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100GbE: QPSK versus OFDM

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ABSTRACT

In this paper the optical performance and complexity of polarization division multiplexed quadrature phase shift keying (PDM-QPSK) is compared to that of PDM orthogonal frequency division multiplexing (PDM-OFDM). The benefits and drawbacks of each modulation format are discussed with respect to the implementation, complexity and transmission performance.

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1. Introduction

In order to increase the robustness and spectral efficiency of high-data rate fiber-optic transmission systems coherent detection in combination with digital signal processing has been proposed so that equalization of these linear distortions can be done at the receiver in the digital domain [1]. The first modulation format that was investigated with such DSP based equalization at the receiver is polarization division multiplexed quadrature phase shift keying (PDM-QPSK). Note that PDM-QPSK is referred to as well as coherently demodulated polarization multiplexed QPSK (CP-QPSK). This single carrier modulation format transmits two QPSK modulated signals over the two polarizations that exist in single-mode fiber. At the receiver, the signal is detected with a polarization diverse detector after which equalization is realized by MIMO processing. It has experimentally been shown that this modulation format provides a superior tolerance towards chromatic dispersion and PMD [2,3]. For the last 2 years a multi-carrier modulation format called PDM-OFDM has been researched as an alternative to PDM-QPSK showing similar tolerances to CD and PMD [4,5]. Like PDM-QPSK, PDM-OFDM uses MIMO processing at the receiver to compensate for the linear distortions, however, with a different MIMO compensation algorithm. PDM-QPSK is a single carrier modulation format that typically employs blind channel estimation for its MIMO processing. In particular, a butterfly structure of complex valued, mul-

tipap adaptive filters has proven to be an effective equalizer for PDM-QPSK [6]. OFDM on the other hand is a multi-carrier modulation format that typically employs many subcarriers (more than 50). Theoretically, the blind channel estimation algorithm could be applied to multicarrier systems as well, but would result in an immense computational complexity as a separate adaptive filter has to be applied for each individual OFDM subcarrier. It has been shown that for multicarrier systems, the computational complexity of a MIMO detector can greatly be reduced by using training symbols (TS) in order to get knowledge about the channel at the receiver [6].

In [7] we made a first assessment on how the optical performance of PDM-OFDM scales with coherently detected single-carrier PDM-QPSK. In this paper, we expand on [7] and report in detail on the latest advances including a comparison of the computational complexity of both modulation formats. Several investigations have been reported comparing the implementation complexity of PDM-OFDM with PDM-QPSK [8–11]. One of the main findings of these works has been that the algorithm complexity is very much dependent on the implementation of the algorithms used for channel estimation, equalization and adaptation of equalizer coefficients. While in [11] only the complexity of equalization is evaluated we also take into account complexity of adaptation of the equalizer in case of blind channel acquisition and complexity of channel estimation from known training symbols. In our evaluation the complexity of PDM-OFDM and PDM-QPSK is compared while optimizing the FFT-size for all system configurations. It is observed that with the help of known training symbols the complexity is considerably lower than if blind channel acquisition is employed. In principle, this reduction of

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computational complexity applies to both PDM–OFDM and PDM–QPSK. However, since the predominant approaches nowadays are PDM–OFDM with training symbols and PDM–QPSK with blind adaptation, this difference is often perceived as advantage of OFDM over single carrier modulation formats.

Note that even though many different OFDM systems have been proposed, we refer to OFDM in this paper as a digital multicarrier technique. We, therefore, exclude coherent WDM systems that sometimes are referred to as OFDM systems [12]. Coherent WDM systems typically have few subcarriers that are generated in the optical domain. These systems typically do not use training symbols, but rely on blind channel estimation instead. Such systems have more in common with single-carrier coherent systems and its evaluation is out of scope of this manuscript.

2. 100GbE OFDM system design

For the transport of 100GbE, overhead is required onto the payload data. Although 100GbE has not been standardized yet, it is foreseen that at least 7% overhead needs to be allocated for FEC and about 4% for the Ethernet line code (64B/66B coding). This results in a raw data rate R_{raw} of 111 Gb/s. Apart from the Ethernet and FEC related overhead, an OFDM system has additional overheads caused by cyclic prefix, training symbols and in some cases pilot subcarriers. Especially at high data rates it is essential to minimize these OFDM related overheads as the system overhead significantly increases the bandwidth requirements at the transmitter and receiver. These overheads will be discussed in the next sections.

2.1. Training symbols (TS)

The overhead for training symbols is dependent on the channel dynamics and the symbol length (or in other words the number of subcarriers that need to be equalized). In a fiber-optic transmission system channel fluctuations are caused for instance by polarization changes. The more stable a channel is the less training symbols are required. In order to minimize the influence of amplifier noise it is common to average subsequent training symbols. If for instance 10 subsequent training symbols are averaged and the fastest polarization change of the transmission link is 50 μs , then a training symbol must be sent every 5 μs . Fig. 1 shows the training symbol overhead (ε_{TS}) for this example as a function of the OFDM symbol size. It can be seen that the training symbol overhead linearly increases with the OFDM symbol size. In most coherent transmission systems reported to date, the training symbol overhead is about 2–4% [4,5].

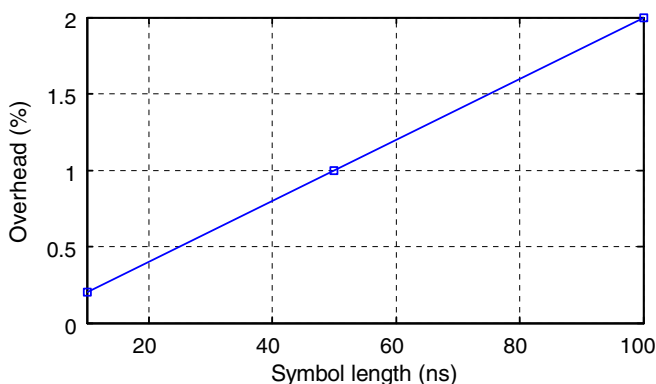


Fig. 1. Training symbol overhead as a function of the symbol length under the assumption that one training symbol is sent every 5 μs .

2.2. Pilot subcarriers (PS)

In a coherent transmission system, a local oscillator is mixed with the received signal and the sum is detected by a photodiode thereby downmixing the received optical signal. In such a system the phase noise of the transmitter and receiver laser have a large influence on the system performance. The influence of phase noise in OFDM systems is twofold, i.e., it generates a common phase rotation (CPR) of all the subcarriers in one symbol and a cross-leakage between the subcarriers named inter-carrier interference (ICI). The former effect is commonly solved in wireless systems using common phase estimation (CPE) [6], aided by inserting dedicated pilot subcarriers. At the receiver these are used to rotate back the received symbols. This method has been used as well in several fiber-optic transmission experiments, see e.g. [5]. The main drawback of CPE, however, is that it does not correct for the ICI, since it inherently assumes the phase of the transmitter (TX) and local oscillator (LO) laser to be constant during one OFDM symbol. Consequently, the OFDM symbol must be short and the phase noise bandwidth of the TX and LO laser must be small to limit the impact of the ICI. Short OFDM symbols allow for smaller FFT size and thus increase the overhead ratio due to the cyclic prefix. In order to minimize laser phase noise, lasers with narrow linewidth must be used. In practice, CPE limits the FFT size to 128 and the linewidth to about 100 kHz [5]. With CPE, typically 10% or more of the subcarriers must be allocated for phase noise compensation [5].

As alternative to CPE we have proposed a different method for phase noise compensation called RF-aided phase noise compensation (RFPNC) [4,13,15]. RFPNC effectively compensates for both the CPR and the ICI. As the name implies, phase noise compensation with RFPNC is realized by placing an RF-pilot tone in the middle of the OFDM spectrum at the transmitter, which is subsequently used at the receiver to revert phase noise impairments. The basic idea behind this PNC scheme is as follows. When an RF-pilot is inserted at the transmitter, this pilot is distorted by phase noise in exactly the same way as the OFDM signal. Therefore, the pilot can be used at the receiver to revert any phase distortions from the OFDM signal. A major advantage of RFPNC is that pilot subcarriers are not required for phase noise compensation, saving about 10% of overhead. Another advantage is that this compensation method is more robust towards phase noise and it has been shown that even PN compensation of conventional DFB lasers can be realized with only a minor penalty [13]. The main drawback, however, is that it requires more complex computations to implement.

2.3. Cyclic prefix (CP)

The required cyclic prefix (CP) is dependent on the chromatic dispersion that is to be compensated for. The required cyclic prefix time τ_g of an OFDM signal is dependent on the bandwidth of the OFDM band and can be defined as:

$$\tau_g = DB_d c / f^2, \quad (1)$$

where D represents the chromatic dispersion of the desired transmission distance [s/m], c is the speed of light [m/s], f represents the center frequency of the OFDM band [Hz] and B_d [Hz] is the effective bandwidth of the modulated OFDM signal:

$$B_d = \frac{R_{\text{gross}}}{\log_2(M)} = \frac{R_{\text{raw}}(1 + \varepsilon_{\text{TS}})(1 + \varepsilon_{\text{CP}})(1 + \varepsilon_{\text{PS}})}{\log_2(M)}, \quad (2)$$

where R_{gross} and R_{raw} are the gross and raw data rate (including FEC and Ethernet overhead) per polarization [bit/s], respectively, M is the constellation size and ε_{CP} , ε_{TS} and ε_{PS} are the cyclic prefix, training symbol and pilot subcarrier overheads, respectively. From (2) it can be concluded that the bandwidth B_d is dependent on the CP overhead, which is defined as [7]:

$$\varepsilon_{CP} = \frac{\tau_g}{\tau_t - \tau_g}, \quad (3)$$

where τ_t is the total OFDM symbol time (including the cyclic prefix). Combining (1)–(3), results in:

$$D = \frac{f^2}{c} \frac{\log_2(M)}{R_{raw}(1 + \varepsilon_{TS})(1 + \varepsilon_{ps})} \frac{\tau_g \tau_t - \tau_g^2}{\tau_t}. \quad (4)$$

From Eq. (4) it can be concluded that for a certain data rate and constellation size, the reach of an OFDM transmission system is dependent on the guard time (cyclic prefix), and the OFDM symbol size. For coherently detected OFDM systems the OFDM symbol size is typically limited by the effectiveness of the phase noise compensation scheme as well as the linewidth of the transmitter and local oscillator laser. For a conventional phase noise compensation scheme (using pilot subcarriers), this results in a maximum OFDM symbol length of about 10–15 ns [5] whereas with RF-aided phase noise compensation, an OFDM symbol length of longer than 100 ns can be realized [4]. The dispersion tolerance, expressed in km of SSMF, as a function of the guard time is shown in Fig. 2 for several OFDM symbol rates. This figure clearly demonstrates the advantage of using long OFDM symbol lengths. For a system with a 10-ns OFDM symbol length the reach is far below 1000-km SSMF whereas by increasing the OFDM symbol length to 100 ns the reach is extended with a factor of ten to more than 5000 km.

3. Performance comparison

In this section we will review the linear and nonlinear performance of coherently detected OFDM and compare this to the performance of coherently detected single carrier systems. The system setup for single carrier PDM-QPSK and the corresponding multi-carrier PDM-OFDM can be taken for example from [2,4], respectively. However, in contrast to [4], which uses multiple OFDM bands, only one single OFDM band is used in this comparison.

3.1. OSNR sensitivity

Both at 40-Gb/s [14,15] and 100-Gb/s [2,4] similar BER sensitivities have been reported for single carrier and OFDM transmission systems. The OSNR sensitivity for coherently detected systems is for a fixed gross data rate independent of the number of subcarriers that is used. It is thus not surprising that the OSNR sensitivity of single carrier and multi-carrier (OFDM) systems are comparable at the same data rate.

3.2. DAC/ADC sampling/bandwidth requirements

In all systems that employ coherent detection with digital equalization ADCs are required at the receiver. The bandwidth

requirements of these receivers are dependent mostly on the data rate and constellation size, not so much on the number of subcarriers. Therefore, single carrier and OFDM transmission systems have practically the same bandwidth requirements for the ADCs.

In order to prevent high frequency distortion and noise components from being mirrored into the baseband signal oversampling is required. A great advantage of OFDM with respect to a single carrier system is that it is straightforward to reduce the required oversampling to for instance a factor of 1.2. Oversampling in an OFDM system is typically realized by inserting unmodulated (i.e. virtual) OFDM subcarriers at high frequencies and thus unlike in single carrier systems no complex re-sampling is required at the receiver. However, the main disadvantage of digital OFDM is that DACs are required at the transmitter for the generation of the signal. The DACs increase the cost and complexity of the system, but at the same time make it also easier to adaptively scale to higher level modulation formats [4].

3.3. Narrowband filtering tolerance

In [16] we have investigated the narrowband filtering tolerance of 120-Gb/s polarization division multiplexed (PDM) OFDM. The simulation results are shown in Fig. 3. In this simulation the optical signal-to-noise ratio (OSNR) is fixed to 10.6-dB, resulting in a BER of 1×10^{-3} . Because of the confined spectrum of the OFDM signal a negligible BER penalty is observed as long as the optical filter is broader than the OFDM band (30-GHz for 120-Gb/s PDM-OFDM). At narrower bandwidths the subcarriers located on the sides of the OFDM signal are attenuated and thereby the SNRs of the subcarriers located at these frequencies are severely degraded. As a result, a steep increase in BER is observed when the filter bandwidth is lower than 30 GHz. The 30 GHz filter tolerance is comparable to that observed for single carrier 100GbE [2]. It can thus be concluded that even though the unfiltered spectrum of PDM-OFDM is significantly smaller, it has a similar tolerance towards narrowband optical filtering compared to single carrier PDM-modulated signals with coherent detection (for the same constellation size M).

3.4. Nonlinear tolerance

A frequently discussed disadvantage of OFDM is that compared to single carrier systems the peak to average power ratio (PAPR) is significantly higher. One would, therefore, expect the nonlinear tolerance of OFDM to be lower resulting in a limited reach. However, on a link without dispersion compensation, the reach of 40 Gb/s PDM-OFDM [16] is similar to that of single carrier 40 Gb/s QPSK [3]. The main reason for this is that although single carrier systems have lower PAPR in back-to-back configuration, the uncompensated chromatic dispersion along the transmission

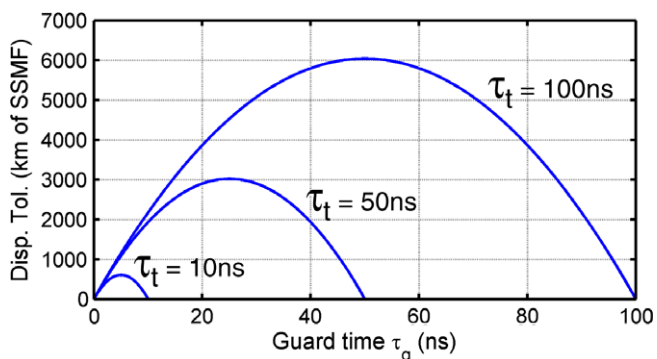


Fig. 2. Dispersion tolerance for 100GbE PDM-OFDM in one single band as a function of the guard time for several OFDM symbol lengths.

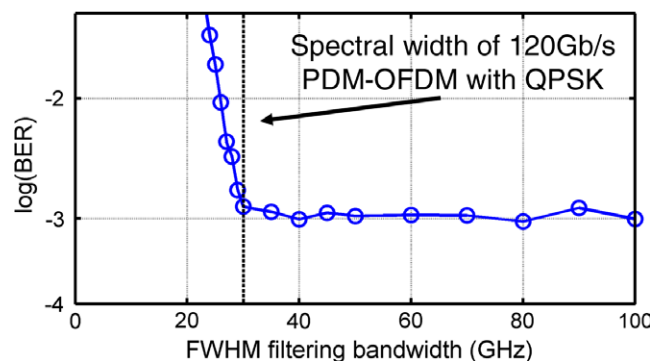


Fig. 3. BER as a function of optical filtering bandwidth of 120-Gb/s PDM-OFDM.

link causes the PAPR of single carrier systems to rise to similar values of OFDM systems, resulting in a similar nonlinear tolerance. Thus for long-haul transmission links that do not use in-line chromatic dispersion compensation, the impact of the high PAPR is small on the transmission reach.

However, in links with chromatic dispersion compensation, the impact of the PAPR is significant. In [17] we have investigated the impact of nonlinear impairments in a periodic compensated dispersion map using DCF after each span for dispersion compensation. In this investigation, three dispersion maps are compared, namely without dispersion compensation, with fully periodic compensation and with a dispersion map that is optimal for 10-Gb/s OOK systems. In order to evaluate the influence of SPM and XPM both single channel and DWDM (50-GHz channel spacing) are simulated. Fig. 4 shows the required OSNR after 1200 km transmission for a BER of 10^{-3} as a function of the launch power. Simulated is one polarization of the 40-Gb/s PDM-OFDM experiment with the OFDM configuration as reported in [15]. In Fig. 4 it can be seen that the highest nonlinear tolerance is observed for the dispersion map without inline dispersion compensation. In the single channel case a reduction in nonlinear tolerance of more than 2 dB is observed for a 1-dB OSNR penalty when dispersion compensation is used. In the WDM simulations, the difference in nonlinear tolerance between the uncompensated and the compensated dispersion maps is increased to about 4 dB of fiber launch power. This clearly shows that both SPM and XPM penalties are significantly higher for periodic compensated dispersion maps.

A possible method to improve the nonlinear tolerance of OFDM for periodic dispersion maps is to apply pre- and post-compensation. Recently it has been shown by Du and Lowery [18] that the nonlinear tolerance of OFDM can be significantly improved for periodically compensated dispersion maps, achieving almost the same nonlinear tolerance of that of single carrier systems. However, the higher the walk-off between the nonlinear regions is, the smaller the gain of pre- and post-compensation. For high dispersive fibers without dispersion compensation the improvement in nonlinear tolerance is negligible. Alternatively the nonlinear performance can be improved by reducing the PAPR at the transmitter [19]. Similar to pre- and post compensation, the effect of PAPR reduction is limited in a dispersion map without dispersion compensation, as chromatic dispersion will cause the PAPR to rise after transmission.

3.5. Chromatic dispersion and PMD tolerance

From (4) it can be concluded that the CD tolerance of an OFDM transmission system is dependent on the bandwidth of the signal (which scales with the gross data rate) and the allocated CP. In an OFDM transmission system practically arbitrary amounts of CD can be compensated for as long as the OFDM symbol length

is not restricted and sufficient CP is allocated. Basically, the increase in CP causes a rise of the CP overhead, which can be mitigated by choosing a long enough OFDM symbol length. It is noted, however, that in practice the length of the OFDM symbol is limited by dynamic effects such as laser phase noise. In single carrier systems, the amount of dispersion tolerance is dependent on the length of the overlap region that is used in the frequency domain equalization [2]. This value scales with the dispersion that is to be compensated for. Thus the receiver complexity scales with dispersion for single carrier systems, whereas the overhead increases in an OFDM system. This holds for the PMD tolerance as well, although PMD compensation is less stringent for a coherent receiver as it generally has a much shorter impulse response. The system complexity will be discussed in more detail in the next section. In coherent detected transmission systems, PMD and CD provide a combined penalty [2,3,15]. PMD, however, causes depolarization as well, thus in order to realize a large PMD tolerance a polarization diverse receiver is required. It has been shown that in combination with a polarization diverse receiver both single carrier [14] and multi-carrier (OFDM) [15] systems can offer a practically unlimited tolerance towards PMD.

3.6. Direct detected OFDM

In the comparison so far we have compared coherent detected QPSK with coherent detected OFDM. However, OFDM can be realized as well with direct detection and it has been shown that similar to coherent detected optical OFDM (CO-OFDM), a large tolerance to linear impairments can be realized with direct detected optical OFDM (DDO-OFDM) [20]. DDO-OFDM is realized by sending the optical carrier along with the OFDM signal so that direct detection with a single photodiode can be used at the receiver to convert the optical field back into the electrical domain. Recently, promising methods have been shown to realize PDM with such a direct detected system [21].

DDO-OFDM is more cost effective as it requires the least optical and electrical components at TX and RX, but does not provide the optimal performance with respect to ASE. Furthermore, DDO-OFDM requires a guard band between the optical carrier and the OFDM signal, which increases the required bandwidth of electrical and optical components and reduces the feasible spectral efficiency. Therefore, DDO-OFDM is not the preferred modulation format for modulation of high data rates (100 Gb/s and higher). CO-OFDM does not require such guardband and additionally provides superior transmission performance. Therefore, this modulation format is has been considered for long-haul transmission systems. The main problem of CO-OFDM is, however, that phase noise compensation is required at the receiver to compensate for the phase variations of the transmitter and receiver laser. As discussed in Section 2.2, the compensation of phase noise increases the data rate overhead or the complexity of the compensation algorithm.

In conclusion it can be said that the superior performance of CO-OFDM with respect to optical signal-to-noise ratio (OSNR) requirements, PMD tolerance and spectral efficiency makes it an excellent candidate for long-haul transmission systems, whereas DDO-OFDM is more suitable for cost-effective short reach applications.

4. System complexity comparison

The algorithm complexity directly affects implementation cost in terms of gate count and power consumption. In this section the complexity of PDM-QPSK and PDM-OFDM are compared. There are many possible equalizer structures for both the single

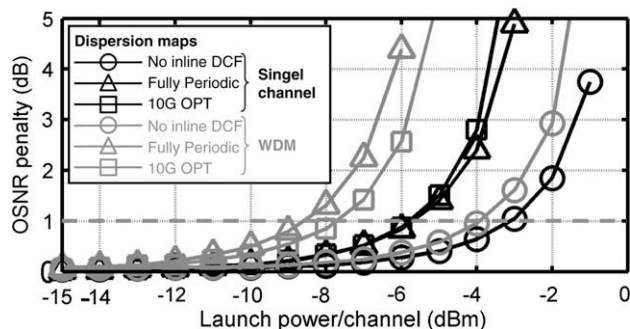


Fig. 4. BER as a function of optical filtering bandwidth of 120-Gb/s PDM-OFDM.

and multi carrier case [10]. However, in this section we confine ourselves to the most commonly used structures. Fig. 5 shows the considered equalizer structures for the single carrier and multi carrier approach. Apart from several functions which are inherently different (clock recovery, OFDM symbol synchronization, cyclic prefix removal, etc.) the equalizers in both approaches primarily equalize CD and PMD.

For the single carrier system (shown in Fig. 5a), the complete equalization structure is located at the receiver. This equalization structure is divided into two parts: a usually static frequency domain equalizer (FDE) followed by an adaptive time domain equalizer (TDE), the rationale behind this two part structure being that the FDE compensates most of the time-invariant chromatic dispersion and the TDE takes care of PMD and polarization rotation. It can easily be shown that at high bit rates the frequency domain implementation is much less complex than the time domain implementation already at moderate chromatic dispersion values like some thousands of ps/nm [10]. Thus, it makes sense to implement the chromatic dispersion equalizer in the frequency domain. The TDE adopts the typical butterfly form consisting of four complex valued adaptive FIR filters that equalize remaining CD and PMD. In Fig. 5b the equalizer structure of the corresponding multi carrier system is shown where the equalizer is distributed between transmitter and receiver. The transmitter contains an inverse Fourier transform while the receiver contains a Fourier transform followed by a one-tap equalizer in the frequency domain consisting of four banks of multipliers, one per subcarrier. This one-tap equalizer takes care of chromatic dispersion and PMD at the same time.

The complexity of the equalizer structures is evaluated in terms of complex multiplications per bit taking into account adaptive update algorithms and channel estimation. For PDM-QPSK we assume that the input signals are sampled at twice the symbol rate. We assume that the single carrier FDE is static and the TDE is adapted using a Least-Mean-Squares (LMS) algorithm. Implementing the LMS update algorithm for the TDE costs one complex multiplication per filter tap. For PDM-OFDM we assume that the channel is estimated periodically every $1/R_{\text{pilot}}$ symbols using a known OFDM pilot symbol in both polarizations and the filter taps are determined by calculating the Moore–Penrose pseudo inverse of the channel matrix for every subcarrier. Independent of the CD to be compensated we restrict all structures to a minimum equivalent length of $9 T/2$ taps in order not to compromise the equalizers' ability to mitigate PMD.

Frequency domain equalization on two polarizations requires the evaluation of four FFTs and $2N$ complex multiplications. Thus, the number of complex multiplications for this is

$$2N + 4 \frac{N}{2} \log_2(N), \quad (5)$$

where N is the FFT size, and the number of useful bits in these two blocks is given by

$$2(N - L_{\text{IR}}) \frac{\log_2(M)}{s_T}, \quad (6)$$

where L_{IR} is the length of the equalizer impulse response which corresponds approximately to the channel impulse response in this case, s_T is the number of samples per symbol, and M is number of points of the signal constellation, e.g. 4 for QPSK.

A complex FIR filter with L taps needs L complex multiplications resulting in $\log_2(M)/s_T$ bits per calculation. Hence, the TDE with four complex FIR filters with L taps needs four L complex multiplications and gives $2 \log_2(M)/s_T$ bits per calculation. Since we are interested only in every second output sample corresponding to a main sample, the effort is cut in half. However, the tap update consumes an additional multiplication per tap which again doubles the effort. Everything put together, we get as number of complex multiplications per bit for the single carrier equalization

$$N_{\text{SC}} = \frac{N + N \log_2(N)}{(N - L_{\text{IR}}) \log_2(M)/s_T} + \frac{2L}{\log_2(M)/s_T}. \quad (7)$$

For PDM-OFDM we assume an oversampling ratio N/U , where N is the number of subcarriers including virtual subcarriers (or FFT size) and U is the number of used subcarriers. OFDM requires four FFTs and four U complex multiplications for $2U \log_2(M)$ useful bits resulting in

$$\left(4 \frac{N}{2} \log_2(N) + 4U \right) / (2U \log_2(M)) \quad (8)$$

complex multiplications per bit. Regarding channel estimation we assume the training symbol setup shown in Fig. 6a. We assume that during time slot 1 a training symbol Tx_1 is sent on polarization 1, while nothing is sent on polarization 2. In time slot 2 a training symbol Tx_2 is sent on polarization 2, while nothing is sent on polarization 1. With the MIMO channel model shown in Fig. 6b the received signal can be described in matrix form

$$\begin{pmatrix} Rx_1 \\ Rx_2 \end{pmatrix} = \begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix} \begin{pmatrix} Tx_1 \\ Tx_2 \end{pmatrix} = \mathbf{G} \begin{pmatrix} Tx_1 \\ Tx_2 \end{pmatrix}. \quad (9)$$

If we denote the received signals in time slot 1 by Rx_{1-T_1} and Rx_{2-T_1} and the received signals in time slot 2 by Rx_{1-T_2} and Rx_{2-T_2} , we can establish the following correspondences

$$\begin{aligned} G_{11} &= Rx_{1-T_1}/Tx_1, \\ G_{12} &= Rx_{2-T_1}/Tx_1, \\ G_{21} &= Rx_{1-T_2}/Tx_2, \\ G_{22} &= Rx_{2-T_2}/Tx_2. \end{aligned} \quad (10)$$

Since the channel matrix has to be evaluated for each used subcarrier, we can deduce from (10) that calculation of all channel matrices requires four U complex multiplications (calculation of $1/Tx_1$ can be carried out in advance).

To obtain estimates of the sent data $(Tx_1 Tx_2)^T$ (9) must be inverted. Calculation of the pseudo inverses $(\mathbf{G}^H \mathbf{G})^{-1} \mathbf{G}^H$ of the U channel matrices \mathbf{G} of dimension 2×2 requires two matrix multiplications of complexity P^3 each and a matrix inversion of complexity $2 + P^2$ (which assumes the same complexity of division and multiplication), where $P = 2$ is the number of polarizations.

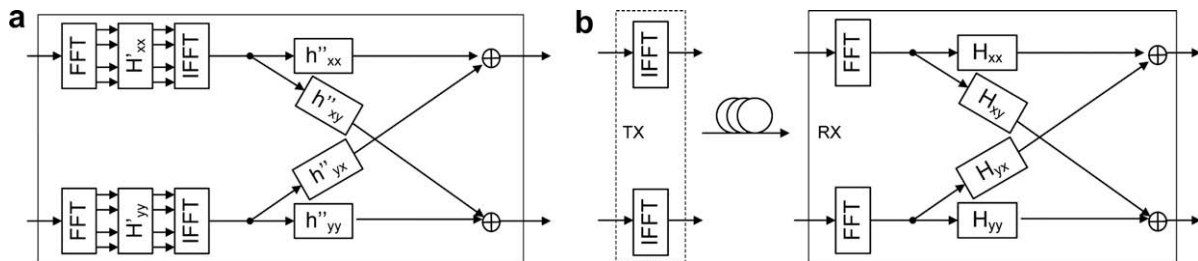


Fig. 5. Equalizer structures for the (a) single carrier and (b) multi carrier system.

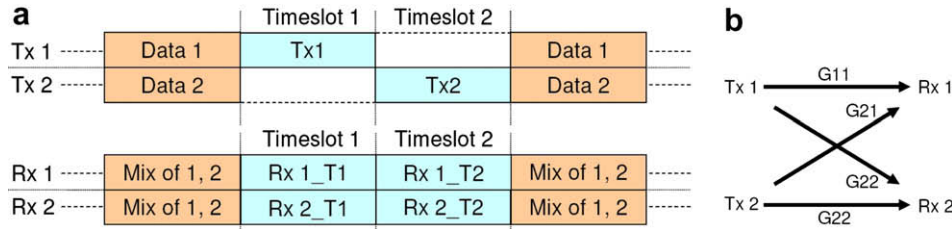


Fig. 6. (a) OFDM training symbol setup. (b) MIMO channel.

Put together, calculation of U channel matrices and calculation of the U channel pseudo inverse matrices requires

$$4U + U(2P^3 + 2 + P^2) = 26U \quad (11)$$

multiplications per estimate. Using (8) and assuming that the channel is estimated every $1/R_{\text{Pilot}}$ symbols, we get

$$N_{\text{MC}} = (N/U \log_2(N) + 2 + 13R_{\text{Pilot}}) / \log_2(M) \quad (12)$$

complex multiplications per bit for channel estimation and equalization for the multi carrier system.

Fig. 7 (left) shows the complexity required to compensate a given amount of CD at 112 Gb/s. The length of the impulse response L_{IR} to be equalized is calculated as the length containing 80% of the power of the fiber impulse response filtered by the receiver low pass filter. L_{IR} is shown dashed in the bottom chart in multiples of $T/2$, T being the symbol duration of PDM-QPSK. The receive filter bandwidth is 0.7 times the symbol rate for PDM-QPSK and 1.1 times the effective bandwidth $B_{\text{eff}} = f_0/2 U/N$ for PDM-OFDM, where f_0 is the sampling frequency and we assume an oversampling ratio $N/U = 1.5$. For PDM-QPSK L_{IR} approximately equals the equalizer length necessary to equalize the distortion with an OSNR penalty smaller than 1 dB. The FFT size is optimized for each value of L_{IR} and shown in the bottom chart as N_{SC} , right axis. A value of 256 is used as minimum limit to assure sufficient frequency resolution. For PDM-OFDM, the parameters N_{MC} , and f_0 are minimized subject to the constraints of sufficient net bit rate and the length of the cyclic prefix being equal to L_{IR} . Furthermore, the sampling rate for the multi carrier approach was constrained to be smaller or equal to the sampling rate in the single carrier approach and the cyclic prefix overhead was constrained to be lower

than 15%. The resulting number of subcarriers is shown in the bottom chart as N_{MC} .

From Fig. 7 it is evident that training symbol based multi carrier equalization is much less complex than the blind single carrier equalization. This is true for all values of chromatic dispersion. The complexity of the SC equalizer increases sharply when the TDE is not sufficient any longer to compensate CD and additional overhead must be spent on the FFT processing. Then, however, complexity grows only logarithmically as with the MC equalizer. The two curves for blind SC equalization correspond to a maximum tolerable DGD value of 50 ps and 100 ps, respectively. Increasing the maximum tolerable DGD from 50 ps to 100 ps increases the computational complexity by about 10 complex multiplications per transmitted bit. Thus, this parameter plays an important role in designing a single carrier system with blind equalization.

The large advantage of multi carrier over single carrier equalization is only caused by the expensive TDE and blind update of the TDE. If a cyclic prefix and training symbols would be used in the single carrier case, the TDE would not be necessary and the complexity of both approaches would be identical [22]. This can easily be seen as both systems then share the same components just in a different order. Therefore, the important difference between the two approaches in Fig. 7 is not single versus multi carrier equalization but rather blind equalization versus equalization with training symbols.

Fig. 7 (right) shows multi carrier system parameters optimized for each value of CD. The constraints on the sampling rate ($f_0 < 56$ GHz) and the CP overhead (<15%) can be seen clearly. For small CD values a small T_{symbol} is sufficient to keep the CP overhead small. As CD grows, the sampling rate f_0 has to be increased to

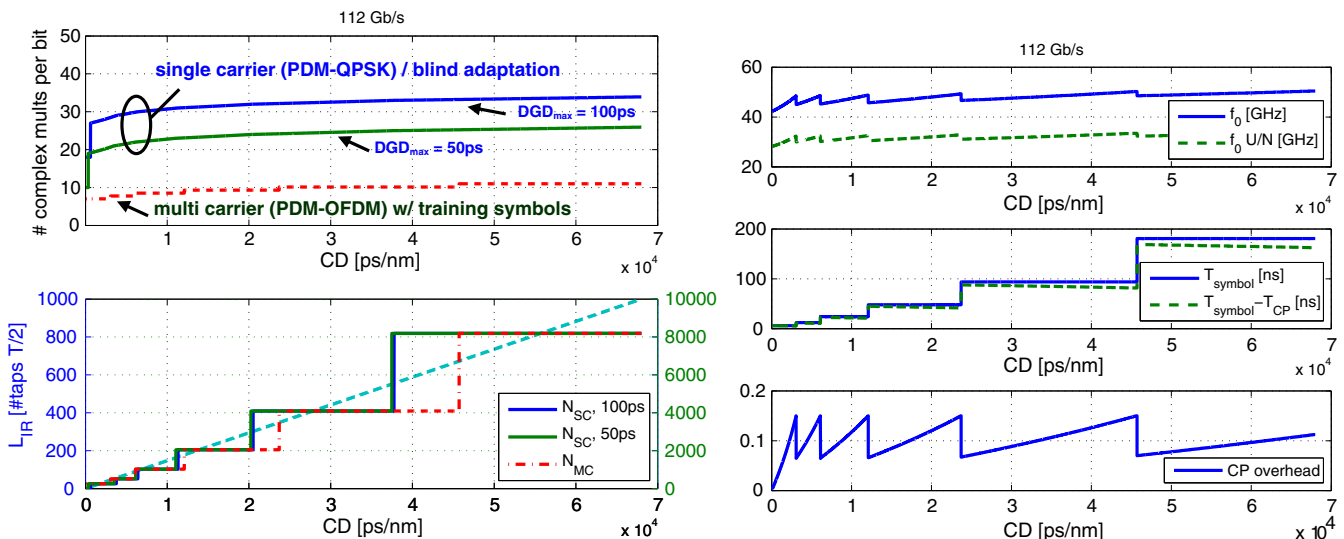


Fig. 7. Left: complexity of blind single carrier equalization and training symbol based multi carrier equalization at 112 Gb/s in terms of complex multiplications per bit for a given CD to be compensated. Right: multi carrier system parameters f_0 , T_{symbol} , $T_{\text{symbol}} - T_{\text{CP}}$, CP overhead.

accommodate the longer cyclic prefix. The result of increasing f_0 is that effectively the OFDM symbol duration stays constant while the symbol duration without cyclic prefix decreases leaving more space for the cyclic prefix. As soon as the CP overhead hits the 15% limit, the number of subcarriers must be increased (typically by a factor of two) and sampling rate and cyclic prefix overhead can be decreased.

Note that the number of carriers and thus the OFDM symbol duration must increase steadily with CD in order to keep the cyclic prefix overhead manageable. A long OFDM symbol duration eventually leads to problems with fast channel variations as training symbols for channel estimation can only be inserted at a certain distance to keep the overhead small. Assuming for instance two pilot symbols every 50 OFDM symbols (corresponding to an overhead of 4%) and a maximum time between channel estimates of $T_s < 100$ ns. This value is exceeded at $CD \approx 46000$ ps/nm. For larger CD values a rapid polarization change within $5 \mu\text{s}$ will incur large penalties if not more overhead is spent on pilot symbols.

5. Conclusions

In this paper we have discussed the design of a 100GbE PDM-OFDM system and compared the performance of PDM-OFDM to that of PDM-QPSK. PDM-OFDM can be easily scaled to higher constellation sizes and allows for flexible oversampling rates, but at the cost of a more complex transmitter (DAC required). Because of its high PAPR, the nonlinear tolerance OFDM requires pre- and post-compensation in links with periodic dispersion compensation. This adds additional complexity to the system. Still, the overall complexity of OFDM is lower than that of QPSK, although we have shown that this is not inherent to the modulation format, but rather to the equalization algorithm.

As long as QPSK is used as modulation format, PDM-QPSK has the great benefit that no DACs are required at the transmitter and, therefore, for the first generation of 100GbE transmission systems it is most likely that this modulation format is used despite the more complex receiver. For higher constellation sizes, however, the complexity of both the modulation at the transmitter as well as equalization at the receiver is significantly increased in the case of single carrier, whereas it remains the same for OFDM. Therefore, it is expected that for these systems OFDM will be used.

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