An Improved MMSE-Based MIMO Detection using Low-Complexity Constellation Search

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Abstract—**The maximum likelihood (ML) detection for multiple-input multiple-output (MIMO) system achieves the optimal performances at the cost of high computational complexities, while the linear detectors attain low complexities with degraded performances. In this paper, we propose two low-complexity detection schemes based on the minimum-mean-square-error (MMSE) detection for MIMO systems. Using on the MMSE-detected data as the starting point, the first scheme searches the constellation subspace using the ML criterion. To further reduce the complexity, the second scheme selects the constellation subspace for search using the reformulated ML criterion. Simulation results show the substantial performance improvements compared to the MMSE detection, with only slightly increased complexities.**

Index Terms—**MIMO detection, maximum likelihood detection, Zero-Forcing, MMSE, wireless communication.**

I. Introduction

The frequency spectrum is the scarce resource for wireless communication systems and the rapid increase of wireless applications has demanded the new techniques to achieve higher spectral efficiency. The multiple-input multiple-output (MIMO) system utilizes the spatial diversity to increase the data rate and spectral efficiency. One of the major challenges to design this system is the high complexity in data detection at the receiver. The commonly practiced MIMO data detection is to pursue the maximum likelihood (ML) criterion. Although the ML detection offers the optimal solution, it encounters difficulties in practical systems due to its high computational complexity. This motivates various variations of the MIMO detections to tradeoff between performances and complexities.

In the literature, there are various studies on optimal and suboptimal MIMO detection techniques. The linear equalization-based MIMO detection includes the zero-forcing (ZF) and minimum-mean-square-error (MMSE) detection. The main advantage of the linear detection is its low complexity and simplicity for implementation. The performances of the linear detectors degrade substantially compared with the optimal ML detectors. Therefore, there exists variations of linear MIMO detection to improve its performance [1]–[5]. Moreover, the sphere decoder (SD) and its variations achieves optimal or near-ML performances with reduced complexity than the exhaustive search. The conventional sphere decoder has two categories : 1) the Fincke-Pohst sphere decoder [6], [7] and 2) the Schnorr-Euchner sphere decoder [8]. Although the conventional SD efficiently reduces the complexity of the ML detection and attains the ML performance [9], its complexity and operations are probabilistic, which complicates the implementations and offers unstable data throughput. Another variation of the sphere decoder with the fixed complexity is studied to avoid the probabilistic uncertainties of the conventional sphere decoders [10], [11]. Certain detectors utilize the statistical property to determine the radius that effectively shrinks the search range of the sphere decoder [9], [12], [13].

In this paper, we propose a simple fixed-complexity MMSE-based MIMO detection. The proposed approaches utilize ML criterion with the MMSE detection as the starting point for search. Based on the results of the MMSE detection, the approach searches the limited range of constellation subspace. Simulation results demonstrate significant performance improvements compared with the MMSE detection. The major advantage of the proposed approach is to attain significant performance improvements of MMSE detections while maintaining the fixed and low computational complexities.

This paper is organized as follows. In section II, the signal model and linear MIMO detectors are briefly described. The proposed approaches based on the MMSE detection and their computational complexities are elaborated in section III. In section IV, simulation results are presented to verify the performance improvements. Section V summarizes and concludes this paper.

II. Signal Model and Review of MIMO Detection Techniques

We consider the baseband flat fading MIMO channel model with M_T transmitting antennas and M_R receiving antennas $(M_R \geq M_T)$ as

$$y = \mathbf{H}x + n \tag{1}$$

where $y, n \in \mathbb{C}^{M_R}$, $\mathbf{H} \in \mathbb{C}^{M_R \times M_T}$, $x \in \mathbb{S}^{M_T}$, $\mathbb{C}$ denotes the set of complex numbers, and $\mathbb{S}$ denotes the set of constellation points of modulations. The cardinality of $\mathbb{S}$, which is denoted as $|\mathbb{S}|$, is countable finite. The $M_T \times 1$ transmitted and $M_R \times 1$ received symbol vector are $x \triangleq [x_1 \dots x_{M_T}]^T$ and $y \triangleq [y_1 \dots y_{M_R}]^T$ respectively, where $[\cdot]^T$ denotes vector transpose. We assume that the symbol vectors x are uncorrelated random with zero mean and covariance matrix $\sigma_x^2\mathbf{I}$, where $\mathbf{I}$ indicates the identity matrix. The additive white Gaussian noise (AWGN) vectors $n \triangleq [n_1 \dots n_{M_R}]^T$ are independent and identically distributed (i.i.d.) complex noise with zero mean and covariance matrix $\sigma_n^2\mathbf{I}$. The complex-valued channel matrix $\mathbf{H}$ has i.i.d. Gaussian entries with zero

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mean and unit variance and we assumed it is perfectly known to the receiver.

The MIMO detection techniques related to this work are briefly discussed in the following subsections. The maximum likelihood (ML) detection achieves the optimal performances; however, it suffers from the exponentially increasing computational complexity. Besides the ML critetion, the zero-forcing (ZF) and minimum mean square error (MMSE) detections are linear equalization-based methods. They are low-complexity and simple to implement with degraded performances. This motivates the major work in this paper focusing on improving the performance of the linear equalization-based methods, at the cost of slightly extra complexities.

A. Maximum Likelihood Detection

From (1), ML detection of a symbol vector can be written as [14]

$$\hat{x}_{ML} = \arg \min_{x \in \mathbb{S}^{M_T}} ||y - \mathbf{H}x||^2 \tag{2}$$

where $\|\cdot\|$ denotes the L2 norm of a vector. The ML criterion is to obtain the solution satisfying (2) in the set $\mathbb{S}^{M_T}$. The major challenge is huge cardinality of $\mathbb{S}^{M_T}$, i.e., $|\mathbb{S}|^{M_T}$, that increases exponentially with M_T.

B. ZF Detection

The zero forcing (ZF) detection multiply the received symbol vector y by an equalization matrix $\mathbf{G}$, i.e. $\hat{x}_{ZF} = \mathbf{G}_{ZF}y$. The zero forcing equalization is derived from the Moore-Penrose pseudo-inverse [15] of $\mathbf{H}$,

$$\mathbf{G}_{ZF} = (\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H \tag{3}$$

where $(\cdot)^{-1}, (\cdot)^H$ denote inverse matrix and hermitian transpose, respectively. We assumed that $M_R \geq M_T$. After equalization, the transmitted symbol vector estimate $\hat{x}_{ZF}$ of the ZF detection is written as

$$\hat{x}_{ZF} = \mathbf{G}_{ZF}\ y = (\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H y = x + \tilde{n} \tag{4}$$

where $\tilde{n} = (\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H n$ disturbs the transmitted symbol vector x. From (4), the crosstalk of x generated by the channel $\mathbf{H}$ in the received y is removed, while the Gaussian noise $\tilde{n}$ is colored noise. The performance of the ZF detection is degraded because of the colored noise $\tilde{n}$.

C. MMSE Detection

The concept that the MMSE detection uses is to minimize the mean square error $E(\|\mathbf{G}y - x\|^2)$, where $E(\cdot)$ denotes the expectation of the random variable. The MMSE detection considers the noise variance and reduces the noise enhancement by using the minimum mean square error equalization matrix obtained from [14]

$$\mathbf{G}_{MMSE} = (\mathbf{H}^H\mathbf{H} + (\sigma_n^2/\sigma_x^2)\mathbf{I})^{-1}\mathbf{H}^H \tag{5}$$

The transmitted symbol vector estimate $\hat{x}_{MMSE}$ of the MMSE detection is written as

$$\hat{x}_{MMSE} = \mathbf{G}_{MMSE}\ y = (\mathbf{H}^H\mathbf{H} + (\sigma_n^2/\sigma_x^2)\mathbf{I})^{-1}\mathbf{H}^H y \tag{6}$$

The results of the ZF and MMSE detection, i.e., $\hat{x}_{ZF}$ and $\hat{x}_{ZF}$, are not necessarily the legal constellation points in $\mathbb{S}$, so they need to be rounded off to the closest constellation point. The function of rounding off the raw detected data to the closest point of the constellation point set $\mathbb{S}$ is denoted as $\lceil\cdot\rceil_{\mathbb{S}}$. We name the operation of $\lceil\cdot\rceil_{\mathbb{S}}$ as quantization. By the quantization process, the quantized symbol vector $\tilde{x}_{ZF}$, $\tilde{x}_{MMSE}$ are expressed as $\tilde{x}_{ZF} = \lceil\hat{x}_{ZF}\rceil_{\mathbb{S}}$ and $\tilde{x}_{MMSE} = \lceil\hat{x}_{MMSE}\rceil_{\mathbb{S}}$.

III. The Proposed Approaches

The major advantage of the equalization-based detection is its low computational complexity; however, their performances are suboptimal. The motivation of this work is to improve the performances of MMSE detector without introducing substantial computational complexities. The general observation is that only few modulation symbols are erroneous in a erroneous MIMO symbol vector when using the MMSE detection. This observation motivates this work to improve the MMSE detection by searching the limited constellation subspace. The details are described in the followings.

A. Subspace Search Based on MMSE Detection

We have the $M_T \times 1$ quantized symbol vector $\tilde{x}_{MMSE} \triangleq [\tilde{x}^1_{MMSE} \dots \tilde{x}^{M_T}_{MMSE}]^T$, where the superscript denotes the symbol index. The likelihood metric D_{MMSE} can be expressed as $D_{MMSE} = \|y - \mathbf{H}\tilde{x}_{MMSE}\|^2$. Since the MMSE detection $\tilde{x}_{MMSE}$ might be erroneous, there might exist another symbol vector $\breve{x}_{MMSE}$ such that its likelihood metric satisfies $\|y - \mathbf{H}\breve{x}_{MMSE}\|^2 \leq D_{MMSE}$. If the symbol vector $\breve{x}_{MMSE}$ minimizes $\|y - \mathbf{H}x\|^2$, it is equivalent to the ML detection, i.e., $\breve{x} = \hat{x}_{ML}$. The exhaustive search induces the cost of high computational complexities because of large number of the candidates in x. Therefore, the main idea of the proposed approach is to limit the search subspace by using the MMSE detection. Since there are only few erroneous symbol entries in the event of MIMO symbol error, the proposed idea is to search the M-dimensional space, where $(M \leq M_T)$, and replace the erroneous symbol entry with the symbol that performs better in terms of likelihood metric. In other words, the proposed approach searches the M-dimensional subspace and replace symbols with smaller likelihood metric. This process is called as the MMSE M-correction. It is noted that when $M = M_T$, the MMSE M-correction is equivalent to the ML detection.

In the case $M = 1$, the MMSE 1-correction searches and intends to correct one symbol entry in the $\tilde{x}_{MMSE}$. The MMSE 1-correction performs a one-dimensional search from 1st to M_T-th symbol locations in the $\tilde{x}_{MMSE}$. When searching the subspace of i-th symbol location, all legal candidates in $\mathbb{S}$, i.e., $\forall x^i_{1c} \in \mathbb{S}$, are tested to replace $\tilde{x}^i_{MMSE}$ and form a new symbol vector $\bar{x}_{1c} \triangleq [\tilde{x}^1_{MMSE} \cdots x^i_{1c} \cdots \tilde{x}^{M_T}_{MMSE}]^T$. The likelihood metric of $\bar{x}_{1c}$, i.e., $\bar{D}_{1c} = \|y - \mathbf{H}\bar{x}_{1c}\|^2$, is then computed and compared with D_{MMSE}. If the likelihood metric of $\bar{x}_{1c}$ improves, i.e., smaller than D_{MMSE}, the $\tilde{x}_{MMSE}$ and D_{MMSE} are updated with $\bar{x}_{1c}$ and $\bar{D}_{1c}$, respectively. The similar process is generalized to MMSE M-correction, where the M symbol locations in the $\tilde{x}^i_{MMSE}$ are searched for better likelihood metric. The steps of the MMSE M-correction are described in the following:

Step 1): Calculate $D_{MMSE} = \|y - \mathbf{H}\tilde{x}_{MMSE}\|^2$.

Step 2): The number of possible M-combinations of an M_T-element set is $\binom{M_T}{M}$. Perform the following three steps for all the $\binom{M_T}{M}$ combinations of possible symbol locations in $\tilde{x}^i_{MMSE}$.

Step 3): These M indices are denoted as $ind_{i+1}, \ldots, ind_{i+M}$, $\forall m, n \in \{i+1, \ldots, i+M\}, ind_m \neq ind_n$. Select one $M \times 1$ symbol vector $x_{Mc} \triangleq [x^{ind_{i+1}}_{Mc} \ldots x^{ind_{i+M}}_{Mc}]^T \in \mathbb{S}^M$ where $\mathbb{S}^M$ denotes the set of M-dimensions of symbols. Replace $\tilde{x}^{ind_{i+1}}_{MMSE}, \ldots, \tilde{x}^{ind_{i+M}}_{MMSE}$ with $x^{ind_{i+1}}_{Mc}, \ldots, x^{ind_{i+M}}_{Mc}$ to form a new symbol vector $\bar{x}_{Mc} \triangleq [\tilde{x}^1_{MMSE} \ldots x^{ind_{i+1}}_{Mc} \ldots x^{ind_{i+M}}_{Mc} \ldots \tilde{x}^{M_T}_{MMSE}]^T$.

Step 4): Calculate $\bar{D}_{Mc} = \|y - \mathbf{H}\bar{x}_{Mc}\|^2$, and if $\bar{D}_{Mc} \leq D_{MMSE}$, let $D_{MMSE} = \bar{D}_{Mc}$ and $\tilde{x}_{MMSE} = \bar{x}_{Mc}$.

Step 5): Select other symbol vector x'_{Mc} to repeat step 3 and step 4 until all $x'_{Mc} \in \mathbb{S}^M$ are tested.

Step 6): The final $\bar{x}_{Mc}$ is the detected MIMO symbol vector.

B. A Reduced-Complexity MMSE M-correction

In this section, we propose an approach to further reduce the complexity of MMSE M-correction by effectively selecting the location of symbol entry in $\tilde{x}_{MMSE}$ for subspace search. The MMSE M-correction conducts exhaustive search within all candidate M-dimensional subspace of $\tilde{x}_{MMSE}$. The objective is to eliminate unnecessary subspace searches by searching only the subspaces where the errors are more likely to occur. In order to achieve this objective, the symbol vector $\bar{x}_{Mc}$ provided from the process of the MMSE M-correction is reformulated as $\bar{D}_{Mc} = \|y - \mathbf{H}(\tilde{x}_{MMSE} + \Delta\bar{x}_{Mc})\|^2$ where $\Delta\bar{x}_{Mc} = \bar{x}_{Mc} - \tilde{x}_{MMSE} = [0 \ldots (x^{ind_{i+1}}_{Mc} - \tilde{x}^{ind_{i+1}}_{MMSE}) \ldots (x^{ind_{i+M}}_{Mc} - \tilde{x}^{ind_{i+M}}_{MMSE}) \ldots 0]^T$. This provides us another perspective to develop a lower-complexity method denoted as the reduced-complexity (RC) MMSE M-correction which improves the MMSE detection performance with lower complexity than MMSE M-correction.

To elaborate the proposed RC-MMSE M-correction, we introduce the notion of deviation vector, denoted as $\Delta\bar{x}_{RcMc} = [\Delta\bar{x}^1_{RcMc} \ldots \Delta\bar{x}^{M_T}_{RcMc}]^T$. Conceptually, the $\Delta\bar{x}_{RcMc}$ is considered as the hypothetical deviation on $\tilde{x}_{MMSE}$, and is used to study the impact of deviation on the likelihood metric. The impact of deviation on the likelihood metric is then used to identify the subspaces where the improvement on likelihood metric by searching is more likely to occur. The reformulated likelihood metric is expressed as

$$\begin{aligned}\bar{D}_{RcMc} &= \|y - \mathbf{H}(\tilde{x}_{MMSE} + \Delta\bar{x}_{RcMc})\|^2 \\ &= \|y - \mathbf{H}\tilde{x}_{MMSE}\|^2 + \|\mathbf{H}\Delta\bar{x}_{RcMc}\|^2 \\ &\quad +2Re[(y - \mathbf{H}\tilde{x}_{MMSE})^H(\mathbf{H}\Delta\bar{x}_{RcMc})], \end{aligned} \quad (7)$$

where $Re[\cdot]$ is the real part of a complex number. Since the first and second terms of (7) are positive, the $\bar{D}_{RcMc}$ can only be possibly reduced through the third term. The third term can be rearranged as

$$\begin{aligned}&(y - \mathbf{H}\tilde{x}_{MMSE})^H(\mathbf{H}\Delta\bar{x}_{RcMc}) \\ &= [(y - \mathbf{H}\tilde{x}_{MMSE})^H\mathbf{H}]\Delta\bar{x}_{RcMc} \\ &= A\Delta\bar{x}_{RcMc} \end{aligned} \quad (8)$$

where $A = [(y - \mathbf{H}\tilde{x}_{MMSE})^H\mathbf{H}] = [A_1 \ldots A_{M_T}]$ is a $1 \times M_T$ row vector. The magnitudes of elements in the set $\{A_1, \ldots, A_{M_T}\}$ are computed, and the M largest magnitudes and their corresponding indices are denoted as $ind^{Rc}_1, \ldots, ind^{Rc}_M$. These M indices, $ind^{Rc}_1, \ldots, ind^{Rc}_M$, are considered the entry indices of the M most possibly erroneous symbols. The reason we select $ind^{Rc}_1, \ldots, ind^{Rc}_M$ is that the M locations of $\tilde{x}_{MMSE}$ corresponding to the largest magnitudes in A are more likely to decrease the overall likelihood metric in (7). If the phases of deviation vector $\Delta\bar{x}_{RcMc}$ is properly aligned with A, the value of the $2Re[(y - \mathbf{H}\tilde{x}_{MMSE})^H(\mathbf{H}\Delta\bar{x}_{RcMc})]$ in (7) is negative, and then $\Delta\bar{D}_{RcMc}$ can possibly be reduced to be less than D_{MMSE}. Therefore, the search in the M subspaces corresponding to the M elements of largest magnitudes in A are more likely to improve the likelihood metric. By searching subspaces indexed by the elements with M largest magnitudes in A, the computational complexity of the RC-MMSE M-correction can be reduced to be lower than the MMSE M-correction. The steps of the RC-MMSE M-correction are described as follows:

Step 1): Calculate $(y - \mathbf{H}\tilde{x}_{MMSE})^H\mathbf{H} = A$.

Step 2): Compute the magnitude of each element of the row vector $A = [A_1 \ldots A_{M_T}]$ and sort them in a descending order.

Step 3): Select the first M elements for which indices $ind^{Rc}_1, \ldots, ind^{Rc}_M$ are assigned.

Step 4): Select one $M \times 1$ symbol vector $x_{RcMc} \triangleq [x^{ind^{Rc}_1}_{RcMc} \ldots x^{ind^{Rc}_M}_{RcMc}]^T \in \mathbb{S}^M$ where $\mathbb{S}^M$ denotes the set of M-dimensions of symbols. Replace $\tilde{x}^{ind_{i+1}}_{MMSE}, \ldots, \tilde{x}^{ind_{i+M}}_{MMSE}$ with $x^{ind^{Rc}_1}_{RcMc}, \ldots, x^{ind^{Rc}_M}_{RcMc}$ to form a new symbol vector $\bar{x}_{RcMc} \triangleq [\tilde{x}^1_{MMSE} \ldots x^{ind^{Rc}_1}_{RcMc} \ldots x^{ind^{Rc}_M}_{RcMc} \ldots \tilde{x}^{M_T}_{MMSE}]^T$.

Step 5): Calculate $\bar{D}_{RcMc} = \|y - \mathbf{H}\bar{x}_{RcMc}\|^2$. If $\bar{D}_{RcMc} \leq D_{MMSE}$, let $D_{MMSE} = \bar{D}_{RcMc}$ and $\tilde{x}_{MMSE} = \bar{x}_{RcMc}$.

Step 6): Select other symbol vector x'_{RcMc} to repeat step 4 and step 5 until all $x'_{RcMc} \in \mathbb{S}^M$ are tested.

Step 7): The final $\bar{x}_{RcMc}$ is the detected MIMO symbol vector.

C. Computational Complexity

The computational complexities are analyzed in this section. Specifically, the computational complexity measured by number of operations is calculated according to the following principles: 1) One multiplication of a complex number is equivalent to four real number multiplications and two real number additions; 2) One addition or subtraction of a complex number is equivalent to two real number additions; 3) One division of a complex number is equivalent to eight real number multiplications and four real number additions; and 4) One addition, subtraction, multiplication, or division of a real number is regarded as one computational operation.

From (2), the computational complexity C_0 of $\|y - \mathbf{H}x\|^2$ is calculated as being $8M_T^2 + 8M_T - 2$. The computational complexities of the ML detection, ZF detection, MMSE detection, MMSE M-correction, and RC-MMSE M-correction are specified and the detailed calculations are described as follows:

a) ML detection: From (2), since the ML detection searches through all possible symbol vectors to find the one that minimizes $\|y - \mathbf{H}x\|^2$, its computational complexity C_{ML} is calculated as being $|\mathbb{S}|^{M_T}(8M_T^2 + 8M_T - 2)$.

b) ZF detection: From (4), by using the Gauss-Jordan Elimination algorithm to compute the matrix inversion, its computational complexity C_{ZF} is calculated as being $(56/3)M_T^3 + 38M_T^2 + (28/3)M_T$.

c) MMSE detection: From (6), the Gauss-Jordan Elimination algorithm is used for the matrix inversion, its computational complexity C_{MMSE} is calculated as being $(56/3)M_T^3 + 40M_T^2 + (34/3)M_T + 1$.

d) MMSE M-correction: Since this method is based on the MMSE detection, its computational complexity C_{Mc} is the sum of the complexity from the MMSE detection and the complexity from correction procedures, which is calculated as being

$$\begin{aligned} &C_{MMSE} + \binom{M_T}{M}(|\mathbb{S}|^M - 1)C_0 \\ &= (56/3)M_T^3 + 40M_T^2 + (34/3)M_T + 1 \\ &+ \binom{M_T}{M}(|\mathbb{S}|^M - 1)(8M_T^2 + 8M_T - 2). \end{aligned}$$

e) Reduced-complexity MMSE M-correction: Similar to the MMSE M-correction, this method is based on the MMSE detection and thus its computational complexity C_{RcMc} is given by

$$\begin{aligned} &C_{MMSE} + (|\mathbb{S}|^M - 1)C_0 \\ &= (56/3)M_T^3 + 40M_T^2 + (34/3)M_T + 1 \\ &+ (|\mathbb{S}|^M - 1)(8M_T^2 + 8M_T - 2). \end{aligned}$$

IV. Simulation Results

In this section, the simulation results are presented to evaluate the performance and computational complexities. We simulate symbol error rate (SER) and compute computational operations shown in Sec. III-C. In terms of these two criteria, the proposed approach and its variations are compared with the ZF detection, the MMSE detection and the ML detection. It is noticed that the performance of sphere decoder is equivalent to the ML detection because the conventional edition of the sphere decoder is used in our simulation. In other words, the

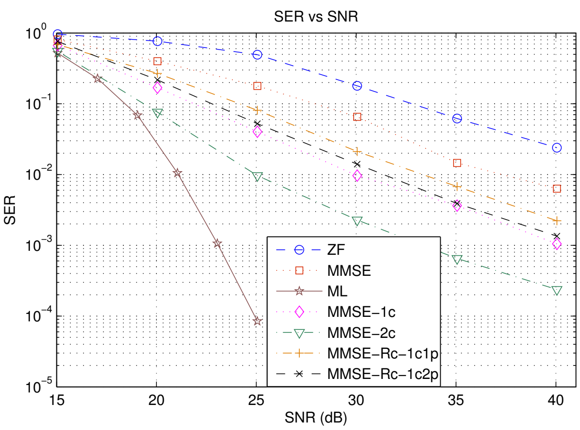

Fig. 1. SER versus SNR performance of the proposed methods and linear detectors using 4-QAM modulation with MIMO channel 8x8.

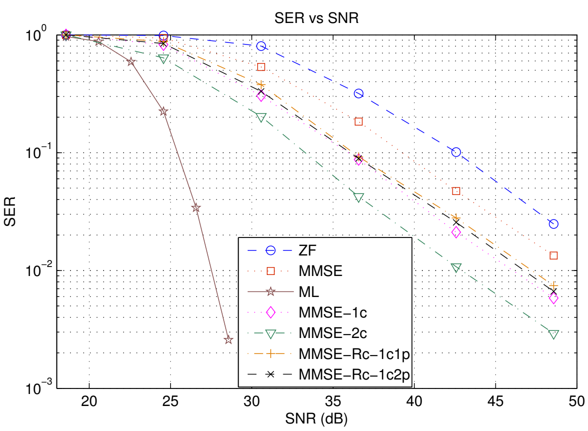

Fig. 2. SER versus SNR performance of the proposed methods and linear detectors using 8-QAM modulation with MIMO channel 12x12.

ML performance also represents the sphere decoder performance in our cases. For SER, two cases are considered: 1) 4-QAM modulation in the 8x8 MIMO system, and 2) 8-QAM modulation in the 12x12 MIMO system. For complexities, four cases are considered: 1) 4-QAM modulation in the 4x4 MIMO system, 2) 4-QAM modulation in the 8x8 MIMO system, 3) 4-QAM modulation in the 12x12 MIMO system, and 4) 8-QAM modulation in the 12x12 MIMO system. In the simulations, the MIMO channel is generated using i.i.d. Gaussian random variables and AWGN model is used. The SNR is defined as $\gamma = E[(\mathbf{H}s)^H(\mathbf{H}s)]/E[n^H n]$. The abbreviations used in the tables and figures are defined as follows.

1) $ML, ZF, MMSE$: They represent the ML , ZF, and MMSE detection, respectively.
2) $MMSE$-$1c$, $MMSE$-$2c$: They represent the MMSE M-correction with $M = 1$ and $M = 2$, respectively.
3) $MMSE$-Rc-$1c1p$, $MMSE$-Rc-$1c2p$: They represent the RC-MMSE M-correction with $M = 1$ and $M = 2$, respectively. It is noted the $MMSE$-Rc-$1c2p$ determines two positions (indices) for correcting the er-

TABLE I
NUMBERS OF COMPUTATIONAL OPERATIONS OF THE PROPOSED METHODS AND LINEAR DETECTORS

Numbers of Computational Operations (unit: 10^4)				
Modulation	4QAM	4QAM	4QAM	8QAM
Channel size	4x4	8x8	12x12	12x12
ML	4.14	3788	2.10×10^6	8.59×10^9
ZF	0.184	1.20	3.78	3.78
MMSE	0.188	1.22	3.82	3.82
MMSE-1c	0.38	2.60	8.32	14.32
MMSE-2c	1.65	25.5	127.57	523.57
MMSE-Rc-1c1p	0.27	1.39	4.19	4.69
MMSE-Rc-1c2p	0.29	1.57	4.57	5.57

roneous symbol entries, and uses each position to execute $MMSE$-Rc-$1c1p$. Namely, it is equivalent to $MMSE$-Rc-$1c1p$ being performed twice.

A. Performance- SER versus SNR

In Fig. 1 and Fig. 2, we present the performance of the MMSE M-correction and RC-MMSE M-correction with $M = 1$ and 2, and compare the performances with the ZF and MMSE detection for the 8x8 and 12x12 MIMO systems, respectively. In Fig. 1, at SER=10^{-2}, the proposed $MMSE$-$1c$ and $MMSE$-$2c$ achieve 7 dB and 12 dB gain over the MMSE detections, respectively. Although the performance of the methods $MMSE$-Rc-$1c1p$ and $MMSE$-Rc-$1c2p$ degrades approximately 3 dB and 1 dB compared with $MMSE$-$1c$, they still achieves 4 dB and 7dB gain over the MMSE detection, respectively. In Fig. 2, we observe the similar performance improvements as Fig. 1. The performances of the $MMSE$-Rc-$1c1p$ and $MMSE$-Rc-$1c2p$ are very close to the method $MMSE$-$1c$.

B. Comparison of Computational Complexity

According to the equations from Sec. III-C, the computational complexities of the methods are calculated and listed for 4x4, 8x8, and 12x12 MIMO systems in Table I. From Table I, the computational complexity of the ML detection is much higher than other methods. We notice that the proposed approaches $MMSE$-$1c$, $MMSE$-Rc-$1c1p$ and $MMSE$-Rc-$1c2p$ cost the small amount of extra computational operations over the MMSE detection, while achieving approximately 7 dB to 12 dB gains. From the algorithmic perspective, the methods $MMSE$-$1c$, $MMSE$-Rc-$1c1p$ and $MMSE$-Rc-$1c2p$ are of complexity $\mathcal{O}(M_T^3)$, while the method $MMSE$-$2c$ is of complexity $\mathcal{O}(M_T^4)$. The complexities are much lower than the ML detection. The numerical results are shown in Table I.

V. CONCLUSION

The improved MMSE-based detection scheme is proposed in this paper. The proposed scheme achieves performance improvements while maintaining the advantage of low complexities in the MMSE detectors. The performance improves significantly compared with the conventional linear MIMO detections, including the ZF and MMSE detector. The proposed MMSE M-correction scheme searches the M-dimensional subspace, and therefore the extra complexity is limited. To further reduce the complexity of MMSE M-correction scheme, the RC-MMSE M-correction is proposed. The RC-MMSE M-correction select the subspaces, which are more likely to improve the likelihood metric, to conduct search. The subspace selection in RC-MMSE M-correction scheme is based on the reformulations of the likelihood metric. The performances and complexities are analyzed and simulated. The results show the performance improvements with only slight extra computational complexities.

REFERENCES

[1] T.-H. Liu, "Some results for the fast MMSE-SIC detection in spatially multiplexed MIMO systems," *IEEE Trans. Wireless Commun.*, vol. 8, pp. 5443–5448, Nov. 2009.

[2] B. Shim, J. W. Choi, and I. Kang, "Towards the performance of ML and the complexity of MMSE - a hybrid approach," in *Proc. IEEE GLOBECOM'08*.

[3] D. Wubben, R. Bohnke, V. Kuhn, and K. D. Kammeyer, "Near-maximum-likelihood detection of MIMO systems using MMSE-based lattice reduction," in *Proc. IEEE ICC'06*, Jun. 2006.

[4] R. Bohnke, D. Wubben, V. Kuhn, and K. D. Kammeyer, "Reduced complexity MMSE detection for BLAST architectures," in *Proc. IEEE GLOBECOM'03*, Dec. 2003.

[5] T.-H. Liu and Y.-L. Y. Liu, "Modified fast recursive algorithm for efficient MMSE-SIC detection of the V-BLAST system," *IEEE Trans. Wireless Commun.*, vol. 7, pp. 3713–3717, Oct. 2008.

[6] E. Viterbo and J. Boutros, "A universal lattice code decoder for fading channel," *IEEE Trans. Inf. Theory*, vol. 45, pp. 1639–1642, Jul. 1999.

[7] B. M. Hochwald and S. ten Brink, "Achieving near-capacity on a multiple-antenna channel," *IEEE Trans. Commun.*, vol. 51, pp. 389–399, Mar. 2003.

[8] M. O. Damen, H. E. Gamal, and N. C. Beaulieu, "On maximum-likelihood detection and the search for the closest lattice point," *IEEE Trans. Inf. Theory*, vol. 49, pp. 2372–2388, Oct. 2003.

[9] B. Hassibi and H. Vikalo, "On the sphere-decoding algorithm I: expected complexity," *IEEE Trans. Signal Process.*, vol. 53, pp. 2806–2818, Aug. 2005.

[10] L. G. Barbero and J. S. Thompson, "Fixing the complexity of the sphere decoder for MIMO detection," *IEEE Trans. Wireless Commun.*, vol. 7, pp. 2131–2142, Jun. 2008.

[11] Z. Guo and P. Nilsson, "Algorithm and implementation of the K-best sphere decoding for MIMO detection," *IEEE J. Sel. Areas Commun.*, vol. 24, pp. 491–503, Mar. 2006.

[12] W. Zhao and G. B. Giannakis, "Sphere decoding algorithms with improved radius search," *IEEE Trans. Commun.*, vol. 53, pp. 1104–1109, Jul. 2005.

[13] B. Shim and I. Kang, "Sphere decoding with a probabilistic tree pruning," *IEEE Trans. Signal Process.*, vol. 56, pp. 4867–4878, Oct. 2008.

[14] S. M. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory*. Englewood Cliffs: NJ: Prentice-Hall, 1993.

[15] G. H. Golub and C. F. V. Loan, *Matrix Computations*, 3rd ed. Baltimore: Johns Hopkins Univ. Press, 1996.