Scale Effects of Shallow Foundation Bearing Capacity on Granular Material

A.B. Cerato and A.J. Lutenegger
University of Massachusetts, Amherst

Introduction

The bearing capacity of shallow foundations on granular material has been studied for years by many different investigators. Although many approaches and additional considerations to the governing criteria of bearing capacity have been presented, the calculation of the ultimate bearing capacity of a footing has changed very little since Terzaghi (1943) presented his general equation for ultimate bearing capacity, q_{ult}. However, current design of shallow foundations on granular soils does not account for the absolute size of the footing, or the scale effect between the soil and the foundation. This may result in an overly conservative design, which in turn results in excessive costs of foundations.

The bearing capacity factor, N_γ, is not a unique value, but depends on the unit weight, γ, and the friction angle, ϕ of the soil. In addition to these elements, there appears to be considerable evidence that for granular materials, the bearing capacity factor N_γ is dependent on the absolute width of the foundation, B; that is, there appears to be a scale effect such that the value of N_γ decreases as the footing width increases, all other variables being constant. Some researchers have suggested that this phenomenon may be related to grain size characteristics of the soil.

This paper presents and discusses the results of square and circular model footing tests conducted on two compacted sands to 1.) determine the influence of Relative Density on N_γ (i.e., for constant foundation width for a given sand while varying the density), and 2.) determine the influence of sand grain size on N_γ (i.e., for a constant foundation width and for different types of sand). The tests were conducted at the University of Massachusetts, Amherst.

Background

Coarse-grained granular soils have absolute scale, relative to the dimensions of most foundation elements. Fine-grained soils, by virtue of their small size, e.g., micron range, are unaware of the dimensions of a foundation element. Whereas there may be millions of individual particles of clay under a footing, by comparison, there may only be a few hundred sand grains under the same footing. This makes the behavior of granular soils unique. Currently, the design techniques for determining the bearing capacity of deep and shallow foundations involving granular soils does not account for any scale effects between the soil and the foundation element. This can result in an overly conservative design, which in turn results in excessive costs of foundations. This scale effect was recognized as early as 1965 by DeBeer (1965). Based on additional observations over the past forty years, there is considerable evidence that for coarse-grained granular soils (i.e., sands and gravels), the bearing capacity factor N_γ is dependent on the absolute width of the foundation, B, (e.g., Hettler and Gudehus 1988; Ueno et al. 1998; 2001; Zhu et al. 2001). This scale effect is illustrated in Figure 1.

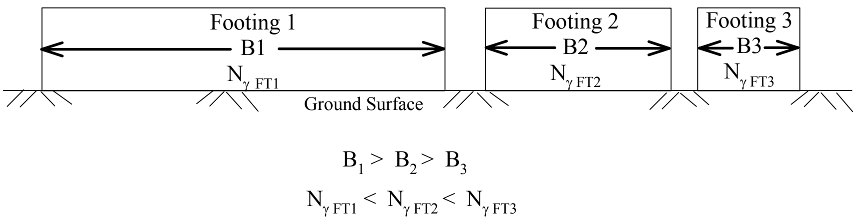

Figure 1. Observed Influence of Foundation Width on the Bearing Capacity Factor, N_γ

While this phenomenon seems to be fairly well recognized by a number of investigators, there is no provision in current design practice to take this behavior into account.

The traditional bearing capacity equation (e.g., Terzaghi 1943; Hansen 1970; Vesic 1973; etc.) for a centrally loaded surface footing on the surface of a granular soil with zero cohesion reduces to:

$$q_{ult} = 0.5\gamma B N_\gamma s_\gamma \qquad [1]$$

where:

q_{ult} = ultimate bearing capacity

B = footing width

γ = density of soil
N_γ = dimensionless bearing capacity factor
s_γ = foundation shape factor

For model or prototype scale loading tests of known geometry, for which an assumed value of s_γ is used, the only unknown in Equation 1 is the bearing capacity factor, N_γ, which, according to all current textbooks, is only dependent on the friction angle, ϕ, of the soil.

Habib (1974) suggested that the value of N_γ be corrected to account for a scale effect and introduced a *modified* bearing capacity factor, N^*_γ, which was related to the number of grains under the footing as:

$$N^*_\gamma = N_\gamma + 400/n \qquad [2]$$

where:
N^*_γ = Modified Bearing Capacity Factor
N_γ = Bearing Capacity Factor
n = number of grains under a footing (B/δ)
B = Footing Width (mm)
δ = Mean Grain Size (mm)

More recently, Shiraishi (1990) suggested that a *modified* bearing capacity factor could be expressed as:

$$N^*_\gamma = 0.71 N_\gamma / B^{0.2} \qquad [3]$$

where:
N^*_γ = Modified Bearing Capacity Factor
N_γ = Reference Bearing Capacity Factor
B = Footing Width

This factor can be incorporated in the bearing capacity equation and expressed in general form as:

$$q_{ult} = 0.5\gamma B N_\gamma s_\gamma (B/B^*)^{-\beta} \qquad [4]$$

where:
B^* = reference footing width = 1.40 m
N_γ = Reference Bearing Capacity Factor
$\beta = 0.2$

The term $(B/B^*)^{-\beta}$ is in effect a dimensionless correction factor to the reference bearing capacity factor N_γ and increases N_γ for $B < 1.4$ m and decreases N_γ for

B > 1.4m. The general shape of this correction factor for β=0.2 is shown in Figure 2. The reference bearing capacity factors used by Shiraishi (1990) were the factors presented by Terzaghi (1943) and are available in most foundation engineering texts.

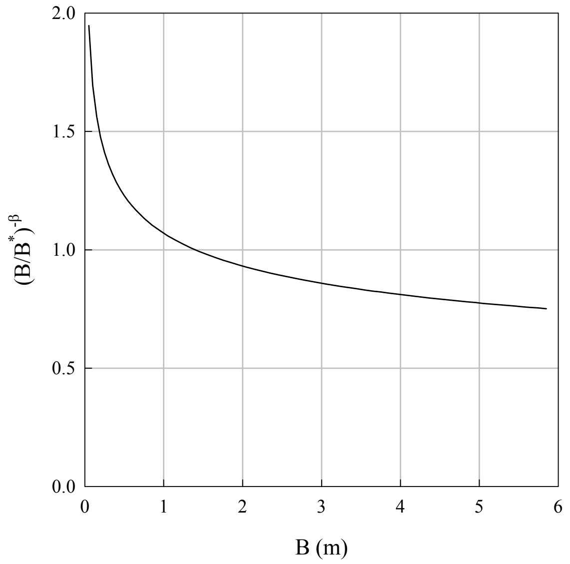

Figure 2. Shiraishi (1990) Bearing Capacity Factor Correction.

A number of bearing capacity studies have suggested different expressions for N_γ, (e.g., Caquot and Kerisel 1953; Lundgren and Mortensen 1953; Feda 1963; Meyerhof 1963; Krizek 1965; Hansen 1970; Vesic 1973; Michalowski 1997). Ingra and Baecher (1983) presented a compilation of model footing tests from the literature and give a recommended equation for determining N_γ from the friction angle of the soil. However, the model scale test results give N_γ values that are all on the upper bound of various proposed theoretical solutions, giving excessively high ultimate bearing capacities which would be conservative.

Current Investigation

In order to evaluate whether the bearing capacity factor N_γ is dependent on the absolute footing width, B, or the grain size, model scale square and circular footing tests were performed on two compacted sands having different characteristics; Brown Mortar Sand (G_s=2.69, ρ_{min}=1.41 Mg/m³, ρ_{max} = 1.70

Mg/m³, D_{50} = 0.6 mm, C_u = 2.1) and Winter Sand (G_s=2.69, ρ_{min}=1.61 Mg/m³, ρ_{max} = 1.96 Mg/m³, D_{50} = 1.6 mm, C_u = 4.5) (Figure 3). Circular and

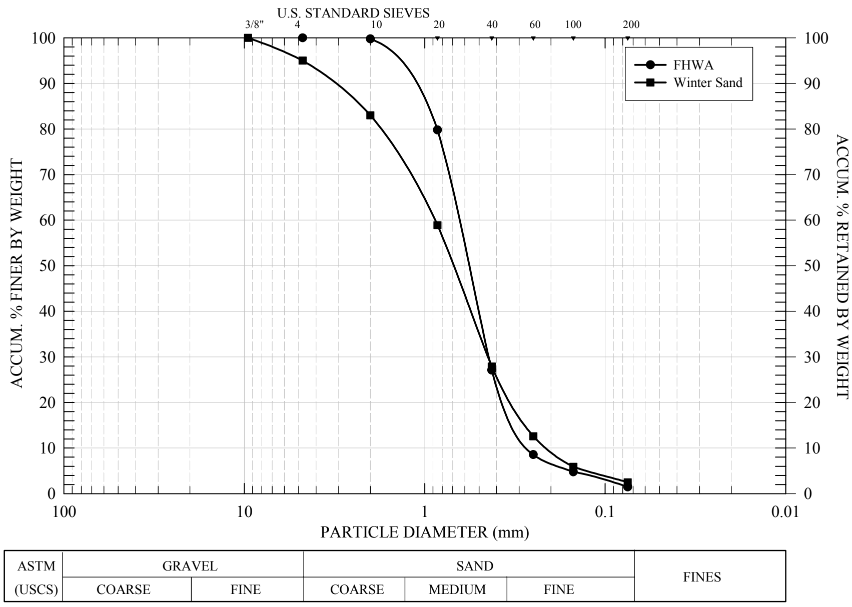

Figure 3. Grain-Size Distribution Curves for Two Test Sands.

square footings were tested to investigate if any difference might be observed relative to scale effects for different foundation shapes. Both slightly moist sands were hand compacted in lifts with a 0.152 m^2 steel tamper to 1.44, 1,52 and 1.60 Mg/m^3 (D_r = 12.6, 42.8, 69.9%) and 1.68, 1.79 and 1.91 Mg/m^3 (D_r = 23.7, 57.2, 86.8%) respectively. The dimensions of the model footings were 25.4, 50.8 and 101.6 mm square and in diameter. Larger footing tests (300 mm, 600 mm and 900 mm) were also performed on the Brown Mortar Sand.

The model footing tests were performed in a 0.762 X 0.762 X 0.305 m steel box with a concrete base (Figure 4). All tests were performed under saturated conditions and with the footings located at the sand surface (D_f = 0) to minimize the terms in the Terzaghi Bearing Equation to $q_{ult} = 0.5\gamma' B N_\gamma s_\gamma$ (Eq. 1). The steel footings were given a rough base by gluing sandpaper to the base and were loaded at a constant rate of 0.001 cm/sec with a Dayton DC Gearmotor until a settlement of at least 0.1 B occurred. The ultimate capacity was interpreted as the bearing stress which produced a relative settlement of 10% B, (i.e. s/B = 0.1). The bearing capacity factor, N_γ was back calculated and plotted versus footing size to attempt to observe the scale effect.

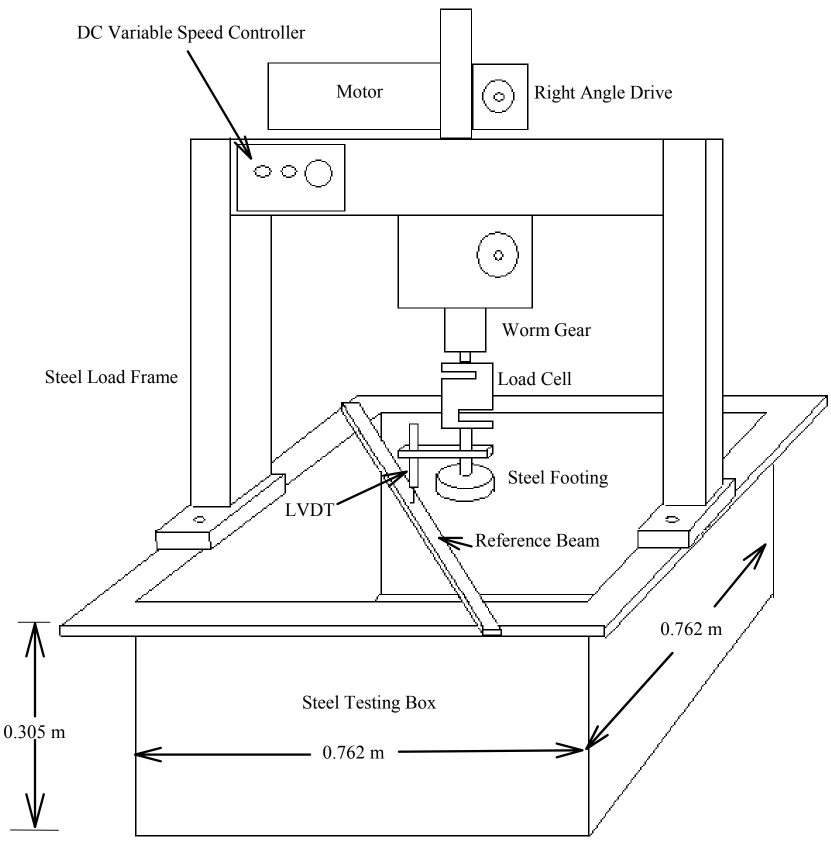

Figure 4. Test Set-up.

Results

Figure 5 presents typical results of load curves for the footing tests. These results are for bearing capacity tests on Winter Sand at a density of 1.91 Mg/m^3 (D_r = 86.8%). The failure modes for each of the footings varied depending on the sand type and density. These curves show that the square footings have a higher bearing capacity than circular footings, which is true for the tests on each sand and at each density. Terzaghi (1943) suggested shape factors of 0.6 for a circular footing and 0.8 for square footings, which indicates that a square footing will have a bearing capacity approximately 0.8/0.6 = 1.33 larger than a circular footing of the same width.

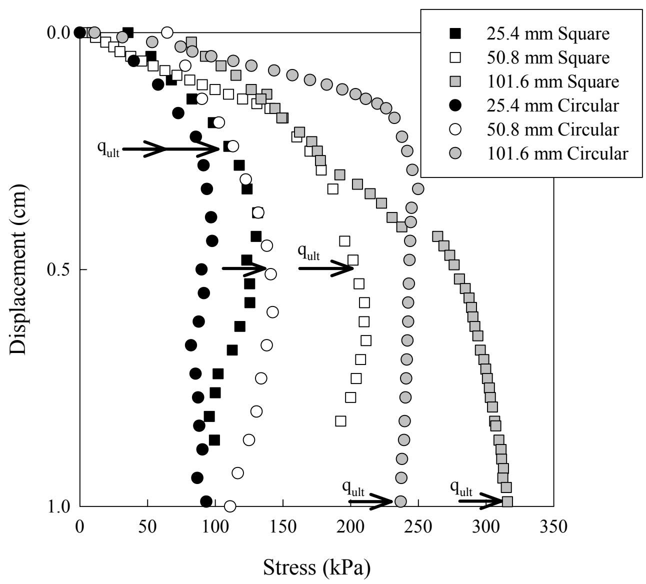

Figure 5. Typical Results of Model Scale Footing Load Tests.

Tests on larger footings (> 0.3 m) were performed in a large test pit. The results of the tests show that values of N_γ for both sands decrease with footing size and increase with increasing relative density. The results also suggest that relative density may have a more pronounced influence on N_γ than grain size (Figure 6). Results from the loose sands show a much less pronounced effect of scale than those from dense sands. However, the results shown in Figure 6 suggest a much more rapid increase in N_γ as the footing width decreases than previously noted.

Conclusions

Results of model scale footing tests on two compacted sands indicate that the bearing capacity factor, N_γ, is dependent on the absolute width of the footing for both square and circular footings. Caution must be used in applying the results of very small-scale model footing tests previously reported in the literature to full-scale behavior.

From the results obtained on different sands at three relative densities, it can be seen that values of N_γ for both sands decrease with footing size and increase with increasing relative density. The results also show that relative density may have a more pronounced influence on N_γ than grain size and the scale effect is more important for dense sands.

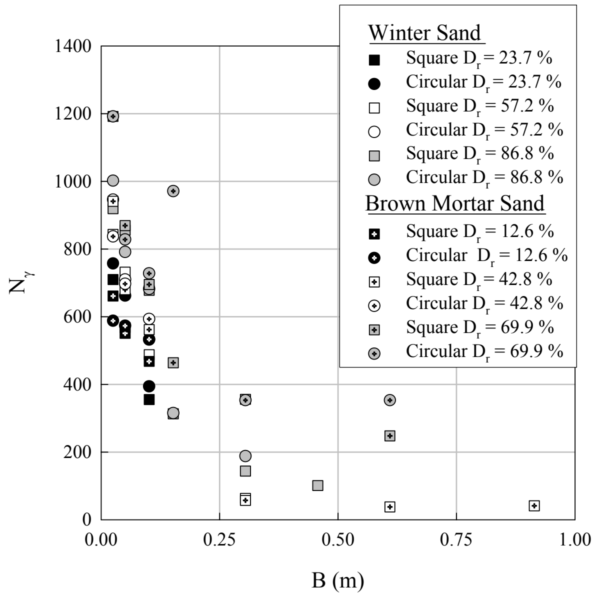

Figure 6. Results of Winter and FHWA Brown Mortar Sand Bearing Capacity Factor, N_γ, and Relative Density, D_r.

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