Rough Sets and Near Sets in Medical Imaging: A Review

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***Abstract*—This paper presents a review of the current literature on rough set and near set-based approaches to solving various problems in medical imaging such as medical image segmentation, object extraction and image classification. Rough set frameworks hybridized with other computational intelligence technologies that include neural networks, particle swarm optimization, support vector machines and fuzzy sets are also presented. In addition, a brief introduction to near sets and near images with an application to MRI images is given. Near sets offer a generalization of traditional rough set theory and a promising approach to solving the medical image correspondence problem as well as an approach to classifying perceptual objects by means of features in solving medical imaging problems. Other generalizations of rough sets such as neighborhood systems, shadowed sets, and tolerance spaces are also briefly considered in solving a variety of medical imaging problems. Challenges to be addressed and future directions of research are identified and an extensive bibliography is also included.**

***Index Terms*—Computational Intelligence, Rough Sets, Near Sets, Medical Imaging, Image Segmentation, Image Classification, Hybrid Rough Image Processing**

I. Introduction

COMPUTATIONAL intelligence techniques and approaches encompass various paradigms dedicated to approximately solving real-world problems in decision making, pattern classification and learning [1]–[3]. Prominent among these paradigms are fuzzy sets, neural networks, genetic algorithms, rough sets, and a generalization of rough sets called near sets. Fuzzy sets provide a natural framework for dealing with uncertainty. It offers a problem-solving tool between the precision of classical mathematics and the inherent imprecision of the real world. For example, imprecision in a segmented image can be represented and analyzed using fuzzy sets. Neural networks provide a robust approach to approximating real-valued, discrete-valued and vector-valued functions. The well-known back propagation algorithm that uses gradient descent to tune network parameters to best fit the training set with input-output pair, has been successfully applied to a variety of problems. Genetic algorithms [3] are stochastic search techniques based on the principles of evolution. Extensive research has been performed exploiting the robust properties of genetic algorithms and demonstrating their capabilities across a broad range of problems. These evolutionary methods have gained recognition as general problem solving techniques in many applications, including function optimization, image processing, classification and machine learning, training of neural networks, and system control. Other approaches like case based reasoning and decision trees [4], [5] are also widely used to solve data analysis problems.

Recently, various published algorithms have been applied to build a computer-aided analysis system in the medical field [6], [7]. The most commonly used algorithms are neural networks, Bayesian classifiers, genetic algorithms, decision trees, and fuzzy theory [8]–[12]. Unfortunately, the techniques developed have not been sufficient to introduce an effective computer-aided analysis in clinical use. A survey of the area can be found in [6].

Rough set theory introduced by Pawlak during the early 1980s [13] spans a quarter century (see, *e.g.*, [14]–[17]). The rough set approach to approximation of sets leads to useful forms of granular computing that are part of computational intelligence [3]. The basic idea underlying the rough set approach to information granulation is to discover to what extent a given set of objects (*e.g.*, pixel windows in an image) approximates another set of objects of interest. Objects are compared by considering their descriptions. A recent generalization of rough set theory has led to the introduction of near sets [18]–[20] and a consideration of the affinities (nearness) of objects [21]. In a near set approach to object classification, an object description is modeled as a vector function that represent object features [20]. Included in the near set approach is a provision for an object feature to be represented by one or more functions, *e.g.*, color represented by functions that measure intensity, hue, and saturation.

Near sets and rough sets are very much like two sides of the same coin. From a rough set point-of-view, the focus is on the approximation of sets with non-empty boundaries. By contrast, in a near set approach, the focus is on the discovery of sets having matching descriptions that does not require a consideration of approximation boundaries. In the context of medical image analysis, an image is viewed as a set of points. That assumption ushers in either a rough set or near

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set approach to medical image analysis. There are a number of practical outcomes of the near set approach, *e.g.*, feature selection [22]–[24], objective evaluation of image segmentations [25], image classification [26]–[30], object recognition in images [19], [26], granular computing [31], and various forms of machine learning [31], [32].

The objective of this article is twofold: present to the rough set and medical imaging research communities the state-of-the art in rough set-based applications to image processing and pattern recognition (in general) with a specific focus on medical imaging (in particular), and motivate research in new trend-setting directions. We review and discuss some representative methods to provide inspiring examples to illustrate how rough sets can be applied to solve medical imaging problems and how medical images can be analyzed, processed, and characterized by rough sets. These examples include, among others, rough representation of a region of interest, rough image entropy, rough c-means clustering, and rough neural intelligent approach for image classification.

This article has the following organization. Section II provides an explanation of the basic framework of rough set theory, along with some of the key definitions. Section III gives an introduction to rough image processing including rough images, rough representation of a region of interest, rough image entropy, and rough-based medical image applications including object extraction and medical image segmentation and clustering. Some useful measures are presented in Section IV. Section V provides a brief review of rough sets combined with other computational intelligence approaches such as rough neural networks, rough fuzzy and rough genetic algorithms as well as Bayesian methods, particle swarm optimization and support vector machines coupled with rough sets. An introduction to near sets, near images and the near set approach to image segmentation is given in Section VI while other generalization approaches of rough sets are presented in Section VII. Finally. challenges and future trends are discussed in Section VIII.

II. ROUGH SETS: FOUNDATIONS

Due to space limitations we provide only a brief explanation of the basic framework of rough set theory, along with some of the key definitions. A more comprehensive review can be found in sources such as [14].

Rough sets theory provides a novel approach to knowledge description and to approximation of sets. Rough theory was introduced by Pawlak during the early 1980s [13] and is based on an approximation space-based approach to classifying sets of objects. In rough sets theory, feature values of sample objects are collected in what are known as information tables. Rows of a such a table correspond to objects and columns correspond to object features.

Let $\mathcal{O}, \mathcal{F}$ denote a set of sample objects and a set of functions representing object features, respectively. Assume that $B \subseteq \mathcal{F}, x \in \mathcal{O}$. Further, let $x_{\sim_B}$ denote

$$x_{/\sim_B} = \{y \in \mathcal{O} \mid \ \forall \phi \in B, \phi(x) = \phi(y)\},$$

i.e., $x \sim_B y$ (description of x matches the description of y). Rough sets theory defines three regions based on the equivalent classes induced by the feature values: lower approximation $\underline{B}X$, upper approximation $\overline{B}X$ and boundary $BND_B(X)$. A lower approximation of a set X contains all equivalence classes $x_{/\sim_B}$ that are proper subsets of X, and upper approximation $\overline{B}X$ contains all equivalence classes $x_{/\sim_B}$ that have objects in common with X, while the boundary $BND_B(X)$ is the set $\overline{B}X \setminus \underline{B}X$, *i.e.*, the set of all objects in $\overline{B}X$ that are not contained in $\underline{B}X$. Any set X with a non-empty boundary is *roughly* known relative, *i.e.*, X is an example of a rough set.

The indiscernibility relation $\sim_B$ (also written as Ind_B) is a mainstay of rough set theory. Informally, $\sim_B$ is a set of all classes of objects that have matching descriptions. Based on the selection of B (*i.e.*, set of functions representing object features), $\sim_B$ is an equivalence relation that partitions a set of objects $\mathcal{O}$ into classes (also called elementary sets [13]). The set of all classes in a partition is denoted by $\mathcal{O}_{/\sim_B}$ (also by $\mathcal{O}/Ind_B$). The set $\mathcal{O}/Ind_B$ is called the quotient set. Affinities between objects of interest in the set $X \subseteq \mathcal{O}$ and classes in a partition can be discovered by identifying those classes that have objects in common with X. Approximation of the set X begins by determining which elementary sets $x_{/\sim_B} \in \mathcal{O}_{/\sim_B}$ are subsets of X.

III. ROUGH IMAGE PROCESSING

Various rough image processing methodologies have been applied to handle the different challenges posed by medical imaging. We can define rough image processing as the collection of all approaches and techniques that understand, represent and process the images, their segments and features as rough sets (see, *e.g.*, [10], [33]–[35]). In this section, we first describe the ability of rough sets to handle and represent images and color images, followed by the various rough based approaches developed for handling the different functional aspects to solve medical imaging problems.

A. The ability of rough sets to handle images

Rough sets provide reasonable structures for the overlap boundary given domain knowledge. The case study for images of the heart on cardiovascular magnetic resonance (MR) images also extends to handling multiple types of knowledge including: myocardial motion, location and signal intensity. A study concerned with distinguishing different picture types of the central nervous system is introduced in [36]. Research involving color images appears in [37]. Histons (*i.e.*, encrustations of a histogram) are used as the primary measure and as a visualization of multi-dimensional color information. The basic idea of a histon is to build a histogram on top of the histograms of the primary color components red, green, and blue. The authors show that the base histogram correlates with the lower approximation, whereas the encrustation correlates with the upper approximation. The problem of a machine vision application where an object is imaged by a camera system is considered in [38]. The object space can be modeled as a finite subset of the Euclidean space when the objects image is captured via an imaging system. Rough sets can bound such sets and provide a mechanism for modeling the spatial uncertainty in the image of the object. This work introduced a rough sets approach for building pattern matching systems that can be applicable with a wide range of images in medical sciences.

B. Rough images

In grayscale images boundaries between object regions are often ill defined because of grayness or spatial ambiguities [39], [40]. This uncertainty can be effectively handled by describing the different objects as rough sets with upper (or outer) and lower (or inner) approximations. Here the concepts of upper and lower approximation can be viewed, respectively, as outer and inner approximations of an image region in terms of granules [40].

Definition 1: (Rough image) Let the universe U be an image consisting of a collection of pixels. Then, if we partition U into a collection of non-overlapping windows of size $m \times n$, each window can be considered as a granule G. Given this granulation, object regions in the image can be approximated by rough sets.

A *rough image* is a collection of pixels along with the equivalence relation induced partition of an image into sets of pixels lying within each non-overlapping window over the image. With this definition, the roughness of various transforms (or partitions) of an image can be computed using image granules for windows of different sizes.

C. Rough representation of a region of interest

A region of interest (ROI), is a selected subset of samples within an image identified for a particular purpose. The concept of ROI is commonly used in medical imaging. For example, the boundaries of a tumor may be defined on an image or in a volume, for the purpose of measuring its size. The endocardial border may be defined on an image, perhaps during different phases of the cardiac cycle, say end-systole and end-diastole, for the purpose of assessing cardiac function.

Hirano and Tsumoto [41] introduced the rough direct representation of ROIs in medical images. The main advantage of this method is its ability to represent inconsistency between the knowledge-driven shape and image-driven shape of a ROI using rough approximations. The method consists of three steps. First, they derive discretized feature values that describe the characteristics of a ROI. Secondly, using all features, they build up the basic regions (categories) in the image so that each region contains voxels that are indiscernible on all features. Finally, according to the given knowledge about the ROI, they construct an ideal shape of the ROI and approximate it by the basic categories. Then the image is split into three sets of voxels, which are:

(1) certainly included in the ROI (positive region),
(2) certainly excluded from the ROI (negative region),
(3) possibly included in the ROI (boundary region).

The ROI is consequently represented by the positive region associated with some boundary regions.

Hirano and Tsumoto [33], [41] described procedures for rough representation of ROIs under single and multiple types of classification knowledge. Usually, the constant variables defined in the prior knowledge, for example some threshold values, do not meet the exact boundary of images due to inter-image variances of the intensity. The approach tries to roughly represent the shape of the ROI by approximating the given shapes of the ROI by the primitive regions derived from feature of the image itself. It is reported that the simplest case occurs when we have only information about the intensity range of the ROI. In this case intensity thresholding is a conventional approach to obtain the voxels that fall into the given range. Let us denote the lower and upper thresholds by Th_L and Th_H, respectively. Then the ROI can be represented by:

$$ROI = \{x(p) \mid Th_L \leq I(x)P \leq Th_H\}, \tag{1}$$

where $x(p)$ denotes a voxel at location p and $I(x(p))$ denotes intensity of voxel $x(p)$.

Fig. 1 illustrates the concept of rough ROI representation. The left image is an original grayscale image. Assume that the ROIs are three black circular regions: ROI_1, ROI_2, and ROI_3. Also assume that we are given a prior knowledge about the ROIs, that is, the lower threshold value Th_L of the ROIs, derived from some knowledge base. With this knowledge we can segment an ideal ROI $X_{R\hat{O}I}$ as follows:

$$X_{R\hat{O}I} = \{x(p) | Th_L \leq I(p)\}. \tag{2}$$

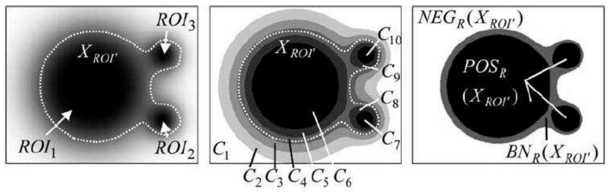


Fig. 1. Rough ROI representation. Left: an original image. Middle: elementary categories. Right: roughly segmented ROI [41]

However, $X_{R\hat{O}I}$ does not correctly match the expected ROIs. This is because Th_L was too small to separate the ROIs. Th_L is a global threshold determined on the other sets, therefore, it should not be directly applied to this image.

D. Rough Image Entropy

Entropy-based information theoretic approaches have received considerable interest in image analysis approaches such as image registration [42]. Previous work on entropic thresholding is based on Shannon's entropy. The idea is to calculate Shannon's entropy based on a co-occurrence matrix and use it as a criterion for selecting an appropriate threshold value. The approach using relative entropy for image thresholding has been shown very competitive compared to Pal's methods, where the relative entropy is chosen to be a thresholding criterion of measuring mismatch between an image and a thresholded image. Currently there are various published approaches using relative entropy and applying it to medical images, multispectral imagery, temporal image sequences, multistage thresholding and segmentation.

Pal *et al.* [40] presented a new definition of image entropy in a rough set theoretic framework, and its application to the problem of object extraction from images by minimizing both object and background roughness. Granules carry local information and reflect the inherent spatial relation of the image by treating pixels of a window as indiscernible or homogeneous. Maximization of homogeneity in both object and background regions during their partitioning is achieved through maximization of rough entropy; thereby providing optimal results for object background classification.

Definition 2: (Rough Image Entropy) [40] Rough image entropy $(R_I E)$ is defined by:

$$R_I E = -\frac{e}{2}[R_{O_T} log_e(R_{O_T}) + R_{B_T} log_e(R_{B_T})]. \quad (3)$$

$R_I E$ lies between 0 and 1 and it has has a maximum value of unity when $R_{O_T} = R_{B_T} = \frac{1}{e}$, and minimum value of zero when $R_{O_T}, R_{B_T} \in \{0, 1\}$. Fig. 2 shows a sample plot of rough entropy for various values of roughness of the object and background [40].

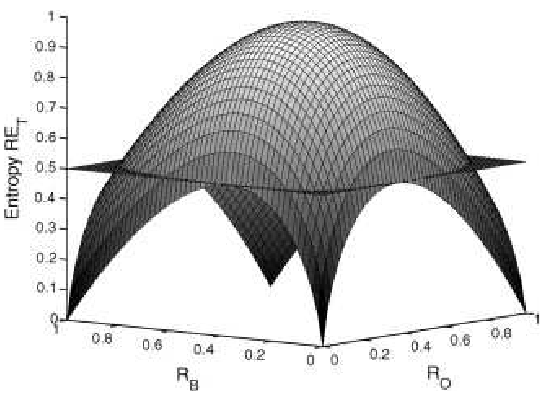


Fig. 2. Rough entropy for various values [40]

Pal *et al.* reported that a maximization of homogeneity in both object and background regions during their partitioning is achieved through maximization of rough entropy; thereby providing optimum results for object-background classification. Also, maximization of the rough entropy measure minimizes the uncertainty arising from vagueness of the boundary region of the object. Therefore, for a given granule size, the threshold for object-background classification can be obtained through its maximization with respect to different image partitions. The rough entropy concepts is applicable for many medical imaging problems such as feature extraction and medical image segmentation problems.

E. Rough Sets for Object Extraction

Identification of anatomical features is a necessary step for medical image analysis. Automatic methods for feature identification using conventional pattern recognition techniques typically classify an object as a member of a predefined class of objects, but do not attempt to recover the exact or approximate shape of that object. For this reason, such techniques are usually not sufficient to identify the borders of organs when individual geometry varies in local detail, even though the general geometrical shape is similar.

Pal *et al.* [40] demonstrated a new application of rough sets for object extraction from grayscale image. In grayscale images boundaries between object regions are often ill-defined. This uncertainty can be handled by describing the different objects as rough sets with upper (outer) and lower (inner) approximations. The set approximation capability of rough sets is exploited in the present investigation to formulate an entropy measure, called rough entropy, quantifying the uncertainty in an object-background image. Let T denote a set of thresholds (which are application dependent). An image object and the background are viewed as two sets with their rough representation by computing the inner approximation of the object ($\underline{Q}_T$), outer approximation of the object ($\overline{Q}_T$), inner approximation of the background ($\underline{B}_T$) and outer approximation of the background ($\overline{B}_T$) as follows:

$$\underline{Q}_T = \bigcup G_i | p_j > T, \forall j = 1, \ldots, mn, \quad (4)$$

$$\overline{Q}_T = \bigcup G_i, \exists j, p_j > T, j = 1, \ldots, mn, \quad (5)$$

$$\underline{B}_T = \bigcup G_i | p_j > T, \forall j = 1, \ldots, mn, \quad (6)$$

$$\overline{B}_T = \bigcup G_i, \exists j, p_j \leq T, j = 1, \ldots, mn, \quad (7)$$

where p_j is a pixel in G_i. The rough set representation of the image for a given $I_{m \times n}$ depends on the value of T.

Pal *et al.* define the roughness (R) of the object O_T and the background B_T as follows:

$$R_{O_T} = 1 - \frac{|\underline{Q}_T|}{|\overline{Q}_T|}, \quad (8)$$

$$R_{B_T} = 1 - \frac{|\underline{B}_T|}{|\overline{B}_T|}, \quad (9)$$

where the notation $|S|$ denotes the cardinality of the set S. This method can be used in many applications in image processing and in particular in medical imaging problems such as automatically identifying the myocardial contours of the heart, segmentation of knee tissues in CT image or segmentation of brain tissues in MR images.

F. Rough Sets in Medical Image Segmentation

The basic idea behind segmentation-based rough sets is that while some cases may be clearly labeled as being in a set X (called the positive region in rough sets theory), and some cases may be clearly labeled as not being in set X (called the negative region), limited information prevents us from labeling all possible cases clearly. The remaining cases cannot be distinguished and lie in what is known as the boundary region.

Among many difficulties in segmenting MRI data, the partial volume effect arises in volumetric images when more than one tissue type occurs in a voxel. In such cases, the voxel intensity depends not only on the imaging sequence and tissue properties, but also on the proportions of each tissue type present in the voxel. Widz *et al.* [11] discussed the partial volume effect problem in the segmentation of magnetic resonance imaging data that entails assigning tissue class labels to voxels. They employ rough sets to automatically identify the partial volume effect, which occurs most often with low resolution imaging.

An interesting strategy for color image segmentation using rough set theory has been presented by Mohabey *et al.* [37]. A new concept of encrustation of the histogram, called histon, has been proposed for the visualization of multi-dimensional color information in an integrated fashion and its applicability in boundary region analysis has been shown. The histon correlates with the upper approximation of a set such that all elements belonging to this set are clarified as possibly belonging to the same segment or segments showing similar color value. The proposed encrustation provides a direct means of segregating a pool of inhomogeneous regions into its components. Experimental results for various images have been presented in their work. They also introduced a hybrid rough set theoretic approximations and fuzzy c-means algorithm for color image segmentation. They segmented natural images with regions having gradual variations in color value. The technique extracts color information regarding the number of segments and the segments' center values from the image itself through rough set theoretic approximations, and presents it as input to a fuzzy c-means block for the soft evaluation of the segments. The performance of the algorithm has been evaluated on various natural and simulated images.

Many clustering algorithms [43] have been developed and applied in medical imaging problems, although most of them cannot process objects in hybrid numerical/nominal feature space or with missing values. In many of them, the number of clusters has to be manually specified while the clustering results are sensitive to the input order of the objects to be clustered. This clearly limits their applicability and reduces the quality of clustering. An improved clustering algorithm based on rough sets and entropy theory was presented by Chena and Wang [44] which aims to avoid the need to pre-specify the number of clusters while also allowing clustering in both numerical and nominal feature space with the similarity introduced to replace the distance index. At the same time, rough set theory endows the algorithm with the function to deal with vagueness and uncertainty in data analysis. Shannon's entropy was used to refine the clustering results by assigning relative weights to the set of features according to the mutual entropy values. A novel measure of clustering quality was also presented to evaluate the clusters. The experimental results confirm that performances of efficiency and clustering quality of this algorithm are improved.

Widz *et al.* [11] introduced an automated multi-spectral MRI segmentation technique based on approximate reducts derived from the theory of rough sets. They utilized T1, T2 and PD MRI images from a simulated brain database as a gold standard to train and test their segmentation algorithm. The results suggest that approximate reducts, used alone or in combination with other classification methods, may provide a novel and efficient approach to the segmentation of volumetric MRI data sets. Segmentation accuracy reaches 96% for the highest resolution images and 89% for the noisiest image volume. They tested the resultant classifier on real clinical data, which yielded an accuracy of approximately 84%.

G. Adaptation of C-Means to Rough Set Theory

C-means clustering is one of the most popular statistical clustering techniques used in segmentation of medical images [45]–[47]. Let us assume that n objects are represented by m-dimensional vectors. The objective is to assign these n objects to k clusters. Each of the clusters is also represented by an m-dimensional vector, which is the centroid or mean vector for that cluster. The process begins by randomly choosing k objects as the centroids of the k clusters. The objects are assigned to one of the k clusters based on the minimum value of the distance $d(v, x)$ between the object vector $v = (v_1, ..., v_j, ..., v_m)$ and the cluster vector $x = (x_1, ..., x_j, ..., x_m)$. After the assignment of all the objects to various clusters, the new centroid vectors of the clusters are calculated as:

$$x_j = \frac{\sum_{v \in x} v_j}{SOC}, \; 1 \le j \le m, \tag{10}$$

where SOC is the size of cluster x. Lingras [48] observes that incorporating rough sets into c-means clustering requires the addition of the concept of lower and upper bounds. Calculation of the centroids of clusters from conventional c-means needs to be modified to include the effects of lower as well as upper bounds. The modified centroid calculations for rough sets are then given by:

$$cen_j = w_{low} \times \frac{\sum_{v \in \underline{R}(x)}}{|\underline{R}(x)|} + w_{up} \times \frac{\sum_{v \in (\underline{BN_R(x)})}}{|\underline{BN_R(x)}|}, \tag{11}$$

where $1 \le j \le m$. The parameters w_{low} and w_{up} correspond to the relative importance of lower and upper bounds, and $w_{low} + w_{up} = 1$. If the upper bound of each cluster were equal to its lower bound, the clusters would be conventional clusters. Therefore, the boundary region $\underline{BN_R(x)}$ will be empty, and the second term in the equation will be ignored. Thus, the above equation will reduce to conventional centroid calculations. The next step in the modification of the c-means algorithms for rough sets is to design criteria to determine whether an object belongs to the upper or lower bound of a cluster.

H. Rough Sets in Feature Reduction and Image Classification

Many researchers have endeavored to develop efficient and effective algorithms to compute useful feature extraction and reduction of information systems, and mutual information and discernibility matrix based feature reduction methods. These techniques have been successfully applied to the medical domain [49], [50].

Wojcik [34] approached the nature of a feature recognition process through the description of image features in terms of rough sets. Since the basic condition for representing images must be satisfied by any recognition result, elementary features are defined as equivalence classes of possible occurrences of specific fragments existing in images. The names of the equivalence classes (defined through specific numbers of objects and numbers of background parts covered by a window) constitute the best lower approximation of window contents (*i.e.*, names of recognized features). The best upper approximation is formed by the best lower approximation, its features, and parameters, all referenced to the object fragments situated in the window. The rough approximation of shapes is robust to accidental changes in the width of contours and lines, to small discontinuities, and, in general, to possible positions or changes in shape of the same feature. The rough sets are utilized also on the level of image processing for noiseless image quantization. This initiative study has many interesting applications in the area of medical image processing including filtering, segmentation, and classification

Swiniarski and Skowron [51] presented applications of rough set methods for feature selection in pattern recognition. They emphasize the role of the basic constructs of rough set approach in feature selection, namely reducts and their approximations, including dynamic reducts. Their algorithm for feature selection is based on an application of a rough set method to the result of principal components analysis (PCA) used for feature projection and reduction. They present various experiments including mammogram recognition.

Hu *et al.* [52] proposed an information measure for computing discernibility power of a crisp equivalence relation or a fuzzy one, which is a key concept in classical rough set and fuzzy-rough set models. Based on the information measure, a general definition of significance of nominal, numeric and fuzzy features is presented.

Lymphoma is a broad term encompassing a variety of cancers of the lymphatic system and is differentiated by the type of cell that multiplies and how the cancer presents itself. It is very important to get an exact diagnosis regarding lymphoma and to determine the treatments that will be most effective for the patient's condition. Milan *et al.* [53] focused on the identification of lymphoma by finding follicles in microscopy images. Their study comprises two stages: in the first stage they did image pre-processing and feature extraction, while in the second stage they used different rough set approaches for pixel classification. These results were compared to decision tree results. The results they got are very promising and show that symbolic approaches can be successful in medical image analysis applications.

Microcalcification on a x-ray mammogram is a significant mark for early detection of breast cancer. Texture analysis methods can be applied to detect clustered microcalcification in digitized mammograms. In order to improve the predictive accuracy of the classifier, the original number of features is reduced into a smaller set using feature reduction techniques. Thangavel *et al.* [54] introduced rough set based reduction algorithms such as Decision Relative Discernibility based reduction, Heuristic approach, Hu's algorithm, Quick Reduct (QR), and Variable Precision Rough Set (VPRS) to reduce the extracted features.

Cyran *et al.* [55] showed how rough sets can be applied to improve the classification ability of a hybrid pattern recognition system. The system presented consists of a feature extractor based on a computer-generated hologram (CGH). Features extracted are shift, rotation, and scale invariant and they can be optimized.

Jiang *et al.* [56] developed a joining associative classifier (JAC) algorithm using rough set theory to mine digital mammography images. Their experimental results showed that the JAC performance was 77.48% in terms of classification accuracy which is higher than 69.11% using conventional associative classifier. At the same time, the number of rules decreased distinctively.

IV. Quantitative Evaluation

This section presents some quantitative measures [57] that are capable of quantifying the relative utility of enhancement techniques in digital imaging, generated rules, and quality of classification measures [58], [59]. This relates to preference criteria and goodness-of-fit chosen for the rules and classifiers.

A key objective of contrast enhancement is to maximize the difference between the background mean and target mean grayscale level, and to ensure that the homogeneity of the mass is increased, both of which aide the visualization of the boundary and location of the mass. Using the ratio of the standard deviation of the grayscales within the image before and after enhancement, we can quantify this improvement using the target-to-background contrast based on the standard deviation. This measure is initially computed by determining the difference between ratios of the mean grayscales in the target and background images in the original and enhanced images using

$$CM_{SD} = \{\frac{(m_t^e/m_b^e) - (m_t^o/m_b^o)}{\sigma_t^e/\sigma_t^o}\}, \tag{12}$$

where $m_t^e, m_b^e, m_t^o, m_b^o$ are the means of the grayscales comprising the target and background respectively of the original image before and after enhancement, and where σ_t^e, σ_t^o are the standard deviations of the grayscales before and after enhancement.

Within the mammogram image, the target has a greater density within the mammogram thus having higher mean grayscale intensity compared to the surrounding background. A good enhancement algorithm should aim to enhance the contrast between target and background by increasing the mean grayscale of the target area and then reducing the mean gray of the background area, thereby increasing the value of CM_{SD}.

The background contrast ratio can also be calculated using the entropy E of target and background areas within an image. This measure is computed in a similar manner to CM_{SD} by determining the difference between ratios of the mean grayscales in the target and background areas in both original and enhanced images using

$$CM_{Entropy} = \{\frac{(m_t^e/m_b^e) - (m_t^o/m_b^o)}{E_t^e/E_t^o}\}, \tag{13}$$

where E_t^e and E_t^o are the entropy of the target in the original and enhancement images, respectively. An effective enhancement algorithm will lead to a large value of $CM_{Entropy}$.

Index of fuzziness and fuzzy entropy are measures for global greyness ambiguity (fuzziness) of an image. They can be regarded as a degree of difficulty in deciding whether a pixel would be treated as black (dark) or white (bright). The index of fuzziness that gives the amount of fuzziness present in an image determines the amount of vagueness by measuring the distance between its fuzzy property plane and the nearest ordinary plane. Accordingly, entropy, H, which makes use of Shannon's function, is regarded as a measure of quality of information in an image in the fuzzy domain. It gives the value of indefiniteness of an image. These quantities [60]–[63] are defined as

$$\gamma = \frac{2}{MN} \sum_M \sum_N min(\mu_{mn}, 1-\mu_{mn}), \tag{14}$$

and

$$H = \frac{1}{MN} ln \sum_M \sum_N ln(\mu_{mn}) - (1-\mu_{mn}).ln(1-\mu_{mn}). \tag{15}$$

It should be noted that the decrease in the index of fuzziness and fuzzy entropy does not ensure proper enhancement of the images. We can only say that a good enhancement algorithm should reduce the greyness ambiguity. However, a low amount of ambiguity does not automatically lead to the desired enhancement effect.

V. Hybrid Intelligent Approaches

Intelligent systems comprise various paradigms dedicated to approximately solving real-world problems, *e.g.*, in decision making, classification or learning; among these paradigms are fuzzy sets, neural networks, decision tree, and rough sets, algorithms. Combination of different computational intelligence techniques in the application area of pattern recognition, and in particular in medical imaging problems, has become one of the most important ways of research in intelligent information processing [64]. In the following subsections we review some of the state-of-the-art in this area.

A. Rough set - Neural network approaches

Neural networks are known for their ability to solve various complex problems in image processing. However, they are unable to determine redundant information from large data sets, which can easily lead to problems such as over complex network structures, long training times, and low converging speeds. Hassanien and Ślęzak [57] introduced a rough neural approach for rule generation and image classification. Hybridization of intelligent computing techniques has lead to an increase in their ability to accurately classify breast images into malignant and benign instances. Algorithms based on fuzzy image processing are first applied to enhance the contrast of the original image, to extract the ROI, and to enhance the edges surrounding that region. Then, features characterizing the underlying texture of the regions of interest are extracted using the gray-level co-occurrence matrix. A rough set approach to feature reduction and rule generation is then applied. Finally, a rough neural network is designed to discriminate different ROIs in order to separate them into malignant and benign cases. The rough neural network employed is built from rough neurons [65], each of which can be viewed as a pair of sub-neurons, corresponding to the lower and upper bounds.

Definition 3: (Rough neuron) A rough neuron R_n is a pair of usual rough neurons $R_n = (U_n, L_n)$, where U_n and L_n are the upper rough neuron and the lower rough neuron, respectively.

Let (Ir_{L_n}, Or_{L_n}) be the input/output of a lower rough neuron and (Ir_{U_n}, Or_{U_n}) be the input/output of an upper rough neuron. The input/output of the lower/upper rough neurons is calculated by

$$Ir_{L_n} = \sum_{j=1}^{n} w_{L_{nj}} On_j, \tag{16}$$

$$Ir_{U_n} = \sum_{j=1}^{n} w_{U_{nj}} On_j, \tag{17}$$

$$Or_{L_n} = min(f(Ir_{L_n}), f(Ir_{U_n})), \tag{18}$$

$$Or_{U_n} = max(f(Ir_{L_n}), f(Ir_{U_n})). \tag{19}$$

The output of the rough neuron (O_{rn}) is then computed as

$$O_{rn} = \frac{Or_{U_n} - Or_{L_n}}{average(Or_{U_n}, Or_{L_n})}. \tag{20}$$

Rough neural networks [65]–[67] consist of one input layer, one output layer and one hidden layer. The number of hidden neurons is determined by

$$N_{hn} \le \frac{N_{ts} * T_e * N_f}{N_f + N_o}, \tag{21}$$

where N_{hn} is the number of hidden neurons, N_{ts} is the number of training samples, T_e is the tolerance error, N_f is the number of features, and N_o is the number of the output [68].

Another successful example introduced by Jiang *et al.* [69] was used to classify digital mammograms where they integrated a neural network with reduction based on rough set theory (which they called the rough neural network (RNN)). The experimental results showed that the RNN performs better than conventional neural networks not only in terms of complexity, but also that it achieves a 92.37% classification accuracy compared to the 81.25% achieved using a normal neural network only.

Swiniarski and Hargis [51] described an application of rough set methods to feature selection and reduction as a front end to a neural-network-based texture image recognition system. Their application included a singular-value decomposition (SVD) for feature extraction, principal components analysis (PCA) for feature projection and reduction, and rough sets methods for feature selection and reduction. For texture classification a feedforward backpropagation neural network was employed. The numerical experiments showed the ability of rough sets to select a reduced set of pattern features, while providing better generalization of neural-network texture classifiers (see also [49]).

B. Rough set - Fuzzy set approaches

Rough-fuzzy sets [70] can be seen as a particular case of fuzzy-rough sets. A rough-fuzzy set is a generalization of a rough set derived from the approximation of a fuzzy set in a crisp approximation space. This corresponds to the case where the conditional values are crisp, and only the decision attribute values are fuzzy. The lower and upper approximations indicate the extent to which objects belong to a target set. Mao *et al.* [71] proposed a new fuzzy Hopfield-model net based on rough-set reasoning for the classification of multispectral images. The main purpose is to embed a rough-set learning scheme into the fuzzy Hopfield network to construct a classification system called a rough-fuzzy Hopfield net (RFHN). The classification system is a paradigm for the implementation of fuzzy logic and rough systems in neural network architecture. Instead of all the information in the image being fed into the neural network, the upper- and lower-bound gray levels, captured from a training vector in a multispectal image, are fed into a rough-fuzzy neuron in the RFHN. Therefore, only 2/N pixels are selected as the training samples if an N-dimensional multispectral image was used.

Wang *et al.* [72] proposed a new nearest neighbor clustering classification algorithm based on fuzzy-rough set theory (FRNNC). First, they make every training sample fuzzy-roughness and use edit nearest neighbor algorithm to remove training sample points in class boundary or overlapping regions, and then use mountain clustering method to select representative cluster center points. Then, Fuzzy-Rough Nearest neighbor algorithm (FRNN) is applied to classify the test data. The new algorithm is applied to hand gesture image recognition, and the results show that it is more effective and performs better than other nearest neighbor methods.

Hassanien [73] introduced a hybrid scheme that combines the advantages of fuzzy sets and rough sets in conjunction with statistical feature extraction techniques. The introduced scheme starts with fuzzy image processing as a pre-processing technique to enhance the contrast of the whole image, to extract the ROI, and then to enhance the edges surrounding the ROI. Further, features from the segmented ROIs are extracted using the gray-level co-occurrence matrix. Rough sets are used for the generation of all reducts that contain minimal number of features and rules. Finally, these rules are passed to a classifier for discrimination of different ROIs to classify images.

Image clustering analysis is one of the core techniques for image indexing, classification, identification and image segmentation. Mitra *et al.* [74] introduced a hybrid clustering architecture, in which several subsets of patterns can be processed together with the objective of finding a common structure. A detailed clustering algorithm is developed by integrating the advantages of both fuzzy sets and rough sets. Further, they provide a measure of quantitative analysis of the experimental results for synthetic and real-world data.

Petrosino *et al.* [75] presented a multi-scale method based on the hybrid notion of rough fuzzy sets. This method comes from the combination of two models of uncertainty: vagueness handled by rough sets and coarseness handled by fuzzy sets. Marrying both notions leads to approximation of sets by means of similarity relations or fuzzy partitions. The most important features are extracted from the scale spaces by unsupervised cluster analysis, to successfully tackle image processing tasks. The approaches in [74], [75] can be applied in many medical imaging clustering problems such as image segmentation of abdomen images, and clustering filter bank response vectors to obtain a compact representation of the image structures obtained by an image quality verification of color retina images in diabetic retinopathy screening.

Sarkar [76] generalizes the concept of rough membership functions in pattern classification tasks to rough-fuzzy membership functions and rough-fuzzy ownership functions. Unlike the rough membership value of a pattern, which is sensitive only toward the rough uncertainty associated with the pattern, the rough-fuzzy membership (or ownership) value of the pattern signifies the rough uncertainty as well as the fuzzy uncertainty associated with the pattern. Various set theoretic properties of the rough-fuzzy functions are exploited to characterize the concept of rough-fuzzy sets. These properties are also used to measure the rough-fuzzy uncertainty associated with the given output class.

C. Rough set - Genetic algorithm approaches

Genetic algorithms and rough set theory have been used in combination in the study of images. Lingras [77] proposed an unsupervised rough set classification method using genetic algorithms, and also illustrated how genetic algorithms can be used to develop rough sets. The proposed rough set theoretic genetic encoding are especially useful in unsupervised learning. A rough set genome consists of upper and lower bounds for sets in a partition. The partition may be as simple as the conventional expert class and its complement or a more general classification scheme.

Mitra *et al.* [78] described a way of designing a hybrid system for detecting the different stages of cervical cancer. Hybridization includes the evolution of knowledge-based subnetwork modules with a genetic algorithm using rough set theory and the ID3 algorithm. Crude subnetworks for each module are initially obtained via rough set theory and the ID3 algorithm. These subnetworks are then combined, and the final network is evolved using genetic algorithms. The evolution uses a restricted mutation operator, which utilizes the knowledge of the modular structure, already generated, for faster convergence. The GA tunes the network weights and structure simultaneously.

D. Rough sets - Swarm Intelligence approaches

Das *et al.* [8] hybridized rough set theory with Particle Swarm Optimization (PSO). The hybrid rough-PSO technique has been used for grouping the pixels of an image in its intensity space. Medical images frequently become corrupted with noise. Fast and efficient segmentation of such noisy images has remained a challenging problem for years. In their work, the authors treated image segmentation as a clustering problem. Each cluster is modeled with a rough set. PSO is employed to tune the threshold and relative importance of upper and lower approximations of the rough sets. Davies-Bouldin clustering validity index is used as the fitness function, which is minimized while arriving at an optimal partitioning.

Another approach that uses rough set with PSO has been proposed by Wang *et al.* [58]. The authors applied rough sets to predict the degree of malignancy in brain glioma. As feature selection can improve the classification accuracy effectively, rough set feature selection algorithms are employed to select features. The selected feature subsets are used to generate decision rules for the classification task. A rough set attribute reduction algorithm that employs a search method based on PSO is proposed and compared with other rough set reduction algorithms. Experimental results show that reducts found by the proposed algorithm are more efficient and can generate decision rules with better classification performance. Moreover, the decision rules induced by rough set rule induction algorithm can reveal regular and interpretable patterns of the relations between glioma MRI features and the degree of malignancy, which are helpful for medical experts.

E. Rough sets - Support vector machines approaches

Support Vector Machines (SVMs) are a general algorithm based on guaranteed risk bounds of statistical learning theory. They have found numerous applications in image processing and pattern recognition and, in particular in medical imaging problems such as in classification of brain PET images, detection of microcalcification (MC) clusters in digital mammograms, lung cancer nodules extraction and classification, *etc*., and are now established as one of the standard computational intelligence tools. To inherit the merits of both rough set theory and SVMs, a hybrid classifier called rough set support vector machines (RS-SVMs) is proposed by Gexiang *et al.* [79] to recognize radar emitter signals. Rough sets are used in a preprocessing step to improve the performances of SVMs. A large number of experimental results showed that RS-SVMs achieve lower recognition error rates than SVMs and RS-SVMs have stronger capabilities of classification and generalization than SVMs, especially when the number of training samples is small.

Lingras and Butz [80] described how binary classification with SVMs can be interpreted using rough sets and how rough set theory may help in reducing the storage requirements of the 1-v-1 approach in the operational phase. Their techniques provided better semantic interpretations of the classification process. The theoretical conclusions are supported by experimental findings involving a synthetic dataset. The presented work is useful for soft margin classifiers in solving medical imaging problems especially a multi-class classification system for medical images [81].

Yun *et al.* [82] have used a rough-support vector machine integration and developed the Improved Support Vector Machine (ISVM) algorithm to classify digital mammography images, where rough sets are applied to reduce the original feature sets and the support vector machine is used classify the reduced information. The experimental results show that the ISVM classifier can get 96.56% accuracy which is higher than 92.94% using SVM, and the error recognition rates are close to 100%.

VI. CLASSIFYING IMAGES: NEAR SET APPROACH

This section gives a brief introduction to a near set approach to classifying images. In this approach, medical images are separated into non-overlapping sets of images that are similar (*descriptively* near to) each other. The near set approach is well suited to investigating medical images. This section introduces recent work on near images by Henry and Peters [26].

Let $\langle O, \mathbb{F}\rangle$ be a perceptual system, *i.e.*, a real valued total deterministic information system where O is a non-empty set of perceptual objects, and $\mathbb{F}$ is a countable set of probe functions. For every $\mathcal{B} \subseteq \mathbb{F}$, the weak nearness relation $\simeq_{\mathcal{B}}$ is defined as follows,

$$\simeq_{\mathcal{B}} = \{(x,y) \in O \times O \mid \exists\, \phi_i \in \mathcal{B},\ \phi_i(x) = \phi_i(y)\}.$$

The relation $\simeq_{\mathcal{B}}$ is considered *weak*, since this nearness relation between the objects (*e.g.*, pixels in an image) in each pair (x, y) requires at least one (not every) probe function satisfying $\phi_i(x) = \phi_i(y)$ to establish that x and y are near each other. Furthermore, let $X, Y \subseteq O$. A set X is weakly near to a set Y within the perceptual system $\langle O, \mathbb{F}\rangle$ $(X \underline{\bowtie}_{\mathbb{F}} Y)$ iff there are $x \in X$ and $y \in Y$ and there is $\mathcal{B} \subseteq \mathbb{F}$ such that $x \simeq_{\mathcal{B}} y$. Finally, define an elementary set (class) as

$$x_{/\simeq_{\mathcal{B}}} = \{x' \in X \mid x' \simeq_{\mathcal{B}} x\},$$

and define a partition of O (quotient set) as

$$O_{/\simeq_{\mathcal{B}}} = \{x_{/\simeq_{\mathcal{B}}} \mid x \in O\}.$$

A nearness measure (NM) useful in determining the degree of resemblance between two images is given in (22). Let the sets X and Y be weakly near each other in $\langle O, \mathbb{F}\rangle$, *i.e.*, there exists $\mathcal{B} \subseteq \mathbb{F}$ such that $x \simeq_{\mathcal{B}} y$. Then, the degree of nearness between X and Y is measured using (22).

$$NM_{\sim_{\mathcal{B}}}(X,Y) = \frac{\sum_{x_{/\simeq_{\mathcal{B}}} \in X_{/\simeq_{\mathcal{B}}}} \sum_{y_{/\simeq_{\mathcal{B}}} \in Y_{/\simeq_{\mathcal{B}}}} \eta\left(x_{/\simeq_{\mathcal{B}}}, y_{/\simeq_{\mathcal{B}}}\right)}{\max(|X_{/\simeq_{\mathcal{B}}}|, |Y_{/\simeq_{\mathcal{B}}}|)}, \tag{22}$$

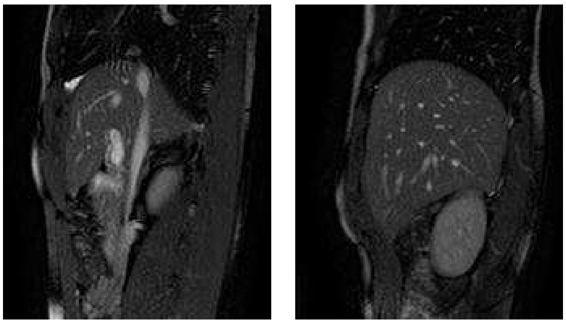
where $\eta\left(x_{/\simeq_{\mathcal{B}}}, y_{/\simeq_{\mathcal{B}}}\right)$ in (22) is defined as follows:

$$\eta\left(x_{/\sim_{\mathcal{B}}}, y_{/\sim_{\mathcal{B}}}\right) = \begin{cases} \min(|x_{/\sim_{\mathcal{B}}}|, |y_{/\sim_{\mathcal{B}}}|) & \text{, if } \phi_i(x) = \phi_i(y)\ \forall \phi_i \in \mathcal{B}, \\ 0 & \text{, otherwise.} \end{cases}$$

In other words, the nearness of two sets can be measured by the cardinality of their elementary (equivalence) classes. Sets that are similar with respect to the probe functions in $\mathcal{B}$ will have equivalence classes with similar numbers of objects producing a nearness degree close to or equal to 1. By contrast, sets that are not similar will have equivalence classes that share little with each other and will produce a nearness degree close to or equal to 0.

The following simple example demonstrates these concepts. Fifteen images from both the Berkeley Segmentation Dataset [83], and a 4DMRI dataset (see, Fig. 3 for example images) [84] were used to show that using this measure, similar images have a higher degree of nearness to each other than images which are not similar. In this example, the MRI images were used to represent images that are similar while the Berkeley Segmentation Dataset contains images which are of many different scenes and objects, and as such, do not have much relation to each other in terms of perceptual content of the images.

Formally, let $\mathbb{F}$ consist of a single probe function, namely the information content of the domain (input image). Further, let X and Y represent images that are partitioned into subimages, and let $O = X \cup Y$. Thus, each $o \in O$ is a perceptual



(a) (b)

Fig. 3. Sample MRI images of Respiratory Organ

object and in this example is given by a subimage of either image X or Y. Thus, we have defined a perceptual information system $\langle O, \mathbb{F} \rangle$.

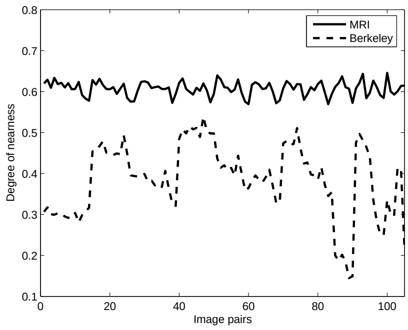


Fig. 4. Degree of nearness between image pairs.

In this example, we are interested in the results of using Eq. 22 on images that are similar to each other versus images that are not. To this end, we performed two different experiments, one for the MRI dataset and one for the Berkeley dataset. For each experiment, we selected all unordered pairs of images and compared them using Eq. 22. This gives $\binom{15}{2} = 105$ comparisons. The results of this experiment are shown in Fig. 4. As can be seen, this measure produces higher values for the MRI images which suggests that they are "nearer" each other than the Berkeley images. Also, there is less variability for the MRI values because the Berkeley images are quite different from each other. These results are promising and will lead to future work in object recognition using the near set approach.

VII. Promising Generalizations of Rough Sets in Medical Imaging

In addition to near sets, there are a number of other generalizations of rough sets that have promise in medical image analysis. This section briefly presents the basic approach in several of these generalizations, namely, tolerance spaces [27]–[29], [85]–[87], neighborhood systems [88]–[90], and shadowed sets [91]–[93].

A. Tolerance Spaces and Neighborhood Systems

The term *tolerance space* was coined by Zeeman in 1961 in modeling visual perception with tolerances [85]. Images are viewed as sets of fixed points. Let O denote a set of perceptual objects (*e.g.*, gray level images) and let $\simeq$ denote a relation that is reflexive (for all $x \in O$, $x \simeq x$) and symmetric (for all $x, y \in O$, $x \simeq y$ and $x \simeq y$) but transitivity of $\simeq$ is not required. Then $(O, \simeq)$ is a tolerance space. A tolerance is directly related to the exact idea of closeness or resemblance or *being within tolerance* in comparing objects. The basic idea is to find objects such as images that resemble each other with a tolerable level of error. The main idea underlying tolerance theory comes from Henri Poincaré [94]. The physical continuum (*e.g.*, measurable magnitudes in the physical world of medical imaging) are contrasted with the mathematical continuum (real numbers) where *almost solutions* are common and given equations have no exact solutions. An *almost solution* of an equation (or a system of equations) is an object which, when substituted into the equat(INSA)ion, transforms it into a numerical 'almost identity', *i.e.*, a relation between numbers which is true only approximately (within a prescribed tolerance) [86]. Equality in the physical world is meaningless, since it can never be verified either in practice or in theory. Hence, the basic idea in a tolerance view of medical imaging, for example, is to replace the indiscernibility relation in rough sets with a tolerance relation.

For example, tolerance near sets were introduced in [28] and elaborated in [27], [29], [30]. Briefly, here is the basic approach.

Definition 4: **Tolerance Relation** [30] Let $\langle O, \mathbb{F} \rangle$ be a perceptual system and let $\epsilon \in \mathbb{R}$ (set of all real numbers). For every $\mathcal{B} \subseteq \mathbb{F}$ the tolerance relation $\cong_{\mathcal{B}}$ is defined as follows:

$$\cong_{\mathcal{B},\epsilon} = \{(x, y) \in O \times O : \parallel \phi(x) - \phi(y) \parallel \leq \epsilon\}.$$

If $\mathcal{B} = \{\phi\}$ for some $\phi \in \mathbb{F}$, instead of $\cong_{\{\phi\}}$ we write $\cong_{\phi}$. Further, for notational convince, we will write $\cong_{\mathcal{B}}$ instead of $\cong_{\mathcal{B},\epsilon}$ with the understanding that ϵ is inherent to the definition of the tolerance relation.

As in the case with the indiscernibility relation, a tolerance class can be defined as

$$x_{/\cong_{\mathcal{B}}} = \{y \in X \mid y \cong_{\mathcal{B}} x\}. \tag{23}$$

From Defn. 4, a tolerance relation defines a covering of O (*i.e.* an object can belong to more than one tolerance class). For this reason, (23) is called a tolerance class instead of an elementary set. In addition, each pair of objects x, y in a tolerance class $x_{/\cong_{\mathcal{B}}}$ must satisfy the condition $\parallel \phi(x) - \phi(y) \parallel \leq \epsilon$. Next, a quotient set for a given a tolerance relation is the set of all tolerance classes and is defined as

$$O_{/\cong_{\mathcal{B}}} = \{x_{/\cong_{\mathcal{B}}} \mid x \in O\}.$$

As was the case with the equivalence relation, tolerance classes reveal relationships in perceptual systems leading to the definition of a tolerance nearness relation.

Definition 5: **Weak Tolerance Nearness Relation** [28], [30]
Let $\langle O, \mathbb{F} \rangle$ be a perceptual system and let $X, Y \subseteq O, \epsilon \in \mathbb{R}$. The set X is perceptually near to the set Y within the perceptual system $\langle O, \mathbb{F} \rangle$ ($X \underset{\mathbb{F}}{\bowtie} Y$) iff there exists $x \in X$, $y \in Y$ and there is a $\phi \in \mathbb{F}, \epsilon \in \mathbb{R}$ such that $x \cong_{\mathcal{B}} y$. If a

perceptual system is understood, then we say shortly that a set X is perceptually near to a set Y in a weak tolerance sense of nearness.

A tolerance nearness measure (tNM) under a tolerance relation is given as

$$tNM_{\cong_{\mathcal{B}}}(X,Y) = \sum_{x_{/\cong_{\mathcal{B}}} \in X_{/\cong_{\mathcal{B}}}} \sum_{y_{/\cong_{\mathcal{B}}} \in Y_{/\cong_{\mathcal{B}}}} \frac{\xi\left(x_{/\cong_{\mathcal{B}}}, y_{/\cong_{\mathcal{B}}}\right)}{\max(|x_{/\cong_{\mathcal{B}}}|, |y_{/\sim_{\mathcal{B}}}|)}, \tag{24}$$

where

$$\xi\left(x_{/\cong_{\mathcal{B}}}, y_{/\cong_{\mathcal{B}}}\right) = \begin{cases} \min(|x_{/\cong_{\mathcal{B}}}|, |y_{/\cong_{\mathcal{B}}}|) & \text{, if } \parallel \phi(x) - \phi(y) \parallel \leq \epsilon, \\ 0 & \text{, otherwise.} \end{cases}$$

Notice the subtle difference between the two nearness measures, namely, NM in (22) and tNM in (24). Since objects can belong to more than one tolerance class, the denominator of Eq. 24 has moved inside the summation. Similarly, Eq.'s 22 & 24 are equivalent when $\epsilon = 0$.

The neighborhood system (NS) paradigm has been widely used in image analysis [95]–[98]. Neighborhood systems were introduced by Sierpenski and Krieger during the mid-1950s [88], adopted by Lin during the late 1980s for describing relationships between objects in database systems [89] and considered in the context of rough sets [90], [99]. Associated with a neighborhood system (NS) is a set of cliques. A clique is either a single site or subset of sites such than any two sites are neighbors of each other [100]. Cliques are uniquely determined by the particular NS chosen. For an element x in a finite universe U, one associates a neighborhood $B(x) \subseteq U$. A *NS(x)* is a nonempty family of neighborhoods of x [90].

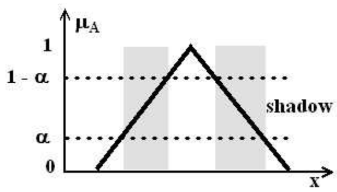


Fig. 5. Sample shadowed set [91].

B. Shadowed Sets

Shadowed sets (ShS) were introduced in 1998 by Pedrycz as a means of simplifying processes carried out with fuzzy sets and in establishing a bridge between rough sets and fuzzy sets [91] and in image processing [92], [93]. A shadowed set is viewed as an approximation of a fuzzy set [91], [101]. A *shadowed set* is a localization of membership values by forming "shadows" and using only 0-1 degrees of membership. Let $A \subseteq X$ (*i.e.*, A is a subset of the universe of discourse X) and let $\mu_A : X \rightarrow [0,1]$ (*i.e.*, μ_A is a membership function associated with the fuzzy set A. In Fig. 5, the membership values belonging to $(\alpha, 1-\alpha)$ are those values characterized by great uncertainty or lack of knowledge and they are considered the "shadow" of the induced shadowed set. In general, a shadowed set on X is any mapping $s : X \rightarrow \{0, 1, (0,1)\}$. For the sample shadowed set in Fig. 5, let $\alpha \in (0, 0.5)$ be a fixed value, then the α-shadowed set of μ_A (denoted by $s_\alpha(\mu_A)$ is defined to be a shadow of A [102], where

$$s_\alpha(\mu_A)(x) = \begin{cases} 0, & \text{if } \mu_A(x) \leq \alpha, \\ 1, & \text{if } \mu_A(x) \geq 1 - \alpha, \\ 0.5, & \text{otherwise.} \end{cases}$$

C. Choosing a Technology for Medical Image Analysis

Rough sets are ideally suited for feature-based image segmentations, image clustering and approximations of medical images. Pal's rough image entropy model is very useful in classifying images relative to the information content of either image regions or entire images and in extracting objects from grayscale images. Notice that the focus in the rough set approach to medical imaging is on approximation methods applied to single images or in grouping parts of an image in terms of equivalence classes. In applications where there is a need to determine the degree of resemblance (nearness) between medical images and to find clusters of medical images that resemble each other, then tolerance spaces in general and tolerance near sets, in particular, are useful.

The conjecture here is that neighborhood systems will be useful in analyzing image sequences found in video microscopy, X-ray cinematrography, 3D laser-scanning cofocal microscopy (LSCM) and magnetic resonance imaging (MRI), where there is an interest in observing shape-change and extracting meaningful information from image sequences. Rather than global information (how a specimen has translated, rotated or changed as a whole, to what extent pairs of images resemble each other) is easily detected using tolerance near sets, whereas neighborhood systems are more suited for extracting local information about shape-change of individual regions within a specimen.

An obvious advantage to shadowed sets is a simplified view of fuzzy sets in medical image analysis. The side-effect of introducing a shadowed set is shifting attention to an α-region of a fuzzy set considered important for a particular application such as medical imaging.

VIII. Challenges and Future Directions

Rough set theory encompasses an extensive group of methods that have been applied in the medical domain and that are used for the discovery of data dependencies, importance of features, patterns in sample data, and feature space dimensionality reduction. Most of the current literature on rough set-based methods for medical imaging focuses on classification and dimensionality reduction issues. A number of papers also deal with medical imaging problems such as image segmentation, image filtering, and voxel representation. From what has been presented in the literature, it is obvious that the rough set approach provides a promising means of solving a number of medical imaging problems. It should be observed that rough set or near set by themselves or in combination with other computational intelligence technologies work remarkably well in separating medical images into approximation regions that facilitate automated image segmentation and object recognition. The challenge now is to develop near set-based methods that offer an approach to classifying perceptual objects by means of features. It is fairly apparent that near set methods can be useful in object recognition, especially in solving medical imaging problems. The near set approach to object description, feature selection, and automatic image segmentation based on the partition of an image into equivalence classes offer a practical as well as straightforward approach to classifying images. It is in the domain of medical image segmentation that the near set approach holds the greatest promise for medical imaging.

A combination of various computational intelligence technologies in pattern recognition and, in particular, medical imaging problems has become one of the most promising avenues in image processing research. From the perspective of rough sets, further explorations into possible hybridizations of rough sets with other technologies are necessary to build a more complete picture of rough or near set-based applications in medical imaging. What can be said at this point is that the rough set and near set approaches pave the way for new and interesting avenues of research in medical imaging and represent an important challenge for researchers.

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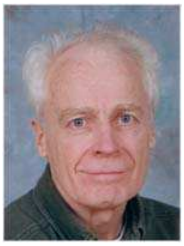

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