its corresponding receiver Rx_i, $i \in \{1,2,3\}$, during the overall communication period of T time slots are decodable with probability one.

III. Proposed Transmission Scheme

In this section, the proposed transmission scheme is described. In the first three subsections, we describe the transmission blocks for each phase of the scheme. In the last subsection, we determine the numbers of the transmission blocks and calculate the achieved number of DoF.

A. First Phase

In phase 1, the original data symbols are transmitted. All transmitters are scheduled to transmit simultaneously, where each transmitter transmits a data vector of b'_1 symbols per single transmission block in T'_1 time slots.

Let us consider the signal received by Rx_1 during the k-th transmission block, $1 \leq k \leq k_1$, which is calculated as

$$\mathbf{y}_1^{(1,k)} = \sum_{i=1}^{3} \mathbf{H}_{1i}^{(1,k)} \mathbf{C}_{[i]}^{(k)} \mathbf{u}_{[i]}^{(k)}. \tag{9}$$

Let us consider the interference term $\mathbf{H}_{12}^{(1,k)} \mathbf{C}_{[2]}^{(k)} \mathbf{u}_{[2]}^{(k)}$ of Tx_2. The channel matrix $\mathbf{H}_{12}^{(1,k)}$ and the precoding matrix $\mathbf{C}_{[2]}^{(k)}$ are distributed independently, thereby the $T'_1 N \times b'_1$ matrix $\mathbf{H}_{12}^{(1,k)} \mathbf{C}_{[2]}^{(k)}$ is almost surely full rank with $(T'_1 N - b'_1)$-dimensional left null space. It means that almost surely, there exists a full rank matrix $\mathbf{W}_{12}^{(1,k)} \in \mathbb{C}^{T'_1 N \times T'_1 N - b'_1}$, for which

$$\mathbf{W}_{12}^{(1,k)\mathrm{H}} \mathbf{H}_{12}^{(1,k)} \mathbf{C}_{[2]}^{(k)} = \mathbf{0}_{T'_1 N - b'_1 \times b'_1} \tag{10}$$

holds. In (10), the columns of the matrix $\mathbf{W}_{12}^{(1,k)}$ are linearly independent vectors lying in the left null space of $\mathbf{H}_{12}^{(1,k)} \mathbf{C}_{[2]}^{(k)}$.

Let us denote by $\mathbf{w}_{12,\xi}^{(1,k)}$ the ξ-th column of the matrix $\mathbf{W}_{12}^{(1,k)}$, $1 \leq \xi \leq T'_1 N - b'_1$. By projecting the received vector $\mathbf{y}_1^{(1,k)}$ onto $\mathbf{w}_{12,\xi}^{(1,k)}$, Rx_1 will cancel the signal of the interferer Tx_2 and will obtain

$$\begin{aligned}\mathbf{w}_{12,\xi}^{(1,k)\mathrm{H}} \mathbf{y}_1^{(1,k)} = \mathbf{w}_{12,\xi}^{(1,k)\mathrm{H}} \mathbf{H}_{11}^{(1,k)} \mathbf{C}_{[1]}^{(k)} \mathbf{u}_{[1]}^{(k)} + \\ \mathbf{w}_{12,\xi}^{(1,k)\mathrm{H}} \mathbf{H}_{13}^{(1,k)} \mathbf{C}_{[3]}^{(k)} \mathbf{u}_{[3]}^{(k)}.\end{aligned} \tag{11}$$

The sum of (11) is comprised of a linear combination of $\mathbf{u}_{[1]}^{(k)}$, which is a signal useful for Rx_1, and a linear combination of $\mathbf{u}_{[3]}^{(k)}$, which is an interference term remaining at Rx_1. The remaining interference term $\mathbf{w}_{12,\xi}^{(1,k)\mathrm{H}} \mathbf{H}_{13}^{(1,k)} \mathbf{C}_{[3]}^{(k)} \mathbf{u}_{[3]}^{(k)}$ is useful for both Rx_1 and Rx_3 as follows:

- it can be subtracted from $\mathbf{w}_{12,\xi}^{(1,k)\mathrm{H}} \mathbf{y}_1^{(1,k)}$ to recover $\mathbf{w}_{12,\xi}^{(1,k)\mathrm{H}} \mathbf{H}_{11}^{(1,k)} \mathbf{C}_{[1]}^{(k)} \mathbf{u}_{[1]}^{(k)}$, which is a term useful for Rx_1;
- it is a term useful for Rx_3.

We further use the notation of order-2 symbols described in [3], where the order-2 symbol is a term which is useful for two receivers simultaneously. We denote by $u_{[l|i,j],\xi}^{(k)}$ the order-2 symbol, which is desired by both Rx_i and Rx_j, and is available at Tx_l, $i \neq j$, $l \in \{i, j\}$, $1 \leq \xi \leq T'_1 N - b'_1$. From (11), the following order-2 symbol is generated:

$$u_{[3|1,3],\xi}^{(k)} = \mathbf{w}_{12,\xi}^{(1,k)\mathrm{H}} \mathbf{H}_{13}^{(1,k)} \mathbf{C}_{[3]}^{(k)} \mathbf{u}_{[3]}^{(k)}. \tag{12}$$

Similarly, Rx_1 can cancel the signal of the interferer Tx_3 and obtain the order 2-symbol

$$u_{[2|1,2],\xi}^{(k)} = \mathbf{w}_{13,\xi}^{(1,k)\mathrm{H}} \mathbf{H}_{12}^{(1,k)} \mathbf{C}_{[2]}^{(k)} \mathbf{u}_{[2]}^{(k)}, \tag{13}$$

where $\mathbf{w}_{13,\xi}^{(1,k)}$ is the ξ-th column of the matrix $\mathbf{W}_{13}^{(1,k)}$, for which the equality $\mathbf{W}_{13}^{(1,k)\mathrm{H}} \mathbf{H}_{13}^{(1,k)} \mathbf{C}_{[3]}^{(k)} = \mathbf{0}_{T'_1 N - b'_1 \times b'_1}$ holds. By projecting the received signal vector onto all available vectors $\mathbf{w}_{12,\xi}^{(1,k)}$ and $\mathbf{w}_{13,\xi}^{(1,k)}$, $1 \leq \xi \leq T'_1 N - b'_1$, the sets of order-2 symbols $\{u_{[3|1,3],\xi}^{(k)}\}_{\xi=1}^{T'_1 N - b'_1}$ and $\{u_{[2|1,2],\xi}^{(k)}\}_{\xi=1}^{T'_1 N - b'_1}$ will be generated, respectively. We further assume $\frac{b'_1}{T'_1 N} \geq \frac{1}{2}$, which ensures all generated order-2 symbols are non-zero values almost surely. By applying similar processing, the sets of order-2 symbols $\{u_{[1|1,2],\xi}^{(k)}\}_{\xi=1}^{T'_1 N - b'_1}$ and $\{u_{[3|2,3],\xi}^{(k)}\}_{\xi=1}^{T'_1 N - b'_1}$ will be generated from the remaining interference terms at Rx_2, and the sets $\{u_{[1|1,3],\xi}^{(k)}\}_{\xi=1}^{T'_1 N - b'_1}$ and $\{u_{[2|2,3],\xi}^{(k)}\}_{\xi=1}^{T'_1 N - b'_1}$ will be generated from the remaining interference terms at Rx_3. This results in overall $q'_{1\Sigma} = 6(T'_1 N - b'_1)$ order-2 symbols generated per transmission block.

Let us consider the decodability requirement of the data vector $\mathbf{u}_{[1]}^{(k)}$ at Rx_1. Out of the available six order-2 symbol sets, there are four sets which can provide Rx_1 with linear combinations of $\mathbf{u}_{[1]}^{(k)}$, resulting in $4\,(T'_1 N - b'_1)$ terms in total. It is possible for Rx_1 to decode $\mathbf{u}_{[1]}^{(k)}$ only if the number of the obtained linear combinations is greater than or equal to the number of the desired unknowns, i.e. $4\,(T'_1 N - b'_1) \geq b'_1$, which can be rewritten as

$$\frac{b'_1}{T'_1 N} \leq \frac{4}{5}. \tag{14}$$

Due to the transmission symmetry among Tx_1, Tx_2 and Tx_3, identical decodability requirements hold for Rx_2 and Rx_3.

Further, depending on the ratio M/N, we consider two cases, which differ in the way the transmission blocks are designed. In each case, the decodability requirements of (3) and (14) override each other, thus only one of them has to be satisfied. The first case corresponds to the region of $M/N > 4/5$, where only (14) has to be fulfilled. In such a case, in order to maximize the amount of the transmitted data, the transmitters adjust b'_1 and T'_1 such that $\frac{b'_1}{T'_1 N} = \frac{4}{5}$ holds. The second case corresponds to the region of $M/N < 4/5$, where only (3) has to be fulfilled. Here, in order to maximize the amount of the transmitted data, the transmitters set $b'_1 = MT'_1$. In this case, the number of linear combinations obtained by each receiver exceeds the number of desired unknowns, thereby only $b'_1/4 = MT'_1/4$ order-2 symbols are necessary to be retransmitted from each order-2 symbol set to ensure decodability. The aforementioned two cases will be treated by two transmission schemes denoted as Scheme 1 and Scheme 2, respectively, which will be introduced next. For the case of $M/N = 4/5$, Scheme 2 will be applied. Due to the results on the achievable DoF already available in the literature [7], we will focus on the region of

$$3/4 < M/N < 1. \tag{15}$$

Scheme 1: $(4/5 < M/N < 1)$

We choose $b'_1 = 4N$ and $T'_1 = 5$ which ensures $\frac{b'_1}{T'_1 N} = \frac{4}{5}$. This results in overall $b'_{1\Sigma} = 12N$ symbols transmitted and $q'_{1\Sigma} = 6N$ order-2 symbols generated per transmission block. The linear independence of the useful linear combinations obtained by each receiver can be shown using the proof similar to the one described in [5], however due to the lack of space it is omitted in the paper.

Scheme 2: $(3/4 < M/N \leq 4/5)$

We choose $T'_1 = 4$ and $b'_1 = 4M$ and take the first $MT'_1/4 = M$ elements from each order-2 symbol set for retransmission in phase 2. This results in overall $b'_{1\Sigma} = 12M$ symbols transmitted and $q'_{1\Sigma} = 6M$ order-2 symbols generated per transmission block. Similarly, the proof of the linear independence is omitted here due to space limitation.

B. Second Phase

In phase 2, the order-2 symbols generated in phase 1 are transmitted. The transmitters are scheduled to transmit simultaneously in pairs, where each of the scheduled transmitters transmits b'_2 order-2 symbols per single transmission block in T'_2 time slots. Both of the scheduled transmitters transmit order-2 symbols useful for the same pair of receivers, i.e. Tx_i and Tx_j transmit order-2 symbols useful for both Rx_i and Rx_j, $1 \leq i, j \leq 3, i \neq j$.

Let us consider the k-th transmission block, $1 \leq k \leq k_2$, where Tx_1 and Tx_2 are scheduled for the transmission. Both transmitters transmit the order-2 symbols that are simultaneously useful for both Rx_1 and Rx_2. The order-2 symbols to be transmitted by Tx_1 and Tx_2 constitute two b'_2-element vectors $\mathbf{u}^{(k)}_{[1|1,2]}$ and $\mathbf{u}^{(k)}_{[2|1,2]}$, for which the random precoding matrices $\mathbf{C}^{(k)}_{[1|1,2]}, \mathbf{C}^{(k)}_{[2|1,2]} \in T'_2 M \times b'_2$ are used, respectively. After the transmission, both Rx_1 and Rx_2 obtain $T'_2 N$ linear combinations of $\mathbf{u}^{(k)}_{[1|1,2]}$ and $\mathbf{u}^{(k)}_{[2|1,2]}$. To decode both $\mathbf{u}^{(k)}_{[1|1,2]}$ and $\mathbf{u}^{(k)}_{[2|1,2]}$, Rx_1 and Rx_2 miss yet $2b'_2 - T'_2 N$ linear combinations.

Let us consider the signal received at Rx_3,

$$\mathbf{y}_3^{(2,k)} = \mathbf{H}_{31}^{(2,k)} \mathbf{C}^{(k)}_{[1|1,2]} \mathbf{u}^{(k)}_{[1|1,2]} + \mathbf{H}_{32}^{(2,k)} \mathbf{C}^{(k)}_{[2|1,2]} \mathbf{u}^{(k)}_{[2|1,2]}, \quad (16)$$

which is a sum of two interference terms. Similarly to phase 1, we define the matrices $\mathbf{W}_{31}^{(2,k)}$ and $\mathbf{W}_{32}^{(2,k)}$, for which

$$\begin{aligned} \mathbf{W}_{31}^{(2,k)\mathrm{H}} \mathbf{H}_{31}^{(2,k)} \mathbf{C}^{(k)}_{[1|1,2]} &= \mathbf{0}_{T'_2 N - b'_2 \times b'_2}, \\ \mathbf{W}_{32}^{(2,k)\mathrm{H}} \mathbf{H}_{32}^{(2,k)} \mathbf{C}^{(k)}_{[2|1,2]} &= \mathbf{0}_{T'_2 N - b'_2 \times b'_2} \end{aligned} \quad (17)$$

hold. Let us denote by $\mathbf{w}_{31,\xi}^{(2,k)}$ and $\mathbf{w}_{32,\xi}^{(2,k)}$, $1 \leq \xi \leq T'_2 N - b'_2$, the ξ-th columns of the matrices $\mathbf{W}_{31}^{(2,k)}$ and $\mathbf{W}_{32}^{(2,k)}$, respectively. By projecting the received signal onto $\mathbf{w}_{31,\xi}^{(2,k)}$ and $\mathbf{w}_{32,\xi}^{(2,k)}$, Rx_3 will get the linear combinations which contain order-2 symbols transmitted by only a single transmitter:

$$\begin{aligned} \mathbf{w}_{31,\xi}^{(2,k)\mathrm{H}} \mathbf{y}_3^{(2,k)} &= \mathbf{w}_{31,\xi}^{(2,k)\mathrm{H}} \mathbf{H}_{32}^{(2,k)} \mathbf{C}^{(k)}_{[2|1,2]} \mathbf{u}^{(k)}_{[2|1,2]}, \\ \mathbf{w}_{32,\xi}^{(2,k)\mathrm{H}} \mathbf{y}_3^{(2,k)} &= \mathbf{w}_{32,\xi}^{(2,k)\mathrm{H}} \mathbf{H}_{31}^{(2,k)} \mathbf{C}^{(k)}_{[1|1,2]} \mathbf{u}^{(k)}_{[1|1,2]}. \end{aligned} \quad (18)$$

We further use the notation of order-(2,1) symbols used in [5], where the order-(2,1) symbol is a term which is desired by two receivers and overheard at a single unintended receiver. We denote by $u^{(k)}_{[l|i_1,i_2;j],\xi}$ the order-(2,1) symbol, which is desired by Rx_{i_1} and Rx_{i_2}, available at Tx_l, and is known at Rx_j, $1 \leq i_1, i_2, l, j \leq 3, i_1 \neq i_2 \neq j$, $l \in \{i_1, i_2\}$. From (18), the following order-(2,1) symbols are generated:

$$\begin{aligned} u^{(k)}_{[2|1,2;3],\xi} &= \mathbf{w}_{31,\xi}^{(2,k)\mathrm{H}} \mathbf{H}_{32}^{(2,k)} \mathbf{C}^{(k)}_{[2|1,2]} \mathbf{u}^{(k)}_{[2|1,2]}, \\ u^{(k)}_{[1|1,2;3],\xi} &= \mathbf{w}_{32,\xi}^{(2,k)\mathrm{H}} \mathbf{H}_{31}^{(2,k)} \mathbf{C}^{(k)}_{[1|1,2]} \mathbf{u}^{(k)}_{[1|1,2]}. \end{aligned} \quad (19)$$

By projecting $\mathbf{y}_3^{(2,k)}$ onto all available vectors $\mathbf{w}_{31,\xi}^{(2,k)}$ and $\mathbf{w}_{32,\xi}^{(2,k)}$, the sets of order-(2,1) symbols $\{u^{(k)}_{[2|1,2;3],\xi}\}_{\xi=1}^{T'_2 N - b'_2}$ and $\{u^{(k)}_{[1|1,2;3],\xi}\}_{\xi=1}^{T'_2 N - b'_2}$ will be generated, respectively. We assume $\frac{b'_2}{NT'_2} \geq \frac{1}{2}$ holds, which ensures all generated order-(2,1) symbols are non-zero values almost surely. Given the sets delivered to both Rx_1 and Rx_2, each of the receivers will obtain additional $2\left(T'_2 N - b'_2\right)$ linear combinations of $\mathbf{u}^{(k)}_{[1|1,2]}$ and $\mathbf{u}^{(k)}_{[2|1,2]}$. It is possible for Rx_1 and Rx_2 to decode both $\mathbf{u}^{(k)}_{[1|1,2]}$ and $\mathbf{u}^{(k)}_{[2|1,2]}$, only if the number of the linear combinations provided to Rx_1 and Rx_2 is greater than or equal to the number of the missing linear combinations, i.e. $2\left(T'_2 N - b'_2\right) \geq 2b'_2 - T'_2 N$, which can be rewritten as

$$\frac{b'_2}{T'_2 N} \leq \frac{3}{4}. \quad (20)$$

Due to the limitation of (15), the requirement of (20) overrides the requirement of (3). To maximize the amount of the transmitted data, we choose $b'_2 = 3N$ and $T'_2 = 4$, which ensures $\frac{b'_2}{T'_2 N} = \frac{3}{4}$. This results in overall $b'_{2\Sigma} = 6N$ order-2 symbols transmitted and $q'_{2\Sigma} = 2N$ order-(2,1) symbols generated per transmission block. Similarly to phase 1, the proof of the linear independence of the linear combinations of the order-2 symbols obtained by Rx_1 and Rx_2 is omitted. All transmitter pairs use transmission blocks with identical parameters b'_2 and T'_2. The transmission blocks of Scheme 1 and Scheme 2 use identical values of b'_2 and T'_2.

C. Third Phase

In phase 3, the order-(2,1) symbols generated in phase 2 are transmitted, where all transmitters are scheduled to transmit simultaneously. Each transmitter transmits b'_3 order-(2,1) symbols comprised of two $b'_3/2$-element vectors per single transmission block in T'_3 time slots. Let us consider the k-th transmission block, $1 \leq k \leq k_3$. The order-(2,1) symbols transmitted by Tx_1, Tx_2 and Tx_3 constitute six vectors $\mathbf{u}^{(k)}_{[1|1,2;3]}$, $\mathbf{u}^{(k)}_{[1|1,3;2]}$, $\mathbf{u}^{(k)}_{[2|1,2;3]}$, $\mathbf{u}^{(k)}_{[2|2,3;1]}$, $\mathbf{u}^{(k)}_{[3|1,3;2]}$ and $\mathbf{u}^{(k)}_{[3|2,3;1]}$, precoded using random matrices $\mathbf{C}^{(k)}_{[1|1,2;3]}$, $\mathbf{C}^{(k)}_{[1|1,3;2]}$, $\mathbf{C}^{(k)}_{[2|1,2;3]}$, $\mathbf{C}^{(k)}_{[2|2,3;1]}$, $\mathbf{C}^{(k)}_{[3|1,3;2]}$ and $\mathbf{C}^{(k)}_{[3|2,3;1]}$, respectively.

Let us consider the signal received by Rx_1, which is

$$\begin{aligned} \mathbf{y}_1^{(3,k)} = \mathbf{H}_{11}^{(3,k)} &\left(\mathbf{C}^{(k)}_{[1|1,2;3]} \mathbf{u}^{(k)}_{[1|1,2;3]} + \mathbf{C}^{(k)}_{[1|1,3;2]} \mathbf{u}^{(k)}_{[1|1,3;2]}\right) + \\ + \mathbf{H}_{12}^{(3,k)} &\left(\mathbf{C}^{(k)}_{[2|1,2;3]} \mathbf{u}^{(k)}_{[2|1,2;3]} + \mathbf{C}^{(k)}_{[2|2,3;1]} \mathbf{u}^{(k)}_{[2|2,3;1]}\right) + \\ + \mathbf{H}_{13}^{(3,k)} &\left(\mathbf{C}^{(k)}_{[3|1,3;2]} \mathbf{u}^{(k)}_{[3|1,3;2]} + \mathbf{C}^{(k)}_{[3|2,3;1]} \mathbf{u}^{(k)}_{[3|2,3;1]}\right). \end{aligned} \quad (21)$$

TABLE I
SUMMARY OF SCHEME 1 AND SCHEME 2

	Phase 1					Phase 2					Phase 3				
M/N	b'_1	T'_1	$b'_{1\Sigma}$	$q'_{1\Sigma}$	k_1	b'_2	T'_2	$b'_{2\Sigma}$	$q'_{2\Sigma}$	k_2	b'_3	T'_3	$b'_{3\Sigma}$	k_3	d
$4/5 < \frac{M}{N} < 1$	$4N$	5	$12N$	$6N$	3	$3N$	4	$6N$	$2N$	3	$2N$	4	$6N$	1	$36N/31$
$3/4 < \frac{M}{N} \leq 4/5$	$4M$	4	$12M$	$6M$	$3N$	$3N$	4	$6N$	$2N$	$3M$	$2N$	4	$6N$	M	$\frac{9MN}{3N+4M}$

Since Rx_1 possess the knowledge about the vectors $\mathbf{u}^{(k)}_{[2|2,3;1]}$ and $\mathbf{u}^{(k)}_{[3|2,3;1]}$, it can subtract them from $\mathbf{y}^{(3,k)}_1$. The remaining vectors $\mathbf{u}^{(k)}_{[1|1,2;3]}$, $\mathbf{u}^{(k)}_{[2|1,2;3]}$, $\mathbf{u}^{(k)}_{[1|1,3;2]}$ and $\mathbf{u}^{(k)}_{[3|1,3;2]}$ contain the terms desired by Rx_1, comprising in total $2b'_3$ unknowns.

The number of the useful linear combinations available to Rx_1 is equal to the size of the vector $\mathbf{y}^{(3,k)}_1$, which equals T'_3N. It is possible for Rx_1 to decode all desired unknowns only if the number of the available linear combinations is greater than or equal to the number of the desired unknowns, i.e. $T'_3N \geq 2b'_3$, which can be rewritten as

$$\frac{b'_3}{T'_3N} \leq \frac{1}{2}. \qquad (22)$$

Due to the symmetry of the transmission among Tx_1, Tx_2 and Tx_3, identical decodability requirements hold for the receivers Rx_2 and Rx_3. Similarly to phase 2, the requirement of (22) overrides the requirement of (3) due to the limitation of (15). To maximize the amount of the transmitted data, we choose $b'_3 = 2N$ and $T'_3 = 4$, which ensures $\frac{b'_3}{T'_3N} = \frac{1}{2}$. This results in overall $b'_{3\Sigma} = 6N$ order-(2,1) symbols transmitted per transmission block. Transmission blocks of Scheme 1 and Scheme 2 use identical values of b'_3 and T'_3.

D. Achieved Number of DoF

The numbers of the transmission blocks of the i-th phase k_i, $i \in \{1, 2, 3\}$, are chosen according to (1). Each receiver will recover useful order-(2,1) symbols and will use them to decode useful order-2 symbols, which in turn will be used to decode the original data symbols. The achieved number of DoF can then be calculated as $d = \frac{3b_1}{T_1+T_2+T_3}$. The parameters of Scheme 1 and Scheme 2 and the achieved number of DoF are summarized in Table I.

The achieved number of DoF is compared to the one of [7] in Fig. 2, where the normalized number of DoF $\frac{d}{3N}$ is plotted as a function of M/N. Additionally, we plot an outer bound

$$\frac{d_{\text{outer}}}{3N} = \begin{cases} \frac{3}{7}, & \text{if } \frac{3}{4} \leq \frac{M}{N} \leq 1, \\ \frac{M}{M+N}, & \text{if } \frac{1}{2} \leq \frac{M}{N} < \frac{3}{4}, \end{cases} \qquad (23)$$

which can be obtained similarly to the outer bound for the 3-user SISO IC of [8] using the results of [9]. For $M/N < 1$, the proposed transmission scheme utilizes all receive antennas, which leads to the larger number of transmitted data symbols as compared to the transmission scheme proposed for the region of $31/32 < M/N \leq 18/13$ in [7]. For $M/N < 4/5$, the performance of the proposed scheme decreases with M/N due to the requirement of (3), which limits the number of data symbols transmitted in phase 1.

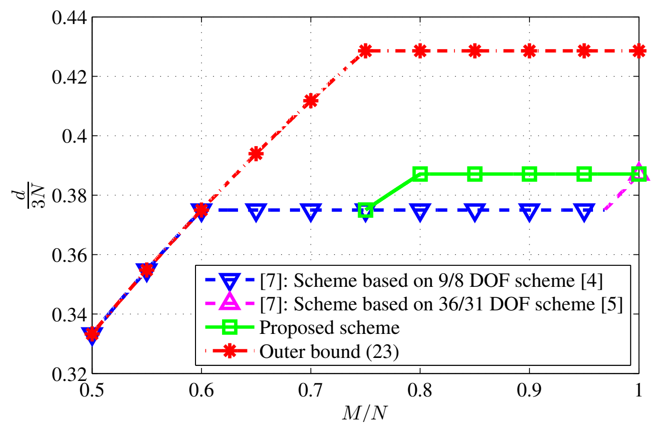

Fig. 2. Number of DoF for 3-user MIMO IC with delayed CSIT

IV. CONCLUSIONS

For the three-user MIMO IC under the delayed CSIT setting where each transmitter has M antennas and each receiver has N antennas, new results on the achievable number of DoF in the region of $3/4 < M/N < 1$ were obtained. The results are based on a three-phase transmission scheme, which compared to the previous approaches uses available transmit and receive antennas in a more effective way.

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