CLOSED-FORM SOLUTION FOR POSITIONING BASED ON ANGLE OF ARRIVAL MEASUREMENTS

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ABSTRACT

This paper deals with the problem of estimating the position of a user equipment operating in a wireless communication network. In particular, we present a new positioning method based on the angles of arrival (AOA) measured in several radio links between that user equipment and different base stations. The proposed AOA-based method leads us to a non-iterative closed-form solution of the positioning problem, and an statistical analysis of that solution is also included. The comparison between this method and the classical AOA-based positioning technique is discussed in terms of computational load, convergence of the solution and also in terms of the bias and variance of the position estimate.

1. INTRODUCTION

Since the FCC announced its mandate for a implementation of progressively increasing emergency services for cellular phone callers in October 1996, a great effort has been done by companies and the research community to provide feasible solutions to the position location (PL) problem [1].

The location of a user equipment (UE) involves two steps: firstly, the measurement of radio parameters useful for location purposes (usually, time delays and/or angles of arrival) and secondly, the combination of those radio parameters to provide both a position estimate and a quality parameter of this estimate. Focussing on the second step, radiolocation methods that estimate the UE position can be classified into three broad categories:

- Direct Finding PL systems, which estimate the position by measuring angles of arrival (AOA) of several radio links.
- Range-based PL systems, which estimate the position of a UE by measuring time of arrival (TOA) or time-difference of arrival (TDOA) of several radio links.
- Hybrid PL systems, that estimate the position by combining AOAs and time delays measurements.

In the recent literature, an special attention has been given to PL methods that involve AOA measurements. This is mainly due to the increasingly presence of antenna arrays in the BSs of wireless communication networks. In fact, existing PL systems range mainly from TDOA systems, AOA methods, TDOA/AOA methods and cellular aided GPS methods [2].

In this paper, we focus on AOA-based PL problem in a wireless communication network although the results we present are also useful for hybrid positioning techniques. In section 2, the classical procedure to estimate the position from AOA measurements is briefly reviewed. Afterwards, we present a new formulation of the AOA-based PL problem that leads to a set of equations which, unlike the classical procedure, depends linearly on the user co-ordinates. A closed-form solution to this linear equations is proposed as well as an statistical analysis of that solution. Section 3 includes some simulations results useful to compare the proposed AOA-based PL technique with the classical one. Finally, we conclude with the summary.

2. AOA-BASED POSITIONING TECHNIQUES

The AOA-based PL technique need the use of antenna arrays in the receivers. With the current technology, it is more feasible that the antenna array is located in the BS and, therefore, the AOA is measured in the uplink. Whether angle measurements are performed in the uplink or in the downlink, an AOA measurement restricts the source location along a line in the direction of the angle that joints the BS and the UE and called Line Of Bearing (LOB). When two or more AOA measurements from multiple receivers are used, the location estimate of a source is obtained as the intersection of the LOBs (a two dimensional problem is considered hereafter without loss of generality). Due to the presence of errors in the AOA measurements, the LOBs not intersect at the same point and some signal processing is needed to provide a solution to the PL problem. Before going into that, let us define the notation of the location problem with BS^i the i^{th} BS located at $\mathbf{r_i}=[x_i\ y_i]^T$, $\mathbf{r}=[x\ y]^T$ the UE co-ordinates and θ_i the AOA measured from BS^i (figure 1). We also denote by θ_i' the actual angle between the UE and BS^i.

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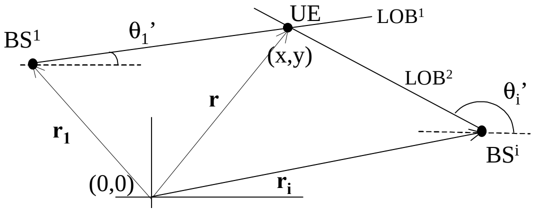

Figure 1. Notation in AOA-based PL techniques

2.1 Approximated Maximum-Likelihood solution for AOA-based positioning

The classical procedure to solve the positioning problem of figure 1 was proposed by Torrieri in [3]. It departs from the nonlinear relation between the bearing angle, θ_i, and the UE position $\mathbf{r}=[x\ y]^T$,

$$\theta_i = f_i(\mathbf{r}) + n_i \tag{1}$$

with n_i the angle measurement error and function $f_i()$ defined as

$$f_i(\mathbf{r}) = \arctan\left(\frac{x-x_i}{y-y_i}\right). \tag{2}$$

Gathering angle measurements from different BSs, we deal with the following nonlinear system of equations

$$\boldsymbol{\theta} = \mathbf{f}(\mathbf{r}) + \mathbf{n} . \tag{3}$$

This triangulation problem can be solved by Torrieri's approach [3]. It consists in linearizing the $\mathbf{f(r)}$ function by expanding it in a Taylor series around a reference point, denoted by $\mathbf{r_0}$. Once the equation system is linearized, the maximum likelihood (ML) estimator is used to provide the following UE position estimate

$$\mathbf{r}^{ML} = \mathbf{r}_0 + \left(\mathbf{G}^T \mathbf{N}^{-1} \mathbf{G}\right)^{-1} \mathbf{G}^T \mathbf{N}^{-1} [\boldsymbol{\theta} - \mathbf{f}(\mathbf{r}_O)]. \tag{4}$$

Matrix $\mathbf{N}$ is the measurement error covariance matrix, $\mathbf{N}$=E[$\mathbf{nn'}$], and matrix $\mathbf{G}$ is the matrix of the resulting equation system after linearizing (3). Matrix $\mathbf{G}$ is equal to

$$\mathbf{G} = \begin{bmatrix} -(\sin\theta_{01})/d_{01} & (\cos\theta_{01})/d_{01} \\ \vdots & \vdots \\ -(\sin\theta_{0n})/d_{0n} & (\cos\theta_{0n})/d_{0n} \end{bmatrix} \tag{5}$$

with angle $\theta_{0i} = f_i(\mathbf{r_0})$ and d_{0i} is the distance between BS^i and $\mathbf{r_0}$.

The bias of the ML position estimate (4) depends both on the mean value of the error measurement and on the linearization error. Assuming $\mathbf{n}$ is zero mean and once Torrieri's approach has converged, the bias of (4) becomes zero. The covariance matrix of the estimation is equal to

$$\mathbf{C}_{\mathbf{r}^{ML}} = \left(\mathbf{G}^T \mathbf{N}^{-1} \mathbf{G}\right)^{-1}. \tag{6}$$

Although Torrieri's approach provides an approximated ML unbiased estimator, the convergence of the iterative process is not always ensured (in fact, it depends both on $\mathbf{r_0}$ and also on the relative position of the UE with respect to the BSs). This undesired behavior of Torrieri's approach motivates our alternative solution of the AOA-based positioning problem.

2.2 Non-iterative solution for AOA-based positioning

It can be seen that the following relation holds for each BS,

$$\mathbf{r} = \mathbf{r_i} + d_i \cdot \mathbf{v_i} \tag{7}$$

with d_i the range from the UE to BS^i and $\mathbf{v_i}$ defined as the unitary vector in the i^{th} LOB direction,

$$\mathbf{v}_i^T = \left[\cos\theta_i' \quad \sin\theta_i'\right]. \tag{8}$$

Since we assume no knowledge about the range, relation (7) is equivalent to

$$-x_i \cdot \sin\theta_i' + y_i \cdot \cos\theta_i' = -x \cdot \sin\theta_i' + y \cdot \cos\theta_i' . \tag{9}$$

In case of having n BSs hearing the user mobile, and considering also errors in the AOA measurements, the following over-conditioned system is obtained

$$\begin{bmatrix} -x_1 \cdot \sin\theta_1' + y_1 \cdot \cos\theta_1' \\ \vdots \\ -x_n \cdot \sin\theta_n' + y_n \cdot \cos\theta_n' \end{bmatrix} \approx \begin{bmatrix} -\sin\theta_1' & \cos\theta_1' \\ \vdots & \vdots \\ -\sin\theta_n' & \cos\theta_n' \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} \tag{10}$$

that leads to the following matrix-vector notation

$$\mathbf{b}(\boldsymbol{\theta}) \approx \mathbf{H}(\boldsymbol{\theta})\ \mathbf{r} . \tag{11}$$

It can be seen that the proposed formulation of the AOA-based PL problem leads us to the equation system (11) that linearly depends on the UE co-ordinates. The solution to (11) we propose is the well-known least squares (LS) solution, defined as

$$\mathbf{r}^{LS} = (\mathbf{H}^T(\boldsymbol{\theta})\mathbf{H}(\boldsymbol{\theta}))^{-1}\mathbf{H}^T(\boldsymbol{\theta})\mathbf{b}(\boldsymbol{\theta}) = \mathbf{H}^{\#}(\boldsymbol{\theta})\ \mathbf{b}(\boldsymbol{\theta}) , \tag{12}$$

with $\mathbf{H}^{\#}(\boldsymbol{\theta})$ the pseudoinverse of $\mathbf{H}(\boldsymbol{\theta})$. The existence of $\mathbf{H}^{\#}(\boldsymbol{\theta})$ depends on the singularity of the $\mathbf{H}^T(\boldsymbol{\theta})\mathbf{H}(\boldsymbol{\theta})$ product. It can be shown that the determinant of this matrix product is equal to

$$\det\left(\mathbf{H}^T(\boldsymbol{\theta})\mathbf{H}(\boldsymbol{\theta})\right) = \sum_{i=1}^{n}\sum_{j=i+1}^{n} \sin^2\left(\theta_i - \theta_j\right), \tag{13}$$

and, in consequence, the $\mathbf{H}^T(\boldsymbol{\theta})\mathbf{H}(\boldsymbol{\theta})$ matrix will not be singular, unless $\theta_i=\theta_j$ for all $i\neq j$ or unless $\theta_i=\theta_j+\pi$, which happens when the LOB^i is the same as the LOB^j.

It is important to notice that θ_i errors in (11) affect not only $\mathbf{b}(\boldsymbol{\theta})$ but also $\mathbf{H}(\boldsymbol{\theta})$. Thus, it would be reasonable to take into account the Total Least Squares (TLS) solution for solving (11) instead of the LS one, see e.g. [4]. In [5], the authors compared both TLS and LS solutions in a certain scenario and concluded that the LS solution outperforms the TLS one. This can be due to the fact that, usually, TLS solutions seem to provide more accurate results than LS whenever the errors in $\mathbf{H}$ are zero mean [4], fact that does not hold in this problem.

A relatively simple statistical analysis of solution (12) arises assuming $\theta_i = \theta_i' + \Delta\theta_i$ with $\Delta\theta_i$ a zero mean Gaussian random variable with variance σ_i^2 and uncorrelated with other $\Delta\theta_j$ for $j \neq i$. Considering $\Delta\theta_i$ small enough, we can approximate

$$\mathbf{b}(\boldsymbol{\theta}) \approx \mathbf{b}(\boldsymbol{\theta}') + \boldsymbol{\delta}\mathbf{b} \equiv \mathbf{b} + \boldsymbol{\delta}\mathbf{b} \quad (14)$$

$$\mathbf{H}(\boldsymbol{\theta}) \approx \mathbf{H}(\boldsymbol{\theta}') + \boldsymbol{\delta}\mathbf{H} \equiv \mathbf{H} + \boldsymbol{\delta}\mathbf{H} \quad (15)$$

with $\mathbf{b}=\mathbf{b}(\boldsymbol{\theta}')$, $\mathbf{H}=\mathbf{H}(\boldsymbol{\theta}')$, $\boldsymbol{\delta}\mathbf{b}$ the error vector in $\mathbf{b}(\boldsymbol{\theta})$ (16) and $\boldsymbol{\delta}\mathbf{H}$ the error matrix in $\mathbf{H}(\boldsymbol{\theta})$ (17).

$$\boldsymbol{\delta}\mathbf{b} = \begin{bmatrix} -x_1 \Delta\theta_1 \cos\theta_1 - y_1 \Delta\theta_1 \sin\theta_1 \\ \vdots \\ -x_n \Delta\theta_n \cos\theta_n - y_n \Delta\theta_n \sin\theta_n \end{bmatrix} \quad (16)$$

$$\boldsymbol{\delta}\mathbf{H} = \begin{bmatrix} -\Delta\theta_1 \cos\theta_1 & -\Delta\theta_1 \sin\theta_1 \\ \vdots & \vdots \\ -\Delta\theta_n \cos\theta_n & -\Delta\theta_n \sin\theta_n \end{bmatrix} \quad (17)$$

Substituting (14) and (15) in expression (11), the LS solution solves in fact the following linear system of equations,

$$(\mathbf{H} + \boldsymbol{\delta}\mathbf{H})^T (\mathbf{b} + \boldsymbol{\delta}\mathbf{b}) = (\mathbf{H} + \boldsymbol{\delta}\mathbf{H})^T (\mathbf{H} + \boldsymbol{\delta}\mathbf{H}) (\mathbf{r} + \boldsymbol{\delta}\mathbf{r}) \quad (18)$$

with $\boldsymbol{\delta}\mathbf{r}$ the position error vector. If 2^{nd} order error terms in (18) are discarded, after arranging the resulting expression, we have that the position error of the proposed LS solution is equal to

$$\boldsymbol{\delta}\mathbf{r} = \mathbf{H}^{\#} (\boldsymbol{\delta}\mathbf{b} - \boldsymbol{\delta}\mathbf{H}\ \mathbf{r}). \quad (19)$$

It can be easily seen from (16), (17) and (19) that the mean value of the position error $\boldsymbol{\delta}\mathbf{r}$ is null, that is, the proposed estimator $\mathbf{r}^{LS}$ in (12) is unbiased. In the same way, but after more tedious work, the covariance matrix of $\boldsymbol{\delta}\mathbf{r}$ becomes equal to

$$\mathbf{C}_{\mathbf{r}^{LS}} = E(\boldsymbol{\delta}\mathbf{r}\ \boldsymbol{\delta}\mathbf{r}^T) = \mathbf{H}^{\#}\ \boldsymbol{\Lambda}\ \mathbf{H}^{\#T} \quad (20)$$

with $\boldsymbol{\Lambda}$ a diagonal matrix with ($\sigma_i^2\ d_i^2$) in the i^{th} position of the main diagonal. These statistical results of the position estimate $\mathbf{r}^{LS}$ are verified experimentally in next section. As it will be seen, both the mean value and covariance matrix of (19) fit quite well the estimated ones, obtained with Monte Carlo realizations.

The comparison of the closed-form estimator (12) and the one proposed by Torrieri (4) becomes a difficult task basically for two reasons. Firstly, in both cases the mean square error of the UE position estimate depends on the geometry of the problem, that is, on the relative position of the UE with respect to the BSs. Secondly, the theoretical results of mean square error that Torrieri presents in [3] do not always hold because the convergence of the iterative procedure is not ensured. Thus, the comparison of the estimators should be done in a given scenario in terms of both the bias and the variance of x and y estimates.

3. SIMULATION RESULTS

The simulation scenario we consider consists of 4 120º-sectored BSs that covered a given area where the cell diameter is about 400 meters. For each (x,y) position of the plane, 500 realizations are used to compute the mean value and covariance of (x,y) estimators, both for Torrieri and for the proposed closed-form solution. The angular measurements are taken as Gaussian with standard deviation equal to 5º for each BS, that is, σ_i=0.0873rad. For Torrieri's estimator, $\mathbf{r_0}$ is set randomly around the true position of the UE by adding a zero mean Gaussian perturbation in x and y co-ordinates with an standard deviation equal to 200 meters (remind the cell diameter is about 400 meters). Then, Torrieri's equations are iterated 6 times. It is worth mentioning that similar results are obtained if Torrieri's approach is initialized in the geometrical centre of the 4 BS.

Figure 2 shows the estimated standard deviation of the position (that is, the square root of the trace of the covariance matrix) attained by the AOA-based Torrieri's method, for those realizations that converge after 6 iterations (the convergence condition is to have placed the UE with an error lower than the cell size). Values over 200 meters for the standard deviation are clipped and hereafter the magnitudes plotted are in meters. With respect to the bias, we have experimented that it is less than 10m for 70% of the positions and less than 20m for 85% of the positions. The non-zero bias values appear mainly around each BS, as it happens with the standard deviation in figure 2.

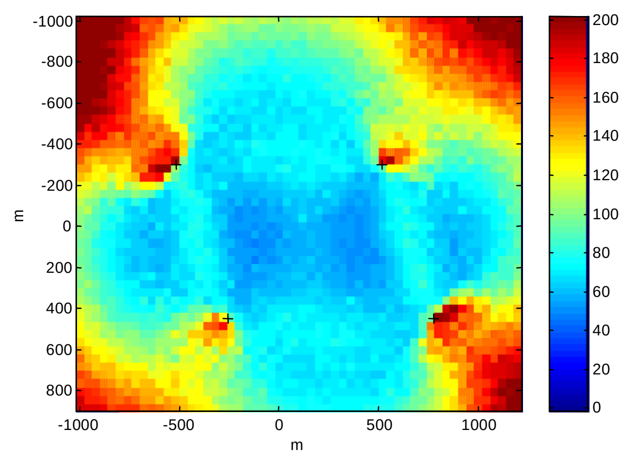

Figure 2. Estimated standard deviation for AOA-Torrieri's method

On the other hand, figure 3 exhibits the estimated standard deviation attained by the AOA-based closed-form LS method. The bias for this technique is of less than 10m for 50% of the positions and less than 20m for 80% of the positions. For this scenario we have also compared the covariance matrix expression in (20) to the estimated one but using 1000 realizations. The relative error value of the $\sqrt{\text{trace}(\mathbf{Cr}^{LS})}$ using (20) with respect to the estimated standard deviation is less than 4% in 85% of the positions, and less than 10% for 97% of the positions. Important to remark is the fact that this relative error is less than 6% for mostly of the positions inside a circle that joins all BSs. In consequence, we can conclude than expression (20) is a good approximation to the covariance matrix of the non-iterative closed-form LS estimator (12).

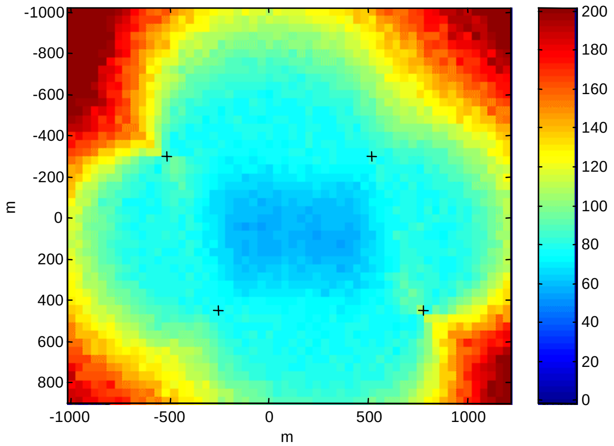

Figure 3. Estimated standard deviation for the closed-form LS method.

From these results it can be observed that Torrieri's method overcomes the closed-form LS method, both in standard deviation and in bias. Nevertheless, it is important to remark that the proposed LS method shows better performance around the BSs and do not suffer from convergence problems. In fact in Torrieri's approach, about 50000 realizations have been discarded due to convergent problems. Figure 4 shows the rate of non-convergent realizations after 6 iterations. It can be observed that this value is important all over the coverage area.

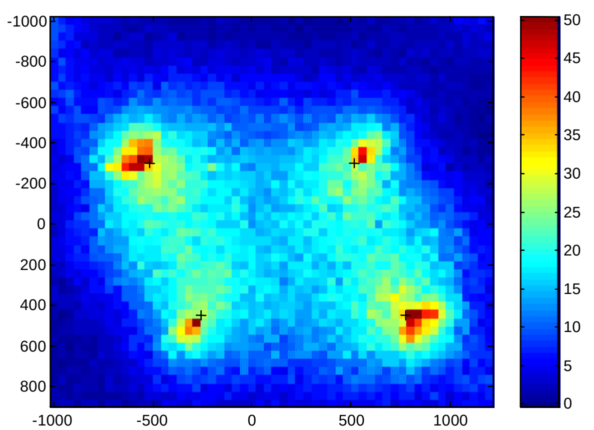

Figure 4. Rate of non-convergent realizations for the AOA-based Torrieri's method

These results indicate that the closed-form LS solution (12) can be useful as a first approach to obtain the initial value $\mathbf{r_0}$ that Torrieri's approach needs. In fact, figure 5 depicts the estimated standard deviation attained by the AOA-based Torrieri's method initialized with $\mathbf{r_0} = \mathbf{r}^{LS}$. With respect to the bias, we have experimented that 80% of the positions is less than 10m whereas 95% of the positions is less than 20m.

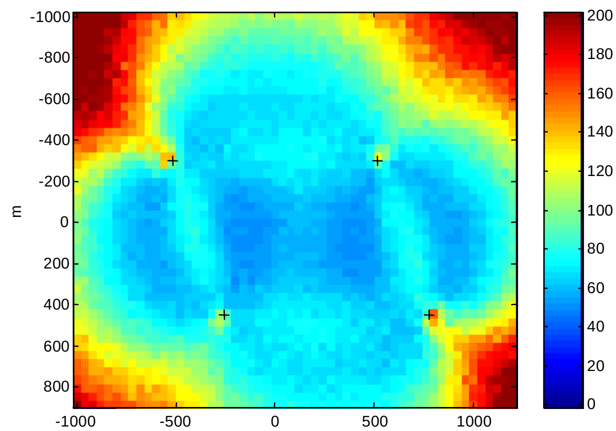

Figure 5. Estimated standard deviation for AOA-Torrieri's method initialized with $\mathbf{r}^{LS.}$

In terms of bias and standard deviation, it is clear that this last procedure overcomes Torrieri's approach randomly initialized. The improvement is mainly achieved in the areas around each BS without a penalization in the performance in other regions. Also remarkable is the fact that the non-convergent realizations have been diminished down to 1500 (see figure 6 that shows the rate of non-convergent realizations after 6 iterations, for the AOA-based Torrieri's method initialized with $\mathbf{r_0} = \mathbf{r}^{LS}$).

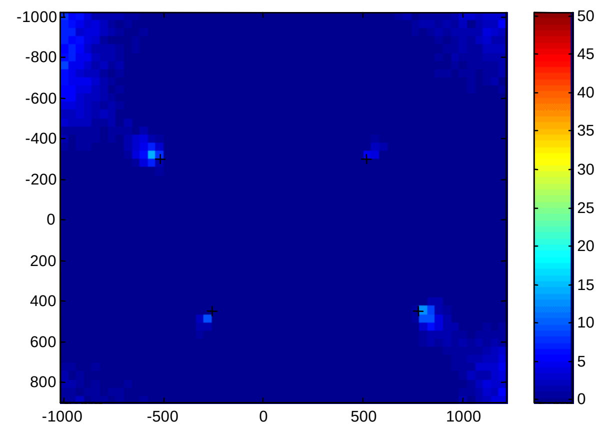

Figure 6. Rate of non-convergent realizations for AOA-based Torrieri's method with $\mathbf{r_0} = \mathbf{r}^{LS}$.

4. SUMMARY

A new AOA-based positioning method has been presented. The most remarkable feature of this method is the fact that it provides a non-iterative closed-form solution to the location problem. In comparison with the classical approach proposed by Torrieri, this method is less computational demanding and do not suffer from convergence problems. Although Torrieri's approach, whenever it converges, overcomes the new method in terms of bias and variance of the position estimate it has been shown that the non-iterative closed-form solution can be useful to initialized the iterative procedure.

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