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Author(s): S. S. STEVENS
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THE PSYCHOPHYSICS OF SENSORY FUNCTION*

By S. S. STEVENS

AN INQUIRY into the nature of sensory communication begins properly with psychophysics, the hundred-year-old discipline concerned with the responses that organisms make to the energies of the environment. We live in a restless world of energetic forces, some of which affect us and some of which, like radio waves, impinge upon us and pass unnoticed because we have no sense organs able to transduce them. But we see lights, hear sounds, taste substances, and smell vapors; and it is these elementary facts of psychophysics that stir our interest in the anatomy and physiology of the mechanisms that make sensation possible. An orderly and systematic account of sensory communication must include a delineation of *what* is perceived as well as an explanation of *how* perception is accomplished. In this sense, psychophysics defines the challenge: it tells what the organism can do and it asks those who are inspired by such mysteries to try, with scalpel, electrode, and test tube, to advance our understanding of how such wonders are performed.

It must be confessed at the outset that psychophysics has often failed to do its part of the job with distinction. Its task is not easy. For one thing, long-standing prejudices, derived in great measure from a chronic dualistic metaphysics, have triggered a variety of stubborn objections whenever it has been proposed that sensation may be amenable to orderly and quantitative investigation. You cannot, the objectors complain, measure the inner, private, subjective strength of a sensation. Perhaps not, in the sense the objectors have in mind, but in a different and very useful sense the strength of a sensation can, as we shall see, be fruitfully quantified. We must forego arguments about the private life of the mind and ask sensible objective questions about the input-output relations of sensory transducers as these relations are disclosed in the behavior of experimental organisms, whether men or animals.

Another difficulty is that psychophysics had an unfortunate childhood. Although Plateau, in the 1850's, made a half-hearted attempt to suggest the proper form of the function relating apparent sensory intensity to stimulus intensity, he was shouted down by Fechner, who saddled the infant discipline with the erroneous "law" that bears his name (see Stevens, 1957b). Perhaps the hardest task before us is to clear the scientific bench-top of the century-old dogma that sensation

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intensity grows as the logarithm of stimulus intensity (Fechner's law). The relation is not a logarithmic function at all. On more than a score of sensory continua it has now been shown that apparent, or subjective, magnitude grows as a *power function* of stimulus intensity, and the exponents of the power function have been found to range from about 0.33 for brightness to about 3.5 for electric shock (60 c.p.s) applied to the fingers. There seems to exist, in other words, a simple and pervasive psychophysical law, a law that was once conjectured by Plateau and later abandoned by him, a law that is congenial not only to the mounting empirical evidence, but also to certain reasonable principles of theory construction (Luce, 1959). There will be more to say about the power law, but first a few words about Fechner.

The misconception began when Fechner, in 1850, espoused the view that error itself provides a unit of measurement. He called it the just noticeable difference (jnd). Under most circumstances the jnd is a statistical concept, a measure of the dispersion or variability of a discriminatory response, in short, a measure of error. In deriving his logarithmic law, Fechner made the erroneous assumption that error is constant all up and down the psychological scale. Although he was willing to assume that at the stimulus level error is relative, i.e., $\Delta\phi = k\phi$ (Weber's law), he assumed that at the psychological level $\Delta\psi$ equals a constant. From these two assumptions he derived the relation, $\psi = k \log \phi$, and thereby caused much mischief.

It is curious indeed that Fechner, a physicist, should have assumed that error, or variability of judgment, is constant all up and down the psychological continuum. Most variables do not behave that way. On the continua with which a physicist most often deals, error is usually not constant but tends to vary with magnitude. It is percentage error that typically stays constant; precision can generally be stated as one part in so many.

Suppose Fechner had taken this as his model, not only for the stimulus jnd $\Delta\phi$, but also for the subjective jnd $\Delta\psi$. He then could have written

$$\Delta\psi/\psi = k\Delta\phi/\phi$$

from which it would follow that the psychological magnitude ψ is a power function of the physical magnitude ϕ. But he fought off this suggestion when it was first made (by Brentano), and, with Fechner's temporary victory, psychophysics entered upon a period of futility during which there seemed to be no more interesting work to do than measure the jnd. And the logarithmic law became "an idol of the den."

So much for the past. Since the 1930's, psychophysics has been staging a comeback. New interest in the age-old problem of sensory response has been kindled by the invention of procedures for assessing the over-all input-output operating characteristics of the intact sensory system.

These methods show that sensory response grows according to a power law. So rarely does it happen in the study of behavior that a simple relation can be shown to hold under many diverse kinds of stimulation, that the widespread invariance of the power law becomes a matter of large significance.

Measurement

The problem of the laws that govern the reactions of sentient organisms is intimately bound up with the problem of measurement. Since the theory of measurement was thoroughly explored in another symposium (Churchman and Ratoosh, 1959), it need not divert us here. It may be helpful, however, to refer to Table I which attempts a systematic classification of scales of measurement in a compact form (Stevens, 1946a, 1951). The four scales listed, *nominal*, *ordinal*, *interval*, and *ratio*, are those most commonly used in the business of science, and all of them get involved in research on sensory communication.

The nominal scale, the most general of the lot, is not always thought of as a form of measurement, mainly because names or letters, rather than numbers, are most often employed to designate the categories or classes used in nominal scaling. Yet, this ubiquitous and important form of measurement goes on constantly, for it includes the process of identifying and classifying. Mostly we take only a casual interest in such problems, but our interest has a way of turning into animated curiosity when it becomes a question of doing the detective work necessary to pin the proper labels on the functional parts of the central nervous system. The identification of the "areas" associated with this or that sensory process constitutes a lively exercise in nominal scaling. And, needless to say, much of our scientific effort in this field never goes beyond the essential and basic nominal level. The ordinary determination of thresholds, which involves the categorization of stimuli into classes (e.g., seen and not seen), is another important instance of nominal scaling (Stevens, 1958).

The key to the nature of the four kinds of scales lies in a powerful but simple principle: the concept of invariance. When we have carried out a series of empirical operations, e.g., comparisons, orderings, balancings, etc., we assign a set of numbers to reflect the outcome of the operations. This is the essence of measurement. But what kind of measurement have we achieved? That depends on the answer to the decisive question: in what ways can the scale numbers be transformed without loss of empirical information? As shown in Table I, each of the scales has its group of permissible transformations.

The ratio scale, the scale of greatest interest, allows only multiplication by a constant, as when we change from inches to centimeters. No more general transformation is allowed. If an arbitrary constant were to be added to the measured diameters of a set of nerve fibers, for example,

TABLE I.—A CLASSIFICATION OF SCALES OF MEASUREMENT.

Measurement is the assignment of numbers to objects or events according to rule. The rules and the resulting kinds of scales are tabulated below. The basic operations needed to create a given scale are all those listed in the second column, down to and including the operation listed opposite the scale. The third column gives the mathematical transformations that leave the scale form invariant. Any number x on a scale can be replaced by another number x' where x' is the function of x listed in column 3. The fourth column lists, cumulatively downward, examples of statistics that show invariance under the transformations of column 3 (the mode, however, is invariant only for discrete variables).

Scale	*Basic Empirical Operations*	*Mathematical Group-Structure*	*Permissible Statistics (Invariantive)*	*Typical Examples*
Nominal	Determination of equality	Permutation group $x' = f(x)$ where $f(x)$ means any one-to-one substitution	Number of cases Mode "Information" measures Contingency correlation	"Numbering" of football players Assignment of type or model numbers to classes
Ordinal	Determination of greater or less	Isotonic group $x' = f(x)$ where $f(x)$ means any increasing monotonic function	Median Percentiles Order correlation (type 0: interpreted as a test of order)	Hardness of minerals Grades of leather, lumber, wool, and so forth Intelligence-test raw scores
Interval	Determination of the equality of intervals or of differences	Linear or affine group $x' = ax + b$ $a > 0$	Mean Standard deviation Order correlation (type I: interpreted as r) Product moment (r)	Temperature (Fahrenheit and Celsius) Position on a line Calendar time Potential energy Intelligence-test "standard scores" (?)
Ratio	Determination of the equality of ratios	Similarity group $x' = cx$ $c > 0$	Geometric mean Harmonic mean Per cent variation	Length, numerosity, density, work, time intervals, and so forth Temperature (Kelvin) Loudness (sones) Brightness (br ils)

the resulting numbers would tell us less than we knew before. We would have lost some valuable information, namely, our knowledge of the *ratios* among the fiber diameters. Which fiber is twice as thick as some given fiber would now, with the altered scale values, be impossible to tell. In general, therefore, the more restricted are the admissible transformations, the more the scale is able to tell us.

As regards the measurement of sensation, the schema of Table I suggests that our aspiration should be to measure, where possible, on a ratio scale. This would call for assigning numbers to sensory magnitudes in such a way that anything more drastic than multiplication by a constant would result in a loss of information. Several variations on such a procedure have been elaborated (Stevens, 1958), but before we consider the resulting scales, certain distinctions need to be made. (Some nominal scaling needs to be done!)

Sensory Qualities

An obvious thing about sensations is that they differ in both kind and amount. Sweet is different from sour, but both may vary from weak to strong. The sensory qualities get named and classified (nominal scaling) but we try to measure the subjective intensities on higher-order scales.

The distinctive quality aroused by a given sensory excitation presents a baffling problem for which no plausible explanation is yet available. Why does a sound differ from a taste in the way it does? This qualitative aspect of the sensory world confronts us with a baffling succession of discontinuous leaps as we go from one sense modality to another, and no one seems to know why. On the other hand, it is at precisely this level that the anatomist and the neurophysiologist join the game and perform some of their most effective work in tracing pathways for the various modalities, and even for some of the separate qualities within a modality. Clearly, these problems of topography need to be clarified before an understanding of the sensory mechanisms can be anchored in the soup and substance of neural process. Connections by themselves may not explain it all, but connections are there, and it seems improbable that they count for nothing.

Two Kinds of Continua

Psychophysics progresses beyond the elementary task of naming sensory qualities as soon as it becomes concerned with sensations that appear to lie on a continuum of some sort. A continuum seems clearly to be involved when sensations vary in strength or intensity, but certain other attributes of sensory response seem also to form continua in the ordinary sense of the term.

It would greatly simplify the mission of psychophysics if all the sensory continua obeyed the same rules and did so in an invariant fashion. It turns out, however, that a basic distinction needs to be made between two kinds of continua, prothetic and metathetic. Loudness, for example, is prothetic; pitch is metathetic. An important difference between the psychophysical functions governing pitch and loudness is this: the jnd for pitch represents a constant distance on the scale of subjective pitch, measured in *mels*, whereas the jnd for loudness represents an increasing distance on the subjective scale of loudness, measured in *sones* (Stevens and Volkmann, 1940). In other words, provided they are measured in subjective units, the jnd for pitch is constant, but the jnd for loudness grows rapidly larger as loudness is increased. The uniformity of sensitivity or resolving power on the pitch continuum, and the nonuniformity on the loudness continuum, entail several other functional differences between pitch and loudness. These are discussed elsewhere (Stevens, 1957b).

The prothetic continua (loudness, brightness, and subjective intensity in general) seem to be concerned with *how much*. The metathetic continua (pitch, apparent azimuth, apparent inclination) have to do with *what kind* or *where* (position). Corresponding to these two functional classes, there seem to be two basic physiological mechanisms. Sensory discrimination can be mediated by either of two processes, the one *additive*, the other *substitutive* (Stevens, 1946b). We detect, for example, an increase in loudness when excitation is added to excitation already present. We detect a change in pitch when new excitation is substituted for excitation that has been removed. Or, to consider another modality, we can tell when a light pressure changes to a strong pressure at a given point on the skin (addition of excitation), and we can also tell when a stimulus is moved from one to another location (substitution of excitation). Whether all perceptual continua that behave in the prothetic manner are mediated by additive physiological processes is not certain, of course, but in at least some instances it seems evident that the existence of two basic kinds of physiological mechanisms is reflected in the behavior of the psychological scales and functions which we construct from subjective measurements in the sensory domain.

Most of what follows is concerned with prothetic continua, for they seem the more interesting and well-behaved. It should be noted, however, that the pitch continuum provides an example of a rather exciting attempt to match up and thereby "explain" several psychophysical functions by means of a physiological substratum. Position of maximal excitation on the basilar membrane appears to relate in a straightforward linear manner to several sensory functions, including the mel scale of subjective pitch, the jnd, and the so-called critical band width (Békésy and Rosenblith, 1951; Zwicker, Flottorp, and Stevens, 1957).

Three Kinds of Sensory Measures

Three separate classes of sensory scales are distinguished (and sometimes confused) in psychophysics:

1. *Discriminability scales.* These are constructed in the tradition of Fechner, or his modern counterpart, Thurstone. Some measure of jnd,

variability, confusion, or resolving power is employed as a unit, and a scale is constructed by counting off such units.

2. *Category scales* (*partition scales*). These are constructed by one or another variation on the procedure that Plateau invented when he required observers to partition a segment of a continuum into equal-appearing intervals. (Plateau had eight artists paint a gray that seemed to lie halfway between black and white.) Bisection is one partitioning

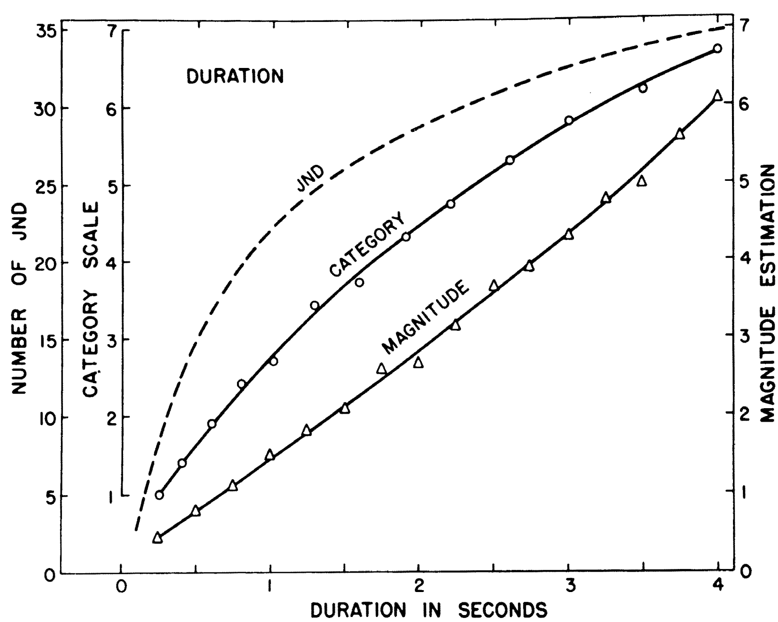

FIG. 1. Three kinds of psychological measures of apparent duration. *Triangles:* mean magnitude estimations by 12 observers who judged the apparent durations of white noises. *Circles:* mean category judgments by 16 observers on a scale from 1 to 7. The two end stimuli (0.25 and 4.0 sec) were presented at the outset to indicate the range, and each observer twice judged each duration on a 7-point scale. *Dashed curve:* discriminability scale obtained by counting off jnd.

procedure; asking a listener to assign a series of tones to n equally spaced categories is another.

3. *Magnitude scales.* These are ratio scales of apparent magnitude, constructed by one or another of four principal methods, of which "fractionation" is perhaps the best known and "magnitude estimation" the most useful (see Stevens, 1956b, 1959b). Under the method of magnitude estimation the observer simply estimates the apparent strength or intensity of his subjective impressions relative to a standard, or modulus, set either by himself or by the experimenter. The power functions obtained by this procedure can, and indeed should, be validated by direct

cross-modality matches—a procedure that does not require the observer to make numerical estimations.

An important difference between prothetic and metathetic continua is this: on metathetic continua all three kinds of scales tend to be linearly related one to another; on prothetic continua the three kinds of scales are always nonlinearly related (Stevens and Galanter, 1957; Stevens, 1959c). Typical examples of the relations among the three kinds of scales on prothetic continua are shown in Figure 1, for apparent duration

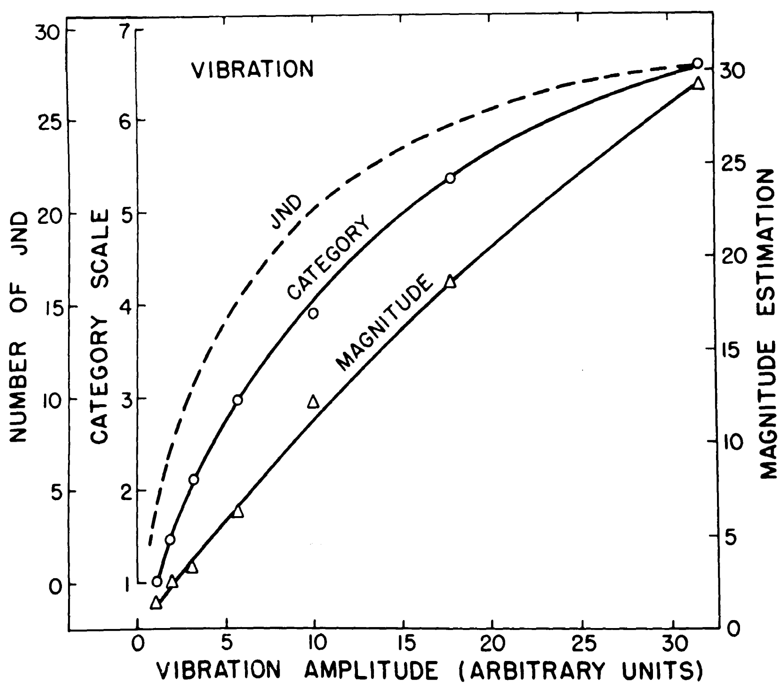

FIG. 2. Three kinds of psychological measures of the apparent intensity of a 60-cycle vibration applied to the fingertip. Procedures were essentially similar to those for Fig. 1. For details, see S. S. Stevens (1959).

(of a noise), and in Figure 2, for apparent intensity of vibration applied to the fingertip. On all prothetic continua the magnitude scale is a power function, the discriminability (jnd) scale approximates a logarithmic function, and the category scale assumes a form intermediate between the other two. Over the different sense modalities, these relations among the three scales are strikingly invariant; they constitute one of the really stable aspects of psychophysics.

Of the three kinds of measures shown in Figures 1 and 2, the one that seems most directly related to the over-all input-output function of a sensory system is the magnitude scale. The scale obtained by counting off jnd is really only that. At most it tells us how resolving power varies with stimulus magnitude. The category scale is at best only an interval scale on which the zero point is arbitrary. It is not a ratio scale. But, since it is nonlinearly related to the ratio scale of apparent magnitude, the category scale turns out, in fact, to be not even a good interval scale. The reasons for the curvature of the category scale have been discussed elsewhere (Stevens and Galanter, 1957). Roughly speaking, it is as though the observer, when he tries to partition a continuum into equal intervals, finds himself biased by the fact that a given *difference* at the low end of the scale is more noticeable or impressive than the same difference at the high end of the scale. This asymmetry is not present on metathetic continua, and therefore the category scale is not systematically curved.

Operating Characteristics

Sense organs serve as transducers that convert the energies of the environment into neural form. Like any transducer, each sense organ has its dynamic operating characteristic, defined by the input-output relation. It is only recently that much attention has been paid to the dynamics of sensory function—the manner in which the sensory system responds to variations in input intensity. Future efforts in this direction promise interesting rewards, however, for the form of the over-all dynamic process is now becoming more fully understood.

Conceivably, of course, all sense organs could have the same operating characteristic. All sensations would then grow at the same rate with increasing stimulus intensity. That this is far from true can be readily verified by a simple comparison. Note, for example, what happens when the luminance of a spot of light is doubled. Then note what happens when a 60-cycle current passing through the fingers is doubled. Doubling the luminance of a spot of light in a dark field has surprisingly little effect on its apparent brightness. As estimated by the typical (median) observer, the apparent increase is only about 25 per cent. But doubling the current through the fingers makes the sensation of shock seem about ten times as strong. The dynamic operating characteristics of these two sensory systems are clearly and dramatically different.

Closer investigation reveals, however, that both brightness and shock have a fundamental feature in common. In both instances the psychological magnitude ψ is related to the physical magnitude ϕ by

$$\psi = k\phi^n$$

The exponent n has the value 0.33 for brightness and 3.5 for shock. The value of k depends merely on one's choice of units. As will be shown below the physical measure used to express ϕ needs also to take account of the threshold.

The power function has the convenient feature that in log-log coordinates it plots as a straight line whose slope is equal to the value of the exponent. Figure 3 illustrates this fact and shows how the slow growth of brightness contrasts with the rapid growth of electric shock. Also included for comparison is the function obtained by asking observers to make magnitude estimations of the apparent length of various lines. Here, as we should expect, the slope (exponent) of the function is not very different from 1.0. This is another way of saying that to most people a length of 100 centimeters looks about twice as long as a length of 50 centimeters.

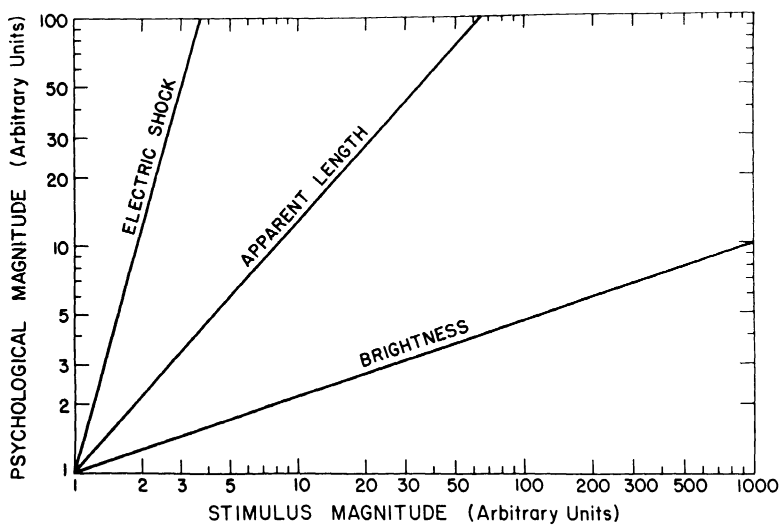

FIG. 3. Scales of apparent magnitude for three prothetic continua plotted in log-log coordinates. The slope of the line corresponds to the exponent of the power function governing the growth of the psychological magnitude.

The same three functions shown in Figure 3 are plotted in linear coordinates in Figure 4. The function for apparent length is almost a straight line (exponent about 1.1), but electric shock grows as an accelerating function and brightness as a decelerating function.

Exponents

The number of prothetic continua on which the psychophysical power law has been shown to hold to at least a first-order approximation now exceeds two dozen. In the author's experience, there appears to be no exception. (Hence the temerity of calling it a *law*.)

Table II lists the exponents of the power functions for some of the continua explored thus far. Although this table extends and revises the list presented earlier (Stevens, 1957b), it must still be regarded as tentative and incomplete, for there is virtually no limit to the number of different combinations of sense organs and stimuli that are waiting to be studied.

All the exponents in Table II were determined by the method of magnitude estimation. Many of them have been confirmed in other

TABLE II—REPRESENTATIVE EXPONENTS OF THE POWER FUNCTIONS RELATING PSYCHOLOGICAL MAGNITUDE TO STIMULUS MAGNITUDE ON PROTHETIC CONTINUA

Continuum	*Exponent*	*Conditions*
Loudness	0.6	Binaural
Loudness	0.55	Monaural
Brightness	0.33	5° target—dark-adapted eye
Brightness	0.5	Point source—dark-adapted eye
Lightness	1.2	Reflectance of gray papers
Smell	0.55	Coffee odor
Smell	0.6	Heptane
Taste	0.8	Saccharine
Taste	1.3	Sucrose
Taste	1.3	Salt
Temperature	1.0	Cold—on arm
Temperature	1.6	Warm—on arm
Vibration	0.95	60 c.p.s.—on finger
Vibration	0.6	250 c.p.s.—on finger
Duration	1.1	White noise stimulus
Repetition rate	1.0	Light, sound, touch, and shocks
Finger span	1.3	Thickness of wood blocks
Pressure on palm	1.1	Static force on skin
Heaviness	1.45	Lifted weights
Force of handgrip	1.7	Precision hand dynamometer
Vocal effort	1.1	Sound pressure of vocalization
Electric shock	3.5	60 c.p.s. through fingers

laboratories and by other methods, such as fractionation. Many of them, as we shall see, have also been validated by cross-modality intercomparisons. Nevertheless, it must be understood that the exact value of an exponent is difficult to determine with precision, and some of those listed in Table II must be regarded as first approximations only. In all cases, of course, the exponent represents an average value and is not necessarily appropriate to a particular individual. At least ten observers were used to determine each of the exponents in Table II, although some exponents (e.g., loudness, brightness, lifted weights) have been determined in several laboratories and on large numbers of observers.

The particular version of the method of magnitude estimation used in our most recent experiments—the one arrived at after some years of trial and error—is extremely simple. In an experiment on loudness, for example, the procedure may be as follows. The experimenter presents a "standard" sound of moderate intensity and tells the observer to consider its loudness to have the value "10." The experimenter then presents in irregular order a series of intensities above and below the standard and instructs the observer to assign to each stimulus a number proportional to the apparent loudness. In other words, the question is: if the standard is 10, what is each of the other stimuli? The observer is told to use any numbers that seem appropriate, fractions, decimals, or whole numbers, and to judge each stimulus as *he* hears it. The standard is usually presented only at the beginning of the series, although a stimulus

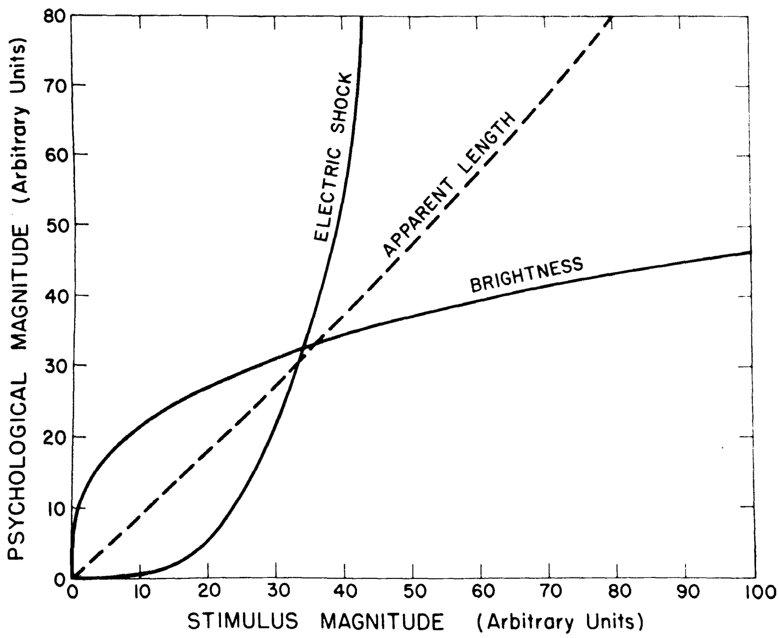

FIG. 4. In linear coordinates the subjective magnitude functions are concave upward or downward depending on whether the power-function exponent is greater or less than 1.0.

having the same intensity as the standard may appear as one of the stimuli to be judged along with the others. With a series of six to ten stimuli, each stimulus is usually presented twice, but the order of the stimuli is made different for each observer. In the averaging of the data from a group of observers it is usual to compute the geometric means of the estimates, although sometimes the median provides a more representative measure. Since the distributions of responses are usually skewed, the arithmetic mean is seldom an appropriate statistic.

Cross-Modality Comparisons

Few scientists fail to sense an uneasy concern about the foregoing procedure, which seems to rely merely on the observer's expression of opinion, and which seems also to depend on his having a moderately sophisticated understanding of the number system. This is a proper concern, because naiveté about numbers, and especially about the concept of proportion, certainly impedes the ability of some observers to perform well in this kind of experiment. The matching of numbers to sensation intensity is not something that a person does with fine precision, or that he feels great certainty about, even though the typical graduate student can usually manage a consistent set of estimates.

The interesting question, however, is not whether we are uneasy about the procedure, but whether the experiments on magnitude estimation can predict other empirical consequences that can be put to test. In particular, can we confirm the power law without asking observers to make any numerical estimations at all? If so, can we proceed to verify the relations among the exponents listed in Table 2? An affirmative answer to these questions is suggested by the results of a method in which the observer equates the apparent strengths of the sensations produced in two different modalities. By means of such cross-modality matches, made at various levels of stimulus intensity, an "equal-sensation function" can be mapped out, and its form can be compared with the form predicted by the magnitude scales for the two modalities involved.

If, given an appropriate choice of units, two modalities are governed by the equations

$$\psi_1 = \phi_1^m$$

and

$$\psi_2 = \phi_2^n$$

and if the subjective values ψ_1 and ψ_2 are equated by cross-modality matches at various levels, then the resulting equal-sensation function will have the form

$$\phi_1^m = \phi_2^n$$

In terms of logarithms

$$\log \phi_1 = n/m\ (\log \phi_2)$$

In other words, in log-log coordinates the equal-sensation function should be a straight line whose slope is given by the ratio of the two exponents.

The experimental question is whether observers can make cross-modality matches, and whether their matches can, in fact, be predicted from the ratio scales of apparent magnitude determined independently by magnitude estimation. The ability of observers to make the simple judgment of apparent equality has been well-established in other contexts. Heterochromatic photometry and the mapping of equal-loudness contours provide two well-known examples of procedures that involve the judgment of apparent equality of sensory intensity—a judgment made in the presence of an obvious qualitative difference. It is but a small step to extend these procedures to cross-modality equations. As a matter of fact, some cross-modality equations seem less difficult than some equations within a single modality.

In principle, of course, cross-modality matches can be made between every sensory continuum and every other one. Since this potential enterprise involves heroic numbers of experiments, only certain illustrative

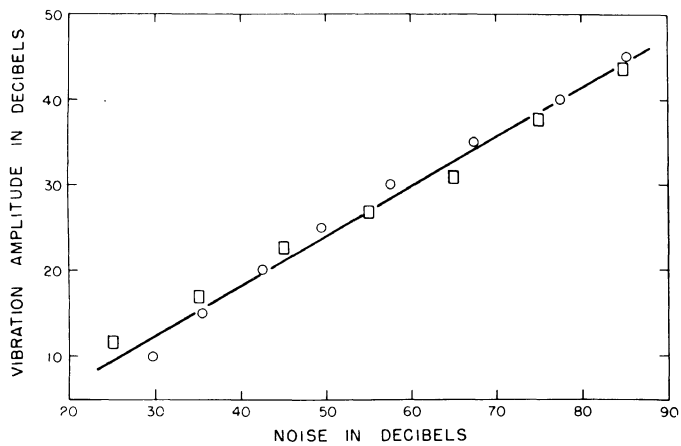

FIG. 5. An equal-sensation function relating 60-cycle vibration on the fingertip to the intensity of a band of noise. The observers adjusted the loudness to match the vibration (circles) and the vibration to match the loudness (squares). The stimulus values are measured in terms of logarithmic scales (decibels).

tests have been completed. They are sufficient, however, to demonstrate the general validity of the ratio scales of subjective magnitude. A few of these cross-modality experiments will be described.

Loudness versus Vibration

Two stimuli that are relatively easy to equate for apparent strength are sound and mechanical vibration. The sound employed was a band of noise of moderately low frequency and the vibration was a single frequency (60 c.p.s.) delivered to the end of the middle finger (Stevens, 1959a).

The matching of the apparent intensities of sound and vibration was carried out in two complementary experiments. In one experiment the level of the sound was adjusted to match the vibration; in the other the level of the vibration was adjusted to match the sound. The sound and vibration were presented simultaneously. (In many experiments the stimuli have been presented successively for one reason or another.) Each of ten observers made two adjustments at each level in each experiment.

The results are shown in Figure 5. The circles represent the means of the decibel levels to which the sound was adjusted, and the squares represent the means of the decibel levels to which the vibration was adjusted. The coordinate scales are in decibels relative to the approximate thresholds of the two kinds of stimuli.

The interesting point to note is that the slope of the line in Figure 5 is 0.6, which is close to the slope that is called for by the ratio of the exponents of the two magnitude functions. It is also apparent that the relation is essentially linear, which is consistent with the fact that, over the ranges of the stimuli involved, both loudness and vibration are governed by power functions.

The departure of some of the points from a straight line in Figure 5 is in large measure due to the interesting fact that, depending on which stimulus is adjusted, the slope turns out to be slightly different. The situation is analogous to the two regression lines in a correlation plot. This "regression" or "centering tendency" is common, if not universal, in matching procedures, and it points up the desirability of a balanced design in which each stimulus is made to serve as both the standard and the variable (*cf.* Stevens, 1955b). The matching of loudness and vibration turned out to be surprisingly easy. Some of the observers, who happened to have served in loudness matching experiments, expressed the opinion that matching loudness to vibration seemed easier than matching the loudnesses of two tones of widely different pitch or quality (*cf.* Stevens, 1956a). The consistency of the judgments seemed to bear this out.

Other Comparisons

Cross-modality matches similar to those between vibration and loudness have been made for other pairs of continua, notably vibration *vs.* electric shock, and electric shock *vs.* loudness (S. S. Stevens, 1959a). This "round robin" of cross-modality comparisons completes an interesting circle in process of validation, for it turns out that all the matches are consistent with the predictions derived from the ratio scales determined by magnitude estimation. Futhermore, from the equal-sensation functions determined for two of the three pairs of continua, the function for the third pair can be predicted, and this prediction is verified to within a good approximation.

Another procedure by which the operating characteristics of sensory systems can be compared is cross-modality *ratio matching*. The observer is asked to make the apparent ratio between one pair of stimuli match the apparent ratio between some other pair. In particular, he may adjust stimulus D so that he achieves the relations: A is to B as C is to D.

This procedure is exemplified by an experiment conducted by J. C. Stevens in which the observer adjusted a pair of loudnesses to match a ratio defined by a pair of brightnesses (see S. S. Stevens, 1957b). Figure 6 shows the median settings made by fifteen observers, each of whom twice adjusted the loudness of the second of two noises until the apparent ratio between the noises equaled the apparent ratio between two luminous targets seen against a dark surround. The tendency of the points in Figure 6 to fall near the 45-degree diagonal means that whatever ratio (number of decibels) the experimenter set between the lights, the observer set approximately the same ratio between the sounds. The largest ratio used (40 db) represents a stimulus ratio of 10,000 to 1.

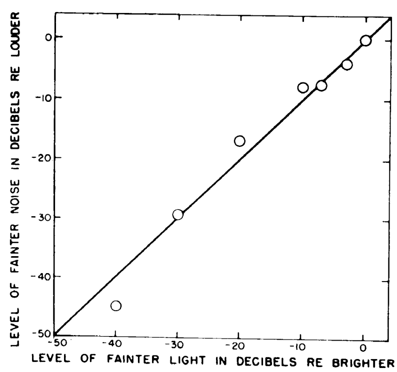

FIG. 6. Results of adjusting a loudness ratio to match an apparent brightness ratio defined by a pair of luminous circles. One of the circles was made dimmer than the other by the amount shown on the abscissa. The observer produced white noises by pressing one or the other of two keys and he adjusted the level of one noise (ordinate) to make the loudness ratio seem equal to the brightness ratio. The brighter light was about 99 db re 10^{-10} lambert and the louder noise was about 92 db re 0.0002 dyne per square centimeter.

The outcome shown in Figure 6 is what would be expected if both brightness and loudness were governed by a power law and if the two exponents were approximately the same size. The consensus of many experiments on loudness (S. S. Stevens, 1955b) shows that the exponent is approximately 0.3 when the stimulus is measured in terms of sound energy (0.6 when measured in terms of sound pressure). The exponent for brightness is approximately 0.33. Thus, in terms of the energy delivered to the sense organs, the exponents for loudness and brightness are approximately the same size. The similarity of these exponents is illustrated by Figure 7, which exhibits the results of two of the many experiments that have been performed to determine ratio scales of subjective magnitude for loudness and brightness.

It should be pointed out that the use of decibel scales simplifies the stimulus specification for vision and audition, and facilitates comparisons between their sensory dynamics. The foregoing examples demonstrate some of these advantages. Although the application of decibel measures to visual stimuli is not yet common practice, there is much to recommend it (see S. S. Stevens, 1955a). As a matter of fact, except for the rigidities of professional custom, the application of the decibel notation to the measurement of light presents less difficulty than its application to sound, because the decibel is defined in terms of energy flow

$$N_{db} = 10 \log \frac{E_1}{E_0}$$

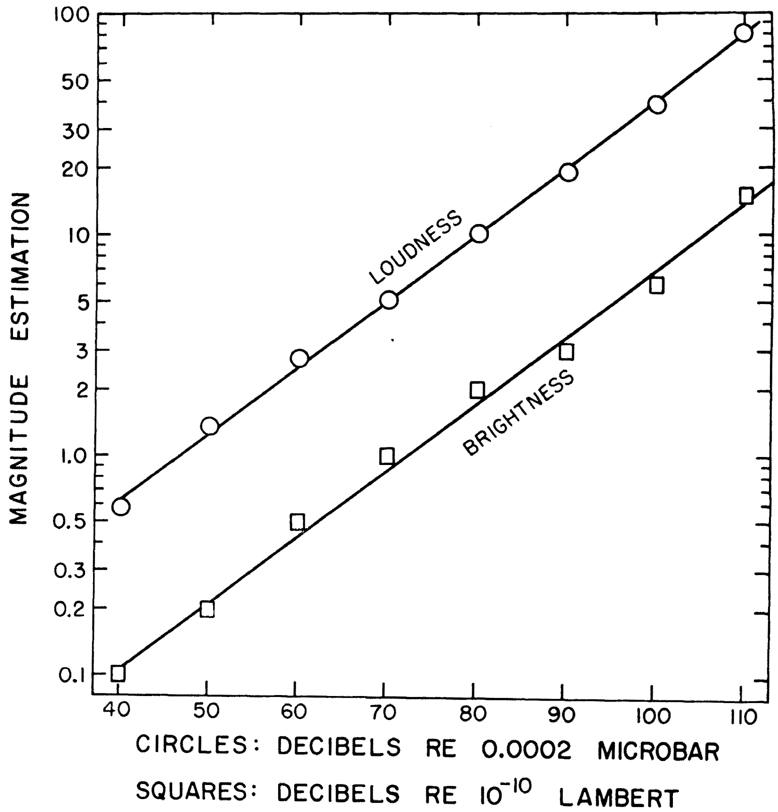

FIG. 7. Median magnitude estimations for loudness and brightness. For the loudness of a 1000-cycle tone, each of 32 observers made two estimates at each level. Since no standard modulus was designated, each observer chose his own, and the resulting numerical estimates were transformed to a common modulus at the 80-db level.

For brightness, each of 28 dark-adapted observers made two estimates of each stimulus level. The target subtended an angle of about 5° and was illuminated for about 3 sec. Once, at the beginning of each session, the observer was shown a stimulus of 70 db (14 observers) or 80 db (14 observers) and told to call it "10." The estimates were transformed to a common modulus at 70 db.

and it is only by a kind of bastardized extension that the decibel gets used with measures of sound pressure. The energy in a sound wave is proportional to the square of the sound pressure, but only under very special conditions. With light, on the other hand, we are concerned only with energy measures, relative or absolute, and there is no need to become entangled in measures that are nonlinearly related to energy.

Force of Handgrip

Like any other sensation, the subjective impression of muscle tension can be measured on a ratio scale of psychological magnitude. By squeezing a precision dynamometer (Fig. 8), an observer can produce a sensation of apparent force and at the same time activate a dial that indicates the actual force exerted. Two pertinent questions pose themselves: (1) How does the feeling of apparent force relate to the physical force exerted? (2) What happens when observers try to report the apparent magnitude of other kinds of sensations by squeezing the dynamometer instead of by making numerical estimations? In other words, we face a problem in scaling and a problem in cross-modality matching.

The scaling problem was attacked by J. C. Stevens and J. D. Mack (1959) who found that the apparent force of handgrip grows as the 1.7

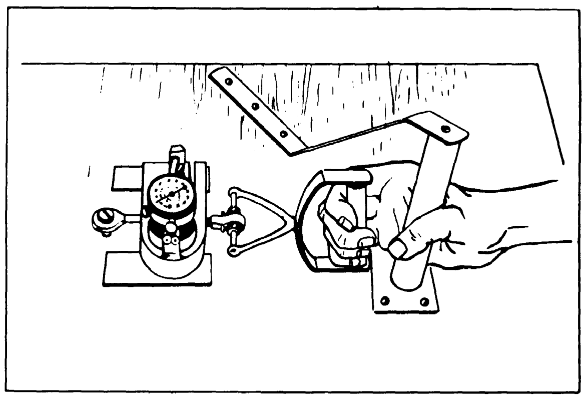

FIG. 8. One of the hand dynamometers used. It consisted of a pair of handles, one of which was connected through a ball-joint to a calibrated force gauge (Dillon).

power of the physical force applied. They used several methods, but principally magnitude estimation and magnitude production. The method of magnitude production, which has not been mentioned before in this article, proved to be an easy and convenient procedure for scaling the continuum of apparent muscular force. Instead of producing stimuli and asking observers to judge their magnitudes (as in magnitude estimation), the experimenter named numbers in an irregular order and the observer exerted forces that seemed to him proportional to the numbers. With magnitude estimation the observer squeezed until the experimenter signaled that a sufficient force had been achieved and then the observer estimated its apparent magnitude. The results of both procedures approximated power functions, but there was a tendency for magnitude production to give a slightly higher exponent than magnitude estimation.

Figures 9 and 10 show the results for the individual observers in two different experiments, each of which employed the two different methods. The two experiments involved different kinds of dynamometers: the one shown in Figure 8, which had a stiff, noncompliant force gauge, and a more compliant dynamometer whose handle moved through about 1.5 inches for a pull of 40 pounds. The two dynamometers gave similar results. It is clear from Figures 9 and 10 that the results for each observer approximate a power function. It is also clear that the slope may vary from one observer to another. It is not clear, however, that this variation in slope (exponent) means anything more than that people differ in what they understand by relative magnitudes. Whether the

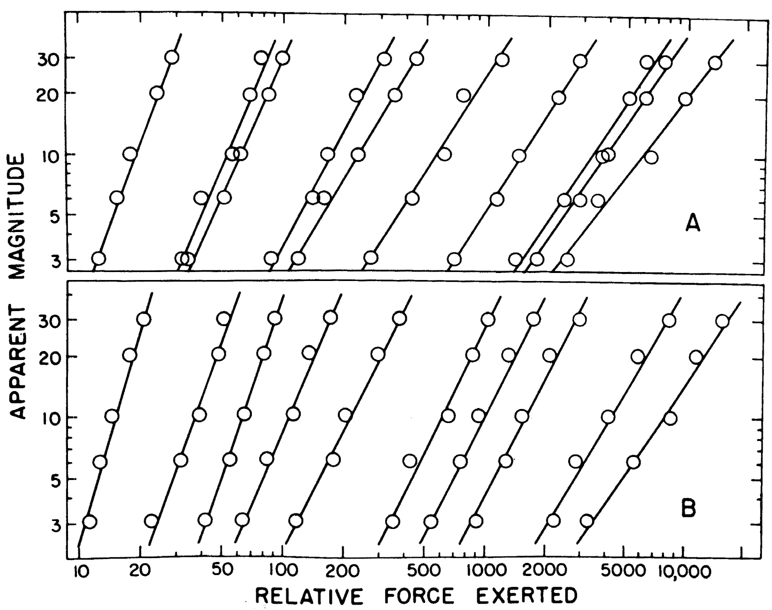

FIG. 9. Functions for apparent force of handgrip obtained by the method of magnitude production. Each curve is for a single observer and the position of the curve on the abscissa is arbitrary. The experimenter designated the numerical values (ordinate) and the observer produced the appropriate squeezes (abscissa). (a) Medians of 7 squeezes by each observer using the dynamometer shown in Fig. 8. (b) Medians of 10 squeezes by each observer using the more compliant dynamometer.

action of one man's sensory transducers is different from another's cannot be told with certainty from a single experiment of this sort. It is possible, on the other hand, that a battery of cross-modality comparisons might well provide definitive evidence of abnormal sensory function. The use of such a battery for the detection of recruitment in hard-of-hearing patients has been proposed elsewhere (Stevens, 1959c).

Handgrip versus Nine Other Continua

Although a subjective scale for force of handgrip proves interesting in its own right, the convenient ability of handgrip to serve as an indicator of other subjective magnitudes has led to even more exciting results. Instead of asking observers to emit numbers in response to stimuli, we

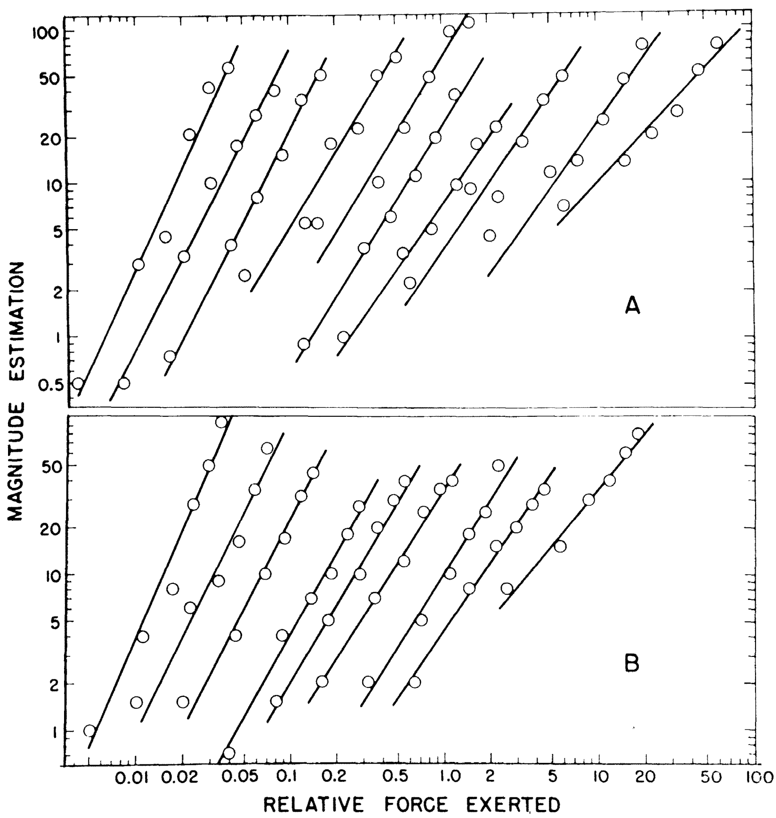

FIG. 10. Functions for apparent force of handgrip obtained by the method of magnitude estimation. Each curve is for a single observer, and the position of the curve on the abscissa is arbitrary. (a) Medians of 6 estimates by each observer using the dynamometer shown in Fig. 8. The forces estimated were 4, 10, 15, 22, 30, and 40 pounds. (b) Medians of 10 estimates by each observer using the more compliant dynamometer. The forces estimated were 5, 11, 17, 23, 29, and 35 pounds.

can ask them to emit squeezes of appropriate sizes. In this manner, observers have matched apparent force to apparent sensory intensity on nine different continua, and have produced the results shown in Figure 11 (J. C. Stevens, Mack, and S. S. Stevens 1960; J. C. Stevens and S. S. Stevens).

Two points are immediately evident. All the data in Figure 11 approximate power functions—straight lines in log-log coordinates—and the slopes stand in the same order as the values of the exponents listed in Table II. Less obvious, but even more interesting, is the exact numerical relation between the slopes determined by matching with handgrip and those determined by matching with numbers (i.e., magnitude estimation). Since the exponent for handgrip itself is approximately 1.7, we should expect that the exponent for a given continuum in Table II would be about 1.7 times as large as the slope of the corresponding line in Figure 11. How nearly this expectation is fulfilled is shown by the comparisons in Table III. Despite the variability inherent in experiments of this sort, the agreement between the obtained and predicted exponent is generally satisfactory. This agreement testifies with a certain eloquence to the basic validity of the ratio scales of sensory magnitude.

The Stimulus Scale

Except for the two continua, warm and cold, the physical stimuli of all the continua discussed above have been measured on the ordinary

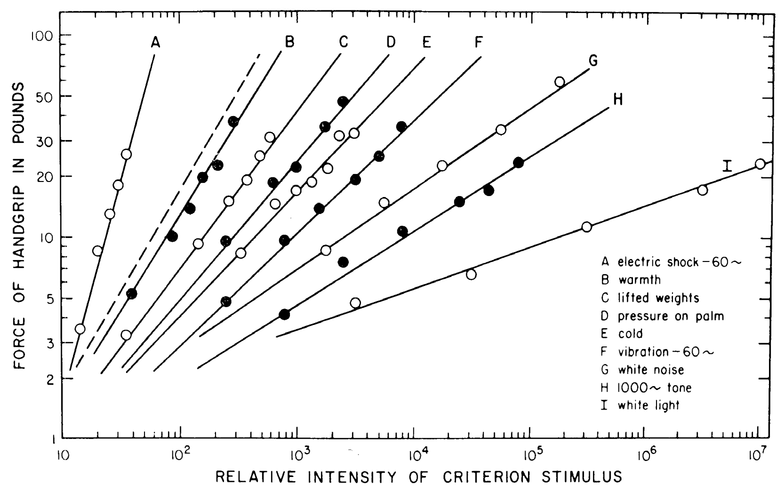

FIG. 11. Equal-sensation functions obtained by matching force of handgrip to various criterion stimuli. Each point stands for the median force exerted by 10 or more observers to match the apparent intensity of a criterion stimulus. The relative position of a function along the abscissa is arbitrary. The dashed line shows a slope of 1.0 in these coordinates.

physical scales of amperes, grams, dynes, etc. This practice is sufficiently accurate for most purposes, but when we look more closely we see that the general form of the power law is

$$\psi = k(\phi - \phi_0)^n$$

where ϕ_0 is a constant value corresponding to "threshold." For ranges of stimuli well above the minimum detectable level, the value of ϕ_0 is usually negligible, but it assumes larger proportions when subjective scales are extended downward toward very low values.

Temperature provides a clear and dramatic example of the importance of measuring stimuli in terms of the ratio scale of *distance* from threshold (J. C. Stevens and S. S. Stevens). The threshold for warmth when aluminum stimulators are applied to the inside of the forearm is about 305.7

degrees above zero on the absolute scale (Kelvin). Compared to the short range of tolerable thermal stimuli, this is indeed a high threshold.

As shown in Figure 12 (log-log plots), when apparent temperature is scaled by magnitude estimation and the results plotted against the Kelvin scale, the data fall on a curve that is sharply concave downward. When plotted in terms of degrees above the neutral or threshold value, however,

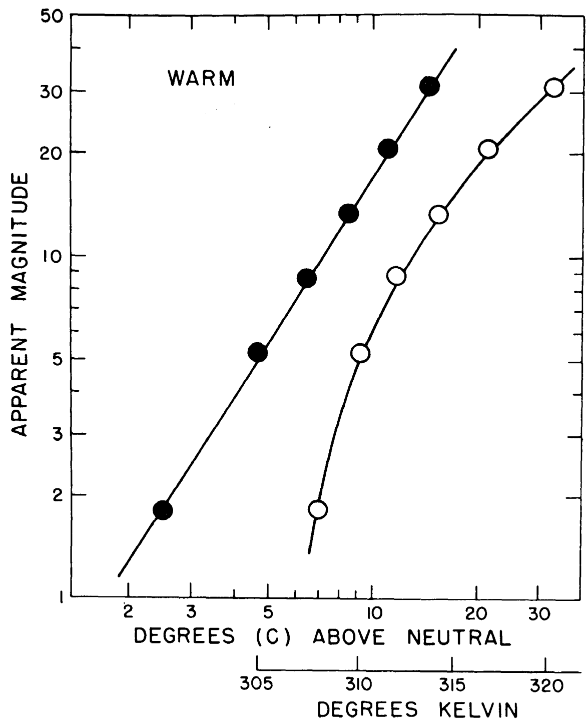

FIG. 12. Magnitude estimation of apparent warmth. Each point is the geometric mean of 36 estimates (12 observers). The upper abscissa, for the filled points, is a log scale of the difference in temperatue (Celsius) between the stimulus and the "physiological zero." The lower abscissa for the unfilled points, is a log scale of the absolute temperature (Kelvin).

the data fit a power function with an exponent of about 1.6. From these measurements it follows that the power-function formula for subjective warmth ψ_w is

$$\psi_w = k(T_K - 305.7)^{1.6}$$

where T_K is absolute temperature.

In a similar type of experiment (aluminum stimulators applied to the arm), the formula for cold ψ_c turned out to be

$$\psi_c = (304.2 - T_K)^{1.0}$$

The difference between the two values, 305.7 and 304.2, corresponding to ϕ_0 is of no particular significance. It presumably means that the observers' average skin temperature was different in the two experiments. On most other continua, the value of the additive constant ϕ_0 is small relative to the usable stimulus range. Nevertheless, in two instances, a revision of the stimulus scale designed to take explicit account of ϕ_0 has transformed otherwise wayward data into well-behaved power functions. The scale for tactile vibration (60 c.p.s.) applied to the arm was corrected in this manner (Stevens, 1959d), and a similar treatment was applied to the loudness scale by Scharf and J. C. Stevens. As Luce (1959) has pointed out, the use of an additive constant to bring the zero of the physical scale into coincidence with the zero of the psychological scale is a proper generalization of the power-function law. (Differences on a ratio scale constitute a ratio scale, as do also differences on an interval scale.)

TABLE III—THE EXPONENTS (SLOPES) OF EQUAL-SENSATION FUNCTIONS, AS PREDICTED FROM RATIO SCALES OF SUBJECTIVE MAGNITUDE, AND AS OBTAINED BY MATCHING WITH FORCE OF HANDGRIP

Ratio Scale		*Scaling by Means of Handgrip*		
Continuum	*Exponent of Power Function*	*Stimulus Range*	*Predicted Exponent*	*Obtained Exponent*
Electric shock (60-cycle current)	3.5	0.29–0.73 milliamp	2.06	2.13
Temperature (warm)	1.6	2.0–14.5°C. above neutral temperature	0.94	0.96
Heaviness of lifted weights	1.45	28–480 grams	0.85	0.79
Pressure on palm	1.1	0.5–5.0 pounds	0.65	0.67
Temperature (cold)	1.0	3.3–30.6°C. below neutral temperature	0.59	0.60
60-cycle vibration	0.95	17–47 db re approximate threshold	0.56	0.56
Loudness of white noise	0.6	55–95 db re 0.0002 dyne/cm²	0.35	0.41
Loudness of 1000-cycle tone	0.6	47–87 db re 0.0002 dyne/cm²	0.35	0.35
Brightness of white light	0.33	56–96 db re 10^{-10} lambert	0.20	0.21

In calling ϕ_0 the "threshold" value, we raise a problem concerning precisely what is meant by the term threshold. It is not necessarily the threshold as measured in some arbitrary manner under arbitrary conditions. Rather, it should probably be thought of as the "effective" threshold that obtains at the time and under the conditions of the experiment in which the magnitude scale is determined. Needless to say, this "effective" threshold cannot be measured very precisely. Consequently, it becomes expedient to take as the value of ϕ_0 the constant value whose subtraction from the stimulus values succeeds in rectifying the log-log plot of the magnitude function. Provided the constant value so chosen is a reasonable threshold value, this procedure seems justified. At any rate, it has worked well for the four continua, vibration, loudness, warmth, and cold.

Variability

Needless to say, the responses people make to sensory intensity are variable. Although an exemplary picture of this variability is shown in Figures 9 and 10, a further statement is in order about it. The statement will be brief, however, because the author confesses to a certain lack of enthusiasm for elaborate statistical analyses.

By and large, the interquartile range encountered when groups of observers undertake magnitude estimation on intensive continua is of the order of 0.2 to 0.3 log unit. (It may, of course, be lower for continua that are easy to judge.) This variability contains certain obvious components, however, the most important of which appear to be the following.

1. *Variability due to the observer's modulus, i.e., his conception of the "standard."* Since we are concerned only with the *form* of the magnitude scale, this source of variability is of no concern. When desired, it can be partialed out in one way or another, with a consequent reduction in the over-all variability.

This component of variability is especially evident in those experiments in which each observer is allowed to choose his own modulus (Stevens, 1956b). It also plays a prominent role in cross-modality matches (Stevens, 1959a; J. C. Stevens, Mack, and S. S. Stevens, 1960). Each observer, for example, has his own conception of what force of handgrip matches what level of loudness, but the absolute values chosen by the observer are irrelevant so far as the form of the equal-sensation function is concerned. It is only the relative values that matter. Concretely, our main concern is with the slopes and not the intercepts of the functions in Figures 9 and 10.

2. *Variability due to the observer's conception of a subjective ratio.* In a method like fractionation or magnitude estimation each person must make up his own mind about what he considers "half as bright," say, and not all observers arrive at the same conclusion. (A plot showing an example of the variability encountered in halving and doubling may be found in Stevens, 1957a). Nothing much can be done about this source of variability, except perhaps to try to avoid biases and constraints in the conditions of observation.

3. *Variability due to differing sense-organ operating characteristics.* This source of variability is biologically the most interesting, but it is probably of only minor magnitude in a group of "normal" observers. Nevertheless, in the hard-of-hearing ear, or the night-blind eye, it may be a factor of considerable consequence. (The state of a sense organ like the eye is also changed, of course, in the process of adaptation.) In order to evaluate this "sense-organ" factor, it may prove useful to apply a battery of cross-modality matching tests. A sufficient battery of these tests should prove capable, for example, of distinguishing the person with auditory recruitment from the person whose conception of a subjective loudness ratio is merely atypical.

An interesting problem related to variability concerns the question of departures from the power-function law. Can we expect that the power law will always hold rigorously (provided, of course, no errors arise in our measurements), or should we look for second-order deviations from it? The data of particular experiments sometimes depart from the power law, but, in most instances, it is not easy to determine whether these defections are due to artificial biases of one kind or another. Nevertheless, since the possibility of genuine departures from the power law is a problem of basic moment, an effort should be made to devise procedures of sufficient accuracy to settle the question. The fact that the power law is closely approximated by so many data in so many different sense-modalities adds interest and significance to any authentic departures from the power-function form.

The Role of Transducers

The foregoing suggestion of a method for determining the individual oudness function in a hard-of-hearing ear assumes that the nature of the sensory transducer largely determines the form of a magnitude function. An opposite assumption has often been made, however, to the effect that the magnitude function merely reflects how observers have learned in the past to associate sensory impressions with some known aspect of the physical stimulus. The "learning" explanation has recently been revived (Warren, 1958) and, under the name "physical correlate theory," it is alleged to provide a "basis for Stevens' empirical law" (Warren, Sersen, and Pores, 1958). If this theory were correct, it would presumably explain why the psychophysical law is a power function, but the evidence that learning accounts for all the exponents in Table II is mostly nonexistent. Familiarity with the stimulus may be a factor in people's judgments on some kinds of continua (although how one would prove it is hard to see). Many of the continua in Table II are quite unfamiliar to the typical observer—at least as regards measures of stimulus intensity. Especially difficult to conceive is how familiarity with the physical stimuli—even if the observers had such familiarity—

could account for the results of cross-modality matching like those shown in Figure 11.

It seems rather more probable that the exponents are what they are because of the nature of the sensory transducers. It is likely, for example, that the exponents for light and sound are smaller than 1.0 because these sensory transducers behave essentially as "compressors"—a characteristic that enables them to handle the enormous dynamic ranges of stimulation to which they are subjected. At the other extreme, in the transduction process involved with electric current applied to the fingers, there is an operation of "expansion" in the sense that the psychological magnitude grows as an accelerating function of stimulus intensity, i.e., the exponent is greater than 1.0. It seems quite improbable that the form of this function was "learned" by the observer.

It is an interesting question whether electrical stimulation of other nerves than those in the fingers would also exhibit an accelerating transduction characteristic. In a study of the "electrophonic effect" (Jones, Stevens, and Lurie, 1940) patients lacking tympanic membranes were stimulated by means of an electrode placed inside the middle-ear cavity. Although some patients heard pure tones, seven of the group heard only a buzzing noise whose quality was more or less independent of the frequency of the stimulating current. Since it is certain that other nerves (e.g., the facial and the vestibular) were occasionally stimulated in the course of these experiments, it seems safe to conclude that the auditory nerve was also sometimes directly affected by the current. Direct, unpatterned stimulation of the auditory nerve fibers would account for the patient's hearing only a noise.

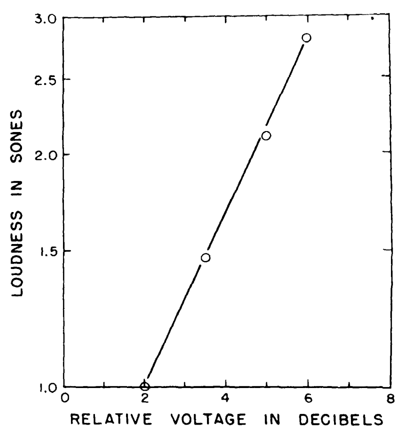

FIG. 13. Showing the steep growth of loudness with increasing electric current applied to the auditory nerve of a patient whose eardrum had been removed. The current was delivered by an electrode placed in the middle-ear cavity. The exponent is about four times as large as the exponent obtained with acoustic stimulation.

Some of the patients noted a large change in loudness when only a small change was made in the stimulating current. This effect was so striking that an attempt was made to measure the loudness change by comparing it with a sound in the opposite (normal) ear. The outcome is shown in Figure 13 (for further explanation, see Stevens, Carton, and Shickman, 1958). Apparently, if the auditory end organ is bypassed, and if a stimulating current acts directly on the eighth nerve, a new transducer process becomes involved and the function describing the growth of loudness acquires a radically different exponent. The implication is that the "compression" observed when the normal ear is stimulated by sound waves is a function of the sense organ, not of some higher center in the nervous system.

When it is discovered that two continua presumed to be rather similar are governed by different exponents (Table II), one suspects that there may be basic differences in the transducer systems involved. Two pairs of such continua are especially interesting: warmth and cold, and taste and smell. Warmth and cold are interesting because the same stimulating device, applied to the same place on the arm produces two scales of sensory intensity, one for stimulus temperatures below neutral and one for temperatures above neutral (J. C. Stevens and S. S. Stevens). Not only do warmth and cold produce sensations of different quality, they also appear to do it by means of transducers with different operating characteristics. The temperature sense is also unique in that the neutral or threshold point is the bottom of one stimulus scale (warmth) and the top of another (cold). Cold increases as the stimulus value decreases.

Taste and smell are often classed together as chemical senses. The mode of action of the stimulus for smell has been such a mystery, however, that mechanisms other than the bathing of end organs by chemical solutions have been hypothesized from time to time. This state of affairs gives added significance to the obvious difference between the operating characteristics of the olfactory and gustatory systems. For the several substances thus far tested the exponents of the power functions for olfactory intensity have run from about 0.5 to 0.6 (Jones, 1958; Reese and Stevens). Some earlier experiments on taste, with the method of fractionation, gave exponents of the order of 1.0 (Beebe-Center and Waddell, 1948). The exponents for taste listed in Table II were obtained by Mary McLean in some exploratory experiments with the method of magnitude estimation. This work is still in progress, but there is little doubt that the exponents for taste are generally about twice as large as the exponents for smell. Does this difference in the dynamics of apparent intensity mean that two wholly different mechanisms underlie the transduction processes in taste and smell?

REFERENCES

Beebe-Center, J. G., and D. Waddell. A general psychological scale of taste *J. Psychol.*, 1948, 26, 517–524.

Békésy, G. v., and W. A. Rosenblith. The mechanical properties of the ear. In S. S. Stevens (Ed.), *Handbook of Experimental Psychology.* New York: Wiley, 1951.

Jones, F. N. Scales of subjective intensity for odors of diverse chemical nature. *Amer. J. Psychol.*, 1958, 71, 305–310.

Jones, R. C., S. S. Stevens, and M. H. Lurie. Three mechanisms of hearing by electrical stimulation. *J. Acoust. Soc. Amer.*, 1940, 12, 281–290.

LUCE, R. D. On the possible psychophysical laws. *Psychol. Rev.*, 1959, 66, 81–95.
REESE, J. S., and S. S. STEVENS. Subjective intensity of coffee odors. *Amer. J. Psychol.*, in press.
SCHARF, B., and J. C. STEVENS. The form of the loudness function near threshold. Proc. 3rd Int. Congr. Acoustics, in press.
STEVENS, J. C., and J. D. MACK. Scales of apparent force. *J. Exp. Psychol.*, 1959, 58, 405–413.
STEVENS, J. C., J. D. MACK, and S. S. STEVENS. Growth of sensation on seven continua as measured by force of handgrip. *J. Exp. Psychol.*, 1960, 59, 60–67.
STEVENS, J. C., and S. S. STEVENS. Warmth and cold—dynamics of sensory intensity. *J. Exp. Psychol.*, in press.
STEVENS, S. S. On the theory of scales and measurement. *Science*, 1946a, 103, 677–680.
STEVENS, S. S. The two basic mechanisms of sensory discrimination. *Fed. Proc.*, 1946b, 5 [Abstract].
STEVENS, S. S. Mathematics, measurement, and psychophysics. In S. S. Stevens (Ed.), *Handbook of Experimental Psychology*. New York: Wiley, 1951.
STEVENS, S. S. Decibels of light and sound. *Physics Today*, 1955a, 8(10), 12–17.
STEVENS, S. S. The measurement of loudness. *J. Acoust. Soc. Amer.*, 1955b, 27, 815-829.
STEVENS, S. S. Calculation of the loudness of complex noise. *J. Acoust. Soc. Amer.*, 1956a, 28, 807–832.
STEVENS, S. S. The direct estimation of sensory magnitudes—loudness. *Amer. J. Psychol.*, 1956b, 69, 1–25.
STEVENS, S. S. Concerning the form of the loudness function. *J. Acoust. Soc. Amer.*, 1957a, 29, 603–606.
STEVENS, S. S. On the psychophysical law. *Psychol. Rev.*, 1957b, 64, 153–181.
STEVENS, S. S. Problems and methods of psychophysics. *Psychol. Bull.*, 1958, 54, 177-196.
STEVENS, S. S. Cross-modality validation of subjective scales for loudness, vibration, and electric shock. *J. Exp. Psychol.*, 1959a, 57, 201–209.
STEVENS, S. S. Measurement, psychophysics, and utility. In C. W. Churchman and P. Ratoosh (Eds.), *Measurement: Definitions and Theories*. New York: Wiley, 1959b.
STEVENS, S. S. On the validity of the loudness scale. *J. Acoust. Soc. Amer.*, 1959c, 31, 995–1003.
STEVENS, S. S., A. S. CARTON, and G. M. SHICKMAN. A scale of apparent intensity of electric shock. *J. Exp. Psychol.*, 1958, 56, 328–334.
STEVENS, S. S., and E. H. GALANTER. Ratio scales and category scales for a dozen perceptual continua. *J. Exp. Psychol.*, 1957, 54, 377–411.
STEVENS, S. S., and J. VOLKMANN. The quantum of sensory discrimination. *Science*, 1940, 92, 583–585.
WARREN, R. M. A basis for judgments of sensory intensity. *Amer. J. Psychol.*, 1958, 71, 675–687.
WARREN, R. M., E. A. SERSEN, and E. B. PORES. A basis for loudness-judgments. *Amer. J. Psychol.*, 1958, 71, 700–709.
ZWICKER, E., G. FLOTTORP, and S. S. STEVENS. Critical bandwidth in loudness summation. *J. Acoust. Soc. Amer.*, 1957, 29, 548–557.