2D Parallel Thinning Algorithms Based on Isthmus-Preservation

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***Abstract*—Skeletons are widely used shape descriptors which summarize the general form of binary objects. A technique to obtain skeletons is the thinning, that is an iterative layer-by-layer erosion in a topology-preserving way. Conventional thinning algorithms preserve line endpoints to provide important geometric information relative to the object to be represented. Bertrand and Couprie proposed an alternative strategy by accumulating isthmus points that are line interior points. In this paper we present six new 2D parallel thinning algorithms that are derived from some sufficient conditions for topology preserving reductions and based on isthmus-preservation.**

I. Introduction

Skeletons play important role in various applications of image processing and pattern recognition [11]. Thinning is an iterative layer-by-layer erosion until only the skeletons of the binary objects are left [6], [12].

A *2D binary picture* [4], [5] is a mapping that assigns a value of 0 or 1 to each point with integer coordinates in the 2D digital space denoted by $\mathbb{Z}^2$. Points having the value of 1 are called *black* points, and those with a zero value are called *white* ones. The objects of a picture are comprised of black points; white points form the background and the cavities of the picture. We consider $(8,4)$–pictures, where 8–adjacency and 4–adjacency are, respectively, used for the objects and their complementary [4]. It is assumed that any picture contains finitely many black points.

A *reduction operator* transforms a binary picture only by changing some black points to white ones (which is referred to as the *deletion* of 1's). A *parallel reduction operator* deletes all points satisfying its condition simultaneously. Parallel thinning algorithms are composed of parallel reduction operations [3].

The first essential requirement for thinning algorithms is the topology preservation [4]. A 2D reduction operation does *not* preserve topology if any object in the input picture is split (into several objects) or completely deleted, any cavity in the input picture is merged with the background or another cavity, or a cavity is created where there was none in the input picture [5]. A *simple* point is an object point whose deletion does not alter the topology of the picture [4].

The second requirement to be complied with by thinning algorithms is the shape preservation. For example, an object having the shape as the letter "b" should not be transformed into an object like an "o". This is why endpoint criteria are generally applied by most of the existing thinning algorithms [3]. These conventional algorithms use operators that delete some simple points which are not endpoints, since preserving endpoints provides important geometrical information relative to the shape of the objects. Note that all endpoints are simple points in each endpoint characterization.

Bertrand and Couprie introduced an alternative strategy to preserve geometric features in thinning [1]. They proposed a sequential thinning scheme based on a generalization of curve/surface interior points that are called *isthmus*es. Isthmuses are dynamically detected and accumulated in a constraint set of non-simple points.

Despite of the topological constraint, Couprie found five existing 2D parallel thinning algorithms that are not topology-preserving [2]. In order to verify that a parallel reduction preserves topology, Ronse introduced the minimal non-simple sets in [9], and Kong gave some sufficient conditions [5]. Németh, Kardos, and Palágyi indroduced modified versions of Kong's sufficient conditions and combined them with the known parallel thinning approaches and endpoint characterizations to generate a family of topology preserving thinning and shrinking algorithms [7], [8].

In this paper we present various 2D parallel thinning algorithms that are derived from some sufficient conditions for topology preserving reductions, parallel thinning strategies, and based on isthmus-preservation.

The rest of this paper is organized as follows. Section 2 gives the basic notions of 2D digital topology and some sufficient conditions for parallel reduction operators to preserve topology. In Section 3, the conventional and the isthmus-based parallel thinning schemes are sketched. Section 4 reviews the proposed isthmus-based parallel thinning algorithms. Finally, we round off the paper with some concluding remarks.

II. Basic Notions and Results

In this paper, we use the fundamental concepts of digital topology as reviewed by Kong and Rosenfeld [4].

Let $p=(p_x,p_y)$ and $q=(q_x,q_y)$ be two points in $\mathbb{Z}^2$ and let us denote by $d(p,q)=\sqrt{(p_x-q_x)^2+(p_y-q_y)^2}$ their Euclidean distance. These two points, p and q, are *4–adjacent* if $d(p,q)\leq 1$ and they are *8–adjacent* if $d(p,q)\leq\sqrt{2}$. Let $N_j(p)$ (for $j=4,8$) denote the set of points *j–adjacent* to point p, and $N_j^*(p) = N_j(p)\backslash\{p\}$ refers to the set consisting of the proper j–adjacent neighbors of p. The sequence of distinct points $\langle x_0, x_1, \ldots, x_n\rangle$ is called a *j–path* (for $j = 4, 8$) of length n from point x_0 to point x_n in a non–empty set of points X if each point of the sequence is in X and x_i is j–adjacent to x_{i-1} for each $1 \le i \le n$. Note that a single point is a j–path of length 0. Two points are said to be *j–connected* in the set X if there is a j–path in X between them. A set of points X is *j–connected* in the set of points $Y \supseteq X$ if any two points in X are j–connected in Y.

The lexicographical order relation $\prec$ between two distinct points $p = (p_x, p_y)$ and $q = (q_x, q_y)$ is defined as follows: $p \prec q \quad \Leftrightarrow \quad p_y < q_y \vee (p_y = q_y \wedge p_x < q_x)$.

The *2D binary (8,4) digital picture* $\mathcal{P}$ is a quadruple $\mathcal{P} = (\mathbb{Z}^2, 8, 4, B)$ [4]. Each element of $\mathbb{Z}^2$ is called a *point* of $\mathcal{P}$. Each point in $B \subseteq \mathbb{Z}^2$ is called a *black point* and has a value of 1 assigned to it. Each point in $\mathbb{Z}^2\backslash B$ is called a *white point* and has a value of 0 assigned to it. 8–adjacency and 4–adjacency are, respectively, used for the black points and the white ones. A *black component* is a maximal 8–connected set of points in B, while a *white component* is a maximal 4–connected set of points in $\mathbb{Z}^2\backslash B$. It is assumed that any picture is finite (i.e., it contains finitely many black points).

A black point is called a *border point* in $(8, 4)$ pictures if it is 4–adjacent to at least one white point. A black point p is called an *interior point* if it is not a border point. A border point p is an *endpoint of type E* if there is one black point in $N_8^*(p)$ or there are two 4–adjacent black points in $N_8^*(p)$.

A black point is called a *simple point* if its deletion preserves the topology of the picture [4]. There are various characterizations of simple points. One of them is stated as follows:

Theorem 1: [4] Black point p is simple in picture $(\mathbb{Z}^2, 8, 4, B)$ if and only if all of the following conditions hold:

1) p is a border point.
2) The set $N_8^*(p)$ contains exactly one black 8–component.

Note that the simplicity of point p in a $(8, 4)$ picture is a local property; it can be decided in view of $N_8^*(p)$.

Parallel reduction operators delete a set of black points and not only a single simple point. Németh, Kardos, and Palágyi gave the following sufficient conditions for topology-preserving parallel reduction operators [8].

Theorem 2: Let $\mathcal{O}$ be a parallel reduction operation. The operation $\mathcal{O}$ is topology preserving for $(8, 4)$ pictures if all of the following conditions hold for any black point p in picture $(\mathbb{Z}^2, 8, 4, B)$ deleted by $\mathcal{O}$:

1) Point p is simple in picture $(\mathbb{Z}^2, 8, 4, B)$.
2) For any simple point $q \in N_4^*(p) \cap B$, p is simple in picture $(\mathbb{Z}^2, 8, 4, B \setminus \{q\})$, or q is simple in the picture $(\mathbb{Z}^2, 8, 4, B \setminus \{p\})$, or $q \prec p$.
3) Point p does not coincide with the points marked "$\star$" in the seven black components depicted in Fig. 1.

III. Parallel Thinning

The conventional parallel thinning scheme can be described by Algorithm 1, where "deletable" points are some simple

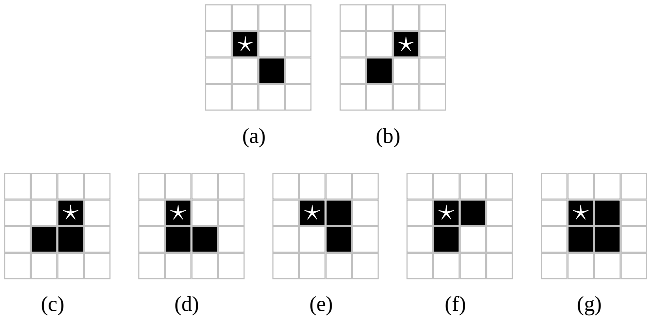


Fig. 1. Black points marked "$\star$" in the seven black components contained in a 2×2 square are designated to be preserved by Condition 3 of Theorem 2.

points that are not endpoints.

Algorithm 1 Conventional Parallel Thinning

Input: picture $(\mathbb{Z}^2, 8, 4, X)$
Output: picture $(\mathbb{Z}^2, 8, 4, Y)$
$Y = X$
repeat
  $D = \{p \mid p$ is "deletable" in $Y\}$
  $Y = Y \setminus D$
until $D = \emptyset$

In Algorithm 1, the kernel of the **repeat** cycle corresponds to an iteration step of the thinning process. Iterations (where all object points that satisfy the deletion condition are removed simultaneously) are repeated until stability is reached.

Endpoint preservation yields that the produced skeletons represent the shapes of the original objects, but it is a double-edged sword. Its risk is that each unwanted endpoint (that appears during the thinning process) corresponds to an unwanted side branch in the skeleton produced by an endpoint-preserving thinning algorithm. That is why Bertrand and Couprie introduced an alternative strategy to preserve geometric features in thinning [1]. They proposed a generalization of curve/surface interior points that are called *isthmus*es. Isthmuses are dynamically detected and accumulated in a constraint set of non-simple points. The isthmus-based parallel thinning scheme is sketched by Algorithm 2.

Algorithm 2 Isthmus-Based Parallel Thinning

Input: picture $(\mathbb{Z}^2, 8, 4, X)$
Output: picture $(\mathbb{Z}^2, 8, 4, Y)$
$Y = X$
$I = \emptyset$
repeat
  $B = \{p \mid p \notin I$ and p is a border point in $Y\}$
  $S = \{p \mid p \in B$ and p is a simple point in $Y\}$
  $I = I \cup (B \setminus S)$
  $D = \{p \mid p \in S$ and p is "deletable" in $Y\}$
  $Y = Y \setminus D$
until $D = \emptyset$

In each iteration step, some border points that are not isthmuses can be deleted and detected isthmuses (i.e., border points that are not simple ones) are accumulated in the constraint set I.

IV. Six Variations on Isthmus-Based Parallel Thinning Algorithms

In this section, six new isthmus-based thinning algorithms composed of parallel reduction operations that satisfy Theorem 2 are reported. The proposed algorithms were tested on objects of different shapes. Here we can present their results superimposed on just two test images. The first one is a 120×45 image of a violin with 2498 object points (see Figs. 2, 5, and 7), and the second test image is a 552×607 image of a salamander containing 108 615 object points (see Fig. 8).

The pairs of numbers in parentheses (see Figs. 2, 5, 7, and 8) are the count of object points in the produced pictures and the parallel speed (i.e., the number of the required parallel reduction operations [3]).

A. A Fully-Parallel Algorithm

In fully parallel algorithms, the same parallel reduction operation is applied in each iteration step [3].

Our isthmus-based fully parallel thinning algorithm FP-Isthmus is sketched by Algorithm 3.

Algorithm 3 FP-Isthmus

Input: picture $(\mathbb{Z}^2, 8, 4, X)$
Output: picture $(\mathbb{Z}^2, 8, 4, Y)$
$Y = X$
$I = \emptyset$
repeat
  $B = \{p \mid p \notin I$ and p is a border point in $Y\}$
  $S = \{p \mid p \in B$ and p is a simple point in $Y\}$
  $I = I \cup (B \setminus S)$
  $D = \{p \mid p \in S$ and p is FP-deletable in $Y\}$
  $Y = Y \setminus D$
until $D = \emptyset$

FP-deletable points are defined as follows:

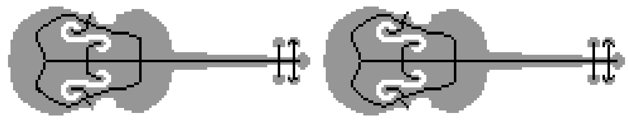
Definition 1: Black point p is FP-deletable if all the conditions of Theorem 2 hold.

Figure 2 presents an illustrative example for a skeleton produced by algorithm FP-Isthmus compared with the existing fully parallel algorithm FP-E (that preserves endpoints of type E) [8]. It is illustrated in Fig. 3 how the fully parallel algorithms FP-E and FP-Isthmus work.

Deletable points of the proposed fully parallel algorithm FP-Isthmus (see Definition 1) are derived directly from conditions of Theorem 2. Hence, it is topology preserving.

B. Subiteration-Based Algorithms

In subiteration-based thinning algorithms, an iteration step is decomposed into k successive parallel reduction operations according to the k deletion directions. If direction d is the current deletion direction, then some d-border points are



FP-Isthmus (294, 17) FP-E (300, 17)

Fig. 2. Skeletons produced by the proposed isthmus-based fully parallel thinning algorithm FP-Isthmus and the corresponding endpoint-based algorithm FP-E.

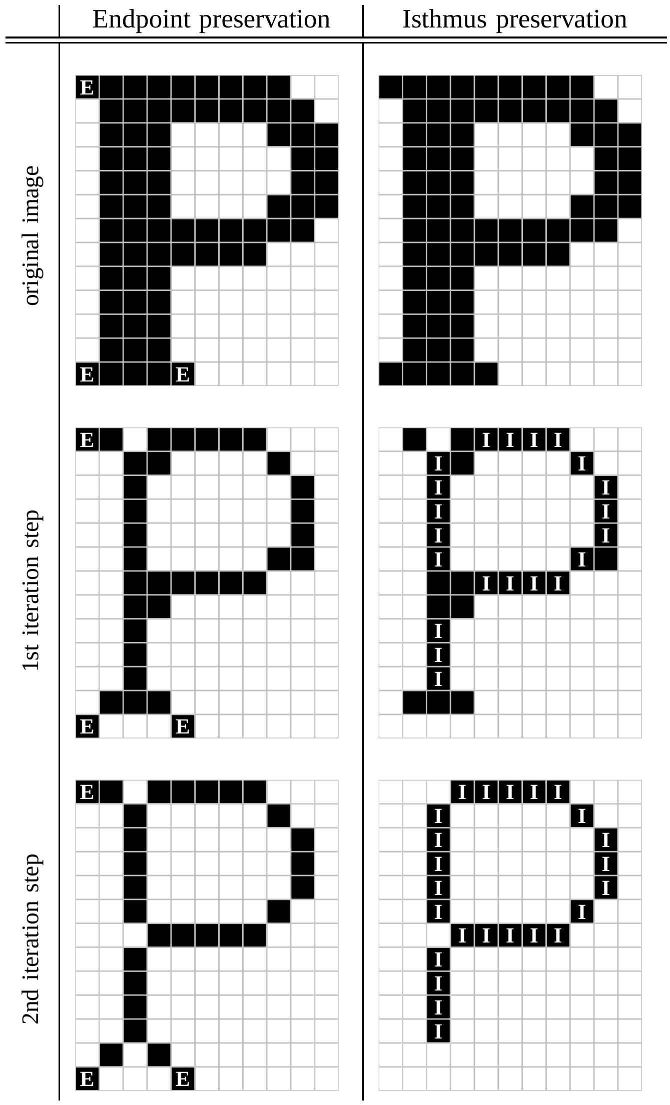


Fig. 3. A sample object to show how the proposed fully parallel thinning algorithms work with endpoint and isthmus preservation. Points marked "E" corresponds to the endpoints of type E, while isthmus points are marked "I".

deleted [3]. Existing 2D subiteration-based parallel thinning algorithms consider $k = 2$ or $k = 4$ directions [3], [7], [8], [12].

A black point p is an N-border point if point p_N (see Fig. 4) is white. The W-, S-, E-border points can be defined similarly. In addition, a black point p is an NE-border point if p_N or p_E is white. Considering another pairs of directions, we can likewise talk about NW-, SW-, SE-border points (see Fig. 4).

p_{NW}	p_N	p_{NE}
p_W	p	p_E
p_{SW}	p_S	p_{SE}

Fig. 4. Notations for the 3×3 neighborhood of point p.

Our new subiteration-based thinning algorithms using a sequence of deletion directions Q are SI-Q-Isthmus (see Algorithm 4), ($Q = \langle NE, SW \rangle$, $\langle N, E, S, W \rangle$, $\langle NE, SW, NW, SE \rangle$).

Algorithm 4 SI-Q-Isthmus

Input: picture $(\mathbb{Z}^2, 8, 4, X)$
Output: picture $(\mathbb{Z}^2, 8, 4, Y)$
$Y = X$
$I = \emptyset$
repeat
  $D = \emptyset$
  for all $d \in Q$ **do**
    $B = \{p \mid p \notin I$ and p is a border point in $Y\}$
    $S = \{p \mid p \in B$ and p is a simple point in $Y\}$
    $I = I \cup (B \setminus S)$
    $D_d = \{p \mid p \in S$ and p is SI-d-deletable in $Y\}$
    $Y = Y \setminus D_d$
    $D = D \cup D_d$
  end for
until $D = \emptyset$

SI-d-deletable points are defined as follows:

Definition 2: Black point p is called SI-d–deletable if all the following conditions hold ($d \in \{N, E, S, W, NE, SW, NW, SE\}$):

1) Point p is a simple and d-border point.
2) For any simple and d-border point $q \in N_4^*(p)$, p is simple in $N_8^*(p) \setminus \{q\}$ or q is simple in $N_8^*(q) \setminus \{p\}$, or $q \prec p$.
3) Depending on the deletion direction d, the following conditions are to be satisfied:
   - if $d \in \{N, E, S, W\}$, then p does not coincide with the point marked "$\star$" depicted in Fig. 1(a) and (b);
   - if $d = NE$, then p does not coincide with the point marked "$\star$" depicted in Fig. 1(a), (b), (c), (e), and (f);
   - if $d = SW$, then p does not coincide with the point marked "$\star$" depicted in Fig. 1(a), (b), (c), (d), and (f);
   - if $d = NW$, then p does not coincide with the point marked "$\star$" depicted in Fig. 1(a), (b), (d), (e), and (f);
   - if $d = SE$, then p does not coincide with the point marked "$\star$" depicted in Fig. 1(a), (b), (c), (d), and (e).

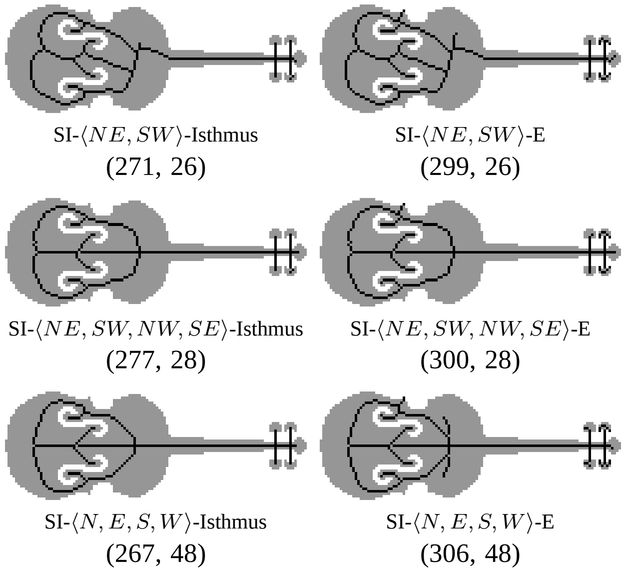


Fig. 5. Skeletons produced by the proposed isthmus-based algorithms SI-$\langle NE, SW \rangle$-Isthmus, SI-$\langle NE, SW, NW, SE \rangle$-Isthmus, and SI-$\langle N, E, S, W \rangle$-Isthmus and the corresponding endpoint-based algorithms SI-$\langle NE, SW \rangle$-E, SI-$\langle NE, SW, NW, SE \rangle$-E, and SI-$\langle N, E, S, W \rangle$-E.

Figure 5 presents some examples for skeletons produced by our new algorithms SI-$\langle NE, SW \rangle$-Isthmus, SI-$\langle NE, SW, NW, SE \rangle$-Isthmus, and SI-$\langle N, E, S, W \rangle$-Isthmus compared with the corresponding subiteration-based algorithms SI-$\langle NE, SW \rangle$-E, SI-$\langle NE, SW, NW, SE \rangle$-E, and SI-$\langle N, E, S, W \rangle$-E (that preserve endpoints of type E) [8].

It can readily be seen that deletable points of the proposed subiteration-based algorithms (see Definition 2) are derived from the conditions of Theorem 2. Hence, all of the three algorithms are topology preserving.

C. *Subfield-Based Algorithms*

Subfield-based algorithms partition the digital space into k subfields. During an iteration step, the subfields are alternatively activated, and a set of border points in the active subfield can be deleted by a parallel reduction operation [3].

The existing 2D subfield-based thinning algorithms partition the digital space $\mathbb{Z}^2$ into two and four subfields, see Fig. 6. In the case of k subfields, the i-th subfield denoted by $S_k(i)$ is defined as follows ($k = 2, 4$; $i = 0, \ldots, k-1$):

$$S_2(i) = \{p = (x, y) \mid (x + y) \equiv i \pmod 2\},$$
$$S_4(i) = \{p = (x, y) \mid 2 \cdot (y \bmod 2) + (x \bmod 2) = i\}.$$

0	1	0	1	0
1	0	1	0	1
0	1	0	1	0
1	0	1	0	1
0	1	0	1	0

(a)

0	1	0	1	0
2	3	2	3	2
0	1	0	1	0
2	3	2	3	2
0	1	0	1	0

(b)

Fig. 6. Partitions of $\mathbb{Z}^2$ into two (a) and four (b) subfields. For the k-subfield case, the points marked i are in the subfield $S_k(i)$ ($k = 2, 4$, $i = 0, \ldots, k-1$).

Our new isthmus-based subfield-based thinning algorithms are SF-2-Isthmus and SF-4-Isthmus (see Algorithm 5).

Algorithm 5 SF-k-Isthmus

Input: picture $(\mathbb{Z}^2, 8, 4, X)$
Output: picture $(\mathbb{Z}^2, 8, 4, Y)$
$Y = X$
$I = \emptyset$
repeat
  $D = \emptyset$
  for $i = 0$ to $k-1$ **do**
    $B = \{p \mid p \notin I$ and p is a border point in $Y\}$
    $S = \{p \mid p \in B$ and p is a simple point in $Y\}$
    $I = I \cup (B \setminus S)$
    $D_i = \{p \mid p \in S$ and p is SF-k-deletable in $Y\}$
    $Y = Y \setminus D_i$
    $D = D \cup D_i$
  end for
until $D = \emptyset$

SF-k-deletable points ($k = 2, 4$) are defined as follows:

Definition 3: Black point p is called SF-k–deletable if all the following conditions hold ($k = 2, 4$):

1) Point p is simple in subfield $S_k(i)$.
2) If $k = 2$, then p does not coincide with the points marked "$\star$" in Fig. 1(a) and 1(b).

Figure 7 presents some examples for skeletons produced by our new algorithms SF-2-Isthmus and SF-4-Isthmus compared with the corresponding subfield-based algorithms SF-2-E and SF-4-E (that preserve endpoints of type E) [8].

It can readily be seen that deletable points of the proposed subfield-based algorithms (see Definition 3) are derived from the conditions of Theorem 2. Hence, both algorithms are topology preserving.

V. Conclusions

This paper presents new parallel thinning algorithms. The major contributions of this work are:

- Six variations for parallel thinning algorithms were constructed (each algorithm differs from the other ones). Deletion rules of the proposed algorithms were not given by matching templates (as it is usual), they were derived from sufficient conditions for topology preserving parallel reductions.

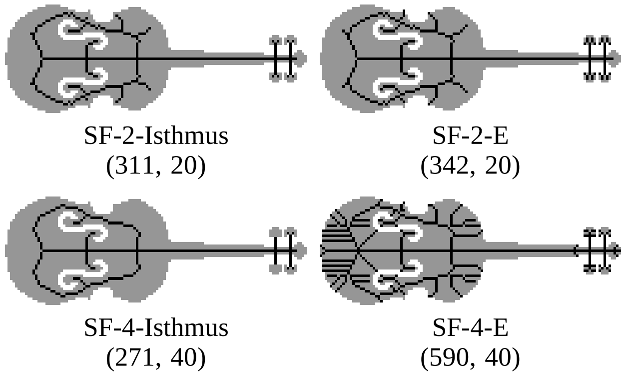


Fig. 7. Skeletons produced by the proposed two isthmus-based algorithms (SF-2-Isthmus and SF-4-Isthmus) and the corresponding endpoint-based algorithms (SF-2-E and SF-4-E).

- The proposed algorithms are based on isthmus-preservation (instead of the conventional endpoint-preservation thinning scheme).
- The topological correctness of all the proposed algorithms is guaranteed.
- Thanks to the isthmus-based approach, our new algorithms produce less unwanted side branches than the conventional ones with endpoint-preservation. Note that each skeletonization technique (including thinning) is rather sensitive to coarse object boundaries. The false segments included by the produced skeletons can be removed by a pruning process (i.e., a post-processing step) [10].

Unfortunately, there is no room to present more examples here, hence we invite the reader to visit the website at `http://www.inf.u-szeged.hu/~gnemeth/thinning_gallery/skeleton_alg2d.php`, where skeletons produced by various existing 2D thinning algorithms are also presented.

Finally, note that the proposed algorithms can be implemented efficiently on conventional sequential computers by adapting the general framework proposed by Németh and Palágyi [7]. Skeletons of large objects containing 1.000.000 points can be produced within one second on a usual PC.

Acknowledgements

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References

[1] G. Bertrand, M. Couprie, *Transformations topologiques discrètes*, In: D. Coeurjolly, A. Montanvert, J. Chassery (Eds.), Géométrie discrète et images numériques, Hermès, 2007, pp. 187-209.
[2] M. Couprie, Note on Fifteen 2D Parallel Thinning Algorithms, *Internal Report, Université de Marne-la-Vallée*, IMG2006-01, 2009.

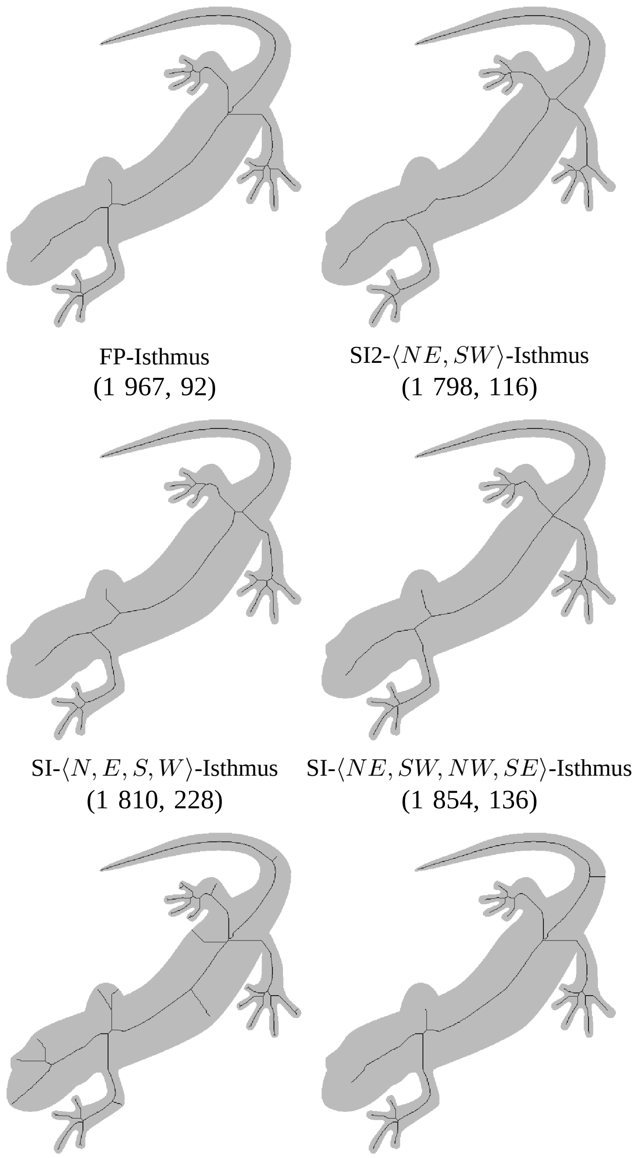

Fig. 8. Skeletons produced by the proposed six algorithms for a 552×607 image of a salamander. We can state that the fully parallel algorithm FP-Isthmus and the 4-subiteration algorithms SI-$\langle N, E, S, W \rangle$-Isthmus and SI-$\langle NE, SW, NW, SE \rangle$-Isthmus can produce "reasonable" skeletons. The 2-subiteration algorithm SI-$\langle NE, SW \rangle$-Isthmus can overshrink the objects and produce asymmetric skeletons for symmetric objects (see Fig. 4). By nature of the subfield-based thinning technique, both algorithms SF-2-Isthmus and SF-4-Isthmus can produce some unwanted side branches.

[3] R.W. Hall, *Parallel connectivity-preserving thinning algorithms*, In: Topological algorithms for digital image processing, Kong, T.Y., Rosenfeld, A. (Eds.), Elsevier Science, 1996, pp. 145-179.
[4] T.Y. Kong, A. Rosenfeld, "Digital Topology: Introduction and survey", *Computer Vision, Graphics, and Image Processing* 48, 1989, pp. 357-393.
[5] T.Y. Kong, "On topology preservation in 2-d and 3-d thinning", *Int. Journal of Pattern Recognition and Artificial Intelligence* 9, 1995, pp. 813-844.
[6] L. Lam, S.-W. Lee, C.Y. Suen, "Thinning Methodologies – A Comprehensive Survey", *IEEE Trans. Pattern Analysis and Machine Intelligence* 14, 1992, pp. 869-885.
[7] G. Németh, K. Palágyi, "Topology Preserving Parallel Thinning Algorithms" *International Journal of Imaging Systems and Technology* 21, 2011, pp. 3744.
[8] G. Németh, P. Kardos, K. Palágyi, "2D Parallel Thinning and Shrinking Based on Sufficient Conditions for Topology Preservation", *Acta Cybernetica* 20, 2011, 127-146.
[9] C. Ronse, "Minimal Test Patterns for Connectivity Preservation in Parallel Thinning Algorithms for Binary Digital Images", *Discrete Applied Mathematics* 21, 1988, pp. 67-79.
[10] D. Shaked, A. Bruckstein, "Pruning medial axes", *Computer Vision and Image Understanding* 69, 1998, pp. 156-169.
[11] K. Siddiqi, S.M. Pizer (Eds.), *Medial Representations – Mathematics, Algorithms, and Applications*, Series in Computational Imaging, Springer, 2008.
[12] C.Y. Suen, P.S.P. Wang (Eds.), *Thinning Methodologies for Pattern Recognition*, Series in Machine Perception and Artificial Intelligence 8, World Scientific, 1994.