Diversity Order Bounds for Wireless Relay Networks

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Abstract **– This paper derives bounds on the diversity order and high SNR probability of outage behavior of wireless relay networks with arbitrary link connectivity between cooperating terminals. Single and multiple antennas per terminal are considered. Two general codebook generation schemes are analyzed and compared. Common codebook generation involves the source and all relays sharing a common randomly generated codebook, and encompasses many practical encoding schemes, including both repetition coding and space-time coding. Independent codebook generation involves the source and all relays employing independent randomly generated codebooks, and provides an information theoretic lower bound on the probability of outage of all achievable codebook generation schemes. The results indicate that although independent codebook generation does not offer any diversity gain over common codebook generation, it can offer a significant probability of outage improvement under certain conditions.**

Index Terms **– cooperative diversity, cooperative relaying, spatial diversity, multihop relaying**

I. INTRODUCTION

Since increased spatial diversity is one of the key benefits expected of cooperative connectivity, it is important to understand the bounds on the maximum achievable diversity order of wireless relay networks with arbitrary, but generally less than full, cooperative connectivity. This paper derives bounds on the maximum achievable diversity order and high signal to noise ratio (SNR) probability of outage of connected wireless relay networks with arbitrary link connectivity between cooperating terminals. Although previous work has derived the diversity order for specific combinations of cooperative connectivity and relaying method (for example [2], [4], [5], [7]), the current paper presents a more general formulation applicable to cooperative networks with any number of relay terminals and any possible combination of links between cooperating terminals. Information theoretic probability of outage formulations are used as they are independent from any particular coding scheme, applicable for arbitrary transmission rates, and enable straightforward derivation of asymptotic diversity order results.

Two general codebook generation schemes are considered and compared. Common codebook generation refers to cooperative encoding schemes where the source and all relays sharing a common randomly generated codebook, and encompasses many practical encoding schemes, including both repetition coding and space-time coding. Independent codebook generation refers to cooperative encoding schemes where the source and all relays employing independent randomly generated codebooks, and provides an information theoretic lower bound on the probability of outage of all achievable codebook generation schemes. As noted in [4], the generation of independent random codebooks corresponds to utilizing parallel channels, and can achieve better bandwidth efficiency at the cost of increased complexity. The contrast in this paper between common and independent codebook generation is a generalization of that between repetition and parallel channel coding presented in [6].

This paper considers the class of relaying methods where only the destination terminal is required to correctly decode the transmitted information signal (destination decoding). This includes cooperative diversity protocols such as adaptive or selective decode-and-forward (DF) relaying and amplify-and-forward (AF) relaying [5], where it has been shown that it is not necessary for all intermediate relay terminals to correctly decode in order for the destination to correctly decode, and that outage events at intermediate relays do not degrade the diversity order of the system. The probability of outage of relay networks where only the destination terminal is required to correctly decode is at least the probability of outage at any cut set in the network, since an outage event at any cut set will result in an outage event at the destination. Fig. 1 shows an example network, annotated with the achievable diversity order of each cut set that is relevant

II. SYSTEM MODEL

Let S_R denote the complete set of distinct cut sets associated with the directed network graph, let L_i denote the set of inter-terminal links associated with a particular cut set S_i, and let each $L_{k,l} \in L_i$ denote the inter-terminal link that joins terminals T_k and T_l across cut set S_i. Also, let $S_{O(m)}$ denote the set of cut sets that are associated with m inter-terminal links (i.e. $|L_i| = m, \forall S_i \in S_{O(m)}$ where $|L_i|$ is the cardinality of L_i). Notation of the form x_{T_i} is abbreviated to x_i for simplicity of exposition. The link SNR at between terminal T_k and terminal T_i is given by

$$\psi_{k,i} = SNR\mu_{k,i}|a_{k,i}|^2, \quad (1)$$

where SNR is a reference signal to noise ratio, $\mu_{k,i}$ is a scaling factor for each link SNR with respect to the reference signal to noise ratio, and $a_{k,i}$ captures the effects of distance-dependant attenuation, shadowing, and fading between T_k and T_i. For the case of mutually independent flat slow Raleigh fading each $a_{k,i}$ is modeled as an iid complex Gaussian random variable with variance $\sigma^2_{k,i}$.

It is assumed that all relays operate in half-duplex mode and that a network with N transmitters in general requires $2 \le K \le N$ orthogonal channels, resulting in a rate factor of

$1/K$. The system model does not imply any particular method by which the set of relay terminals, or set of active links between the pairs of terminals, are chosen. The method used in the remainder to calculate the diversity order based on the $SNR \to \infty$ behavior of the probability of the maximum average mutual information falling below a target rate R is similar to that used in [4]. Different from [1], for practicality the model is constrained to non-overlapping symbol periods.

III. Common Codebook Generation

This section derives bounds on the maximum achievable diversity order and high SNR probability of outage of wireless relay networks employing common codebook generation. This form of codebook generation encompasses practical combination schemes that do not require a separate orthogonal channel for each inter-terminal link, including repetition coding and space-time coding. The maximum average mutual information when multiple inter-terminal links are combined takes the form of a log-sum, $I = \log(1+\sum\psi)$, and the probability of outage at the destination terminal is at least the probability of outage at any cut set in the network.

The maximum average mutual information at cut set S_i (across all the inter-terminal links associated with cut set S_i) is given by

$$I_i = \frac{1}{K}\log\left(1+SNR\sum_{L_{k,l}\in L_i}\mu_{k,l}\left|a_{k,l}\right|^2\right), \quad (2)$$

and the probability of outage at S_i is the probability of the maximum average mutual information falling below a target rate R, and is given by

$$p_i^{out}(SNR,R) = \Pr\left[\sum_{L_{k,l}\in L_i}\mu_{k,l}\left|a_{k,l}\right|^2 < \frac{2^{KR}-1}{SNR}\right]. \quad (3)$$

Since outage events at different cut sets are not necessarily independent due to the possibility of shared inter-terminal links, the total probability of outage at any cut set in the network is given by

$$p^{out}(SNR,R) = \Pr\left[\bigcup_{S_i\in S_R}\left(\sum_{L_{k,l}\in L_i}\mu_{k,l}\left|a_{k,l}\right|^2 < \frac{2^{KR}-1}{SNR}\right)\right], \quad (4)$$

which when expanded to show all possible terminal outage event combinations is expressed as

$$\begin{aligned} p^{out}(SNR,R) = &\sum_{S_i\in S_R}\left(\Pr\left[\sum_{L_{k,l}\in L_i}\mu_{k,l}\left|a_{k,l}\right|^2 < \frac{2^{KR}-1}{SNR}\right]\right) \\ &-\sum_{\substack{S_i,S_j\in S_R\\ S_i\neq S_j}}\left(\begin{array}{l}\Pr\left[\sum_{L_{k,l}\in L_i}\mu_{k,l}\left|a_{k,l}\right|^2 < \frac{2^{KR}-1}{SNR}\right] \\ \times\Pr\left[\left(\sum_{L_{k,l}\in L_j}\mu_{k,l}\left|a_{k,l}\right|^2 < \frac{2^{KR}-1}{SNR}\right) \middle| \left(\sum_{L_{k,l}\in L_i}\mu_{k,l}\left|a_{k,l}\right|^2 < \frac{2^{KR}-1}{SNR}\right)\right]\end{array}\right) \\ &+\sum \textit{other terms with an odd \# of outage events} \\ &-\sum \textit{other terms with an even \# of outage events} \end{aligned} \quad (5)$$

This can be further expanded to separate out terms involving cut sets with the minimum number of inter-terminal links M_S, and taken to the limit as $SNR \to \infty$ to result in

$$\begin{aligned} &\frac{p^{out}(SNR,R)}{\left(\frac{2^{KR}-1}{SNR}\right)^{M_S}} \\ &= \left(\begin{array}{l} \sum_{\substack{S_i\in S_R\\ S_i\in S_{O(M_S)}}}\left(\underbrace{\left(\frac{2^{KR}-1}{SNR}\right)^{-M_S}\Pr\left[\sum_{L_{k,l}\in L_i}\mu_{k,l}\left|a_{k,l}\right|^2 < \frac{2^{KR}-1}{SNR}\right]}_{\to (M_S!)^{-1}\prod_{L_{k,l}\in L_i}\left(\mu_{k,l}\sigma_{k,l}^2\right)^{-1}}\right) \\ +\sum_{\substack{S_i\in S_R\\ S_i\notin S_{O(M_S)}}}\left(\underbrace{\left(\frac{2^{KR}-1}{SNR}\right)^{-M_S}\Pr\left[\sum_{L_{k,l}\in L_i}\mu_{k,l}\left|a_{k,l}\right|^2 < \frac{2^{KR}-1}{SNR}\right]}_{\to 0}\right) \end{array}\right) \\ &- \left(\begin{array}{l} \sum_{\substack{S_i,S_j\in S_R\\ S_i\neq S_j\\ S_i\in S_{O(M_S)}}}\left(\begin{array}{l}\underbrace{\left(\frac{2^{KR}-1}{SNR}\right)^{-M_S}\Pr\left[\sum_{L_{k,l}\in L_i}\mu_{k,l}\left|a_{k,l}\right|^2 < \frac{2^{KR}-1}{SNR}\right]}_{\to (M_S!)^{-1}\prod_{L_{k,l}\in L_i}\left(\mu_{k,l}\sigma_{k,l}^2\right)^{-1}} \\ \times\underbrace{\Pr\left[\left(\sum_{L_{k,l}\in L_j}\mu_{k,l}\left|a_{k,l}\right|^2 < \frac{2^{KR}-1}{SNR}\right)\middle|\left(\sum_{L_{k,l}\in L_i}\mu_{k,l}\left|a_{k,l}\right|^2 < \frac{2^{KR}-1}{SNR}\right)\right]}_{\to 0}\end{array}\right) \\ +\sum_{\substack{S_i,S_j\in S_R\\ S_i\neq S_j\\ S_i\notin S_{O(M_S)}}}\left(\begin{array}{l}\underbrace{\left(\frac{2^{KR}-1}{SNR}\right)^{-M_S}\Pr\left[\sum_{L_{k,l}\in L_i}\mu_{k,l}\left|a_{k,l}\right|^2 < \frac{2^{KR}-1}{SNR}\right]}_{\to 0} \\ \times\underbrace{\Pr\left[\left(\sum_{L_{k,l}\in L_j}\mu_{k,l}\left|a_{k,l}\right|^2 < \frac{2^{KR}-1}{SNR}\right)\middle|\left(\sum_{L_{k,l}\in L_i}\mu_{k,l}\left|a_{k,l}\right|^2 < \frac{2^{KR}-1}{SNR}\right)\right]}_{\to 0}\end{array}\right) \end{array}\right) \\ &+\underbrace{\sum \textit{other terms with an odd \# of outage events}}_{\to 0} \\ &-\underbrace{\sum \textit{other terms with an even \# of outage events}}_{\to 0} \\ &\to \sum_{\substack{S_i\in S_R\\ S_i\in S_{O(M_S)}}}\left(\frac{1}{M_S!}\prod_{L_{k,l}\in L_i}\frac{1}{\mu_{k,l}\sigma_{k,l}^2}\right) \end{aligned}, \quad (6)$$

where the asymptotic approximation uses CDF results for the sum of independent exponential random variables similar to the method in [4].

The probability of outage at high SNR can therefore be approximated by

$$p^{out}(SNR,R) \approx \sum_{\substack{S_i\in S_R\\ S_i\in S_{O(M_S)}}}\left(\frac{1}{M_S!}\prod_{L_{k,l}\in L_i}\frac{1}{\mu_{k,l}\sigma_{k,l}^2}\right)\left(\frac{2^{KR}-1}{SNR}\right)^{M_S}, \quad (7)$$

and the maximum diversity order of the network is given by

$$d = -\lim_{SNR\to\infty}\frac{\log p^{out}(SNR,R)}{\log SNR} = M_S = \min_{S_i\in S_R}\{|L_i|\}, \quad (8)$$

the minimum number of inter-terminal links across all cut sets in the network. This is equivalent to the number of disjoint paths through the network joining the source and destination. Furthermore, inter-terminal links that are not associated with cut sets with the minimum number of inter-terminal links do not asymptotically (at high SNR) affect the probability of outage.

IV. Independent Codebook Generation

This section derives bounds on the maximum achievable diversity order and high SNR probability of outage of wireless relay networks employing independent codebook generation. This form of codebook generation provides an information theoretic lower bound on the probability of outage of all achievable codebook generation schemes. The maximum average mutual information when multiple inter-terminal links are combined takes the form of a sum-of-logs, $I = \sum \log(1+\psi)$, and the probability of outage at the destination terminal is at least the probability of outage at any cut set in the network.

The maximum average mutual information at cut set S_i is given by

$$I_i = \frac{1}{K}\sum_{L_{k,l}\in L_i}\log\left(1+SNR\mu_{k,l}|a_{k,l}|^2\right) = \frac{1}{K}\log\prod_{L_{k,l}\in L_i}\left(1+SNR\mu_{k,l}|a_{k,l}|^2\right), \quad (9)$$

and the probability of outage at S_i is the probability of the maximum average mutual information falling below a target rate R, and is given by

$$p_i^{out}(SNR,R) = \Pr\left[\prod_{L_{k,l}\in L_i}\left(1+SNR\mu_{k,l}|a_{k,l}|^2\right) < 2^{KR}\right]. \quad (10)$$

This can be lower bounded by

$$p_i^{out}(SNR,R) \ge \prod_{L_{k,l}\in L_i}\Pr\left[\left(1+SNR\mu_{k,l}|a_{k,l}|^2\right) < 2^{KR/|L_i|}\right] \ge \prod_{L_{k,l}\in L_i}\Pr\left[\mu_{k,l}|a_{k,l}|^2 < \frac{2^{KR/|L_i|}-1}{SNR}\right], \quad (11)$$

where the lower bound results from applying the relation $\Pr\left[\prod_k x_k < \prod_k y_k\right] \ge \prod_k \Pr[x_k < y_k]$ for independent x_k and y_k. This lower bound is used instead of the more precise result (17) of [6] as it produces a much simpler, non-recursive expression for the probability of outage at high SNR. However, result (17) of [6] could readily be applied.

Since outage events at different cut sets are not necessarily independent due to the possibility of shared inter-terminal links, the total probability of outage at any cut set in the network is given by

$$p^{out}(SNR,R) = \Pr\left[\bigcup_{S_i\in S_R}\left(\prod_{L_{k,l}\in L_i}\left(1+SNR\mu_{k,l}|a_{k,l}|^2\right) < 2^{KR}\right)\right], \quad (12)$$

which when expanded to show all possible terminal outage event combinations and using the lower bound of (11) is expressed as

$$\begin{aligned} p^{out}(SNR,R) \ge & \sum_{S_i\in S_R}\left(\prod_{L_{k,l}\in L_i}\Pr\left[\mu_{k,l}|a_{k,l}|^2 < \frac{2^{KR/|L_i|}-1}{SNR}\right]\right) \\ & - \sum_{\substack{S_i,S_j\in S_R\\ S_i\neq S_j}}\left(\begin{array}{l}\prod_{L_{k,l}\in L_i}\Pr\left[\mu_{k,l}|a_{k,l}|^2 < \frac{2^{KR/|L_i|}-1}{SNR}\right] \\ \times\Pr\left[\bigcap_{L_{k,l}\in L_j}\left(\mu_{k,l}|a_{k,l}|^2 < \frac{2^{KR/|L_j|}-1}{SNR}\right) \Big| \bigcap_{L_{k,l}\in L_i}\left(\mu_{k,l}|a_{k,l}|^2 < \frac{2^{KR/|L_i|}-1}{SNR}\right)\right]\end{array}\right) \\ & + \sum \text{other terms with an odd \# of outage events} \\ & - \sum \text{other terms with an even \# of outage events} \end{aligned}. \quad (13)$$

This can be further expanded to separate out terms involving cut sets with the minimum number of inter-terminal links M_S, and taken to the limit as $SNR \to \infty$ to result in

$$\begin{aligned} \frac{p^{out}(SNR,R)}{\left(\frac{2^{KR/M_S}-1}{SNR}\right)^{M_S}} \ge & \left(\begin{array}{l}\sum_{\substack{S_i\in S_R\\ S_i\in S_{O(M_S)}}}\left(\underbrace{\left(\frac{2^{KR/M_S}-1}{SNR}\right)^{-M_S}\prod_{L_{k,l}\in L_i}\Pr\left[\mu_{k,l}|a_{k,l}|^2 < \frac{2^{KR/|L_i|}-1}{SNR}\right]}_{\to\prod_{L_{k,l}\in L_i}(\mu_{k,l}\sigma_{k,l}^2)^{-1}}\right) \\ + \sum_{\substack{S_i\in S_R\\ S_i\notin S_{O(M_S)}}}\left(\underbrace{\left(\frac{2^{KR/M_S}-1}{SNR}\right)^{-M_S}\prod_{L_{k,l}\in L_i}\Pr\left[\mu_{k,l}|a_{k,l}|^2 < \frac{2^{KR/|L_i|}-1}{SNR}\right]}_{\to 0}\right)\end{array}\right) \\ & - \left(\begin{array}{l}\sum_{\substack{S_i,S_j\in S_R\\ S_i\neq S_j\\ S_i\in S_{O(M_S)}}}\left(\begin{array}{l}\underbrace{\left(\frac{2^{KR/M_S}-1}{SNR}\right)^{-M_S}\prod_{L_{k,l}\in L_i}\Pr\left[\mu_{k,l}|a_{k,l}|^2 < \frac{2^{KR/|L_i|}-1}{SNR}\right]}_{\to\prod_{L_{k,l}\in L_i}(\mu_{k,l}\sigma_{k,l}^2)^{-1}} \\ \times\underbrace{\Pr\left[\bigcap_{L_{k,l}\in L_j}\left(\mu_{k,l}|a_{k,l}|^2 < \frac{2^{KR/|L_j|}-1}{SNR}\right) \Big| \bigcap_{L_{k,l}\in L_i}\left(\mu_{k,l}|a_{k,l}|^2 < \frac{2^{KR/|L_i|}-1}{SNR}\right)\right]}_{\to 0}\end{array}\right) \\ + \sum_{\substack{S_i,S_j\in S_R\\ S_i\neq S_j\\ S_i\notin S_{O(M_S)}}}\left(\begin{array}{l}\underbrace{\left(\frac{2^{KR/M_S}-1}{SNR}\right)^{-M_S}\prod_{L_{k,l}\in L_i}\Pr\left[\mu_{k,l}|a_{k,l}|^2 < \frac{2^{KR/|L_i|}-1}{SNR}\right]}_{\to 0} \\ \times\underbrace{\Pr\left[\bigcap_{L_{k,l}\in L_j}\left(\mu_{k,l}|a_{k,l}|^2 < \frac{2^{KR/|L_j|}-1}{SNR}\right) \Big| \bigcap_{L_{k,l}\in L_i}\left(\mu_{k,l}|a_{k,l}|^2 < \frac{2^{KR/|L_i|}-1}{SNR}\right)\right]}_{\to 0}\end{array}\right)\end{array}\right) \\ & + \sum \underbrace{\text{other terms with an odd \# of outage events}}_{\to 0} \\ & - \sum \underbrace{\text{other terms with an even \# of outage events}}_{\to 0} \\ \to & \sum_{\substack{S_i\in S_R\\ S_i\in S_{O(M_S)}}}\left(\prod_{L_{k,l}\in L_i}\frac{1}{\mu_{k,l}\sigma_{k,l}^2}\right) \end{aligned}, \quad (14)$$

where the asymptotic approximation again uses CDF results for the sum of independent exponential random variables.

The probability of outage at high SNR can therefore be approximated by

$$p^{out}(SNR,R) \approx \sum_{\substack{S_i \in S_R \\ S_i \in S_{O(M_S)}}} \left(\prod_{L_{k,l} \in L_i} \frac{1}{\mu_{k,l}\sigma_{k,l}^2} \right) \left(\frac{2^{KR/M_S}-1}{SNR} \right)^{M_S}, \quad (15)$$

and the maximum diversity order of the network is given by

$$d = -\lim_{SNR\to\infty} \frac{\log p^{out}(SNR,R)}{\log SNR} = M_S = \min_{S_i \in S_R} \{|L_i|\}, \quad (16)$$

the minimum number of inter-terminal links across all cut sets in the network. We see that the maximum diversity order of independent codebook generation is identical to that of common codebook generation. This result indicates that for networks where only the destination terminal is required to correctly decode the transmitted information signal, independent codebook generation does not offer any diversity gain over common codebook generation. This is a generalization of the similar result in [6] to cooperative networks with any number of relay terminals and any possible combination of links between cooperating terminals.

There is a difference in the probability of outage at high SNR of independent codebook generation in comparison to that of common codebook generation. The high SNR probability of outage gain factor of independent codebook generation over common codebook generation can be upper bounded by taking the ratio of (7) and (15), and is given by

$$\frac{p_C^{out}(SNR,R)}{p_I^{out}(SNR,R)} \le \frac{\sum_{\substack{S_i \in S_R \\ S_i \in S_{O(M_S)}}} \left(\frac{1}{M_S!} \prod_{L_{k,l} \in L_i} \frac{1}{\mu_{k,l}\sigma_{k,l}^2} \right) \left(\frac{2^{K_C R}-1}{SNR} \right)^{M_S}}{\sum_{\substack{S_i \in S_R \\ S_i \in S_{O(M_S)}}} \left(\prod_{L_{k,l} \in L_i} \frac{1}{\mu_{k,l}\sigma_{k,l}^2} \right) \left(\frac{2^{K_I R/M_S}-1}{SNR} \right)^{M_S}} \le \frac{1}{M_S!} \left(\frac{2^{K_C R}-1}{2^{K_I R/M_S}-1} \right)^{M_S}, \quad (17)$$

where again K_C is the number of channels required for common codebook generation and K_I is the number of channels required for independent codebook generation. The high SNR probability of outage gain factor indicates that independent codebook generation can offer a significant probability of outage improvement over common codebook generation, but only under the conditions that the minimum number of inter-terminal links across all cut sets in the network is high and the number of channels required for independent codebook generation is not significantly larger than the number of channels required for common codebook generation. If independent codebook generation requires more channels than common codebook generation and the diversity order is low, then the high SNR probability of outage gain factor may be less than unity (increased outage). If the same number of channels is required for both independent codebook generation and common codebook generation then the high SNR probability of outage gain factor will always be greater than unity (decreased outage).

V. Extension To Multiple Antennas

Recent wireless base-stations, whether they are the source terminal in downlink transmissions or the destination terminal in uplink transmissions, are already starting to incorporate multiple antennas. This trend is expected to become even more prevalent in the future. Additionally, the incorporation of cooperative diversity techniques in fixed relay networks raises the possibility that some wireless relay terminals may have sufficient physical geography to support multiple antennas with reasonable correlation characteristics. Therefore, wireless relay networks where some or all of the cooperating terminals employ multiple antennas are of significant interest.

We now extend the system model to the case where there may be more than one physical antenna at each terminal. A redefinition of terminology is required. Let A_i denote the set of antennas at terminal T_i, let A_{ij} denote the j^{th} antenna of terminal T_i, let L_i denote the set of inter-antenna links associated with a particular cut set S_i, let $S_{O(m)}$ denote the set of cut sets that are associated with m inter-antenna links (i.e. $|L_i| = m, \forall S_i \in S_{O(m)}$).

The link SNR between the j^{th} antenna of terminal T_i and the l^{th} antenna of terminal T_k is given by

$$\psi_{kl,ij} = SNR\mu_{kl,ij}|a_{kl,ij}|^2 / |A_k|, \quad (18)$$

where $\mu_{kl,ij}$ is a scaling factor for each link SNR with respect to the reference signal to noise ratio, and $a_{kl,ij}$ is the composite signal amplitude attenuation factor for the link $L_{kl,ij} \in L_i$ between the l^{th} antenna of terminal T_k and the j^{th} antenna of terminal T_i. Each $a_{kl,ij}$ is modeled as an iid complex Gaussian random variable with variance $\sigma_{kl,ij}^2$. It is assumed that the total transmit power at each terminal is kept constant with respect to the single antenna case such that the set of transmit antennas equally partition the transmit power. With this revised terminology, the results when there may be more than one physical antenna at each terminal are straightforward generalizations of the results for exactly one physical antenna at each terminal.

When there are an arbitrary number of physical antennas per terminal in networks with common codebook generation, the process for calculating the maximum achievable diversity order is identical to that with a single antenna per terminal with the exception that the log-sum is over the links between all pairs of antennas. For networks with common codebook generation the probability of outage at high SNR can be approximated by

$$p^{out}(SNR,R) \approx \sum_{\substack{S_i \in S_R \\ S_i \in S_{O(M_S)}}} \left(\frac{1}{M_S!} \prod_{L_{kl,ij} \in L_i} \frac{|A_k|}{\mu_{kl,ij}\sigma_{kl,ij}^2} \right) \left(\frac{2^{KR}-1}{SNR} \right)^{M_S}, \quad (19)$$

and the maximum diversity order of the network is given by

$$d = -\lim_{SNR \to \infty} \frac{\log p^{out}(SNR, R)}{\log SNR} = M_S = \min_{S_i \in S_R} \{|L_i|\}, \quad (20)$$

where M_S is the minimum number of inter-antenna links across all cut sets. The derivation of (19) follows that for the case of a single antenna per terminal.

When there are an arbitrary number of physical antennas per terminal in networks with independent codebook generation, the process for calculating the maximum achievable diversity order is identical to that with a single antenna per terminal with the exception that the sum-of-logs is over the links between all pairs of antennas. For networks with independent codebook generation the probability of outage at high SNR can be approximated by

$$p^{out}(SNR, R) \approx \sum_{\substack{S_i \in S_R \\ S_i \in S_{O(M_S)}}} \left(\prod_{L_{kl,ij} \in L_i} \frac{|A_k|}{\mu_{kl,ij} \sigma^2_{kl,ij}} \right) \left(\frac{2^{KR/M_S} - 1}{SNR} \right)^{M_S}, \quad (21)$$

and the maximum diversity order of the network is given by

$$d = -\lim_{SNR \to \infty} \frac{\log p^{out}(SNR, R)}{\log SNR} = M_S = \min_{S_i \in S_R} \{|L_i|\}, \quad (22)$$

where M_S is the minimum number of inter-antenna links across all cut sets. The derivation of (21) follows that for the case of a single antenna per terminal. This result indicates that for networks where only the destination terminal is required to correctly decode the transmitted information signal, independent codebook generation does not offer any diversity gain over common codebook generation even when there are an arbitrary number of physical antennas per terminal.

The high SNR probability of outage gain factor of independent codebook generation over common codebook generation can be upper bounded by taking the ratio of (19) and (21), and is given by

$$\frac{p_C^{out}(SNR, R)}{p_I^{out}(SNR, R)} \le \frac{\displaystyle\sum_{\substack{S_i \in S_R \\ S_i \in S_{O(M_S)}}} \left(\frac{1}{M_S!} \prod_{L_{kl,ij} \in L_i} \frac{|A_k|}{\mu_{kl,ij} \sigma^2_{kl,ij}} \right) \left(\frac{2^{K_C R} - 1}{SNR} \right)^{M_S}}{\displaystyle\sum_{\substack{S_i \in S_R \\ S_i \in S_{O(M_S)}}} \left(\prod_{L_{kl,ij} \in L_i} \frac{|A_k|}{\mu_{kl,ij} \sigma^2_{kl,ij}} \right) \left(\frac{2^{K_I R/M_S} - 1}{SNR} \right)^{M_S}} . \quad (23)$$

$$\le \frac{1}{M_S!} \left(\frac{2^{K_C R} - 1}{2^{K_I R/M_S} - 1} \right)^{M_S}$$

Again, we note that independent codebook generation can offer a significant probability of outage improvement over common codebook generation under certain conditions. This has a higher likelihood of being the case when there are multiple antennas per terminal as the value of M_S will in general be larger.

VI. CONCLUSION

This paper has derived bounds on the maximum achievable diversity order and high SNR probability of outage of wireless relay networks with arbitrary link connectivity between cooperating terminals. Two general codebook generation schemes are considered and compared: common codebook generation, where the source and all relays share a common randomly generated codebook, and independent codebook generation, where the source and all relays employ independent randomly generated codebooks. This paper considers the idealized class of relaying methods where only the destination terminal is required to correctly decode the transmitted information signal, and therefore the presented probability of outage results for independent codebook generation are a lower bound over possible relaying methods and codebook generation schemes.

When there is a single antenna per terminal, the maximum achievable diversity order is constrained by the minimum number of inter-terminal links across all cut sets in the network. When there are an arbitrary number of antennas per terminal it is shown that the maximum achievable diversity order is constrained by the minimum number of associated inter-antenna links across all cut sets in the network. Furthermore, links that are not associated with cut sets with the minimum number of inter-terminal or inter-antenna links do not asymptotically (at high SNR) affect the probability of outage. These diversity results do not depend on the codebook generation scheme.

The results indicate that although independent codebook generation does not offer any diversity gain over common codebook generation, it can offer a significant probability of outage improvement under the conditions that the minimum number of inter-terminal or inter-antenna links across all cut sets in the network is high and the number of channels required for independent codebook generation is not significantly larger than the number of channels required for common codebook generation.

REFERENCES

[1] K. Azarian, H. Gamal, and P. Schniter, "On the Achievable Diversity-Multiplexing Tradeoff in Half-Duplex Cooperative Channels," Proc. of *Allerton Conference on Communications, Control, Computing*, pp. 4152-4172, 2004.

[2] J. Boyer, D. D. Falconer, and H. Yanikomeroglu, "Multihop Diversity in Wireless Relaying Channels," *IEEE Trans. on Communications*, vol. 52, no. 10, pp. 1820-1830, 2004.

[3] J. Boyer, D. Falconer, and H. Yanikomeroglu, "Cooperative Connectivity Models for Wireless Relay Networks," submitted to *IEEE Trans. on Wireless Communications*.

[4] J. Laneman and G. Wornell, "Distributed Space-Time-Code Protocols for Exploiting Cooperative Diversity in Wireless Networks," *IEEE Trans. on Information Theory*, vol. 49, no. 10, pp. 2415-2425, 2003.

[5] J. Laneman, D. Tse, and G. Wornell, "Cooperative Diversity in Wireless Networks: Efficient Protocols and Outage Behavior," *IEEE Trans. on Information Theory*, vol. 50, no. 12, pp. 3062-3080, 2004.

[6] J. Laneman, "Limiting Analysis of Outage Probabilities for Diversity Schemes in Fading Channels," *IEEE Global Telecommunications Conference*, pp. 1242-1246, 2003.

[7] A. Sendonaris, E. Erkip, and B. Aazhang, "User Cooperation Diversity – Part I: System Description / Part II: Implementation Aspects and Performance Analysis," *IEEE Trans. on Communications*, vol. 51, no. 11, pp. 1927-1948, 2003.

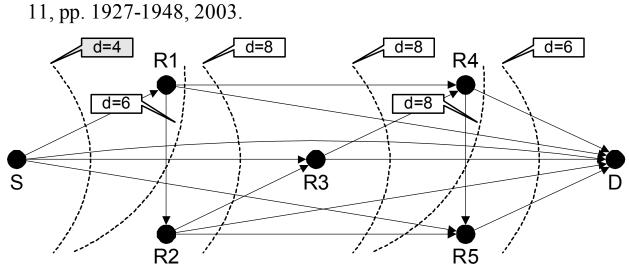

Fig. 1. Example Network with Cut Sets and Diversity Order (d=4)