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ELSEVIER Applied Mathematics and Computation 173 (2006) 902–915
APPLIED MATHEMATICS AND COMPUTATION
www.elsevier.com/locate/amc

DEA efficiency assessment using ideal and anti-ideal decision making units ☆

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Abstract

This paper introduces two virtual decision making units (DMUs) called ideal DMU (IDMU) and anti-ideal DMU (ADMU) into the data envelopment analysis (DEA). The resultant DEA models are, respectively, referred to as the data envelopment analysis with ideal and anti-ideal decision making units. One evaluates DMUs from the viewpoint of the best possible relative efficiency, while the other evaluates them from the perspective of the worst possible relative efficiency. The two distinctive efficiencies are combined to form a comprehensive index called the relative closeness (RC) to the IDMU just like the well-known TOPSIS approach in multiple attribute decision making (MADM). The RC index is then used as the evidence of overall assessment of each DMU, based on which an overall ranking for all the DMUs can be obtained. Two numerical examples are provided to illustrate the applications of the proposed DEA models and the RC index.

☆ This research was supported by the project on Human Social Science of MOE, PR China under the Grant No. 01JA790082, and by Fok Ying Tung Education Foundation under the Grant No. 71080.
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doi:10.1016/j.amc.2005.04.023

Keywords: Data envelopment analysis (DEA); Ideal decision making unit (IDMU); Anti-ideal decision making unit (ADMU); Relative closeness (RC); Ranking

1. Introduction

Data envelopment analysis (DEA), developed by Charnes et al. [1], usually evaluates decision making units (DMUs) from the angle of the best possible relative efficiency. If a DMU is evaluated to have the best possible relative efficiency of unity, then it is said to be DEA efficient; otherwise it is said to be DEA inefficient. DEA efficient DMUs are always thought to perform better than DEA inefficient DMUs. If a DEA efficient DMU, however, also has a poorer relative efficiency than a DEA inefficient DMU when they are both evaluated from the angle of the worst possible relative efficiency, can we still say that the DEA efficient DMU performs better than the DEA inefficient DMU? In this situation, the conclusion is obviously uncertain. So, there is a clear need to combine the best and the worst possible relative efficiencies to give an overall assessment of each DMU.

Entani et al. [2] considered DEA efficiencies from both the optimistic and the pessimistic viewpoints. In their DEA models, the worst and the best possible relative efficiencies were utilized to constitute an interval. Their model for the computation of the worst possible relative efficiency, however, has a deadly drawback that it lost some information on inputs and outputs because only one input and one output data of the DMU under evaluation were effectively utilized and all the other input and output data did not work.

Wang et al. [3] proposed a bounded DEA model for precise data. The bounded DEA model makes the most of all input and output information to measure both the best and the worst possible relative efficiencies of each DMU by introducing a virtual anti-ideal DMU (ADMU), which consumes the most inputs only to produce the least outputs. It can therefore identify both the efficiency and inefficiency frontiers.

In this paper, DEA efficiency evaluation problems will be handled in a different way. A virtual ideal DMU (IDMU) will be further introduced into DEA model. The two virtual DMUs, IDMU and ADMU, are used to construct two DEA models for the calculation of the best possible and the worst possible relative efficiencies, respectively. The two distinctive efficiencies are integrated using the well-known TOPSIS approach in multiple attribute decision making (MADM) to generate a composite index called the relative closeness (RC) to the IDMU. The RC index will be used as the evidence of overall assessment of each DMU, based on which an overall ranking for all the DMUs can be generated very easily.

The paper is organized as follows. In Section 2, we develop two DEA models with a virtual IDMU or ADMU to capture the best and the worst possible relative efficiencies, respectively. Section 3 defines the relative closeness index to combine the best and the worst possible relative efficiencies of each DMU. Two numerical examples are provided in Section 4 to illustrate the applications of the proposed DEA models and the RC index. The paper is concluded in Section 5.

2. DEA models with IDMU or ADMU

Assume that there are n DMUs to be evaluated, each DMU with m inputs and s outputs. We denote by x_{ij} ($i = 1,\ldots,m$) and y_{rj} ($r = 1,\ldots,s$) the values of inputs and outputs of DMU_j ($j = 1,\ldots,n$), which are all known and positive. An IDMU and an ADMU can be defined as follows:

Definition 1. An IDMU is a virtual DMU, which can use the least inputs to generate the most outputs. While an ADMU is a DMU, which consumes the most inputs only to produce the least outputs.

Note that a virtual IDMU may not exist in practical production activity at least at current technical level, while a virtual ADMU may exist in practical production activity because the waste of resources is always allowed in the theory of production possibility set.

According to the above definition, we denote by $x_i^{\min}$ ($i = 1,\ldots,m$) and $y_r^{\max}$ ($r = 1,\ldots,s$) the inputs and outputs of the IDMU, and by $x_i^{\max}$ ($i = 1,\ldots,m$) and $y_r^{\min}$ ($r = 1,\ldots,s$) the inputs and outputs of the ADMU, respectively, where $x_i^{\min}$ and $x_i^{\max}$ are the minimum and the maximum of the ith input, $y_r^{\min}$ and $y_r^{\max}$ are the minimum and the maximum of the rth output. They are determined by the following formulae:

$$x_i^{\min} = \min_j\{x_{ij}\} \quad \text{and} \quad x_i^{\max} = \max_j\{x_{ij}\}, \qquad i = 1,\ldots,m,$$
$$y_r^{\min} = \min_j\{y_{rj}\} \quad \text{and} \quad y_r^{\max} = \max_j\{y_{rj}\}, \qquad r = 1,\ldots,s.$$

Although the IDMU is a virtual DMU, its production behavior should become the goal of each DMU's pursuing. According to the implication of efficiency, the efficiency of the IDMU can be defined as

$$\theta_{\text{IDMU}} = \frac{\sum_{r=1}^{s} u_r y_r^{\max}}{\sum_{i=1}^{m} v_i x_i^{\min}},$$

where u_r and v_i are the factor weights assigned to the rth output and the ith input. It is obvious that the IDMU should be able to achieve the highest/best possible relative efficiency. Therefore, we may construct the following fractional programming model:

$$\begin{aligned} \text{Maximize} \quad & \theta_{\mathrm{IDMU}} = \frac{\sum_{r=1}^{s} u_r y_r^{\max}}{\sum_{i=1}^{m} v_i x_i^{\min}} \\ \text{subject to} \quad & \theta_j = \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leqslant 1, \quad j = 1, \ldots, n, \\ & u_r, v_i \geqslant \varepsilon, \quad \forall r, i, \end{aligned} \tag{1}$$

where u_r and v_i are decision variables and ε is the non-Archimedean infinitesimal.

Using Charnes and Cooper transformation, the above fractional programming model can be solved through the following linear programming model:

$$\begin{aligned} \text{Maximize} \quad & \theta_{\mathrm{IDMU}} = \sum_{r=1}^{s} u_r y_r^{\max} \\ \text{subject to} \quad & \sum_{i=1}^{m} v_i x_i^{\min} = 1, \\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leqslant 0, \quad j = 1, \ldots, n, \\ & u_r, v_i \geqslant \varepsilon, \quad \forall r, i. \end{aligned} \tag{2}$$

Let θ_{IDMU}^* be the optimum efficiency of the IDMU. Since there exists such a possibility that the above LP model (2) may have multiple optima, we utilize the following fractional programming model to determine the best possible relative efficiency of DMU_0 under the condition that the best possible relative efficiency of the IDMU remains unchanged:

$$\begin{aligned} \text{Maximize} \quad & \theta_{j_0} = \frac{\sum_{r=1}^{s} u_r y_{rj_0}}{\sum_{i=1}^{m} v_i x_{ij_0}} \\ \text{subject to} \quad & \theta_{\mathrm{IDMU}}^* = \frac{\sum_{r=1}^{s} u_r y_r^{\max}}{\sum_{i=1}^{m} v_i x_i^{\min}}, \\ & \theta_j = \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leqslant 1, \quad j = 1, \ldots, n, \\ & u_r, v_i \geqslant \varepsilon, \quad r = 1, \ldots, s, \quad i = 1, \ldots, m, \end{aligned} \tag{3}$$

where j_0 is the DMU under evaluation (usually denoted by DMU_0) and θ_{IDMU}^* is the best possible relative efficiency of the IDMU. The fractional programming problem (3) can be solved through the following linear programming model:

$$\begin{aligned} \text{Maximize} \quad & \theta_{j_0} = \sum_{r=1}^{s} u_r y_{rj_0} \\ \text{subject to} \quad & \sum_{i=1}^{m} v_i x_{ij_0} = 1, \end{aligned} \tag{4}$$

$$\sum_{r=1}^{s} u_r y_j^{\max} - \sum_{i=1}^{m} v_i(\theta_{\mathrm{IDMU}}^* x_i^{\min}) = 0,$$
$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leqslant 0, \quad j = 1, \ldots, n,$$
$$u_r, v_i \geqslant \varepsilon, \quad \forall r, i.$$

As such, the efficiency of the ADMU can be defined as

$$\varphi_{\mathrm{ADMU}} = \frac{\sum_{r=1}^{s} u_r y_r^{\min}}{\sum_{i=1}^{m} v_i x_i^{\max}}.$$

As an ADMU, its efficiency is evidently worse than any other DMUs. The following fractional programming model is thus constructed:

$$\begin{aligned} \text{Minimize} \quad & \varphi_{\mathrm{ADMU}} = \frac{\sum_{r=1}^{s} u_r y_r^{\min}}{\sum_{i=1}^{m} v_i x_i^{\max}} \\ \text{subject to} \quad & \varphi_j = \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \geqslant 1, \quad j = 1, \ldots, n, \\ & u_r, v_i \geqslant \varepsilon, \quad \forall r, i, \end{aligned} \tag{5}$$

which can be solved through the following linear programming model:

$$\begin{aligned} \text{Minimize} \quad & \varphi_{\mathrm{ADMU}} = \sum_{r=1}^{s} u_r y_r^{\min} \\ \text{subject to} \quad & \sum_{i=1}^{m} v_i x_i^{\max} = 1, \\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \geqslant 0, \quad j = 1, \ldots, n, \\ & u_r, v_i \geqslant \varepsilon, \quad \forall r, i. \end{aligned} \tag{6}$$

Let $\varphi_{\mathrm{ADMU}}^*$ be the worst efficiency of the ADMU. Then the following fractional programming model can be used to determine the worst possible relative efficiency of DMU_0 under the condition that the worst possible relative efficiency of the ADMU keeps unchanged:

$$\begin{aligned} \text{Minimize} \quad & \varphi_{j_0} = \frac{\sum_{r=1}^{s} u_r y_{rj_0}}{\sum_{i=1}^{m} v_i x_{ij_0}} \\ \text{subject to} \quad & \varphi_{\mathrm{IDMU}}^* = \frac{\sum_{r=1}^{s} u_r y_r^{\min}}{\sum_{i=1}^{m} v_i x_i^{\max}}, \end{aligned} \tag{7}$$

$$\varphi_j = \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \geqslant 1, \quad j = 1, \ldots, n,$$
$$u_r, v_i \geqslant \varepsilon, \quad r = 1, \ldots, s, \quad i = 1, \ldots, m.$$

The above fractional programming model (7) can be solved through the following linear programming model:

$$\begin{aligned}
\text{Minimize} \quad & \varphi_{j_0} = \sum_{r=1}^{s} u_r y_{rj_0} \\
\text{subject to} \quad & \sum_{i=1}^{m} v_i x_{ij_0} = 1, \\
& \sum_{r=1}^{s} u_r y_j^{\min} - \sum_{i=1}^{m} v_i (\varphi_{\mathrm{IDMU}}^* x_i^{\max}) = 0, \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \geqslant 0, \quad j = 1, \ldots, n, \\
& u_r, v_i \geqslant \varepsilon, \quad \forall r, i.
\end{aligned} \tag{8}$$

Let $\theta_{j_0}^*$ and $\varphi_{j_0}^*$ be the best and the worst possible relative efficiencies of DMU_0, respectively, which are the optimal objective function values of the models (4) and (8), then we have the following definitions.

Definition 2. DMU_0 is said to be DEA efficient if and only if $\theta_{j_0}^* = 1$, otherwise it is said to be non-DEA efficient.

Definition 3. DMU_0 is said to be DEA inefficient if and only if $\varphi_{j_0}^* = 1$, otherwise it is said to be non-DEA inefficient.

Note that the conventional DEA approach does not strictly distinguish between non-DEA efficient and DEA inefficient units and uses them interchangeably. In the above definitions, however, non-DEA efficient, DEA inefficient and non-DEA inefficient units are all strictly distinguished because each of them represents different meanings. Non-DEA efficient units do not necessarily mean they are DEA inefficient. As such, non-DEA inefficient units do not necessarily mean they are DEA efficient.

It also must be pointed out that the efficiencies in DEA models (5) and (8) based on ADMU are defined to be greater than or equal to one, which is quite different from the traditional DEA efficiency or inefficiency that is defined to be less than or equal to unity. As a matter of fact, the traditional DEA efficiency has the ability to differentiate those DMUs that perform poorly, but has no ability to identify which DMU to perform best among those DEA efficient DMUs, while the efficiencies defined in the models (5) and (8) have the capability of identifying which DMU to perform best, but have no capability of discerning which DMU to perform worst. So, the two definitions of efficiency complement each other.

3. The RC index for combining the best and the worst possible relative efficiencies

From the last section it is known that DEA models (1)–(4) based on IDMU measure the best possible relative efficiencies of IDMU and the n real DMUs, while DEA models (5)–(8) based on ADMU measure the worst possible relative efficiencies of ADMU and the n real DMUs. These two distinctive efficiency assessments may lead to quite different conclusions. Therefore, there is a need to consider them together to give an overall assessment of each DMU. In order to do so, we introduce the concept of relative closeness, which is widely used in the TOPSIS approach, a well-known MADM methodology [4].

Definition 4. Let θ^*_{IDMU} and $\theta^*_{j_0}$ be the best possible relative efficiencies of IDMU and DMU_0, respectively, determined by DEA models (1)–(4), and φ^*_{IDMU} and $\varphi^*_{j_0}$ be the worst possible relative efficiencies of ADMU and DMU_0, respectively, determined by DEA models (5)–(8), the relative closeness index of DMU_0 to IDMU is defined as

$$\text{RC}_{j_0} = \frac{\varphi^*_{j_0} - \varphi^*_{\text{ADMU}}}{(\varphi^*_{j_0} - \varphi^*_{\text{ADMU}}) + (\theta^*_{\text{IDMU}} - \theta^*_{j_0})}. \tag{9}$$

It is obvious that the bigger difference between $\varphi^*_{j_0}$ and φ^*_{ADMU} and the smaller difference between θ^*_{IDMU} and $\theta^*_{j_0}$ mean the better performance of DMU_0. So, the bigger the RC_{j_0} value, the better the performance of DMU_0.

Note that the TOPSIS approach employs the distances of utility to define the relative closeness, while the RC index in this paper is defined using the distances of efficiency.

Since the RC index integrates both the best and the worst possible relative efficiencies of each DMU, it thus provides an overall assessment for each DMU, based on which an overall ranking for the n real DMUs can be easily obtained.

4. Numerical examples

We now illustrate the applications of the proposed DEA models and the RC index using two numerical examples. One is a simple DEA efficiency-rating problem, in which the overall ranking for the DMUs can be achieved intuitively. The other is a complicated performance-rating problem, where the overall ranking for the DMUs cannot be achieved intuitively and can only be determined by the proposed RC index. All the models were implemented in MS-Excel worksheets and were solved using the Excel Solver. The non-Archimedean infinitesimal was set as $\varepsilon = 10^{-10}$.

Example 1. Consider a DEA efficiency evaluation problem with five DMUs, each DMU with two inputs and one output. The data set is taken from Andersen and Petersen [5] and is shown in Table 1. The CCR efficiency of each DMU is presented in the last column of Table 1.

As can be seen from the rating results of Table 1 that the conventional CCR model identifies DMU_1 through DMU_4 as DEA efficient units, which means they perform equally well. But in fact, DMU_2 obviously outperforms DMU_1 because DMU_2 consumes less resource of input 2 to generate the same output as DMU_1. In order to rank the four DEA efficient units, Andersen and Petersen [5] suggested a ranking approach that compares the DMU under evaluation with a linear combination of all the other DMUs, i.e., the DMU itself is excluded. Based on their approach, the following ranking order was obtained: $DMU_2 \succ DMU_4 \succ DMU_3 \succ DMU_1 \succ DMU_5$, where the symbol "$\succ$" means "performs better than" or "is superior to". Such a ranking order considers obviously only the best possible relative efficiency of each DMU. Therefore, it is somewhat one-sided.

Now, we use the proposed DEA models with IDMU and ADMU to reevaluate these five DMUs. The virtual IDMU and ADMU are defined in the last two rows of Table 1. The resulting efficiency ratings and the RC values are presented in Table 2.

It is clear from Table 2 that the DEA models based on IDMU and ADMU both evaluate the original four DEA efficient units DMU_1 through DMU_4 to be not completely the same. The DEA model based on IDMU assesses both DMU_2 and DMU_3 to be DEA efficient, but DMU_1 and DMU_4 to be no longer DEA efficient although they are rated to be equally well. The DEA model based on ADMU evaluates DMU_3 to be the best DMU, which is followed by DMU_2

Table 1
Data for five DMUs with two inputs and one output

DMU	x_{1j}	x_{2j}	y_{1j}	CCR efficiency
1	2	12	1	1
2	2	8	1	1
3	5	5	1	1
4	10	4	1	1
5	10	6	1	0.75
IDMU	2	4	1	–
ADMU	10	12	1	–

Table 2
Efficiency ratings and the RC values for the five DMUs

DMU	CCR/IDMU efficiency	CCR/ADMU efficiency	RC	Ranking
1	0.714	1	0.244	4
2	1	1.421	0.522	2
3	1	1.543	0.561	1
4	0.714	1.174	0.336	3
5	0.625	1	0.228	5
IDMU	1.667	–	–	–
ADMU	–	0.692	–	–

and DMU_4, and both DMU_1 and DMU_5 to be the worst DMUs. Observing the rating results in Table 2, we may find that the DEA model based on IDMU identifies DEA efficient units, but cannot differentiate them, while the DEA model based on ADMU is quite the contrary, it identifies those DEA inefficient units, but fails to distinguish them further. When the rating results obtained by the two different DEA models are considered together, a fully ranking order may be achieved intuitively for this simple example. The final overall ranking order should be $DMU_3 \succ DMU_2 \succ DMU_4 \succ DMU_1 \succ DMU_5$. Such a ranking can be consistently achieved by using the systematic RC index, whose values for the five DMUs are presented in the fourth column of Table 2.

It is obvious that the overall ranking is different from the ranking obtained by Andersen and Petersen. This is because the overall ranking considers both the best and the worst possible relative efficiencies of each DMU. It is therefore more convincing.

Example 2. Consider a complicated performance rating case with 48 DMUs, each DMU with eight inputs and one output. The data set is presented in Table 3.

The traditional CCR model evaluates 13 of 48 DMUs to be DEA efficient and cannot differentiate them further. The decision maker (DM) is not very happy with such a rating result with so many DMUs being rated as DEA efficient. In order to obtain an improved performance assessment result and generate a reliable overall ranking for the 48 DMUs, the DEA models with IDMU and ADMU are chosen to re-evaluate the performances of the 48 DMUs. The two virtual IDMU and ADMU are defined in the last two column of Table 3. The best possible relative efficiency of IDMU is 5.088169732 and the worst possible relative efficiency of ADMU is 0.380911. Both the best and the worst possible relative efficiencies and the RC value for each DMU are shown in the third through the fifth columns of Table 4, respectively.

It is clear from Table 4 that the DEA model with IDMU evaluates only DMU_{35} and DMU_{43} to be DEA efficient and all the other 46 DMUs to be

Table 3
Data set for 48 DMUs with eight inputs and one output

DMU	x_{1j}	x_{2j}	x_{3j}	x_{4j}	x_{5j}	x_{6j}	x_{7j}	x_{8j}	y_{1j}
1	2,660,822	971,204	43,448	459,434	258,221	625,167	277,250	259,917	4,884,491
2	2,774,620	1,033,617	48,215	185,103	258,221	525,167	277,250	259,917	4,723,829
3	2,773,076	973,773	35,620	190,553	258,221	625,167	277,250	259,917	4,820,005
4	2,662,835	891,078	111,162	371,477	258,221	625,167	277,250	259,917	7,334,145
5	2,710,895	952,892	89,375	410,834	258,254	556,417	323,083	635,333	6,539,885
6	2,814,342	1,056,791	91,785	560,577	258,216	611,417	286,417	335,000	5,682,479
7	2,694,134	951,085	19,650	669,802	258,216	611,417	286,417	335,000	6,277,111
8	2,703,563	958,898	182,601	469,104	258,216	611,417	286,417	335,000	5,926,099
9	2,662,888	967,651	26,203	445,510	258,216	611,417	286,417	335,000	5,718,147
10	2,722,038	1,046,131	134,546	431,736	258,216	611,417	286,417	335,000	6,147,416
11	2,627,079	958,100	68,976	891,432	258,216	611,417	286,417	335,000	5,252,912
12	2,603,678	968,610	151,717	467,505	166,056	612,517	286,417	335,000	6,382,501
13	3,590,599	1,183,646	35,816	693,357	265,488	763,083	1,004,750	322,833	12,230,868
14	3,746,409	1,239,481	169,941	793,554	265,488	763,083	1,004,750	322,833	7,557,249
15	3,722,455	1,169,433	95,663	618,490	265,488	763,083	1,004,750	322,833	8,423,893
16	3,556,553	1,046,161	150,299	1,322,993	265,488	763,083	1,004,750	322,833	10,463,901
17	3,825,155	1,140,802	206,573	1,527,287	356,065	1,058,917	1,110,583	493,250	8,300,902
18	3,748,541	1,160,788	143,555	620,031	283,603	822,250	1,025,917	356,917	9,806,308
19	3,741,728	1,169,037	161,516	1,939,978	283,603	822,250	1,025,917	356,917	10,153,383
20	3,805,723	1,157,862	426,305	1,066,707	283,603	822,250	1,025,917	356,917	8,827,252
21	3,778,409	1,167,120	96,013	1,432,726	283,603	822,250	1,025,917	356,917	8,182,909
22	3,726,112	1,183,541	173,885	1,106,348	283,603	822,250	1,025,917	356,917	10,201,794
23	3,778,060	1,184,726	209,389	772,914	283,603	822,250	1,025,917	356,917	7,331,807
24	3,667,016	1,177,481	470,110	1,450,891	283,603	823,550	1,025,917	356,917	9,250,741
25	2,511,517	928,030	81,930	155,258	296,448	576,667	264,917	264,917	5,362,253
26	2,370,750	911,107	145,754	152,872	296,448	576,667	264,917	264,917	6,038,145
27	2,493,884	911,350	184,321	97,189	296,448	576,667	264,917	264,917	5,756,788
28	2,608,280	919,261	163,187	128,175	296,448	563,750	309,083	309,083	5,974,819
29	2,536,797	921,268	159,789	284,472	296,448	563,750	309,083	309,083	5,974,819

(*continued on next page*)

Table 3 (*continued*)

DMU	x_{1j}	x_{2j}	x_{3j}	x_{4j}	x_{5j}	x_{6j}	x_{7j}	x_{8j}	y_{1j}
30	2,568,108	921,494	123,910	102,268	296,448	574,083	273,750	273,750	6,010,925
31	2,482,813	911,157	34,742	172,243	296,448	574,083	273,750	273,750	6,219,717
32	2,514,367	912,589	259,143	194,135	296,448	574,083	273,750	273,750	6,221,237
33	2,488,061	918,867	77,373	114,083	296,448	574,083	273,750	273,750	5,517,886
34	2,374,287	882,950	152,342	131,287	296,448	574,083	273,750	273,750	6,712,105
35	2,397,961	872,903	107,454	30,049	296,448	574,083	273,750	273,750	6,281,112
36	2,670,327	964,782	367,450	765,893	296,448	574,983	273,750	273,750	6,687,954
37	3,697,678	1,266,102	33,966	350,386	511,774	768,583	396,250	339,167	7,522,988
38	3,259,807	1,349,119	72,199	244,636	511,774	768,583	396,250	339,167	9,210,630
39	3,578,978	1,256,740	43,394	262,109	511,774	768,583	396,250	339,167	8,103,308
40	3,379,848	1,139,177	45,053	229,030	511,774	768,583	396,250	339,167	8,695,839
41	3,480,194	1,205,935	38,111	181,698	511,806	626,917	437,917	259,584	8,238,774
42	3,531,172	1,310,750	27,932	440,126	511,769	740,250	404,583	323,250	8,485,877
43	3,528,843	1,266,230	19,973	158,691	511,769	740,250	404,583	323,250	8,990,070
44	3,585,412	1,227,974	76,672	382,891	511,769	740,250	404,583	323,250	9,081,558
45	3,640,698	1,223,032	60,770	994,747	511,769	740,250	404,583	323,250	9,392,471
46	3,809,906	1,319,310	157,755	552,095	511,769	740,250	404,583	323,250	10,054,721
47	3,710,539	1,287,333	64,258	1,837,216	511,769	740,250	404,583	323,250	9,098,902
48	3,575,403	1,227,284	181,347	854,932	511,769	741,550	404,583	323,250	9,524,008
IDMU	2,370,750	872,903	19,650	30,049	166,056	525,167	264,917	259,584	12,230,868
ADMU	3,825,155	1,349,119	470,110	1,939,978	511,806	1,058,917	1,110,583	635,333	4,723,829

Table 4
Efficiency ratings and the RC values for the 48 DMUs

DMU	CCR efficiency	CCR/IDMU efficiency	CCR/ADMU efficiency	RC	Rank
1	0.7880	0.1924	1.4534	0.1797	31
2	0.7937	0.4077	1.5923	0.2056	23
3	0.8226	0.4262	1.6791	0.2178	20
4	1	0.3070	1.9377	0.2456	11
5	0.9296	0.2623	1	0.1137	45
6	0.7988	0.17383	1.2634	0.1522	40
7	1	0.1789	1.5422	0.1913	27
8	0.8140	0.1850	1.1600	0.1371	42
9	0.9375	0.2391	1.5168	0.1898	28
10	0.8444	0.2196	1.3221	0.1620	37
11	0.7703	0.1081	1.0874	0.1242	43
12	1	0.2087	1.3437	0.1648	35
13	1	0.3305	2.9408	0.3498	2
14	0.6179	0.1573	1.3908	0.1700	33
15	0.7332	0.2352	1.8560	0.2331	15
16	0.9678	0.1410	1.7135	0.2122	22
17	0.7040	0.0953	1	0.1103	48
18	0.8414	0.2580	1.8786	0.2367	14
19	0.8404	0.0956	1.3617	0.1642	36
20	0.7377	0.1204	1.0662	0.1212	44
21	0.6784	0.1057	1.3394	0.1613	38
22	0.8341	0.1589	1.6345	0.2028	25
23	0.5989	0.1505	1.2200	0.1453	41
24	0.7602	0.0975	1	0.1104	47
25	0.8462	0.4643	1.6353	0.2134	21
26	0.9213	0.4238	1.5864	0.2054	24
27	0.9039	0.4393	1.4253	0.1834	29
28	0.8932	0.4344	1.4057	0.1805	30
29	0.8465	0.2768	1.3369	0.1658	34
30	0.9200	0.5619	1.6641	0.2209	19
31	1	0.6019	2.0866	0.2755	6
32	0.9137	0.2912	1.2742	0.1570	39
33	0.8701	0.5968	1.7012	0.2272	17
34	1	0.4996	1.7235	0.2264	18
35	1	1	1.8689	0.2669	10
36	0.9442	0.1208	1	0.1108	46
37	0.8377	0.3878	1.8852	0.2424	13
38	1	0.5872	2.2242	0.2905	5
39	0.8913	0.5294	2.0623	0.2694	9
40	1	0.6352	2.2352	0.2940	4
41	1	0.7513	2.6166	0.3402	3
42	0.9377	0.3578	2.1428	0.2714	7
43	1	1	2.6066	0.3525	1
44	0.9795	0.3958	2.1242	0.2709	8
45	1	0.1756	1.8379	0.2287	16

(*continued on next page*)

Table 4 (*continued*)

DMU	CCR efficiency	CCR/IDMU efficiency	CCR/ADMU efficiency	RC	Rank
46	1	0.2859	1.9270	0.2435	12
47	0.9662	0.0941	1.4060	0.1703	32
48	0.9837	0.1843	1.6163	0.2012	26
IDMU	–	5.0882	–	–	–
ADMU	–	–	0.3809	–	–

non-DEA efficient. The number of DEA efficient units is significantly reduced. The DEA model with ADMU evaluates DMU_5, DMU_{17}, DMU_{24} and DMU_{36} to be DEA inefficient and the other 44 DMUs to be non-DEA inefficient. The RC index shows that DMU_{43} has the best overall performance, which is followed by DMU_{13}, DMU_{41}, DMU_{40} and DMU_{38}, while DMU_{17} has the worst overall performance followed by DMU_{24}, DMU_{36} and DMU_5. The overall ranking order for all the 48 DMUs is presented in the last column of Table 4, from which it can be found that the 48 DMUs are all ranked and distinguished according to their overall performances. This is a significant advantage of the proposed DEA approach over the other DEA methodologies.

5. Conclusions

In this paper we have developed two DEA models, one is based on virtual IDMU and the other is based on virtual ADMU. The former evaluates DMUs using the best possible relative efficiency and can be used to identify DEA efficient units; while the latter evaluates DMUs using the worst possible relative efficiency and can be used to identify those DEA inefficient units. The two distinctive efficiencies are integrated using a relative closeness index, which provides the overall performance assessment of each DMU and can thus be used as the basis of comparing and ranking the DMUs. Comparing with the existing DEA methodologies, the proposed DEA approach has the capability of distinguishing each DMU from the others according to their respective overall performances. Two numerical examples have illustrated the advantages, potential and applications of the proposed DEA models and the RC index.

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