Design Principles for Multi-Hop Wavelength and Time Division Multiplexed Optical Passive Star Networks*

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Abstract

We propose a graph model called the *Receiving Graph* model to describe optical networks where *passive star couplers* are used to interconnect stations, each of which has *one fixed wavelength transmitter* and *one fixed wavelength receiver*, and *Wavelength- and Time-Division Multiplexing (WTDM)* protocols are employed. Based on the model, the inherent characteristics of such WTDM networks can be fully understood and alternative designs can be compared. We discuss several design principles and present some theoretical performance limitations for the networks.

1 Introduction

It has been recognized that the *Wavelength Division Multiplexing (WDM)* mechanism is one of the most promising ways to improve bandwidth utilization in optical networks, where signals are modulated into different wavelengths of light in the electromagnetic spectrum, each of which provides a bandwidth compatible with electronic interfaces [1, 2, 3]. Stations are tapped onto optical fibers via optical transmitters and receivers which are tuned on specific wavelengths. Optical fibers are interconnected by a broadcast optical switch such as a *passive star coupler.* A transmission from a station to another station is accomplished by first tuning a transmitter of the sender and a receiver of the recipient to the same wavelength, and then proceeding transmission. Several transmissions may occur simultaneously as long as transmitters use different wavelengths. More detailed description can be found in [8].

Connectivity of stations can be logically presented in a directed graph, called *logical (connectivity) graph*, where each station corresponds to a node and each possible transmission corresponds to a directed edge. A WDM network with a completely connected logical graph is referred to as *single-hop WDM network*; otherwise, it is referred to as *multihop WDM network*. Surveys can be found in [4, 5]. The multihop WDM networks become attractive because of the fact that they usually are implementable by only fixed wavelength transmitters and receivers, which are economic and reliable. Usually each wavelength is shared by several transmitters according to a *Time-Division Multiplexing (TDM)* protocol. In this paper we are interested in using only *one fixed wavelength transmitter* and *one fixed wavelength receiver* in each station to construct multihop WDM networks employing TDM on each wavelength. We also refer such networks to WTDM networks. (Unless explicitly specified, transmitters (receivers) are referred to as the ones of fixed wavelengths in the rest of this paper.)

[6] used ShuffleNet, a recirculating multi-stage perfect shuffle, as a logical graph. One wavelength is assigned to those edge patterns forming a fully connected bi-pratite graph and each station has one transmitter and one receiver. Note that logical graphs only describe the connectivity of stations and are lack of flexibility of describing the WTDM network unique characteristics including the number of wavelengths used, multicast and wavelength sharing activities and the portion of bandwidth a transmitter can get. Bus-Mesh[7] with the same hardware assumption used a different graph model where each wavelength is treated as a bus. Although the bus concept captures the multicast and wavelength sharing activities, the model only describe a specific topology and the number of wavelengths which can be exploited is bounded by $\sqrt{N}$.

In this paper we propose a new graph model, called *Receiving Graph*, which efficiently reveals the unique characteristics of the WTDM networks. Oppose to logical graphs describing connectivity of stations, receiving graphs show connectivity of wavelengths. We propose a general methodology, called *virtual graph embedding*, to construct receiving graphs based on any given graphs. The model offers a wide range of design alternatives which includes ShuffleNet and Bus-Mesh as well. Furthermore, based on the model we are able to answer the following performance related questions which have not been fully answered before:

- What are the fundamental relationships between the number of stations, the number of wavelengths, the number of stations transmitting (receiving) on a wavelength and the average distance?
- How does propagation delay effect design?
- What are the performance limitations of the WTDM networks?
- What are the best design strategies in different environments?

This paper is organized as follows. Section 2 proposes the receiving graph model. Section 3 shows the process of the virtual graph embedding for constructing a receiving graph based on a given graph. Section 4 describes transmission cycle and routing. Section 5 defines two performance

*Partially supported by a grant from Bellcore

metrics and derive their approximated analytical models. The best design strategies and theoretical bounds of the metrics are also presented. Section 6, finally, draws some conclusions.

2 Receiving Graph Model

First we describe the basic functions of the WTDM networks as follows. All wavelengths are assumed to have the same bandwidth which is bounded by the maximum signal modulation/demodulation speed of a station. A basic data unit, called a *packet*, is of fixed size. The time domain is divided into time slots of equal duration with a slot long enough to contain a packet. A *transmission cycle* can be drawn like a $(W \times \frac{N}{W})$ matrix with relative time slot numbers as column indices, denoted by t_i, and wavelength id numbers as row indices, denoted by w_j. An entry $k \rightarrow l, m$ in column t_i and row w_j means that node n_k has the right to transmit at wavelength w_j in time slot t_i and this transmission can be simultaneously received by nodes n_l and n_m directly. Each station can transmit only once in a cycle and the same cycle is repeated forever. Each station is equipped with an output queue to temporarily buffer outgoing packets. Each station can send out only one packet in its reserved time slot.

Next we define a *receiving graph* as follows. Suppose we have a set of stations $\mathcal{N} = \{n_0, n_1, \cdots n_N\}$ and a set of wavelengths $\mathcal{W} = \{w_0, w_1, \cdots w_W\}$. Each station corresponds to a *node*. Nodes are partitioned into W sets, called *receiving nodes*, according to their receiving wavelengths. The receiving node associated with wavelength w_i is denoted by rn_i. If a node n_s transmits on wavelength w_j, there is a directed edge originating from n_s (inside a receiving node) to receiving node rn_j. Since each node has only one transmitter tuned on a fixed wavelength, each node has only one outgoing edge pointing to a receiving node. The outgoing degree of a receiving node equals the number of nodes inside. The incoming degree of a receiving node equals the number of nodes who share the same transmission wavelength associated with the receiving node. We call this set of nodes the *transmitting group* of the wavelength. Since each node only transmits once in a transmission cycle, the cycle length, denoted by Δ, equals the number of nodes who share this transmission wavelength (i.e., the size of the associated transmitting group).

We assume a uniform communication pattern where each node has equal probability of generating packets for any other node at any given time. And we intend to achieve a fair wavelength access in terms that any wavelength is used by the same number of nodes for transmission and the same number of nodes for receiving. This leads to that each receiving nodes has Δ incoming (outgoing) edges, and $\Delta = \frac{N}{W}$. Considering receiving nodes along, the receiving graph is a regular directed graph of degree Δ and size W, which describes the connectivity of wavelengths.

For example, Figure 1 shows a 4-wavelength 12-node receiving graph and the corresponding transmission cycle ($\Delta = 3$). The nodes are shown by boxes (only node indices are shown) and the receiving nodes are shown by circles. (Note that this receiving graph is essentially equivalent to a Bus-Mesh.) Other alternative designs are also possible such as a 2-wavelength 12-node receiving graph ($\Delta = 6$) and a 3-wavelength 12-node receiving graph ($\Delta = 4$).

From the above examples, it can be seen that there is a trade-off between the number of wavelengths exploited and the average distance of a receiving graph. As W increases, the receiving graph has more receiving nodes, each of which has a lower degree. So the graph becomes sparser and a longer average distance is expected. Nevertheless, each node have more bandwidth to transmit. What is the best design in terms of W, Δ and a receiving graph topology will be discussed in more details in Section 5.

3 Virtual Graph Embedding

We propose a systematic process to construct a receiving graph, called *virtual graph embedding*, as follows. Assume the number of stations N and the number of wavelengths W follow the relation of $N = C \cdot \alpha \cdot W$, where C and α are positive integers (Discussion for C being a positive real can be found in [8]). We choose a regular directed graph with W nodes and degree α for basic wavelength connectivity and call it a *virtual graph*, its nodes *virtual nodes* and its edges *virtual edges*. For each virtual edge, we attach a box (node) at the starting point. Like a receiving graph, each virtual node corresponds to a receiving node (wavelength) with a set of nodes inside. Therefore, a node can be distinguished by (*tran_id* , *rec_id*), where *tran_id* and *rec_id* denote its transmission and receiving wavelengths respectively. For example, Figure 2(a) shows a virtual graph of size 3 and degree 2. After the above process, it is transformed into a 3-wavelength 6-node receiving graph in Figure 2(b).

Now we have $\alpha \cdot W$ nodes in total. To extend to N nodes we stack C copies of a virtual graph together and attaching a node at each edge starting point. Then each node is labeled by a triplet (*stack_id* , *tran_id* , *rec_id*), where *stack_id* is an integer between 0 and $C-1$. All nodes with the same *stack_id* are considered in the same *group* (*stack*). Each virtual edge is corresponding to C nodes (directed edges) in the corresponding receiving graph. We also call these C nodes *candidate nodes* of the virtual edge. For example, a 3-wavelength 6-node receiving graph in Figure 2(b) is extended to a 3-wavelength 12-node receiving graph in Figure 2(c). The candidate nodes for the virtual edge from virtual node 0 to virtual node 1 are (0,1,0) and (1,1,0).

4 Transmission Cycle and Routing

Since $N = C \cdot \alpha \cdot W$, the *transmission cycle length* $\Delta = \frac{N}{W} = C \cdot \alpha$. The only necessary information which needs to be stored in each station is when to transmit. Receivers listen on fixed wavelengths all the time, so no schedules are needed for reception. We suggest a *stack-by-stack transmission* as follows. Basically, all nodes with the same stack id number are scheduled in a period of contiguous time slots. We refer it to *transmission subcycle*. Since each virtual node in a stack of a virtual graph is of degree α, the *transmission subcycle length* $\Delta_s = \alpha$. Within a transmission subcycle, the order of transmissions for all nodes in the same row is arbitrary, but the same pattern is repeated in every transmission subcycle.

Routing a packet from a node to another can be viewed as a traversal of receiving nodes (a sequence of transmitting wavelengths changes). The first move is fixed and after then we have the freedom of choosing the next receiving node (wavelength) by picking anyone of the C *candidate nodes* which transmit on the wavelength. Routing can be simplified if virtual graphs are in forms of certain well-known regular topologies, such as perfect shuffle and hypercube.

The diameter of a virtual graph plus one equals the diameter of the corresponding receiving graph. Two routing algorithms, one aiming at minimizing packet delay by taking advantage of the stack-by-stack transmission (suitable for special control packets or a light traffic load situation) and the other aiming at balancing traffic load (suitable for a moderate or heavy traffic load situation), can be found in [8].

5 Design Strategies

In this section we evaluate the performance of WTDM networks in terms of design parameters such as the number of nodes, the number of wavelengths, and a virtual graph topology by examining two performance metrics, the *average maximum network throughput*, crucial for high traffic load, and the *average minimum packet delay*, crucial for low traffic load. These two metrics are believed to be able to provide useful information for projecting behavior of networks in a general sense.

Analysis is based on a general graph model, so results can be served as theoretical performance bounds. First let us define some terms related to a graph. The *distance*, denoted by H, between two nodes is the minimum number of edges required for going from one node to the other. The *diameter*, denoted by D, is the the longest distance between any pair of nodes. Considering a simple[1] nonselflooping virtual graph with size of W and degree of α, where $2 \leq \alpha \leq (W-1)$, each virtual node can reach at most α virtual nodes with one hop and α^2 with two hops and so on. Therefore,

$$W \leq 1 + \alpha + \alpha^2 + \alpha^3 + \ldots + \alpha^D = \frac{\alpha^{D+1} - 1}{\alpha - 1}. \quad (1)$$

Suppose we consider the case of equality holding for (1), then it follows that

$$D = \log_\alpha(W(\alpha - 1) + 1) - 1. \quad (2)$$

Moreover, the average distance, denoted as $\overline{H}$, can be written as follows.

$$\overline{H} = \frac{\sum_{i=1}^{D} i \cdot \alpha^i}{W-1} = \frac{D\alpha^{D+2} - (D+1)\alpha^{D+1} + \alpha}{(W-1)(\alpha-1)^2}. \quad (3)$$

Recall for any path in a receiving graph, the first move is mandatory and then follow a path in the corresponding virtual graph. Therefore, the average distance, denoted by $\overline{H_{RG}}$, of a receiving graph can be approximated by $\overline{H_{RG}} \approx \overline{H} + 1$.

5.1 Average Maximum Network Throughput

The *average maximum network throughput*, denoted by *Thpt*, is defined as the average number of packets which can be successfully delivered from a source to a destination in one time slot assuming the network is heavily loaded (i.e., each node always has packets to send). If we assume the routing algorithm can always find the shortest length path and each intermediate node has infinite number of buffer spaces, *Thpt* can be approximated by

$$Thpt = \frac{W}{\overline{H_{RG}}} \approx \frac{W}{\overline{H} + 1}. \quad (4)$$

Let the number of available wavelengths be denoted as W_{avl}, then our objective is to

$$\text{Maximize } Thpt$$

$$\text{subject to } \begin{cases} N = C \cdot \alpha \cdot W, \\ 2 \leq \alpha \leq (W-1), \\ W \leq W_{avl} \text{ and } C \geq 1, \\ \text{where } N, W, W_{avl}, \alpha, C \in Z^+. \end{cases} \quad (5)$$

That is, for given N and W_{avl}, we are interested in knowing the values of C, α, and W which maximize *Thpt*.

For ease of computation, we consider C and $\alpha \in R^+$. From (4) we observe that, for a fixed W, *Thpt* increases with α (since the divisor, $\overline{H_{RG}}$, decreases). For a fixed α, as W increases, both dividend, W, and divisor, $\overline{H_{RG}}$, increase. However, the growth of W is much faster than $\overline{H_{RG}}$. Thus, overall *Thpt* increases with W. Intuitively, we should choose both W and α as large as possible to maximize *Thpt*. This also implies that C should be as small as possible.

Since $N = C \cdot \alpha \cdot W$ implies $\alpha \leq \frac{N}{W}$, α should be bounded by $\min\{W-1, \frac{N}{W}\}$. First we consider the case of $W_{avl} - 1 \leq \frac{N}{W_{avl}}$ (i.e., $W_{avl} \leq \sqrt{N}$ when N is large). This implies $W - 1 \leq \frac{N}{W}$ and $2 \leq \alpha \leq \min\{W-1, \frac{N}{W}\} = W - 1$. To maximize *Thpt*, we should choose α as large as possible (i.e., α=$W-1$). It can be shown when $W = W_{avl}$, $\alpha = W_{avl} - 1$ and $C = \frac{N}{W_{avl}(W_{avl}-1)}$, $Thpt = O(W_{avl})$ is the maximum.

Next we consider the case of $W_{avl} - 1 > \frac{N}{W_{avl}}$ (i.e., $W_{avl} > \sqrt{N}$ when N is large). Since the range of $W \leq \sqrt{N}$ has been discussed before, the range of interest left is $W_{avl} \geq W > \sqrt{N}$. Within this range, it implies $W - 1 > \frac{N}{W}$ and $2 \leq \alpha \leq \min\{W-1, \frac{N}{W}\} = \frac{N}{W}$. Similarly, to maximize *Thpt*, α should be chosen as large as possible (i.e., α=$\frac{N}{W}$). This will force C to be 1. By substituting $W = \frac{N}{\alpha}$ into (4), the α maximizing *Thpt* can be computed. Unfortunately, the computation is fairly complicated. Therefore, we simplify (4) as follows. Originating from a specific node, the number of nodes k-hop away is bounded by α^k. Consider α is small and k is large, most of nodes are away from the node with the distance close to the diameter. This implies the average distance $\overline{H_{RG}}$ is approaching to the diameter (D+1) which can be approximated by $\log_\alpha N$ when $W = \frac{N}{\alpha}$ (i.e., C=1). Thus *Thpt* becomes

$$Thpt \approx \frac{N}{\alpha \log_\alpha N}. \quad (6)$$

It can be proven (6) is maximized at $\alpha = e$ ($W = \frac{N}{e}$), where e is the natural number. We verify this value by numerical computation. In Figures 3, we draw the curves of *Thpt* (4) vs. α with $W = \frac{N}{\alpha}$. We observe that the best α, α_{opt}, maximizing *Thpt* occurs between 2 and 3 for all N's. For small to moderate N (e.g., 100, 500 or 1,000) α_{opt} is close to 2 and for large N (e.g., 2,000) α_{opt} is close to e.

To sum up, the general design principles can be stated as follows:

[1] We refer a simple graph to a graph with no duplicated edges.

- First, exploit as many wavelengths as possible as long as it is no greater than $\frac{N}{\alpha_{opt}}$. For small N, $\alpha_{opt} = 2$ and for large N, $\alpha_{opt} = 3$.
- Then let degree α be as large as possible (i.e., C is as small as possible).

5.2 Average Minimum Packet Delay

We assume the propagation delay, denoted by τ, is the same from any station to any other station, and a packet takes the shortest length path with the shortest waiting time (i.e., In each move, we always pick the node from C candidate nodes which will be entitled to transmit in the next transmission subcycle after receiving the packet). Then the *average minimum packet delay*, denoted by L, is the average delay from being generated by a source node to arriving at a destination node assuming there is no queuing in each node (i.e, under a extremely light traffic load).

The first part of L is for waiting the source station's transmission turn (i.e., $\frac{\Delta}{2}$ in average). After transmission, a packet takes τ slots to propagate to an intermediate station and then wait about a half of a transmission subcycle (i.e., $\frac{\Delta_s}{2}$ slots) before the next transmission. The same delay is needed for each intermediate node. Finally, the destination node receives the packet (no waiting time is needed). Thus L can be expressed as

$$\begin{aligned} L &= \frac{\Delta}{2} + (\tau + \frac{\Delta_s}{2})\overline{H_{RG}} - \frac{\Delta_s}{2} \\ &= \frac{N}{2W} + (\tau + \frac{\alpha}{2})(\overline{H} + 1) - \frac{\alpha}{2}. \end{aligned} \quad (7)$$

Our objective is to minimize L subject to the same set of constraints in (4) along with the constraint, $\tau \in R^+$.

First, for a given W, as α increases, the subcycle Δ_s in the second and third terms increase, but $\overline{H_{RG}}$ in the second term decreases. In general, the decreasing of $\overline{H_{RG}}$ is much faster than the increasing of α. That will make the second and third terms decrease , and so does L. Therefore, for a given W, we prefer to increase α as much as possible (i.e., C is as small as possible). Next, for a given α, as W increases, Δ in the first term decreases, but $\overline{H_{RG}}$ in the second term increases. Clearly, the decreasing of Δ is much faster than the increasing of $\overline{H_{RG}}$. However, since the propagation delay τ is also involved in the second term and if it is large, the value of the second term may be magnified. Intuitively, for a small τ, we prefer to enlarge the first term (i.e., choose W (C) as large (small) as possible) to reduce L. On the other hand, as τ is getting larger, L becomes more sensitive to $\overline{H_{RG}}$. Therefore, we should choose a smaller W to maintain a small $\overline{H_{RG}}$.

As we did before, two ranges of W_{avl} are considered, separately. For the range of $W_{avl} \leq \sqrt{N}$, $2 \leq \alpha \leq W - 1$. It can be shown that when $\alpha = W_{avl} - 1$, $W = W_{avl}$ and $C = \frac{N}{W_{avl}(W_{avl}-1)}$, we have the minimum $L = O(\frac{N}{W_{avl}+\tau})$. Another range is $\sqrt{N} < W_{avl}$. Thus $2 \leq \alpha \leq \frac{N}{W}$. To minimize L, $\alpha = \frac{N}{W}$ and C=1. Again, it is hard to compute the α minimizing L. Therefore, we can simplify (7) by substituting $\overline{H_{RG}}$ with $D+1 = \log_\alpha N$ by assuming $W = \frac{N}{\alpha}$ (i.e., C=1), and then obtain

$$L = (\tau + \frac{\alpha}{2}) \log_\alpha N. \quad (8)$$

We further differentiate (8) with respect to α and obtain τ in terms of the best α, α_{opt}, as follows by letting the differentiated equation to be zero.

$$\tau = f(\alpha_{opt}) = \frac{\alpha_{opt}(\ln \alpha_{opt} - 1)}{2}. \quad (9)$$

Note that τ is only associated with α_{opt} and irrelevant to N or W. The inverse of $f()$, $f^{-1}(\tau)$, returns α_{opt} for a given τ. Then $W = \frac{N}{\alpha_{opt}}$, $C = 1$ and $L = O(\tau \cdot \frac{\log_{\alpha_{opt}} N}{\alpha_{opt}} + \log_{\alpha_{opt}} N + \alpha_{opt})$. Figure 4 shows the curve of $f(\cdot)$. When $\tau = 0$, $\alpha_{opt} = e$. (Note: this choice of α_{opt} agrees with that in maximizing *Thpt*.) α_{opt} increases with τ. This matches with our previous observation.

Likewise, we present numerical result to verify the analysis. Figure 5, shows the L vs. α curves for N=100, 500, 1000 and 2000 subject to $\tau = 0$ and $\tau = 5$. We observe that α_{opt} is a small number close to e for $\tau \approx 0$ and as τ increases to 5, α_{opt} is increased to 9 which is close to $f^{-1}(5)$. Also α_{opt} seems to be independent of N and W.

ShuffleNet[6] pointed out that for long propagation delay (e.g., τ=50 time slots) 2-stage design is preferred, since the diameter is bounded by 3. However, the transmission cycle length was not considered. The above discussion reveals that the best design is closely related to propagation delay, transmission cycle length and the average distance, all of which should be considered at the same time.

$f(\cdot)$ is related to topologies of virtual graphs and routing algorithms. The one shown here is for the purpose of demonstrating the relationships between parameters. In reality, $f(\cdot)$ can be derived for a specific topology like ShuffleNet or Bus-Mesh. In general, to minimize L for given τ, W_{avl} and N, we should obey the following rules:

- First, derive $f^{-1}(\cdot)$ and figure out $f^{-1}(\tau) = \alpha_{opt}$ for a given τ.
- Second, exploit as many wavelengths as possible as long as it is not greater than $\frac{N}{\alpha_{opt}}$.
- Third, let α be as large as possible (i.e., C is as small as possible).

6 Conclusion

In this paper we have proposed the receiving graph model to describe the WTDM networks with one fixed wavelength transmitter and one fixed wavelength receiver in each station and demonstrated the relationships between design parameters. And we have also proposed the best design strategies in different environments. A centralized architecture providing a short and uniform propagation delay, which is essential to make WTDM protocols more efficient and practical, can be found in [9].

This work can be extended to allow *dynamic bandwidth allocation* in each wavelength by adding an extra fixed wavelength receiver in each station and tuning it on the transmitting wavelength of the station. As long as a slot is unused by a station, other stations in the same transmitting group will sense it and then use the one in the following cycle (very likely to be still unused by the owner) on a contention basis. Several such TDM protocols based on a single broadcast bus have been proposed [10] and they can be adopted in the WTDM networks directly.

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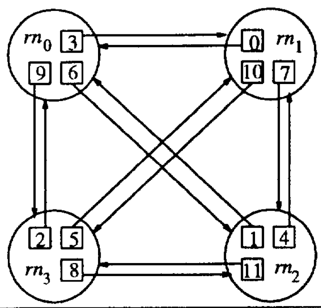

	t_0	t_1	t_2
w_0	0 → 3, 6, 9	1 → 3, 6, 9	2 → 3, 6, 9
w_1	3 → 0, 7, 10	4 → 0, 7, 10	5 → 0, 7, 10
w_2	6 → 1, 4, 11	7 → 1, 4, 11	8 → 1, 4, 11
w_3	9 → 2, 5, 8	10 → 2, 5, 8	11 → 2, 5, 8

Figure 1: A 4-wavelength 12-node receiving graph and its transmission cycle.

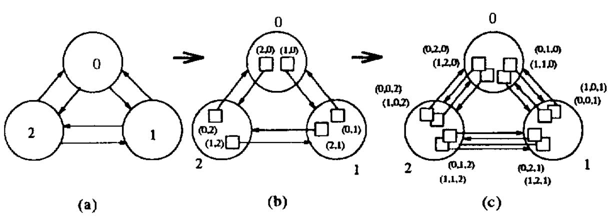

Figure 2: The process of the virtual graph embedding.

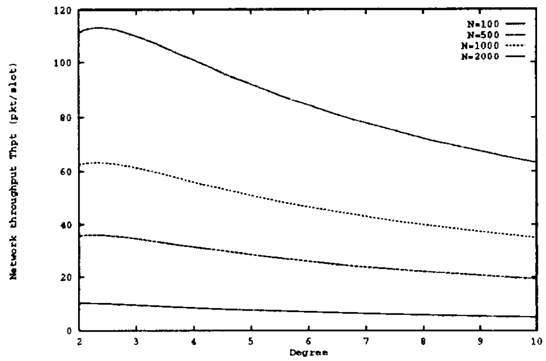

Figure 3: The curves of the average maximum network throughput *Thpt* vs. α for different network sizes.

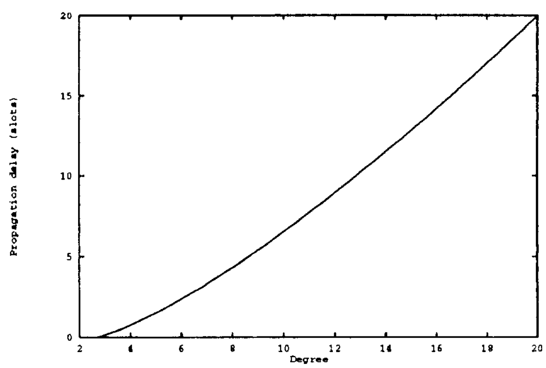

Figure 4: The curves of propagation delay τ vs. the best α (α_{opt}).

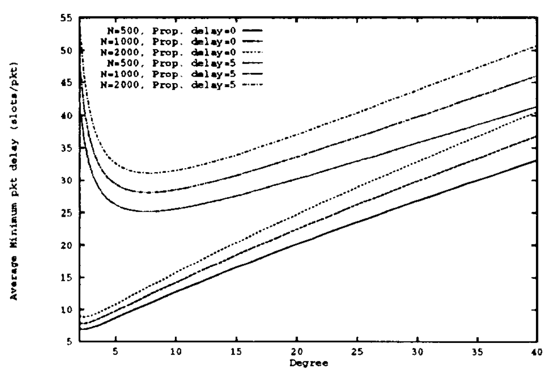

Figure 5: The curves of the average minimum packet delay L vs. α for N=100, 500, 1000 and 2000 subject to $\tau = 0$ and $\tau = 5$.