Unified Performance Analysis of Two-Hop Amplify-and-Forward Relay Systems with Antenna Correlation

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***Abstract*—We present a unified performance analysis of a system in which the source and the destination are equipped with multiple antennas and communicating via a single antenna relay. Our studies can be divided into two parts. First we consider a system with maximal ratio transmission (MRT) at the source and maximal ratio combining (MRC) at the destination by assuming general correlation structures with arbitrary eigenvalue multiplicities at the source and the destination. Then a system with transmit antenna selection (TAS) at the source with uncorrelated antennas and MRC at the destination with correlated antennas is investigated. The exact closed form expressions for outage probability, average symbol error rate (SER), generalized higher moments of SNR for both channel state information (CSI)-assisted and fixed gain relaying are derived and an analysis of the ergodic capacity is provided. Hence, our new results cover several previously reported cases as well as new additional ones. Further, we present the asymptotic analysis which gives an insight of the system performance and the diversity gain in each case. To verify the analytical results we provide Monte Carlo simulations at the end.**

***Index Terms*—CSI-assisted relaying, fixed gain relaying, MRT, TAS, antenna correlation.**

I. Introduction

BOTH CSI-assisted and fixed gain two-hop amplify-and-forward (AF) relays with a source having multiple antennas communicating with multiple antenna destination via a single antenna relay have been investigated in numerous previous literature. The analysis [1]–[9] considered MRT and MRC to improve the system performance.

A. Related Previous Work

Authors in [2] presented the performance of two-hop CSI-assisted relay network over independent Rayleigh fading channels with MRT at the source and MRC at the destination. The performance of the same system, considering the antenna correlation effects at the source and the destination has been analyzed in [3]. Recently, in [6] and [7], the performance of a dual-hop fixed gain network over independent Rayleigh and Nakagami-m fading environments has been analyzed. The fixed gain relaying with exponential correlation is reported in [8].

Authors in [10] introduced a different system model of a single antenna source communicating with a destination equipped with multiple antennas through a CSI-assisted relay having multiple receive antennas employing selection combining (SC) and a single transmit antenna. They have derived the closed form solutions for two cases; dual correlated and multiple uncorrelated antennas at the relay, with no closed form solution for multiple correlated antennas case. For the above cases they have considered correlated antennas at the destination, where correlation matrices have distinct eigenvalues.

B. Contributions

Use of multiple antennas increases the system performance, where as it decreases with the antenna correlation. It is important to investigate this performance loss due to antenna correlation. To the best of our knowledge, the previous studies have only considered specific cases of correlation in CSI-assisted and/or fixed gain relaying.

This paper aims to provide a unified analysis of MRT/MRC for a two-hop AF relay system with arbitrary correlation at the source and the destination. Further, we investigate the performance of the TAS/MRC for the same system with uncorrelated antennas at the source and correlated antennas at the destination. We extend our analysis and results in [9] which was limited to fixed gain to CSI-assisted relaying. A general fading model with arbitrary eigenvalue distribution for the correlation matrices is assumed in which eigenvalues need not be distinct, which allows us to investigate new correlation matrix structures [e.g. Uniform correlation [11, Eq.6.b]]. Specifically, we derive the exact closed form solutions to outage probability, average SER, generalized higher moments of SNR and provide ergodic capacity analysis for both CSI-assisted and fixed gain relay schemes. In order to gain insights, we derive asymptotic expressions in high SNR.

II. System Model

Consider a communication system where a source ($\mathtt{S}$) equipped with n_{T} antennas, communicates with a destination ($\mathtt{D}$), equipped with n_{R} antennas, via a single antenna relay ($\mathtt{R}$).

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We assume that S does not have a direct link to D. Hence there is no cooperative diversity and the diversity gain expected from multiple antennas is expected to play an important role.

The communication from S to D via R takes place in two time slots. In the first time slot, S beamforms its signal to R. The received signal at R can be written as,

$$y_R = \sqrt{P}_1 \boldsymbol{\Delta}^\dagger \mathbf{w}_\mathrm{T} x + n_1, \tag{1}$$

where x is the transmit signal at S and n_1 is zero mean additive white Gaussian noise (AWGN) at R, satisfying $E(|x|^2) = 1$ and $E(|n_1|^2) = N_{01}$, P_1 denotes the transmit power. $\boldsymbol{\Delta} = \boldsymbol{\Phi}_\mathrm{T}^{\frac{1}{2}} \mathbf{h}_1$ for MRT/MRC system and $\boldsymbol{\Delta} = h_1^s$ for TAS/MRC system, $n_\mathrm{T} \times 1$ channel vector from S to R is $\mathbf{h}_1 = [h_1^1, \ldots, h_1^{n_\mathrm{T}}]^T$, where h_1^j is a Rayleigh fading entry and $(\cdot)^T$ and $(\cdot)^\dagger$ denotes the transpose and Hermitian transpose, respectively and $|h_1^s|$ = $\max_{[1<i<n_\mathrm{T}]} |h_1^i|$. $\boldsymbol{\Phi}_\mathrm{T}$ is the $n_\mathrm{T} \times n_\mathrm{T}$ matrix, which models the spatial correlation at S. The weight vector $\mathbf{w}_\mathrm{T}$ is chosen for transmit beamforming as, $\frac{\boldsymbol{\Delta}}{||\boldsymbol{\Delta}||_F}$.

After the first time slot, the received signal y_R at the R is multiplied by a gain, G and then retransmitted to the D during the second time slot. The received signal at D is given by

$$\begin{aligned} y_D &= \sqrt{P}_2 \boldsymbol{\Phi}_\mathrm{R}^{\frac{1}{2}} \mathbf{h}_2 G y_R + \mathbf{n}_2 \qquad (2) \\ &= \sqrt{P}_2 \boldsymbol{\Phi}_\mathrm{R}^{\frac{1}{2}} \mathbf{h}_2 G \left(\sqrt{P}_1 \boldsymbol{\Delta}^\dagger \mathbf{w}_\mathrm{T} x + n_1 \right) + \mathbf{n}_2. \end{aligned}$$

Here P_2 is the relay transmit power, $\mathbf{h}_2 = [h_2^1, \ldots, h_2^{n_\mathrm{R}}]^T$ is $n_\mathrm{R} \times 1$ channel vector from R to D, where h_2^j is a Rayleigh fading entry and $\mathbf{n}_2$ is the AWGN $n_\mathrm{R} \times 1$ noise vector having the variance of $N_{02} \mathbf{I}_{n_\mathrm{R}}$, $\mathbf{I}_n$ is the identity matrix of size n. $\boldsymbol{\Phi}_\mathrm{R}$ is the $n_\mathrm{R} \times n_\mathrm{R}$ matrix, which models the spatial correlation at D.

At D, for MRC, we multiply the received signal by $1 \times n_\mathrm{R}$ weight vector $\mathbf{w}_\mathrm{R}$, $\mathbf{w}_\mathrm{R}^\dagger = \frac{\boldsymbol{\Phi}_\mathrm{R}^{\frac{1}{2}} \mathbf{h}_2}{||\boldsymbol{\Phi}_\mathrm{R}^{\frac{1}{2}} \mathbf{h}_2||_F}$, and after some manipulations, the resulting end-to-end SNR, can be written as [3]

$$\gamma_\text{end} = \frac{\frac{P_1 ||\boldsymbol{\Delta}||_F^2}{N_{01}} \frac{P_2 ||\boldsymbol{\Phi}_\mathrm{R}^{\frac{1}{2}} \mathbf{h}_2||_F^2}{N_{02}}}{\frac{P_2 ||\boldsymbol{\Phi}_\mathrm{R}^{\frac{1}{2}} \mathbf{h}_2||_F^2}{N_{02}} + \frac{1}{G^2 N_{01}}}. \tag{3}$$

Let the distinct real eigenvalues of the transmit correlation matrix, $\boldsymbol{\Phi}_\mathrm{T}$ be denoted by $\phi_1, \phi_2, \ldots, \phi_t$ with multiplicities $\nu_1, \nu_2, \ldots, \nu_t$ respectively such that $\sum_{i=1}^t \nu_i = n_\mathrm{T}$ and the distinct real eigenvalues of the receive correlation matrix, $\boldsymbol{\Phi}_\mathrm{R}$ be denoted by $\sigma_1, \sigma_2, \ldots, \sigma_r$ with multiplicities $\eta_1, \eta_2, \ldots, \eta_r$ respectively where $\sum_{u=1}^r \eta_u = n_\mathrm{R}$.

A. CSI-Assisted Relaying

Based on the previous literature, in the CSI-assisted case we select the variable gain as,

$$G = \sqrt{\frac{1}{P_1 \|\boldsymbol{\Delta}\|_F^2 + N_{01}}} \tag{4}$$

Then, the end-to-end SNR given in (3) has the following form:

$$\gamma_\text{end1} = \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2 + c}. \tag{5}$$

where $\gamma_1 = \|\boldsymbol{\Delta}\|_F^2 \rho_1$ and $\gamma_2 = \|\boldsymbol{\Phi}_\mathrm{R}^{\frac{1}{2}} \mathbf{h}_2\|_F^2 \rho_2$ respectively, with $c = 1$ if we consider the channel noise, $c = 0$ otherwise. $\rho_1 = P_1/N_{01}$ and $\rho_2 = P_2/N_{02}$.

B. Fixed Gain Relaying

When we use the fixed gain, it can be shown that the end-to-end SNR takes the following form:

$$\gamma_\text{end2} = \frac{\gamma_1 \gamma_2}{\gamma_2 + C}. \tag{6}$$

where γ_1 and γ_2 are the same as described above and C is the fixed gain. There are two methods of selecting a fixed gain as described in the technical literature. With the following fixed gain,

$$G = \sqrt{E_{\|\boldsymbol{\Delta}_1\|_F^2} \left[\frac{1}{P_1 \|\boldsymbol{\Delta}_1\|_F^2 + N_{01}} \right]}, \tag{7}$$

the constant C_1 is given by

$$C = C_1 = \left(E_{\gamma_1} \left[\frac{1}{(\gamma_1 + 1)} \right] \right)^{-1}. \tag{8}$$

If the fixed gain factor is selected as

$$G = \sqrt{\frac{1}{P_1 E_{\|\boldsymbol{\Delta}_1\|_F^2} [\|\boldsymbol{\Delta}_1\|_F^2] + N_{01}}}, \tag{9}$$

the constant C_2 can be written as,

$$C = C_2 = E_{\gamma_1}(\gamma_1) + 1. \tag{10}$$

C. MRT/MRC system

The probability density function (pdf) of the random variable (RV), $\gamma_1 = \|\boldsymbol{\Phi}_\mathrm{T}^{\frac{1}{2}} \mathbf{h}_1\|_F^2 \rho_1$ can be obtained by using [12] as,

$$p_{\gamma_1}(z) = \sum_{i=1}^t \sum_{j=1}^{\nu_i} \frac{\omega_{i,j}}{(j-1)! \, (\rho_1 \phi_i)^j} z^{j-1} e^{-\frac{z}{\rho_1 \phi_i}} \tag{11}$$

and the cumulative density function (cdf), $F_{\gamma_1}(\Lambda) = \int_0^\Lambda p_{\gamma_1}(z) dz$ can be derived with the help of [13, Eq.2.321.2] as

$$F_{\gamma_1}(\Lambda) = 1 - \sum_{i=1}^t \sum_{j=1}^{\nu_i} \sum_{k=0}^{j-1} \frac{\omega_{i,j}}{k!} \left(\frac{\Lambda}{\rho_1 \phi_i} \right)^k e^{-\frac{\Lambda}{\rho_1 \phi_i}} \tag{12}$$

where $\omega_{i,j}$ as defined in [12, Eq.7]. $\omega_{i,j}$ can be simplified to any correlation model and in section II-E we present the simplification of $\omega_{i,j}$ for exponential correlation, uniform correlation and independent cases. Pdf and cdf of γ_2 is similar to pdf and cdf of γ_1 and it can be obtained from replacing t, ν_i, ρ_1 and ϕ_i by r, η_u, ρ_2 and σ_u, respectively.

Performing the required expectation in (8) with the help of [13, Eq. (9.211.4)] gives us the following closed form solution for C_1,

$$C_1 = \left(\sum_{i=1}^t \sum_{j=1}^{\nu_i} \frac{\omega_{i,j}}{(\rho_1 \phi_i)^j} \Psi \left(j, j; \frac{1}{\rho_1 \phi_i} \right) \right)^{-1}. \tag{13}$$

where $\Psi(a, b; c)$ denotes the Tricomi confluent hypergeometric function defined in [13, Eq. (9.210.2)]. In the high SNR regime

$(\rho_1 \to \infty)$, using [14, Lemma 1] we can show that C_1 takes the following simplified form given by

$$C_1 = \rho_1 \left[\sum_{i=1}^{t} \left(\frac{\omega_{i,1} \ln(\rho_1 \phi_i)}{\phi_i} + \sum_{j=2}^{\nu_i} \cdot \frac{\omega_{i,j}}{(j-1)\phi_i} \right) \right]^{-1} \quad (14)$$

Similarly we can find the C_2 in (10) as,

$$C_2 = \rho_1 \sum_{i=1}^{t} \sum_{j=1}^{\nu_i} \omega_{i,j} j \phi_i + 1 \quad (15)$$

D. TAS/MRC system

Cdf of γ_1 for TAS can be derived using [15] and the binomial expansion as,

$$F_{\gamma_1}(z) = 1 - \sum_{p=1}^{n_\mathrm{T}} (-1)^{p+1} \binom{n_\mathrm{T}}{p} e^{\frac{-zp}{\rho_1}} \quad (16)$$

By differentiating (16), w.r.t z, we obtain the pdf of γ_1 as,

$$p_{\gamma_1}(z) = \frac{1}{\rho_1} \sum_{p=1}^{n_\mathrm{T}} (-1)^{p+1} \binom{n_\mathrm{T}}{p} p e^{\frac{-zp}{\rho_1}} \quad (17)$$

Using (17) to find the required expectation mentioned in (8) and (10) and with [13, Eq.3.352.4 and 3.326.2], we can find the fixed gain C_{T1} and C_{T2} as,

$$C_{T1} = \rho_1 \left[\sum_{p=1}^{n_\mathrm{T}} \binom{n_\mathrm{T}}{p} p(-1)^p e^{\frac{p}{\rho_1}} \mathrm{Ei}\left(-\frac{p}{\rho_1}\right) \right]^{-1} \quad (18)$$

where Ei(x) is the exponential integral of x defined in [13, Eq. 8.21]

$$C_{T2} = \rho_1 \sum_{p=1}^{n_\mathrm{T}} \binom{n_\mathrm{T}}{p} \frac{1}{p} (-1)^{p+1} + 1 \quad (19)$$

E. Correlation model

Exponential Correlation: In the special case of an exponential correlation matrix, we have all the eigenvalues distinct. For the source we have $t = n_\mathrm{T}$, $\nu_i = 1; \forall i$ and for the destination $r = n_\mathrm{R}$, $\eta_u = 1; \forall u$. Simplification of [12, Eq. (7)] yields

$$\omega_{i,j} = \phi_i^{t-1} \prod_{k=1, k\neq i}^{t} (\phi_i - \phi_k)^{-1} \quad (20)$$

$$\omega_{u,v} = \sigma_u^{r-1} \prod_{k=1, k\neq u}^{r} (\sigma_u - \sigma_k)^{-1} \quad (21)$$

Uniform Correlation: Uniform correlation [11, Eq. 6a] with parameter ρ, we have $\phi_1 = 1 + (n_\mathrm{T} - 1)\rho$ with multiplicity $\nu_1 = 1$ and $\phi_2 = 1 - \rho$ with multiplicity $\nu_2 = n_\mathrm{T} - 1$. $\omega_{i,j}$ can be obtained by simplifying [12, Eq. (7)] as,

$$\omega_{i,j} = \begin{cases} \frac{1}{(1-\delta)^{n_\mathrm{T}-1}}, & i = 1, j = 1 \\ -\frac{\delta}{(1-\delta)^{n_\mathrm{T}-j}}, & i = 2, j = 1, \ldots, n_\mathrm{T} - 1. \end{cases} \quad (22)$$

where $\delta = \frac{\phi_2}{\phi_1}$. Similarly $\omega_{u,v}$ can be written by replacing ϕ_i and n_T with σ_u and n_R respectively.

Independent: In this case we have all the eigenvalues equal to 1. For the source we have $t = 1$ with $\nu_i = n_\mathrm{T}$ and for the destination $r = 1$ with $\eta_u = n_\mathrm{R}$. [12, Eq. (7)] can be simplified as,

$$\omega_{i,j} = \begin{cases} 1, & i = 1, j = n_\mathrm{T} \\ 0, & i = 1, j = 1, \ldots, n_\mathrm{T} - 1. \end{cases} \quad (23)$$

Similarly $\omega_{u,v}$ can be written by replacing n_T with n_R.

III. Exact Outage Probability

The outage probability, P_{out} is a very important statistic which can be used to find several performance parameters of the system. It is defined as the probability that γ_{end} drops below a predefined SNR threshold Λ. The outage probability can be expressed as

$$P_{\text{out}} = \Pr(\gamma_{\text{end}} < \Lambda) = F_{\gamma_{\text{end}}}(\Lambda) \quad (24)$$

A. MRT/MRC system

1) CSI-Assisted Relaying: The cdf of the RV, γ_{end1} can be calculated as [2, Eq. (24)]

$$F_{\gamma_{\text{end1}}}(\Lambda) = 1 - \int_0^\infty \left(1 - F_{\gamma_1}\left(\Lambda + \frac{\Lambda^2 + c\Lambda}{z}\right)\right) p_{\gamma_2}(\Lambda + z) dz \quad (25)$$

Substituting (11) (for γ_2) and (12) in to (25) and with mathematical simplification, we obtain,

$$F_{\gamma_{\text{end1}}}(\Lambda) = 1 - \sum_{i=1}^{t} \sum_{j=1}^{\nu_i} \sum_{u=1}^{r} \sum_{v=1}^{\eta_u} \sum_{k=0}^{j-1} \frac{\omega_{i,j} \omega_{u,v} e^{-\left(\frac{1}{\rho_1 \phi_i} + \frac{1}{\rho_2 \sigma_u}\right)\Lambda}}{(v-1)!(\rho_1 \phi i)^k (\rho_2 \sigma_u)^v} \quad (26)$$

$$\times \frac{\Lambda^k}{k!} \int_0^\infty \left(\frac{c + \Lambda + z}{z}\right)^k (\Lambda + z)^{v-1} e^{-\frac{\Lambda^2 + c\Lambda}{\rho_1 \phi_i z}} e^{-\frac{z}{\rho_2 \sigma_u}} dz$$

Expanding the terms using the binomial expansion and with the help of [13, Eq. (3.471.9)], the integral in (26) can be solved in closed form. Therefore, the outage probability can be expressed as

$$F_{\gamma_{\text{end1}}}(\Lambda) = 1 - \Phi e^{-\left(\frac{1}{\rho_1 \phi_i} + \frac{1}{\rho_2 \sigma_u}\right)\Lambda} \Lambda^{\frac{2v+m+k-n-1}{2}} \quad (27)$$

$$\times (c + \Lambda)^{\frac{k+n-m+1}{2}} K_{m+n-k+1}\left(2\sqrt{\frac{\Lambda^2 + c\Lambda}{\rho_1 \rho_2 \phi_i \sigma_u}}\right)$$

where

$$\Phi = 2 \sum_{i=1}^{t} \sum_{j=1}^{\nu_i} \sum_{u=1}^{r} \sum_{v=1}^{\eta_u} \sum_{k=0}^{j-1} \frac{\omega_{i,j} \omega_{u,v}}{k!(v-1)!} \sum_{m=0}^{k} \sum_{n=0}^{v-1} \binom{k}{m} \binom{v-1}{n} \quad (28)$$

$$\times \frac{(\rho_2 \sigma_u)^{\frac{m+n+1-k-2v}{2}}}{(\rho_1 \phi_i)^{\frac{k+m+n+1}{2}}}$$

and K_v is the v-th order modified Bessel function of the second kind.

2) Fixed Gain Relaying: For fixed gain, cdf of γ_{end2} can be calculated as, [8, Eq.5]

$$F_{\gamma_{\text{end2}}}(\Lambda) = \int_0^\infty \Pr\left(\gamma_1 < \Lambda + \frac{C\Lambda}{\gamma_2} \middle| \gamma_2\right) p_{\gamma_2}(z) dz \quad (29)$$

Substituting (11) (for γ_2) and (12) into (29) and afterwards applying the Binomial expansion yields

$$F_{\gamma_{\text{end2}}}(\Lambda) = 1 - \sum_{i=1}^{t}\sum_{j=1}^{\nu_i}\omega_{i,j}e^{-\frac{\Lambda}{\rho_1\phi_i}}\sum_{u=1}^{r}\sum_{v=1}^{\eta_u}\frac{\omega_{u,v}}{(v-1)!(\rho_2\sigma_u)^v} \tag{30}$$

$$\times\sum_{k=0}^{j-1}\frac{\Lambda^k}{k!(\rho_1\phi_i)^k}\int_0^\infty\left(1+\frac{C}{z}\right)^k z^{v-1}e^{-\frac{C\Lambda}{\rho_1\phi_i z}-\frac{z}{\rho_2\sigma_u}}dz$$

Using again Binomial expansion and then with [13, Eq. (3.471.9)], the outage probability can be expressed in closed form as,

$$F_{\gamma_{\text{end2}}}(\Lambda) = 1 - \Psi e^{-\frac{\Lambda}{\rho_1\phi_i}}\Lambda^{\frac{v+n+k}{2}}K_{n+v-k}\left(2\sqrt{\frac{C\Lambda}{\rho_1\rho_2\phi_i\sigma_u}}\right) \tag{31}$$

where,

$$\Psi = 2\sum_{i=1}^{t}\sum_{j=1}^{\nu_i}\omega_{i,j}\sum_{u=1}^{r}\sum_{v=1}^{\eta_u}\frac{\omega_{u,v}}{(v-1)!}\sum_{k=0}^{j-1}\frac{1}{k!}\sum_{n=0}^{k}\binom{k}{n} \times\frac{C^{\frac{v+k-n}{2}}}{(\rho_1\phi_i)^{\frac{v+k+n}{2}}(\rho_2\sigma_u)^{\frac{v-n+k}{2}}} \tag{32}$$

B. TAS/MRC system

1) CSI-assisted Relaying: Here we analyze the system with TAS at S with S-R independent link while having correlation at D. By employing (11) (for γ_2) and (16) in (25), and using the Binomial expansion with some simplifications we obtain,

$$F_{\gamma_{\text{end1}}}(\Lambda) = 1 - \sum_{p=1}^{n_{\text{T}}}\binom{n_{\text{T}}}{p}\sum_{u=1}^{r}\sum_{v=1}^{\eta_u}\frac{\omega_{u,v}e^{-\Lambda\left(\frac{p}{\rho_1}+\frac{1}{\rho_2\sigma_u}\right)}}{(v-1)!(\rho_2\sigma_u)^v} \tag{33}$$

$$\times(-1)^{p+1}\sum_{q=0}^{v-1}\binom{v-1}{q}\Lambda^{v-q-1}\int_0^\infty z^q e^{-\frac{(\Lambda^2+c\Lambda)p}{\rho_1 z}-\frac{z}{\rho_2\sigma_u}}dz$$

This can be written using [13] as,

$$F_{\gamma_{\text{end1}}}(\Lambda) = 1 - \Xi e^{-\Lambda\left(\frac{p}{\rho_1}+\frac{1}{\rho_2\sigma_u}\right)}\Lambda^{\frac{2v-q-1}{2}}(\Lambda+c)^{\frac{q+1}{2}} \times K_{q+1}\left(2\sqrt{\frac{(\Lambda^2+c\Lambda)p}{\rho_1\rho_2\sigma_u}}\right) \tag{34}$$

where,

$$\Xi = 2\sum_{p=1}^{n_{\text{T}}}(-1)^{p+1}\binom{n_{\text{T}}}{p}\sum_{u=1}^{r}\sum_{v=1}^{\eta_u}\frac{\omega_{u,v}}{(v-1)!}\sum_{q=0}^{v-1}\binom{v-1}{q} \times\left(\frac{1}{\rho_2\sigma_u}\right)^{\frac{2v-q-1}{2}}\left(\frac{p}{\rho_1}\right)^{\frac{q+1}{2}} \tag{35}$$

2) Fixed Gain Relaying: Here we derive the closed form solution for the outage probability of TAS/MRC fixed gain system. By substituting (11) (for γ_2) and (16) in (29), then simplifying, we obtain,

$$F_{\gamma_{\text{end2}}}(\Lambda) = 1 - \sum_{p=1}^{n_{\text{T}}}\binom{n_{\text{T}}}{p}(-1)^{p+1}e^{-\frac{p\Lambda}{\rho_1}}\sum_{u=1}^{r}\sum_{v=1}^{\eta_u}\frac{\omega_{u,v}}{\Gamma(v)(\rho_2\sigma_u)^v} \tag{36}$$

$$\times\int_0^\infty z^{v-1}e^{-\frac{pC_T\Lambda}{\rho_1 z}-\frac{z}{\rho_2\sigma_u}}dz$$

By using [13, Eq. (3.471.9)], we can obtain (36) as,

$$F_{\gamma_{\text{end2}}}(\Lambda) = 1 - \Pi e^{-\frac{p\Lambda}{\rho_1}}\Lambda^{\frac{v}{2}}K_v\left(2\sqrt{\frac{pC_T\Lambda}{\rho_1\rho_2\sigma_u}}\right) \tag{37}$$

where,

$$\Pi = 2\sum_{p=1}^{n_{\text{T}}}\binom{n_{\text{T}}}{p}(-1)^{p+1}\sum_{u=1}^{r}\sum_{v=1}^{\eta_u}\frac{\omega_{u,v}}{\Gamma(v)}\left(\frac{pC_T}{\rho_1\rho_2\sigma_u}\right)^{\frac{v}{2}} \tag{38}$$

IV. Average Symbol Error Rate

In this section we derive the average symbol error rate for both γ_{end1} and γ_{end2}. The average SER, which is valid for different modulation schemes can be written as,

$$P_s = aE_{\gamma_{\text{end}}}[Q(\sqrt{2b\gamma_{\text{end}}})] \tag{39}$$

where $Q(z) = \frac{1}{\sqrt{2\pi}}\int_z^\infty e^{-\frac{y^2}{2}}dy$. a and b define the modulation schemes [e.g. for BPSK, $a = b = 1$]. Taking integration by parts of (39), we obtain,

$$P_s = \frac{a\sqrt{b}}{2\sqrt{\pi}}\int_0^\infty\frac{F_{\gamma_{\text{end}}}(z)}{\sqrt{z}}e^{-bz}dz \tag{40}$$

A. MRT/MRC system

1) CSI-assisted relaying: Employing (27) in (40) and using [13, Eq. 6.621.3] we obtain the average SER as,

$$P_s = \frac{a}{2} - \frac{a\sqrt{b}\Phi}{2}\frac{(2\beta)^\nu}{(\alpha+\beta)^{\lambda+\nu}}\frac{\Gamma(\lambda+\nu)\Gamma(\lambda-\nu)}{\Gamma(\lambda+\frac{1}{2})} \times F\left(\lambda+\nu,\nu+\frac{1}{2};\lambda+\frac{1}{2};\frac{\alpha-\beta}{\alpha+\beta}\right) \tag{41}$$

where, $\alpha = \frac{1}{\rho_1\phi_i}+\frac{1}{\rho_2\sigma_u}+b$, $\lambda = v+k+\frac{1}{2}$, $\nu = m+n-k+1$, $\beta = \frac{2}{\sqrt{\rho_1\rho_2\sigma_u\phi_i}}$, $F(\lambda,\nu;\mu;z)$ defined in [13, Eq. 9.10-9.12] and Φ as in (28).

2) Fixed gain relaying: With the substitution of (31) to (40) and performing the required integration by using [13, Eq.6.631.3], we can derive the average SER as,

$$P_s = \frac{a}{2} - \frac{a\Psi}{2\sqrt{\pi}b^{\frac{v+n+k}{2}}}\alpha^{-\frac{\lambda}{2}}\beta^{-1}\Gamma\left(\frac{1+\nu+\lambda}{2}\right) \times\Gamma\left(\frac{1-\nu+\lambda}{2}\right)e^{\frac{\beta^2}{8\alpha}}\text{W}_{-\frac{\lambda}{2},\frac{\nu}{2}}\left(\frac{\beta^2}{4\alpha}\right) \tag{42}$$

where, $\alpha = 1+\frac{1}{\rho_1\phi_i b}$, $\lambda = v+n+k$, $\nu = v+n-k$, $\beta = 2\sqrt{\frac{C}{\rho_1\rho_2\sigma_u\phi_i b}}$, $\text{W}_{\lambda,\nu}(z)$ defined in [13, Eq. 9.22] and Ψ as in (32).

B. TAS/MRC

1) CSI-assisted relaying: We use (34) in (40) and carry out the integration with the help of [13, Eq. 6.621.3] to obtain the average SER as,

$$P_s = \frac{a}{2} - \frac{a\sqrt{b}\Xi}{2}\frac{(2\beta)^\nu}{(\alpha+\beta)^{\lambda+\nu}}\frac{\Gamma(\lambda+\nu)\Gamma(\lambda-\nu)}{\Gamma(\lambda+\frac{1}{2})} \times F\left(\lambda+\nu,\nu+\frac{1}{2};\lambda+\frac{1}{2};\frac{\alpha-\beta}{\alpha+\beta}\right) \tag{43}$$

where, $\alpha = \frac{p}{\rho_1}+\frac{1}{\rho_2\sigma_u}+b$, $\lambda = v+\frac{1}{2}$, $\nu = q+1$, $\beta = 2\sqrt{\frac{p}{\rho_1\rho_2\sigma_u}}$ and Ξ as in (35).

2) Fixed gain relaying: We employ (37) in (40) and then integrate using [13, Eq.6.631.3] to derive the average SER as,

$$P_s = \frac{a}{2} - \frac{a\Pi}{2\sqrt{\pi}b^{\frac{v}{2}}}\alpha^{-\frac{\lambda}{2}}\beta^{-1}\Gamma\left(\frac{1+\nu+\lambda}{2}\right) \times \Gamma\left(\frac{1-\nu+\lambda}{2}\right)e^{\frac{\beta^2}{8\alpha}}\mathrm{W}_{-\frac{\lambda}{2},\frac{\nu}{2}}\left(\frac{\beta^2}{4\alpha}\right) \tag{44}$$

where, $\alpha = 1 + \frac{p}{\rho_1 b}$, $\lambda = v$, $\nu = v$, $\beta = 2\sqrt{\frac{pC_T}{\rho_1\rho_2\sigma_u b}}$ and Π as in (38).

V. Generalized Higher Moment of SNR and Ergodic Capacity

Here we derive the closed form solution for generalized higher moments of γ_{end1} and γ_{end2}. The SNR moments are important in measuring the performance of the system which can be used to find the average output SNR and the variance. Also it is essential in evaluating the system ergodic capacity.

$$E[\gamma_{\text{end}}^h] = \int_0^\infty z^h p_{\gamma_{\text{end}}}(z)dz \tag{45}$$

After integrating by parts, we obtain,

$$E[\gamma_{\text{end}}^h] = h\int_0^\infty z^{h-1}(1 - F_{\gamma_{\text{end}}}(z))dz \tag{46}$$

A. Generalized higher moments of SNR for MRT/MRC system

1) CSI-relaying: We substitute (27) to (46) and then perform the integration to obtain the desired result. Further by using [13, Eq. 6.621.3], we can obtain the higher moments of γ_{end1} as,

$$E[\gamma_{\text{end1}}^h] = h\Phi\sqrt{\pi}\frac{(2\beta)^\nu}{(\alpha+\beta)^{\lambda+\nu}}\frac{\Gamma(\lambda+\nu)\Gamma(\lambda-\nu)}{\Gamma(\lambda+\frac{1}{2})} \times F\left(\lambda+\nu, \nu+\frac{1}{2}; \lambda+\frac{1}{2}; \frac{\alpha-\beta}{\alpha+\beta}\right) \tag{47}$$

where, $\alpha = \frac{1}{\rho_1\phi_i} + \frac{1}{\rho_2\sigma_u}$, $\lambda = h+v+k$, $\nu = m+n-k+1$, $\beta = \frac{2}{\sqrt{\rho_1\rho_2\sigma_u\phi_i}}$ and Φ as in (28).

2) Fixed gain relaying: We can derive the higher moments of γ_{end2} by employing (37) in (46) and some mathematical manipulation with the help of [13, Eq.6.631.3] as,

$$E[\gamma_{\text{end2}}^h] = h\Psi\alpha^{-\frac{\lambda}{2}}\beta^{-1}\Gamma\left(\frac{1+\nu+\lambda}{2}\right)\Gamma\left(\frac{1-\nu+\lambda}{2}\right) \times e^{\frac{\beta^2}{8\alpha}}\mathrm{W}_{-\frac{\lambda}{2},\frac{\nu}{2}}\left(\frac{\beta^2}{4\alpha}\right) \tag{48}$$

where, $\alpha = \frac{1}{\rho_1\phi_i}$, $\lambda = 2h+v+n+k-1$, $\nu = v+n-k$, $\beta = 2\sqrt{\frac{C}{\rho_1\rho_2\sigma_u\phi_i}}$ and Ψ as in (32). In a similar way, the generalized higher moments of SNR can be found for TAS/MRC system, however, due to the space limitation, we omit it.

B. Ergodic capacity

Ergodic capacity is useful in describing the system performance. Ergodic capacity can be expressed as follows,

$$C_{erg} = \frac{1}{2}E_{\gamma_{\text{end}}}\left[\log_2(1+\gamma_{\text{end}})\right] \tag{49}$$

We can give an approximation to ergodic capacity [16, Eq. 6] as,

$$C_{erg} \approx \frac{1}{2}\log_2(e)\left[\ln(1+E[\gamma_{\text{end}}]) - \frac{E[\gamma_{\text{end}}^2] - E[\gamma_{\text{end}}]^2}{2(1+E[\gamma_{\text{end}}])^2}\right] \tag{50}$$

We can use (47) and (48) in (50) to evaluate the ergodic capacity of CSI-assisted and fixed gain relay systems respectively.

VI. High SNR Analysis

A. MRT/MRC system

1) CSI-Assisted Relaying: Here we obtain the solution for the outage probability at high SNR. We can express the cdf of γ_1 and γ_2 given in (12) by using Maclaurin Series and letting $\frac{\Lambda}{\rho_1} = z$ and $\rho_2 = \mu\rho_1$ as follows,

$$F_{\gamma_1}(z) = 1 - \sum_{i=1}^{t}\sum_{j=1}^{\nu_i}\sum_{k=0}^{j-1}\frac{\omega_{i,j}}{k!}\left(\frac{z}{\phi_i}\right)^k\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}\left(\frac{z}{\phi_i}\right)^n \tag{51}$$

$$F_{\gamma_2}(z) = 1 - \sum_{u=1}^{r}\sum_{v=1}^{\eta_u}\sum_{k=0}^{v-1}\frac{\omega_{u,v}}{k!}\left(\frac{z}{\mu\sigma_u}\right)^k\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}\left(\frac{z}{\mu\sigma_u}\right)^n \tag{52}$$

By substituting the $\omega_{i,j}$ and $\omega_{u,v}$ values and expanding the above summation, it is observed that $[z^n; n \le n_{\text{T}}]$ order terms sum to zero for $n_{\text{T}} < n_{\text{R}}$ and $[z^n; n < n_{\text{R}}]$ order terms sum to zero for $n_{\text{R}} < n_{\text{T}}$. Therefore we can derive the above cdf of γ_1 and γ_2 as,

$$F_{\gamma_1}(z) = \frac{z^{n_{\text{T}}}}{n_{\text{T}}!}\sum_{i=1}^{t}\sum_{j=1}^{\nu_i}\sum_{k=0}^{j-1}\frac{\omega_{i,j}(-1)^{n_{\text{T}}-k+1}}{\phi_i^{n_{\text{T}}}}\binom{n_{\text{T}}}{k} + o(z^{n_{\text{T}}+1}) \tag{53}$$

$$F_{\gamma_1}(z) = \frac{z^{n_{\text{R}}}}{n_{\text{R}}!\mu^{n_{\text{R}}}}\sum_{u=1}^{r}\sum_{v=1}^{\eta_u}\sum_{k=0}^{v-1}\frac{\omega_{u,v}(-1)^{n_{\text{R}}-k+1}}{\sigma_u^{n_{\text{R}}}}\binom{n_{\text{R}}}{k} + o(z^{n_{\text{R}}+1}) \tag{54}$$

We represent the function $f(x)$ of x as $o(x)$ if $\lim_{x\to 0}\frac{f(x)}{x} = 0$. By using [17, Eqs. (A.09) and (A.10)] we can express outage probability of γ_{end1} at high SNR as,

$$F_{\gamma_{\text{end1}}}(z) = \begin{cases} \frac{\alpha_{n_{\text{T}}}z^{n_{\text{T}}}}{n_{\text{T}}} + o(z^{n_{\text{T}}+1}), & n_{\text{T}} < n_{\text{R}} \\ \frac{\alpha_{n_{\text{Eq}}}z^{n_{\text{Eq}}}}{n_{\text{Eq}}} + o(z^{n_{\text{Eq}}+1}), & n_{\text{T}} = n_{\text{R}} = n_{\text{Eq}} \\ \frac{\alpha_{n_{\text{T}}}z^{n_{\text{R}}}}{n_{\text{R}}} + o(z^{n_{\text{R}}+1}), & n_{\text{T}} > n_{\text{R}} \end{cases} \tag{55}$$

where,

$$\alpha_{n_{\text{T}}} = \frac{1}{(n_{\text{T}}-1)!}\sum_{i=1}^{t}\sum_{j=1}^{\nu_i}\sum_{k=0}^{j-1}\frac{\omega_{i,j}(-1)^{n_{\text{T}}-k+1}}{\phi_i^{n_{\text{T}}}}\binom{n_{\text{T}}}{k} \tag{56}$$

$$\alpha_{n_R} = \frac{1}{(n_R-1)!\mu^{n_R}} \sum_{u=1}^{r}\sum_{v=1}^{\eta_u}\sum_{k=0}^{v-1} \frac{\omega_{u,v}(-1)^{n_R-k+1}}{\sigma_u^{n_R}} \binom{n_R}{k} \quad (57)$$

$$\alpha_{n_{Eq}} = \alpha_{n_T} + \alpha_{n_R} \quad (58)$$

From above expression we can say that the diversity order = $\min[n_T, n_R]$. We now simplify the general asymptotic expression in to several special cases.

Exponential Correlation: With the substitution of desired values of $\omega_{i,j}$ of (20), (21) and with some simplification we can convert (56),(57) and (58) to [3, Eq. (17)].

Uniform Correlation: By using the $\omega_{i,j}$ values of (22), we can simplify (56) and (57) for the special case of uniform correlation as follows,

$$\alpha_{n_T} = \frac{(-1)^{n_T+1}}{(n_T-1)!}\left[\frac{1}{\phi_1(\phi_1-\phi_2)^{n_T-1}} - \sum_{j=1}^{n_T-1}\sum_{k=0}^{j-1}(-1)^k\binom{n_T}{k} \times \frac{\phi_1^{n_T-1-j}}{\phi_2^{n_T-1}(\phi_1-\phi_2)^{n_T-j}}\right] \quad (59)$$

$$\alpha_{n_R} = \frac{(-1)^{n_R+1}}{(n_R-1)!\mu^{n_R}}\left[\frac{1}{\sigma_1(\sigma_1-\sigma_2)^{n_R-1}} - \sum_{v=1}^{n_R-1}\sum_{k=0}^{v-1}(-1)^k \times \binom{n_R}{k}\frac{\sigma_1^{n_R-1-v}}{\sigma_2^{n_R-1}(\sigma_1-\sigma_2)^{n_R-v}}\right] \quad (60)$$

Independent: By using the $\omega_{i,j}$ values of (23), we obtain (56) and (57), for the independent case as follows,

$$\alpha_1 = \frac{1}{(n_T-1)!}\sum_{k=0}^{n_T-1}(-1)^{n_T-k+1}\binom{n_T}{k} = \frac{1}{(n_T-1)!} \quad (61)$$

$$\alpha_2 = \frac{1}{(n_R-1)!\mu^{n_R}}\sum_{k=0}^{n_R-1}(-1)^{n_R-k+1}\binom{n_R}{k} = \frac{1}{(n_R-1)!\mu^{n_R}} \quad (62)$$

2) Fixed Gain Relaying: At high SNR we can express fixed gain $C = \rho_1 D$ and we can rewrite (31) by substituting $\rho_2 = \mu\rho_1$, $z = \frac{\Lambda}{\rho_1}$ as follows,

$$F_{\gamma_{\text{end2}}}(z) = 1 - 2\sum_{i=1}^{t}\sum_{j=1}^{\nu_i}\omega_{i,j}e^{-\frac{z}{\phi_i}}\sum_{u=1}^{r}\sum_{v=1}^{\eta_u}\frac{\omega_{u,v}}{\Gamma(v)}\sum_{p=0}^{j-1}\frac{1}{p!}\sum_{n=0}^{p} \times \binom{p}{n}\left(\frac{D}{\mu\sigma_u}\right)^{\frac{v+p-n}{2}}\left(\frac{z}{\phi_i}\right)^{\frac{v+n+p}{2}}K_{n+v-p}\left(2\sqrt{\frac{Dz}{\mu\phi_i\sigma_u}}\right) \quad (63)$$

Further we can rewrite the $F_{\gamma_{\text{end2}}}(z)$ by expanding the modified Bessel function using [13, Eq. 8.446, 8.447.3] and the even property of modified Bessel function $K_v(z) = K_{-v}(z)$ and using Maclaurin Series to expand the $e^{\frac{-z}{\phi_i}}$. Then substituting $\omega_{i,j}$ and $\omega_{u,v}$ values, it is observed that $z^n; n < n_T$ terms sum to zero for $n_T < n_R$ case, $z^n; n < n_R$ terms sum to zero for $n_R < n_T$ case and similarly for $n_R = n_T = n_{Eq}$. Therefore $F_Z(z)$ can be derived avoiding lower order terms as,

$$F_Z(z) = \begin{cases} \frac{\beta_{n_T}z^{n_T}}{n_T} + o(z^{n_T+1}), & n_T < n_R \\ \frac{\beta_{n_{Eq}}z^{n_{Eq}}}{n_{Eq}} + o(z^{n_{Eq}+1}), & n_T = n_R = n_{Eq} \\ \frac{\beta_{n_R}z^{n_R}}{n_R} + o(z^{n_R+1}), & n_T > n_R \end{cases} \quad (64)$$

where

$$\beta_N = N\sum_{i=1}^{t}\sum_{j=1}^{\nu_i}\sum_{u=1}^{r}\sum_{v=1}^{\eta_u}\sum_{p=0}^{j-1}\frac{\omega_{i,j}\omega_{u,v}}{\Gamma(v)p!}\sum_{n=0}^{p}\binom{p}{n}\left(\frac{1}{\phi_i}\right)^N\Theta \quad (65)$$

for $v+n-p>0$

$$\Theta = \sum_{k=0,N\geq p+k}^{v+n-p-1}\frac{(-1)^{N-p+1}(v+n-p-k-1)!D^{p-n+k}}{k!(N-p-k)!(\mu\sigma_u)^{p-n+k}} + \sum_{k=0}^{N-v-n}\left(\frac{D}{\mu\sigma_u}\right)^{v+k}\frac{(-1)^{N-p-k}\Upsilon_{v+n-p}}{k!(v+n-p+k)!(N-v-n-k)!}$$

for $v+n-p=0$

$$\Theta = \sum_{k=0}^{N-p}\left(\frac{D}{\mu\sigma_u}\right)^{v+k}\frac{(-1)^{N-p-k}}{k!k!(N-p-k)!}\Upsilon_0$$

for $v+n-p<0$

$$\Theta = \sum_{k=0,N\geq v+n+k}^{p-v-n-1}\frac{(-1)^{N-v-n+1}(p-v-n-k-1)!D^{v+k}}{k!(N-v-n-k)!(\mu\sigma_u)^{v+k}} + \sum_{k=0}^{N-p}\left(\frac{D}{\mu\sigma_u}\right)^{p-n+k}\frac{(-1)^{N-v-n-k}\Upsilon_{p-v-n}}{k!(p-n-v+k)!(N-p-k)!}$$

where

$$\Upsilon_x = \ln\left(\frac{Dz}{\mu\sigma_u\phi_i}\right) - \psi(k+1) - \psi(x+k+1) \quad (66)$$

and $\psi(x)$ is the Euler psi function. From the above expression we can say that the diversity order = $\min[n_T, n_R]$. Simplification of general case into several special cases are as follows.

Exponential Correlation: With the substitution of desired values of (20) and (21), the general expression, (65) can be reduced to the exponential correlation as,

$$\beta_N = N\sum_{i=1}^{n_T}\sum_{u=1}^{n_R}\frac{\omega_{i,1}\omega_{u,1}}{\phi_i^N}\left[\frac{(-1)^{N+1}}{N!} + \sum_{k=0}^{N-1}\left(\frac{D}{\mu\sigma_u}\right)^{k+1} \times \frac{(-1)^{N-k}\Upsilon_1}{k!(N-k-1)!(k+1)!}\right] \quad (67)$$

In [8], they considered approximated modified Bessel function expansion which lead them to omit important part of equation and lead them to decide diversity order = n_T.

Independent: With the substitution of $\omega_{i,j}$ and $\omega_{u,v}$ of independent case mentioned in (23), the general solution (65) can be simplified as,

$n_T < n_R$:

$$\beta_{n_T} = n_T\sum_{n=0}^{n_T}\frac{(n_R-n-1)!}{(n_R-1)!n!(n_T-n)!}\left(\frac{D}{\mu}\right)^n \quad (68)$$

$\underline{n_T > n_R}$:

$$\beta_{n_\mathrm{R}} = n_\mathrm{R}\frac{(n_\mathrm{T}-n_\mathrm{R}-1)!}{n_\mathrm{R}!(n_\mathrm{T}-1)!}\left(\frac{D}{\mu}\right)^{n_\mathrm{R}} \tag{69}$$

$\underline{n_T = n_R = n_{Eq}}$:

$$\beta_{n_\mathrm{Eq}} = n_\mathrm{Eq}\left[\sum_{n=0}^{n_\mathrm{Eq}-1}\frac{\left(\frac{D}{\mu}\right)^n}{\Gamma(n_\mathrm{Eq})n!(n_\mathrm{Eq}-n)} + \frac{1}{(n_\mathrm{Eq}!)^2}\right. \tag{70}$$
$$\left.\times\left(1-n_\mathrm{Eq}\left(\ln\left(\frac{Dz}{\mu}\right)-2\psi(1)\right)\right)\left(\frac{D}{\mu}\right)^{n_\mathrm{Eq}}\right]$$

B. TAS/MRC system

1) CSI-assisted Relaying: Here we obtain the high SNR outage probability for CSI-assisted relaying for TAS/MRC system. We can rewrite the cdf of γ_1 in (16) by substituting $\rho_2 = \mu\rho_1$, $z = \frac{\Lambda}{\rho_1}$ and using the Maclaurin Series series to expand e^{-pz} as,

$$F_{\gamma_1}(z) = 1 + \sum_{p=1}^{n_\mathrm{T}}\binom{n_\mathrm{T}}{p}(-1)^p\sum_{m=0}^{\infty}\frac{(-1)^m}{m!}p^m z^m \tag{71}$$

It is observed that $z^n; n < n_\mathrm{T}$ terms sum to zero, hence we can rewrite the above expression by doing some simplification for high SNR as,

$$F_{\gamma_1}(z) = \frac{z^{n_\mathrm{T}}}{n_\mathrm{T}}\sum_{p=1}^{n_\mathrm{T}}\binom{n_\mathrm{T}}{p}\frac{(-1)^{n_\mathrm{T}+p}}{(n_\mathrm{T}-1)!}p^{n_\mathrm{T}} + o(z^{n_\mathrm{T}+1}) \tag{72}$$
$$= z^{n_\mathrm{T}} + o(z^{n_\mathrm{T}+1})$$

Analysis for γ_2 is the same as CSI-assisted MRT/MRC high SNR analysis case and high SNR cdf of γ_2 derived in (54). Using a similar approach as given in [17, Eqs. (A.09) and (A.10)], we can rewrite the high SNR outage probability as,

$$F_Z(z) = \begin{cases} \frac{\theta_{n_\mathrm{T}} z^{n_\mathrm{T}}}{n_\mathrm{T}} + o(z^{n_\mathrm{T}+1}), & n_\mathrm{T} < n_\mathrm{R} \\ \frac{\theta_{n_\mathrm{Eq}} z^{n_\mathrm{Eq}}}{n_\mathrm{Eq}} + o(z^{n_\mathrm{Eq}+1}), & n_\mathrm{T} = n_\mathrm{R} = n_\mathrm{Eq} \\ \frac{\theta_{n_\mathrm{R}} z^{n_\mathrm{R}}}{n_\mathrm{R}} + o(z^{n_\mathrm{R}+1}), & n_\mathrm{T} > n_\mathrm{R} \end{cases} \tag{73}$$

where

$$\theta_{n_\mathrm{T}} = n_\mathrm{T} \tag{74}$$

$$\theta_{n_\mathrm{R}} = \frac{1}{(n_\mathrm{R}-1)!\mu^{n_\mathrm{R}}}\sum_{u=1}^{r}\sum_{v=1}^{\eta_u}\sum_{k=0}^{v-1}\frac{\omega_{u,v}(-1)^{n_\mathrm{R}-k+1}}{\sigma_u^{n_\mathrm{R}}}\binom{n_\mathrm{R}}{k} \tag{75}$$

$$\theta_{n_\mathrm{Eq}} = \theta_{n_\mathrm{T}} + \theta_{n_\mathrm{R}} \tag{76}$$

This shows that the diversity order is $\min[n_\mathrm{T}, n_\mathrm{R}]$.

2) Fixed Gain Relaying: For high SNR we can express fixed gain obtained in (18) and (19) as, $C_T = \rho_1 D_T$. We can rewrite (37) by substituting $\rho_2 = \mu\rho_1$, $z = \frac{\Lambda}{\rho_1}$ and expanding the modified Bessel function and using Maclaurin Series series to expand e^{-pz}. Then by substituting the corresponding $\omega_{u,v}$ values, it is observed that $z^n; n < n_\mathrm{T}$ terms sum to zero for $n_\mathrm{T} < n_\mathrm{R}$ case, $z^n; n < n_\mathrm{R}$ terms sum to zero for $n_\mathrm{R} < n_\mathrm{T}$ case and similarly for $n_\mathrm{R} = n_\mathrm{T} = n_\mathrm{Eq}$. Therefore $F_Z(z)$ can be derived avoiding lower order terms as,

$$F_Z(z) = \begin{cases} \frac{\vartheta_{n_\mathrm{T}} z^{n_\mathrm{T}}}{n_\mathrm{T}} + o(z^{n_\mathrm{T}+1}), & n_\mathrm{T} < n_\mathrm{R} \\ \frac{\vartheta_{n_\mathrm{Eq}} z^{n_\mathrm{Eq}}}{n_\mathrm{Eq}} + o(z^{n_\mathrm{Eq}+1}), & n_\mathrm{T} = n_\mathrm{R} = n_\mathrm{Eq} \\ \frac{\vartheta_{n_\mathrm{R}} z^{n_\mathrm{R}}}{n_\mathrm{R}} + o(z^{n_\mathrm{R}+1}), & n_\mathrm{T} > n_\mathrm{R} \end{cases} \tag{77}$$

where

$$\vartheta_N = N\sum_{p=1}^{n_\mathrm{T}}\binom{n_\mathrm{T}}{p}\sum_{u=1}^{r}\sum_{v=1}^{\eta_u}\frac{\omega_{u,v}p^N}{\Gamma(v)}\left[\sum_{k=0}^{v-1}\frac{(v-k-1)!D_T^k}{k!(N-k)!(\mu\sigma_u)^k}\right. \tag{78}$$
$$\left.\times(-1)^{N+p} + \sum_{k=0}^{N-v}\frac{(-1)^{N-k+p+1}}{k!(v+k)!(N-v-k)!}\left(\frac{D_T}{\mu\sigma_u}\right)^{v+k}\Upsilon_v^T\right]$$

where

$$\Upsilon_v^T = \ln\left(\frac{pD_T z}{\mu\sigma_u}\right) - \psi(k+1) - \psi(v+k+1) \tag{79}$$

From (77) we can conclude that diversity order = $\min[n_\mathrm{T}, n_\mathrm{R}]$

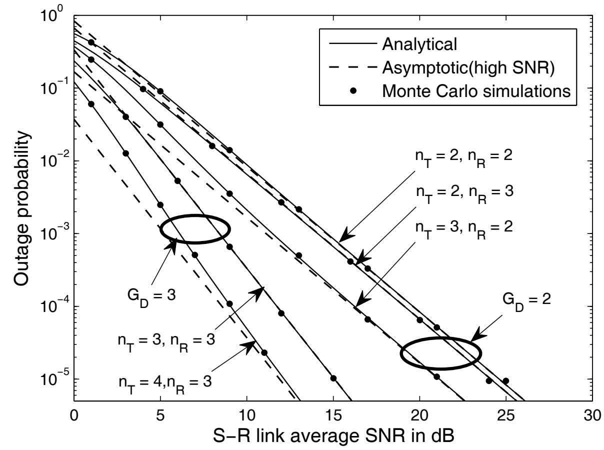

Fig. 1. CSI-assisted outage probability for $\mu = 2, \Lambda = 0$dB, exponential correlation with $\rho = 0.5$. MRT/MRC system

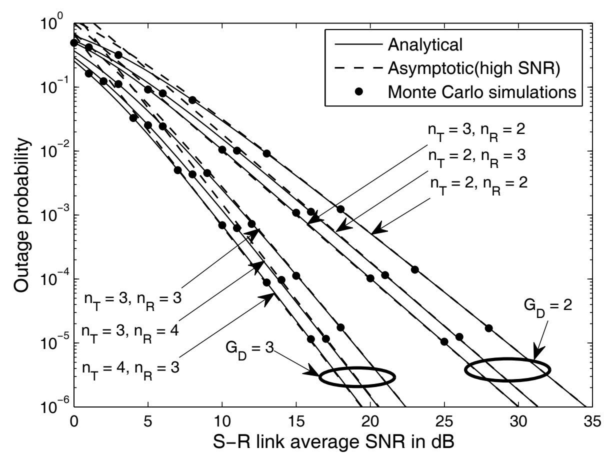

Fig. 2. Fixed Gain outage probability for $\mu = 2, \Lambda = 0$dB, exponential correlation with $\rho = 0.5$. Fixed gain C_2. MRT/MRC system.

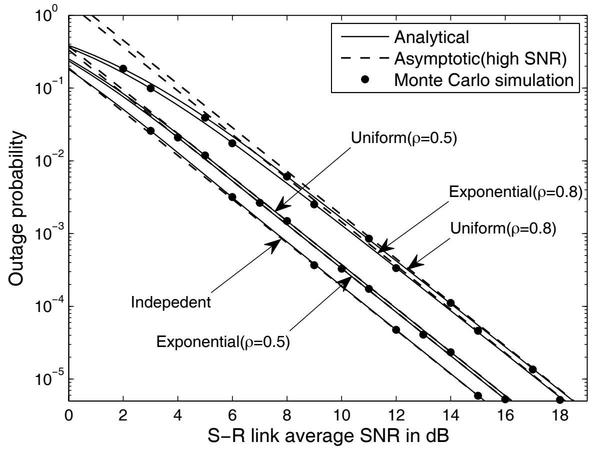

Fig. 3. CSI-assisted outage probability for $\mu = 2, \Lambda = 0$dB, for different correlation cases, $n_{\mathrm{T}} = n_{\mathrm{R}} = 3$. MRT/MRC system

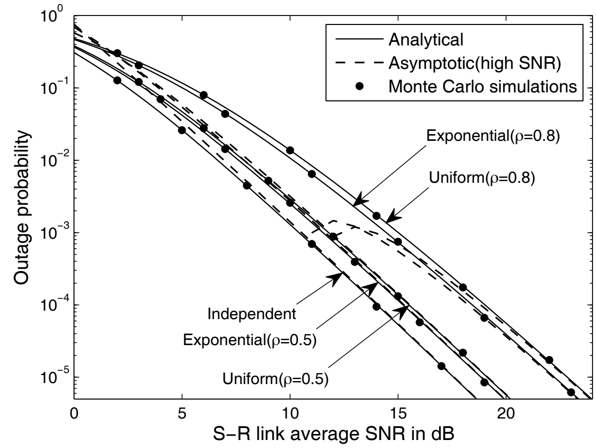

Fig. 4. Fixed Gain outage probability for $\mu = 2, \Lambda = 0$dB, for different correlation cases. Fixed gain C_2. MRT/MRC system.

C. Asymptotic average SER

Here we derive the asymptotic SER. From [18] we can write the asymptotic SER as,

$$P_s^{\infty} = \frac{2^t a \Omega \Gamma\left(t + \frac{3}{2}\right)(2b\rho_1)^{-(t+1)}}{\sqrt{\pi}(t+1)} + o(\rho_1^{-(t+1)}) \tag{80}$$

where a and b define the modulation scheme and $t = \min[n_{\mathrm{T}}, n_{\mathrm{R}}] - 1$. For CSI-assisted MRT/MRC relay, Ω is given in (56),(57) and (58) and for fixed gain MRT/MRC relay it is given in (65). For TAS/MRC CSI-assisted relay, Ω can be obtained from (74),(75) and (76) and for fixed gain relay in (78).

1) Diversity gain: Diversity gain of the system is given as, [18]

$$G_D = t + 1 \tag{81}$$

where $t = \min[n_{\mathrm{T}}, n_{\mathrm{R}}] - 1$.

2) Array gain: Array gain of the system is given as, [18]

$$G_A = 2b\left(\frac{2^t a \Omega \Gamma\left(t + \frac{3}{2}\right)}{\sqrt{\pi}(t+1)}\right)^{-\frac{1}{t+1}} \tag{82}$$

where Ω and t are as defined in (80).

VII. Numerical Results and Discussion

In this section we analyze and verify the presented theoretical results for both CSI-assisted and fixed gain systems in comparison with Monte Carlo simulations. Without loss of generality, we use identical correlation parameter (ρ) at both source and destination. It is observed from the figures that the Monte Carlo simulations are exactly matching with the analytical results and verifying those.

Fig. 1 and Fig. 2 show the CSI-assisted and fixed gain relay system outage probability variation with different antenna configurations for MRT/MRC system. As expected, the outage probability decreases with the increase of average SNR. We see an improvement in the outage when the number of antennas is large. The rate of outage probability improvement is higher when number of antennas at source is greater than that at the destination, this is mainly due to the fact that $\rho_1 = 2\rho_2$. The asymptotic high SNR results are plotted and they coincide with the analytical results. Moreover both figures show that the diversity order is equal to $\min[n_{\mathrm{T}}, n_{\mathrm{R}}]$.

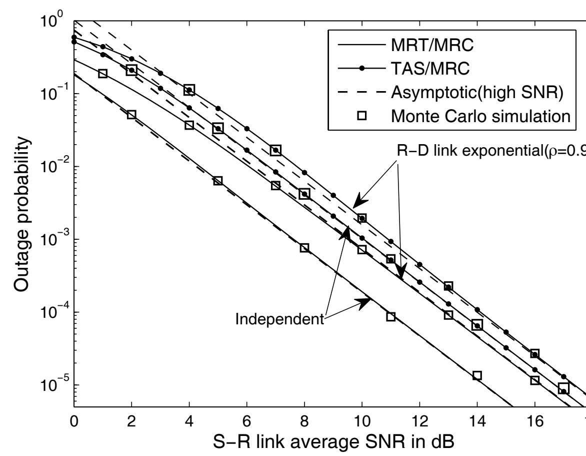

Fig. 5. Comparison of outage probability of TAS and MRT, CSI-assisted systems, for $\mu = 2$, $\Lambda = 0$dB.

The outage probabilities for different correlation cases for the CSI-assisted and fixed gain relaying are depicted in Fig. 3 and Fig. 4 respectively. We can clearly see that a reduction in correlation improves the outage probability. The independent case has the highest improvement in the outage probability and the exponential correlation has a slight improvement in the outage probability than the uniform case for the same correlation parameter. The asymptotic results are drawn and they show that the diversity order does not change with antenna correlation. When comparing Fig. 3 and Fig. 4, we can see that the CSI-assisted case has a higher improvement in outage probability as opposed to the fixed gain relay case.

The outage probability comparison between MRT/MRC and TAS/MRC for CSI assisted systems is shown in Fig. 5 for $n_{\mathrm{T}} = n_{\mathrm{R}} = 3$. Clearly the MRT/MRC system outperforms the other system and as seen from the figure, even the correlated MRT/MRC case performs better than the TAS/MRC inde-

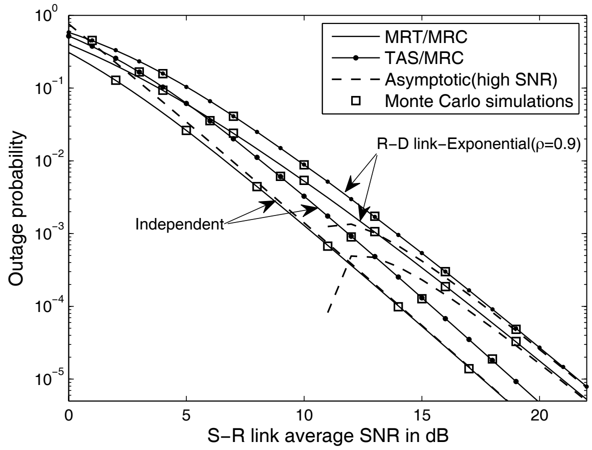

Fig. 6. Comparison of outage probability of TAS and MRT, Fixed Gain systems, for $\mu = 2$, $\Lambda = 0$dB. Fixed gain C_2 and C_{T2}

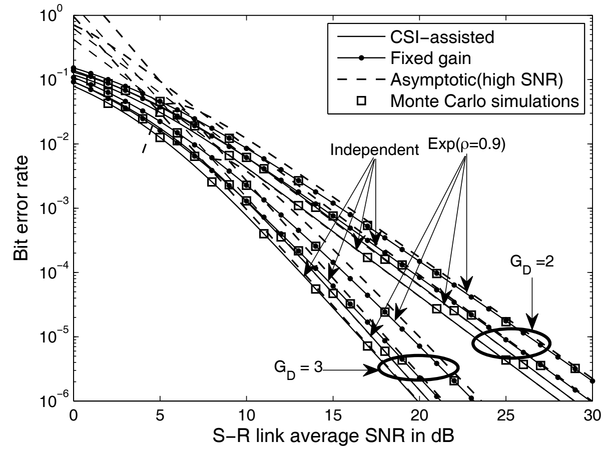

Fig. 8. BER of BPSK for TAS/MRC sytem for $\mu = 2$, $\Lambda = 0$dB.

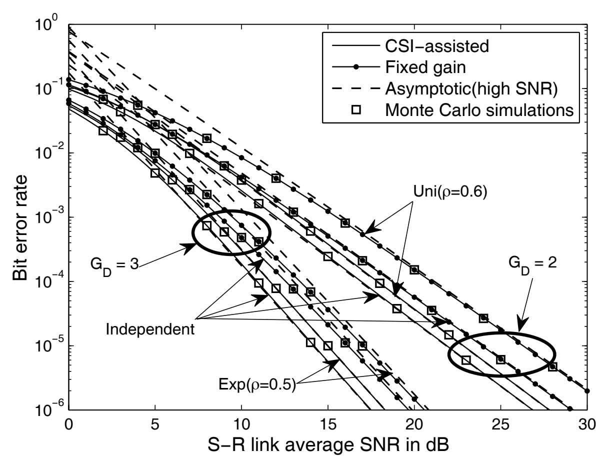

Fig. 7. BER of BPSK for MRT/MRC sytem for $\mu = 2$, $\Lambda = 0$dB.

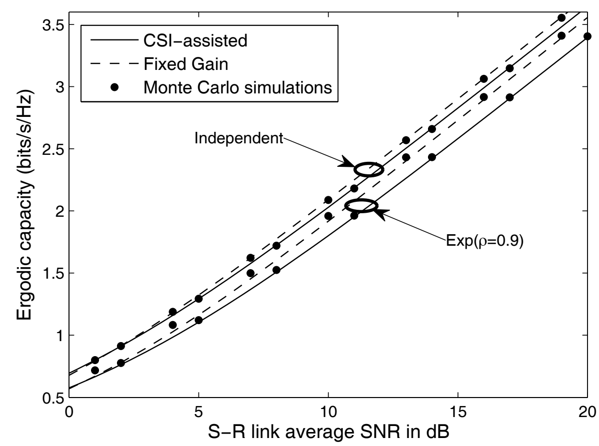

Fig. 9. Ergodic capacity for MRT/MRC sytem for $\mu = 2$, $\Lambda = 0$dB.

pendent case. Overall, both systems have outage probability improvements when the correlation decreases. The asymptotic results show that both systems have the same diversity order. Similarly, the Fig. 6 shows outage probability comparison for fixed gain systems with the same antenna configuration. It is noticed that the MRT/MRC system performs better than the TAS/MRC system. Moreover, when correlation decrease, both systems improve outage probability. When $\mathtt{S}-\mathtt{R}$ and $\mathtt{R}-\mathtt{D}$ channels are independent, the outage probabilities have the highest improvement.

Fig. 7 and Fig. 8 show the BPSK $(a = b = 1)$ BER curves for MRT/MRC and TAS/MRC systems respectively. Fixed gain C_2 and C_{T2} are used. They illustrate the variation of BER with average $\mathtt{S}-\mathtt{R}$ link SNR for both CSI-assisted and fixed gain systems. A higher correlation produces an increased BER. Further, it is observed that the CSI-assisted case outperforms the fixed gain in terms of BER. The asymptotic results are plotted and those verify the diversity order.

Ergodic capacity variation for both CSI-assisted and fixed gain systems is illustrated in Fig. 9. Both approximate solution and simulation results are plotted. As seen in the figure, the fixed gain system has a higher ergodic capacity. The capacity is higher for high SNR, and becomes lower at higher correlation values. Fig.10 shows the capacity variation with the correlation coefficient (ρ). It is observed from the figure that the capacity is higher with exponential correlation compared to the uniform correlation case and with increased correlation, the capacity decreases.

VIII. Conclusion

We have considered a unified performance analysis of both CSI-assisted and fixed gain AF relay for MRT/MRC and TAS/MRC systems. We have investigated the effects of antenna correlation at the source and destination for MRT/MRC system and those at the destination for TAS/MRC system. Our analysis provide a general solution to several previous special cases mentioned in the literature and new additional ones. To obtain the performance of the systems, we have derived the exact closed form solutions for outage probability, average SER, higher moments of SNR and provided an ergodic capacity analysis. To gain the insight of the system performance and diversity gain, we have presented the asymptotic results. The Monte Carlo simulations verified our analytical results. It is observed that system performance decreases with the increase of antenna correlation and CSI-assisted systems outperform

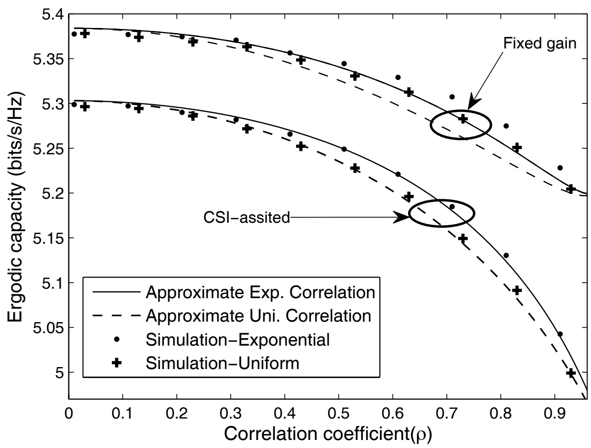

Fig. 10. Ergodic capacity variation with correlation, for $\mu = 2$, $\Lambda = 0$dB.

the fixed gain systems for most of the cases. Further, we can conclude that MRT/MRC system gives a better performance than TAS/MRC system.

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